E C O N O M I C S   B U L L E T I N

Price Discrimination As Portfolio Diversification

Parikshit Ghosh
Indian Statistical Institute

Abstract

A seller seeking to sell an indivisible object can post (possibly different) prices to each of n buyers. Buyers' valuations are private information and drawn independently from the same distribution. If the seller can choose who to sell to in the event there are several willing buyers, her optimal strategy is to post different prices to different buyers. For some distributions, price discrimination may be profitable even if excess demand must be resolved through a uniform lottery.

Citation: Ghosh, Parikshit, (2008) "Price Discrimination As Portfolio Diversification." *Economics Bulletin,* Vol. 4, No. 5 pp. 1-9
Submitted: January 24, 2008. **Accepted:** March 7, 2008.
URL: http://economicsbulletin.vanderbilt.edu/2008/volume4/EB-08D40018A.pdf

1 Introduction

A person is trying to sell a piece of antique furniture by placing classified ads in five different newspapers. Assume the readership of these newspapers are disjoint but have similar income and taste profiles. Should the owner ask for the same price in all five ads, or should she quote five different asking prices (price discrimination)?

This note shows that in many plausible scenarios, the seller's optimal policy is to price discriminate. If she can wait long enough to gather all responses and choose the highest advertised price that received a positive response, it is always profitable to set different prices rather than a uniform price. If she must choose the first positive response that comes her way, it may still be profitable to depart from a uniform price depending on the distribution of buyers' values.

Standard theories of price discrimination hinge on either *observable* heterogeneity among buyers or self selection mechanisms that reveal buyer types. The latter is achieved by making the price conditional on factors whose effect on buyers depends on their willingness to pay. For example, intertemporal price discrimination (Stokey (1979)) exploits the fact that high valuation buyers suffer bigger delay costs, quantity discounts utilize differential marginal utility of high and low valuation buyers (Maskin and Riley (1984)), price variation across outlets relies on less price sensitive buyers having higher search costs (Salop (1977)) and mixed bundling sorts consumers by preference (Adams and Yellen (1976)).[1] In all these cases, price discrimination rides on some method of extracting information, and there is positive correlation between the price a consumer pays and her utility from consuming the good.

In contrast, price discrimination arises here as a way of maximizing seller's return through a diversified portfolio of options. By asking for a high price from some customers, the seller takes a high risk, high payoff gamble. By setting a low asking price for others, she creates a low risk fallback option in case the risky gamble doesn't pay off. Not only are observationally identical agents treated differently, there is no correlation between the price they face and

[1] For a fuller discussion of various kinds of price discriminaion, see Tirole (1988). There is also a large literature on price *dispersion*, but these papers focus not on monopolies but competitive markets with search friction (see for example, Salop and Stiglitz (1977) and Burdett and Judd (1983)).

their private value.

A critical assumption behind these results is that sellers are capacity constrained and the good is indivisible. This means various buyers are substitutes from the seller's perspective, as in an auction. With unlimited capacity as in standard models, the interaction with each buyer works like a separate monopoly problem and price discrimination cannot arise if buyers are *ex ante* identical.

2 The Model

A seller wants to sell an indivisible object. Its value to herself for personal use is normalized to 0. There are n potential buyers, indexed $1, 2, ..., n$. Buyer i's private value of the object is x_i, which is a random variable drawn from a distribution with a density function $f(.)$ on the domain $[\underline{x}, \overline{x}]$ and corresponding c.d.f $F(.)$. Buyer's values are independent and private information.[2]

It is well known that first or second price auctions with appropriate reserve prices constitute an optimum (revenue maximizing) mechanism in this simple environment (Myerson (1979)). Auctions do better than a single posted price because they exploit the private information of interested buyers. However, auctions also present some coordination and credibilty issues which may make them difficult to administer in many practical situations. For example, buyers may be sufficiently separated in space and time to bring them together in a single auction. In some formats like the second price auction, the seller has an incentive to misrepresent the bids of others to the winner. Without going very deeply into why the set of feasible mechanisms may be restricted, I will focus on a particularly simple and commonly observed selling mechanism—posted prices. However, instead of confining ourselves to a *single* posted price, I allow the seller to price discriminate, i.e., post possibly different prices to different buyers or sub-markets. The question of interest is whether it may be profitable to exercise such price discrimination even if buyers are *ex ante* identical.

Suppose the seller can post a price p_i to the ith buyer. Each p_i is interpreted as a

[2]An alternative interpretation is that these are n separate sub-markets, as opposed to individuals, in which case the random variable x_i is to be interpreted as the highest willingness to pay among all customers in sub-market i.

contingent price—it is the price i has to pay only if he receives the good.[3] Buyers can respond with a yes or no, where 'yes' implies an agreement to buy the good at the posted price if it is made available. To complete the description, we need to specify how the good gets allocated in the event of excess demand. I will explore two scenarios: (i) *seller discretion*: among the willing buyers, the seller can choose who should get the good (ii) *lottery*: one of the willing buyers is chosen at random through a uniform lottery.

2.1 Seller Discretion

In this case, the solution to the problem of posting prices (simultaneously) is the same as that to a related sequential pricing problem which is described below.

Suppose buyers arrive sequentially (in the sequence $1, 2, ..., n$). To buyer i, the seller can make a take-it-or-leave-it offer at some price p_i, which this buyer can either accept or reject. If he accepts, the transaction is carried out and the game ends. If he rejects, the seller moves on to the next buyer. If none of the buyers agree to their respective prices, the good is not sold and the seller's payoff is zero.

Buyers' optimal strategies being very simple (says 'yes' if and only if the posted price is less than his value), the problem reduces to a simple dynamic programming problem for the seller. Her feasible plans are price sequences $\{p_i\}_{i=1}^n$. The *ex ante* optimal plan $\{p_i^*\}_{i=1}^n$ is the price sequence which solves

$$\max_{p_i} \pi(p_1, p_2, ..., p_n) \equiv \sum_{i=1}^{n} \left(\prod_{j=1}^{i-1} F(p_j) \right) [1 - F(p_i)]\, p_i \tag{1}$$

where $\prod_{j=1}^{i-1} F(p_j)$ is defined to be 1 for $i = 1$. The sale takes place at price p_i if all buyers j, with $j < i$, decline to buy at the prices posted to them, but i accepts. The probability of this event is $\prod_{j=1}^{i-1} F(p_j).\,[1 - F(p_i)]$. The objective function is the average of the posted prices, weighted by their respective probabilities of materializing as the transaction price.

Lemma 1 *The* ex ante *optimal plan is time consistent.*

[3]The posted prices may be thought of as price offers that are "valid till stocks last".

Proof. Suppose not. Then, for some k, after the first k buyers decline, the seller will find it profitable to deviate from the price sequence $\{p_i^*\}_{i=1}^n$ and offer an alternative subsequence $\{p_i'\}_{i=k+1}^n$ instead. This subsequence yields higher expected profit from the $(k+1)$th decision node onwards, i.e.,

$$\sum_{i=k+1}^{n}\left(\prod_{j=k+1}^{i-1}F(p_j')\right)[1-F(p_i')]\,p_i' > \sum_{i=k+1}^{n}\left(\prod_{j=k+1}^{i-1}F(p_j^*)\right)[1-F(p_i^*)]\,p_i^*$$

Multiplying both sides by $\prod_{j=1}^{k}F(p_j)$ and adding $\sum_{i=1}^{k}\left(\prod_{j=1}^{i-1}F(p_j^*)\right)[1-F(p_i^*)]\,p_i^*$, we get

$$\pi(p_1^*,...,p_k^*,p_{k+1}',...,p_n') > \pi(p_1^*,...,p_k^*,p_{k+1}^*,...,p_n^*)$$

which contradicts the fact that $\{p_i^*\}_{i=1}^n$ is *ex ante* optimal. ■

Time consisency of the seller's optimal plan in the sequential problem allows us to find it using dynamic programming techniques. Let v_i denote the value function when the first i buyers have refused and exactly $n-i$ buyers remain. Then optimal prices are given by the Bellman equation:

$$p_i^* \in \arg\max_{p_i}\,[1-F(p_i)]\,p_i + F(p_i)v_{i+1} \tag{2}$$

and the value functions satisfy the recursive relation

$$v_i = [1-F(p_i^*)]\,p_i^* + F(p_i^*)v_{i+1} \tag{3}$$

Lemma 2 *Suppose for all $v_{i+1}\in[\underline{x},\overline{x}]$, p_i^* as defined by (2) is unique.*[4] *Then the optimal price sequence is strictly decreasing, i.e., $p_1^* > p_2^* > ... > p_n^*$.*

Proof. First, observe that since each value function is an expected transaction price, it must lie in the interior of the domain of buyers' values, i.e., $\underline{x} < v_i < \overline{x}$. Further, the sequence of value functions is strictly decreasing: for all i, $v_i > v_{i+1}$. This is because on examining the

[4] If $f(.)$ is differentiable, a sufficient condition for uniqueness is that the hazard rate $\frac{f(.)}{1-F(.)}$ is increasing. The first order condition for the maximization problem in (2) is: $p_i - \frac{1-F(p_i)}{f(p_i)} = v_{i+1}$. With increasing hazard rate, the left hand side is monotone increasing in p_i, while the right hand side is a constant, implying a unique solution.

objective function of the ith period problem (right hand side of (2)), it becomes clear that $v_i > v_{i+1}$ can be guaranteed by choosing any price $p_i \in (v_{i+1}, \overline{x})$.

For all i, since $v_i \neq v_{i+1}$, unique optimality of p_i^* and p_{i-1}^* implies the following inequalities:

$$\begin{aligned} \left[1 - F(p_i^*)\right] p_i^* + F(p_i^*) v_{i+1} &> \left[1 - F(p_{i-1}^*)\right] p_{i-1}^* + F(p_{i-1}^*) v_{i+1} \\ \left[1 - F(p_{i-1}^*)\right] p_{i-1}^* + F(p_{i-1}^*) v_i &> \left[1 - F(p_i^*)\right] p_i^* + F(p_i^*) v_i \end{aligned}$$

Adding the two inequalities and rearranging terms, we get

$$\left[F(p_{i-1}^*) - F(p_i^*)\right](v_i - v_{i+1}) > 0$$

Since $v_i > v_{i+1}$, it follows that $F(p_{i-1}^*) > F(p_i^*) \Rightarrow . p_{i-1}^* > p_i^*$. ■

The intuition behind Lemma 2 is straightforward. When there are still many buyers to come, the option value of continuing search is high and rejection is not very costly. Hence the seller will set a high price. When she is down to the last few buyers, the reasoning is reversed and the seller faces a steep trade-off between price and probability of sale. She will, consequently, lower the price.

Finally, note that the simultaneous price posting problem described originally has essentially the same solution (except for permutations) as the sequential price posting game characterized above, i.e., $\{p_i^*\}_{i=1}^n$ is also the solution to the former. To see this, consider simultaneous price posting and without loss of generality, impose the following ordering: $p_1 \geq p_2 \geq ... \geq p_n$. Given seller's discretion, she will choose to sell at the highest price which receives a positive response. Her objective function, then, is the same as (1), the *ex ante* payoff in the sequential pricing problem, with the added constraint that $p_1 \geq p_2 \geq ... \geq p_n$. Since the *ex ante* optimal plan of the sequential problem satisfies the constraint (Lemma 2), it is also the solution to the simultaneous pricing problem.

Proposition 1 *In the simultaneous price posting problem, if excess demand is resolved through seller's discretion and p_i^* as defined by* (2) *is unique, the optimal prices are distinct from one another.*

Proof. Follows straight from the preceding discussion and Lemma 2. ■

2.2 Random Allocation

An alternative way of modeling the price posting problem would depart from the previous framework in how excess demand is handled. Here I will consider a scenario in which one of the willing buyers is chosen at random by a uniform lottery rather than by seller's discretion. This is a parsimonious way of capturing the following situation—after the seller has posted all her prices, responses arrive stochastically over time and the seller may be compelled by legal, institutional or self interest reasons to serve the first buyer to express interest.

In such situations, whether optimal pricing is uniform or discriminatory depends on the distribution of buyers' types. To illustrate this, I will consider a simpler discrete model with two buyers, 1 and 2. Each buyer could have a high value x_H, with probability q, or a low value x_L, with probability $1-q$. Buyers' types are independent. The seller simultaneously posts prices p_1 and p_2 to the two buyers, and if both respond 'yes', the good is allocated by the toss of a fair coin and the recipient pays the price that was posted to him. Let $\pi(p_1, p_2)$ denote the expected profit from the posted price pair (p_1, p_2) under these rules.

Assume buyers always want to buy when indifferent. The seller's optimum must be one of three pricing strategies: (i) uniformly low pricing ($p_1 = p_2 = x_L$) (ii) uniformly high pricing ($p_1 = p_2 = x_H$) (iii) discriminatory pricing ($p_1 = x_H, p_2 = x_L$). The next result shows discriminatory pricing is optimum in some but not all situations.

Proposition 2 *In the simultaneous price posting problem, if excess demand is resolved through an uniform lottery, optimal prices are discriminatory ($p_1^* = x_H, p_2^* = x_L$) if and only if*

$$\frac{x_L}{x_H} > \frac{(1-q)(1+2q)}{1+q} \tag{4}$$

Proof. The expected payoffs from the three candidate strategies are as follows:

$$\begin{aligned} \pi(x_L, x_L) &= x_L & (5)\\ \pi(x_H, x_H) &= (1-q^2)x_H & (6)\\ \pi(x_H, x_L) &= \frac{1}{2}(1-q)(x_H + x_L) + qx_L & (7) \end{aligned}$$

When both prices are low, a sale is guaranteed. When both prices are high, the probability that at least one buyer will accept is $1-q^2$. With the discriminatory (high-low) pricing strategy, the buyer facing the low price is always willing to buy. If the buyer facing the high price is also willing to buy (probability $1-q$), the expected price the seller will receive is the average, $\frac{1}{2}(x_H + x_L)$. Otherwise, she receives the low price, x_L.

Discriminatory pricing is optimum when the expression in (7) dominates both (5) and (6). It is easily checked that the first is always true, while (7) dominates (6) if and only if (4) is satisfied. ■

The set of parameters satisfying (4) is of positive measure, since for any $q \in (0,1)$, the right hand side of (4) also lies between 0 and 1. The condition places a lower bound on x_L relative to x_H, because if x_L is too low, it is worth taking a gamble with both customers instead of securing a "bottomline" by keeping one price low and gambling with the other.

3 Conclusion

The literature on price discrimination emphasizes consumer heterogeneity, either directly observed or revealed through a menu of contracts and buyer self selection. This note raises another possibility—sellers' desire to diversify their portfolio of selling options. The effect is extremely robust when the environment allows a seller to wait and choose from the set of willing buyers. It may also arise when this selection is effectively random, but depends on the distribution of buyers' private values in that case.

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