E C O N O M I C S B U L L E T I N

On the consistent use of linear demand systems if not all varieties are available

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Abstract

Linear demand formulations for price competition in horizontally differentiated products are sometimes used to compare situations where additional varieties become available, e.g. due to market entry of new firms. We derive a consistent demand system to analyze such situations and highlight potential problems that can arise from an inconsistent approach.

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1 Introduction

In industrial economics, often the following expression is used to model price competition in horizontally differentiated products: There are n product varieties with demand for variety $j = 1, .., n$, given by:

$$D_j(p) = \frac{1}{n}\left(1 - p_j - \gamma\left(p_j - \frac{\sum_{i=1}^n p_i}{n}\right)\right). \tag{1}$$

This formulation goes back to Shubik and Levitan (1971), is used in many models, and can be found in text books like e.g. Vives (2001), p. 163. The formulation is analytically tractable and has a very intuitive interpretation: Demand decreases directly in the own price but additionally if the own price increases above the price average, where the parameter γ describes how closely the different markets are linked. An important feature of this demand system is that it can be derived from a representative consumer with quasi-linear preferences that can be represented by the following utility function (where w denotes the initial wealth of the consumer):[1]

$$U = \sum_{j=1}^{n} q_j - \frac{1}{2}\left(\sum_{j=1}^{n} q_j\right)^2 - \frac{n}{2(1+\gamma)}\left[\sum_{j=1}^{n} q_j^2 - \frac{\left(\sum_{j=1}^{n} q_j\right)^2}{n}\right] + \left[w - \sum_{j=1}^{n} q_j p_j\right]. \tag{2}$$

2 Varying the number of goods

It is often interesting to compare situations where different numbers of varieties are available. Consider, for instance, the decision problem of a consumer with preferences for different varieties of cereals who goes to the supermarket and realizes that the varieties $m+1, ..., n$ are sold out. How much does she buy from the varieties that are available? Or consider a market where some varieties will be offered if and only if new firms enter the market. How do prices, quantities and welfare change if such market entry occurs? Suppose that $q_j = 0$ for $j = m+1, ...n$, i.e., some varieties are not available because they are sold out or entry did not occur. In this case, a consumer with preferences according to (2) maximizes:

$$U = \sum_{j=1}^{m} q_j - \frac{1}{2}\left(\sum_{j=1}^{m} q_j\right)^2 - \frac{n}{2(1+\gamma)}\left[\sum_{j=1}^{m} q_j^2 - \frac{\left(\sum_{j=1}^{m} q_j\right)^2}{n}\right] + \left[w - \sum_{j=1}^{m} q_j p_j\right]. \tag{3}$$

[1] See Vives (2001), p. 163. Note that there is a typo, where for the last term in the utility function it reads $\sum_j q_j$, while correctly it should be $\left(\sum_j q_j\right)^2$.

Note that (2) differs from the utility of a consumer who is not interested in goods $j = m+1, ...n$. Such a consumer would also consume $q_j = 0$ for $j = m+1, ...n$, but in his utility function m instead on n appears in the product $\frac{n}{2(1+\gamma)} \sum_{j=1}^{m} q_j^2$. This subtle change affects the demand for all the other goods. A consumer with utility function (2) who cares about goods $j = m+1, ..., n$ that are not available and is constrained not to buy them has the following demand functions for the available varieties $j \leq m$:

$$D_j(p) = \frac{1+\gamma}{n}\left[1 - p_j - \frac{\gamma}{n+m\gamma}\left(m - \sum_{i=1}^{m} p_i\right)\right] \text{ for } j \leq m \leq n. \tag{4}$$

On the other hand, a consumer who does not care about goods $j = m+1, ..., n$ and would not buy them even if he could has different demand functions for varieties $j \leq m$:

$$D_j(p) = \frac{1}{m}\left(1 - p_j - \gamma\left(p_j - \frac{\sum_{i=1}^{m} p_j}{m}\right)\right) = \frac{1}{m}\left(1 - p_j - \gamma\left(p_j - \overline{p}\right)\right) \text{ for } j = 1, ..., m, \tag{5}$$

where $\overline{p}$ denotes the "average price".

It is a standard approach to model competition in horizontally differentiated products by assuming that there are n firms, and each firm $i = 1, ..., n$, produces a different good. Obviously, there are many interesting questions concerning the number of firms active in an industry. In this analytical framework, varying the number of firms is often modeled as varying the number of goods available, i.e. by comparing situations in which some goods $j = m+1, ..., n$ are not available to a situation in which they are available and in which the consumer would buy them. One might therefore be tempted to use (5) and perform comparative statics with respect to the number of products m. Typical examples using (5) or some structurally identical formulations, are the analysis of exclusion of firms (e.g. Ordover and Shaffer (2007), Kovenock and Roy (2005), or Bourreau et. al. (2007)) or incentives to merge (Inderst and Wey (2004)) or other forms of comparative statics with respect to the number of firms (Fries et. al. (2006)).

This approach, however, seems problematic. It makes a difference for the demand system whether a consumer is constrained not to buy some goods or whether he voluntarily abstains from buying them. Therefore, if one would use the formulation (5) to evaluate market outcomes for m and $m+1$ (e.g. in order to analyze the effect of the entry of one additional firm), not only the number of products is changed but also the underlying demand structure. Therefore, it will not be possible to easily disentangle which of the two changes drives the results.

3 Example

As an illustration, consider the following problem of exclusion. There are $n = m = 3$ firms in a market, each charging a price of $\frac{1}{2}$. According to (4) with $m = n$ (and according to (5)), each firm could sell $q_i = \frac{1}{6}$. Now imagine that firm 1 and firm 2 manage to exclude firm 3 from the market while still charging $p_1 = p_2 = \frac{1}{2}$ (e.g. by raising firm 3's marginal production cost above $\frac{7}{8}$, since $q_3\left(\frac{1}{2}, \frac{1}{2}, p_3 \geq \frac{7}{8}\right) = 0$). To consistently analyze the effect of such an exclusion, we need to use demand function (4) with $m = 2$ and $n = 3$ for determining the new equilibrium quantities, which are given by $q_1 = q_2 = \frac{3}{16}$, implying an increase in sales for the excluding firms of $\Delta q_1 = \Delta q_2 = \frac{1}{48}$

However, if we would use (5) for the analysis and just reduce m from 3 to 2, we would implicitly assume that consumers no longer care about the third product — which seems strange, since in the initial situation they actually bought it. Furthermore, the prediction of such an approach would be that the new quantities after the exclusion would be $\widehat{q}_1 = \widehat{q}_2 = \frac{1}{4}$, implying a larger gain in sales for the two excluding firms of $\frac{1}{12}$ each. However, this change is a combined result of the exclusion and the change of the underlying demand system, while the difference derived in the previous paragraph can be attributed exclusively to exclusion. Thus, when using (5) and just varying m, one might make mistakes in the positive analysis; in this example by overestimating the incentive for such an exclusionary practice.

Furthermore, a consistent welfare analysis[2] requires that the two demand functions in the two situations are derived from the same consumer, while the demand functions (4) and (5) are derived from different consumers. This is important, in particular when evaluating normatively the effects of exclusion, market entry and entry deterrence.

4 Conclusion

Therefore, if linear demand systems shall be employed to analyze the effect of variations in the number of available products, we propose to use the consistent formulation (4).

References

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[2] The authors of the papers cited do not undertake a welfare analysis.

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Appendix

How to derive (4):

Assume, utility is given by (2), but impose that $q_j = 0$ for $j = m+1, ..., n$.Thus, the consumer maximizes:

$$U = \sum_{j=1}^{m} q_j - \frac{1}{2}\left(\sum_{j=1}^{m} q_j\right)^2 - \frac{n}{2(1+\gamma)}\left[\sum_{j=1}^{m} q_j^2 - \frac{\left(\sum_{j=1}^{m} q_j\right)^2}{n}\right] + \left[w - \sum_{j=1}^{m} q_j p_j\right].$$

The first order conditions for $j = 1, ..., m$ are:

$$\begin{aligned} p_j &= \frac{\partial U}{\partial q_j} \\ p_j &= 1 - \sum_{j=1}^{m} q_j - \frac{n}{2(1+\gamma)}\left[2q_j - \frac{2\left(\sum_{j=1}^{m} q_j\right)}{n}\right] \\ q_j &= \frac{1+\gamma}{n} - \frac{1+\gamma}{n} p_j + \frac{\gamma}{n} \underbrace{\sum_{j=1}^{m} q_j}_{=:X} \end{aligned} \tag{6}$$

Summing over all m yields:

$$X = \frac{m(1+\gamma)}{n} - \frac{1+\gamma}{n}\sum_{j=1}^{m} p_j + \frac{m\gamma}{n}X \tag{7}$$

$$X = \frac{m(1+\gamma)}{n-\gamma m} + \frac{1+\gamma}{n-\gamma m}\sum_{j=1}^{m} p_j. \tag{8}$$

Plugging (8) into (6) then yields the result in the paper.