E C O N O M I C S   B U L L E T I N

Market power, the multiplier and economic policy under oligopolistic competition

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Abstract

In this note, we consider a multisector macroeconomic model under oligopolistic competition. We analyze the effect of an increase of the number of sectors on equilibrium price and on allocations, when the number of oligopolits of each type is constant. We also show that a tax policy has more impact on aggregate activity when the economy has many sectors. Additionally, the tax multiplier is higher than the expenditure multiplier.

This note is a macroeconomic piece of a more general research devoted to cooperation and coordination failures under strategic interactions.
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1 Introduction

In this note, we extend the two-sector Cournot-Nash model proposed by Cooper (1999)[1]. We thus consider a general oligopoly equilibrium macroeconomic model with L sectors in the spirit of Hart (1982), Jones and Manuelli (1992) or Roberts (1987). In order to simplify, we only focus on strategic interactions on the output markets and do not develop the labor market analysis[2]. Thus the environment is the same as in Cooper (1999), the only modification is the introduction of a large but finite number of sectors. Introducing several sectors has two advantages. Firstly, the model proposed generalizes the two goods framework and puts forward the strategic interactions among many markets[3]. Secondly, the macroeconomic consequences of market distorsions caused by imperfect competition on equilibrium prices and allocations can easily be captured by extending the size of the economy, instead of replicating it. In the model, we henceforth keep constant the number of agents within each sector.

This paper therefore captures the consequences of market power on equilibrium allocation within a macroeconomic perspective. We restrict the analysis to symmetric Nash equilibrium[4]. Additionally, we study welfare and economic policy under balanced-budget rule (Schmitt-Grohé and Uribe (1997)). Within this framework, four kinds of results are obtained. Firstly, when the number of sectors increases unboundedly, with a constant number of agents competing in each sector, the market prices and the equilibrium allocations do not concide with the competitive ones. Secondly, welfare increases with consumption when the economy becomes large, with a constant number of agents per sector. Thirdly, it is shown that the multiplier increases with the number of sectors. Fourthly, introducing a uniform taxation policy, we notably establish that the tax rate multiplier is higher than the expenditure one. Additionally, the tax rate increases with the number of sectors in the economy.

The paper is organized as follows. In section 1, we describe the basic economy. In section 2, we consider the optimal strategic plans and determine the symmetric macroeconomic equilibrium. In section 3, we study welfare and the multiplier through economic policy under balanced-budget rule. We notably compare the magnitude of the tax multiplier with the strengh of the expenditure multiplier. In section 4, we conclude.

[1] See Cooper, section 1, Chapter 3. See also Cooper and John (1988), Hart (1982), Jones and Manuelli (1992) or Roberts (1987).

[2] The labor market could be introduced without changing the main features of the model.

[3] The model analyzed here can be represented as a two-step game that arises between producers: in a first step, equilibrium prices are determined for given strategies; in a second step, the equilibrium strategies are determined at these equilibrium prices. This framework has some connections with the concept of oligopoly equilibrium for pure exchange economies notably developed by Gabszewicz (2002).

[4] Imperfect competition holds in all markets. We thus do not consider different degrees of competition between markets. This assumption precludes asymmetric equilibrium concepts.

2 The economy

Consider a L-sector economy with Ln agents indexed h, $h = 1, ..., Ln$, with n agents per sector. All agents are identical within a sector. There are L consumption goods indexed ℓ, with $\ell = 1, ..., L$, and money, which is the *numéraire* commodity for which all agents have an endowment $\bar{m}_h$, $\forall h$. The price of money is then equal to 1, while the prices of good ℓ is denoted p_ℓ, $\forall \ell$. Each good ℓ, $\ell = 1, ..., L$, is produced in quantities $y_{h\ell}$ according to the same constant returns to scale technology, so total costs are a linear function of production[5]. We assume that any agent of one type consumes only the goods produced by the agents of the other types. This feature captures the decentralization of economic activities: the specialization in production and the generalization in consumption[6]. Additionally, each agent h holds real balances m_h. The preferences of agent h are represented by the following utility function:

$$U_h = \prod_{k \neq \ell} \left(\frac{c_{hk}}{\alpha_{hk}} \right)^{\alpha_{hk}} \left(\frac{m_h}{1 - \sum_{k \neq \ell} \alpha_{hk}} \right)^{1 - \sum_{k \neq \ell} \alpha_{hk}} - \beta_\ell y_{h\ell} \text{ , } \forall h \text{ for } k \neq \ell \text{ ,} \quad (1)$$

where $\alpha_{hk} \in (0, 1)$, $\forall k$ measures the strengh of the demand linkage across all sectors and $\beta_\ell \in (0, 1)$, $\forall \ell$ is the marginal disutility of production for good ℓ.

Each oligopolist determines first as a consumer his demand for the good and money. After, he determines as a producer his strategic supply. The program of any consumer h, $h = 1, ..., n$, writes:

$$\underset{(c_h \in IR_+^{L-1}, m_h)}{Max} U_h(c_h, m_h, y_{h\ell}) \text{ s.t. } \sum_{k \neq \ell} p_k c_{hk} + m_h \leq p_\ell y_{h\ell} + \bar{m}_h \text{ , } \forall h \text{ for } k \neq \ell. \quad (2)$$

For given $y_{h\ell}$, the demand functions are $c_{hk} = \alpha_{hk} \frac{\Omega_h}{p_k}$, $\forall k \neq \ell$ and $m_h = (1 - \sum_k \alpha_{hk}) \Omega_h$, where $\Omega_h \equiv p_\ell y_{h\ell} + \bar{m}_h$.

Each oligopolist then maximizes his indirect utility function in order to determine his strategic supply, taking as given the supply of other oligopolists within their sector, i.e. $\sum_{-h} y_{-h\ell}$, the price of the other goods p_k, $\forall k \neq \ell$ and the income of the other sectors Ω_k, with $\Omega_k = \sum_h (p_k y_{hk} + \bar{m}_h)$, $\forall k \neq \ell$. The program of any producer h, for $k \neq \ell$, thus writes:

$$\underset{\{y_{h\ell}\}}{Arg \max} \; p_k^{-\sum_k \alpha_{hk}} \left[p_\ell \left(y_{h\ell} + \sum_{-h \neq h} y_{-h\ell} \right) y_{h\ell} + \bar{m}_h \right] - \beta_\ell y_{h\ell}. \quad (3)$$

This leads to the n first-order conditions, where marginal revenue balances marginal cost:

[5] The distribution of profits and its effects in the economy are thus deleted.
[6] See Diamond (1982), Roberts (1987) or Weitzman (1982).

$$p_k^{-\sum_k \alpha_{hk}} \left(p_\ell + \frac{\partial p_\ell}{\partial y_{h\ell}} y_{h\ell} \right) - \beta_\ell = 0 \text{ , for } \ell \neq k. \tag{4}$$

These n optimality conditions puts into perspective sectoral and intersectoral strategic interactions and involve market equilibrium. The other $(L-1)n$ optimality conditions are similarly defined.

3 Symmetric macroeconomic equilibrium

A symmetric macroeconomic equilibrium is a price level $\tilde{p}$, with $\tilde{p} = \tilde{p}_\ell$, $\forall \ell$, and an allocation per oligopolist $\tilde{y}_{h\ell}$, with $\tilde{y}_h = \tilde{y}_{h\ell}$, $\forall h$, $\forall \ell$ such that all markets clear and each oligopolist optimizes at this allocation for these prices.

The equilibrium concept is a two-step subgame perfect equilibrium. In the first step, each agent determines his best supply strategy taking as given the equilibrium price system (the market clearing conditions) and the strategies of all other oligopolists. In the second step, the equilibrium prices which clear all markets are determined. The game is solved by backward-induction, so the price system, which clears all markets, is firstly determined, and after oligopolists interact in quantity spaces in order to determine their equilibrium strategies. Thus, the equilibrium prices which clear all markets is determined for given strategies and the equilibrium level of activity is determined through strategic interactions (between reaction functions) in quantity spaces.

Within each sector, each oligopolist rationally expects the equilibrium price that clears the market when he determines his optimal plan. The market-clearing condition for good ℓ rationally expected by oligopolists is $p_\ell = \Xi / y_\ell$, $\forall \ell$, where $\Xi \equiv \sum_{-h \neq h} \alpha_{h\ell} \Omega_{-h}$ represents total expenditure[7], and $y_\ell = \sum_{h=1}^{h=n} y_{h\ell}$. From (4), we have:

$$p_\ell p_k^{-\sum_k \alpha_h} z - \beta_\ell = 0 \text{ , for } \ell \neq k \text{ ,} \tag{5}$$

where $z = 1 - \frac{1}{n}$ represents the mark-up in sector ℓ[8]. Hence, the extent of imperfect competition in each market depends on the number of firms and also on the number of sectors in the economy.

At a symmetric general oligopoly equilibrium, we obviously have $\alpha_{h\ell} = \alpha_\ell$, $\forall h$, $\forall \ell$, $\beta_\ell = \beta$, $\forall \ell$, and $p_\ell = \tilde{p}$, $\forall \ell$. Considering that $\frac{1}{L-1} \sum_k \alpha_k = \bar{\alpha}$, the equilibrium price level follows:

$$\tilde{p} = \left(\frac{\beta}{z} \right)^{\frac{1}{1-(L-1)\bar{\alpha}}} . \tag{6}$$

We deduce the reaction functions of any oligopolist h, $h = 1, ..., Ln$, in the symmetric case:

[7] The market-clearing conditions involve monetary prices *à la* Shapley-Shubik (1977).

[8] The term $-1/n$ represents the inverse of the price elasticity of demand evaluated at the equilibrium (-1) times the market share of oligopolist h, i.e. $1/n$, $\forall h = 1, ..., n$.

$$y_{h\ell} = (L-1)\bar{\alpha}\left(y_{hk} + \frac{\bar{m}_h}{\tilde{p}}\right), \forall \ell, \forall k \neq \ell. \quad (7)$$

The equilibrium level of output per firm in each sector $\tilde{y}_{h\ell}$ is given by $\tilde{y}_{hk} = \tilde{y}_{h\ell}, \forall k \neq \ell$:

$$\tilde{y}_{h\ell} = \left[\frac{(L-1)\bar{\alpha}}{1-(L-1)\bar{\alpha}}\right]\frac{\bar{m}_h}{\tilde{p}}, \forall h, \forall \ell. \quad (8)$$

The equilibrium level of activity per sector is then given by averaging (10) over the n agents within each sector:

$$\tilde{y}_{\ell} = \left[\frac{(L-1)\bar{\alpha}}{1-(L-1)\bar{\alpha}}\right]\frac{\bar{m}}{\tilde{p}}, \forall \ell. \quad (9)$$

where $\bar{m} = \sum_h \bar{m}_h$. Under the assumption $(L-1)\bar{\alpha} > 1/2$, the multiplier is greater than one[9].

Proposition 1 *When the number of sectors increases unboundedly, with a constant number of agents per sector, the equilibrium market price and the equilibrium allocations do not coincide with the competitive ones.*

Proof. We compute the macroeconomic equilibrium, i.e. the price level $\tilde{p}$ and the allocation reached by any oligopolist, i.e. $\tilde{y}_{h\ell}$, $\forall h$, $\forall \ell$, and compare it with the result that would be obtained in a competitive environment. When the economy becomes large, i.e. $L \to \infty$, with $\bar{\alpha} \simeq 1/L$, the price and the allocation reached by any oligopolist become respectively $\tilde{p} = \left(\frac{\beta}{z}\right)^{\frac{1}{1-\bar{\alpha}}}$ and $\tilde{y}_{h\ell} = \left(\frac{\bar{\alpha}}{1-\bar{\alpha}}\right)\frac{\bar{m}_h}{\tilde{p}}$, $\forall \ell$. Under perfect competition, each agent maximizes his utility function under his budget constraint, taking all the prices as given, i.e. $Max \prod_{k\neq\ell}\left(\frac{c_{hk}}{\alpha_{hk}}\right)^{\alpha_{hk}}\left(\frac{m_h}{1-\alpha_{hk}}\right)^{1-\sum_{k\neq\ell}\alpha_{hk}} - \beta_\ell y_{h\ell}$ s.t. $\sum_{k\neq\ell} p_k c_{hk} + m_h \leq \bar{m}_h + p_\ell y_{h\ell}$, $\forall h$ for $\ell \neq k$. This gives the demand $c_{hk} = \alpha_{hk}\frac{\Omega_h}{p_k}$, $\forall h$, $\forall k$ and $m_h = (1-\sum_{k\neq\ell}\alpha_{hk})\Omega_h$, $\forall h$, where $\Omega_h \equiv p_\ell y_{h\ell} + \bar{m}_h$. The resolution of (3) yields $p^{-\sum_{k\neq\ell}\alpha_{hk}} p_\ell = \beta_\ell$, $\forall \ell$. Since by symmetry, $\alpha_{h\ell} = \bar{\alpha}$, $\forall h$, $\forall \ell$ and $\beta_\ell = \beta$, $\forall \ell$, the competitive price p^*_ℓ and the corresponding allocation $y^*_{h\ell}$ are $p^*_\ell = \beta^{\frac{1}{1-\alpha}}$, $\forall \ell$ and $y^*_{h\ell} = \frac{\bar{\alpha}}{1-\bar{\alpha}}\left(\frac{\bar{m}}{\beta^{\frac{1}{1-\bar{\alpha}}}}\right)$, $\forall \ell$, $\forall h$. Finally, $(p^*, y^*_{h\ell}) \neq (\tilde{p}, \tilde{y}_{h\ell})$. *QED.* ■

Proposition 1 means that growing the economy does not lead to mimic the results obtained in an environment where the number of agent becomes arbitrarily large, or when the economy is replicated an infinite number of times. Intrasectoral market power is not equivalent to intersectoral market power: the effects of market shares generally differ from the effects of market size[10].

[9] In Cooper's model, the same condition holds when $L = 2$.

[10] When $\alpha \to \infty$ in each sector, the equilibrium price and the corresponding allocations coïncide with the competitive ones (see Cooper (1999) for $L = 2$).

Proposition 2 *Welfare increases with consumption when the economy becomes large, with a constant number of agents per sector.*

Proof. *Consider the utility* $\tilde{U}_h$ of *any agent* h, $h = 1, ..., Ln$, with $\tilde{U}_h = \left[\frac{\tilde{y}_{h\ell}}{(L-1)\bar{\alpha}}\right]^{(L-1)\bar{\alpha}} \left[\frac{\bar{m}_h}{1-(L-1)\bar{\alpha}}\right]^{1-(L-1)\bar{\alpha}} - \beta_\ell y_{h\ell}$. *We have to show that* $\frac{\partial \tilde{U}_h}{\partial \tilde{y}_{h\ell}} > 0$, $\forall h = 1, ..., Ln$. *Differentiating* $\tilde{U}_h$ *at equilibrium with respect to* $\tilde{y}_{h\ell}$ *yields* $\frac{\partial \tilde{U}_h}{\partial \tilde{y}_{h\ell}} = \left(\frac{\bar{m}_h}{\tilde{y}_{h\ell}}\right)^{1-(L-1)\bar{\alpha}} \left(\frac{(L-1)\bar{\alpha}}{1-(L-1)\bar{\alpha}}\right)^{1-(L-1)\bar{\alpha}} - \beta_\ell, \forall \ell$. *From* (8), *we deduce* $\frac{\partial \tilde{U}_h}{\partial \tilde{y}_{h\ell}} = [\tilde{p}(\nu)]^{1-(L-1)\bar{\alpha}} - \beta_\ell > 0$, $\forall \ell$ for $h = 1, ..., Ln$, *if* $z < 1$. *QED.* ■

As in Cooper (1999), the payoff of any agent increases with the degree of competition in markets. This positive externality within a sector is associated with an increase in the degree of competition in the other sectors: more trades imply lower markups and thus lower price for agents of the remaining sector[11].

Proposition 3 *The expenditure multiplier increases with the number of sectors.*

Proof. Immediate from (9): with $\frac{\partial\left[\frac{(L-1)\bar{\alpha}}{1-(L-1)\bar{\alpha}}\right]}{\partial L} = \frac{\bar{\alpha}}{[1-(L-1)\bar{\alpha}]^2} > 0$ since $\frac{\partial \tilde{y}}{\partial(\bar{m}/p)} = \frac{(L-1)\bar{\alpha}}{1-(L-1)\bar{\alpha}}$. *QED.* ■

The preceding proposition examplifies that the impact of the multiplier is higher when the number of commodity increases in the economy.

As an example, consider $L = 2$ and $L = 3$. Then, we respectively have $\frac{\partial \tilde{y}}{\partial(\bar{m}/p)} = \frac{\bar{\alpha}}{1-\bar{\alpha}}$ and $\frac{\partial \tilde{y}}{\partial(\bar{m}/p)} = \frac{2\bar{\alpha}}{1-2\bar{\alpha}} > \frac{\bar{\alpha}}{1-\bar{\alpha}}$ since here $\bar{\alpha} < 1/2$.

4 Economic policy

In order to regulate market distorsions caused by imperfectly competitive behaviors, consider a taxation policy on strategic supplies under balanced-budget rule[12]. We thus assume that a uniform tax τ, with $\tau \in (0, 1)$, must be paid by each strategic supplier in order to finance some government expenditure G_ℓ, $\forall \ell$, with $G_\ell = G$, $\forall \ell$. At the symmetric general equilibrium[13], one has $G = \tau \tilde{y}$.

The program of any oligopolist h who supplies good ℓ can now be written $Max \prod_{k \neq \ell} \left(\frac{c_{hk}}{\alpha_{hk}}\right)^{\alpha_{hk}} \left(\frac{m_h}{1-\alpha_{hk}}\right)^{1-\sum_{k\neq \ell}\alpha_{hk}} - \beta_\ell y_{h\ell}$ s.t. $\sum_{k \neq \ell} p_k c_{hk} + m_h \leq p_\ell (1 - \tau) y_{h\ell} + \bar{m}_h$, $\forall h$ for $k \neq \ell$. The corresponding market-clearing conditions write $p_\ell = \frac{\Omega(\tau)}{y_\ell} + p_\ell \tau y_\ell$, $\forall \ell$. Following the same procedure as in section 3 allows us to determine the price of each commodity ℓ, $\ell = 1, ..., L$, and the aggregate output per sector:

[11] There is also a congestion effect due to the increase of competitiveness in each sector, which decreases the utility of each oligopolist. But such an effect is dominated by the preceding (see Cooper (1999)).

[12] See Schmitt-Grohé and Uribe (1997) for a dynamic perspective in a one sector general equilibrium model.

[13] The competitive equilibrium tax rate is determined at the end of the section.

$$\tilde{p}(\tau) = \left[\frac{\beta(1-\tau)n}{(1-\tau)n-1}\right]^{\frac{1}{1-(L-1)\bar{\alpha}}}, \tag{10}$$

$$\tilde{y}(\tau) = \frac{(L-1)\bar{\alpha}}{[1-(L-1)\bar{\alpha}](1-\tau)}\frac{\bar{m}}{\tilde{p}(\tau)}. \tag{11}$$

Proposition 4 *The level of activity increases with the tax when the tax rate exceeds the markup.*

Proof. *Consider an equilibrium tax* $\tau \in (0,1)$*. We have to show* $\frac{\partial \tilde{y}}{\partial \tau} > 0$ *if* $\tau > z$*, where* $z = 1 - 1/n$*. From (10)-(11):* $\frac{\partial \tilde{p}}{\partial \tau} = \frac{\beta n}{\beta n(1-\tau)[1-(L-1)\bar{\alpha}][(1-\tau)n-1]}\tilde{p} > 0$ *and* $\frac{\partial \tilde{y}}{\partial \tau} = \left[\frac{1}{1-\tau} - \frac{1}{\tilde{p}}\frac{\partial \tilde{p}}{\partial \tau}\right]\tilde{y}$*. Little algebra give* $\frac{\partial \tilde{y}}{\partial \tau} > 0$ *if* $\tau > 1 - \frac{1}{n}$*. QED.* ■

The effect of a tax policy depends here on the extent of market power: when the economy becomes less imperfectly competitive, the markup decreases and provides an incentive for oligopolists, through the multiplier, to increase their supply. When $\tau \leq z$, the taxation policy has no effect; it is (entirely) 'absorbed' in the markup.

Proposition 5 *The tax multiplier is stronger than the expenditure multiplier.*

Proof. Immediate. From (9) and (11), we have $\frac{\partial \tilde{y}}{\partial(\bar{m}/p)} = \frac{(L-1)\bar{\alpha}}{[1-(L-1)\bar{\alpha}](1-\tau)} > \frac{(L-1)\bar{\alpha}}{1-(L-1)\bar{\alpha}} = \frac{\partial \tilde{y}}{\partial(\bar{m}/p)}$ since $\tau \in (0,1)$. *QED.* ■

When the magnitude of the multiplier depends not only on α but also on τ, the impact of an increase in autonomous expenditure $\bar{m}/p$ is reinforced by the government additional demand, which tends to excite economic activity.

Proposition 6 *The tax multiplier increases with the number of sectors.*

Proof. This is an immediate consequence of Prop. (3) and (5): $\frac{\partial\left[\frac{(L-1)\bar{\alpha}}{[1-(L-1)\bar{\alpha}](1-\tau)}\right]}{\partial L} = \frac{\bar{\alpha}}{(1-\tau)[1-(L-1)\bar{\alpha}]^2} > 0$. *QED.* ■

Let us now consider the equilibrium tax rate. Using (10) and (11), the balanced-budget rule given by $\tau\tilde{y}(\tau) = G$ leads to:

$$\frac{\tau}{(1-\tau)}\left[1 - \frac{1}{n(1-\tau)}\right]^{\frac{1}{1-(L-1)\bar{\alpha}}} = \left[\frac{1-(L-1)\bar{\alpha}}{(L-1)\bar{\alpha}}\right]\beta^{\frac{1}{1-(L-1)\bar{\alpha}}}\frac{G}{\bar{m}}. \tag{12}$$

Equation (12) has multiple solutions. So, consider the competitive case for which $n \to \infty$ in each sector. Then (12) gives $\tau^* = \frac{\left[\frac{1-(L-1)\bar{\alpha}}{(L-1)\bar{\alpha}}\right]\beta^{\frac{1}{1-(L-1)\bar{\alpha}}}\frac{G}{\bar{m}}}{1+\left[\frac{1-(L-1)\bar{\alpha}}{(L-1)\bar{\alpha}}\right]\beta^{\frac{1}{1-(L-1)\bar{\alpha}}}\frac{G}{\bar{m}}}$.

5 Conclusion

The preceding model notably shows that the impact of the (tax) multiplier is magnified when the economy becomes large. Additionally, the tax multiplier is stronger than the expenditure multiplier. These results could be compared with those obtained under monopolistic competition with free entry.

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