E C O N O M I C S B U L L E T I N

Dynamic political economy of redistribution policy: the role of education costs

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Abstract

This paper focuses on how education costs affect the political determination of redistribution policy via individual decision making on education. For cases of high costs, there are multiple equilibria: the poor-majority equilibrium featured by the minority of highly educated individuals and a high level of redistribution, and the rich-majority equilibrium featured by the majority of highly educated individuals and a low level of redistribution. For cases of low costs, there is a unique rich-majority equilibrium featured by the majority of highly educated individuals and a high level of redistribution.

The full version of this paper was formally circulated under the title gMarkov Perfect Political Equilibrium with Social Security: The Role of Education Cost. h We would like to thank Hideki Konishi, Ryuichi Tanaka and participants at the Seminar on Economic Theory and Econometrics, Osaka Prefecture University, February 2007; the Macroeconomics Conference for Young Professionals, Osaka, March 2007; Japanese Economic Association Annual Spring Meeting at Osaka Gakuin University, June 2007; Osaka University Forum at Groningen, June 2007; Far Eastern Meeting of the Econometric Society at Taipei, July 2007; and European Meeting of the Econometric Society at Budapest, August 2007 for their useful comments and suggestions. Arawatari acknowledges financial support from the Research Fellowships for Young Scientists of the Japan Society for the Promotion of Science (JSPS). Ono acknowledges financial support from the JSPS through a Grant-in-Aid for Young Scientists (B) (No. 20730208). All remaining errors are ours.
Citation: Ono, Tetsuo and Ryo Arawatari, (2008) "Dynamic political economy of redistribution policy: the role of education costs." *Economics Bulletin,* Vol. 4, No. 13 pp. 1-13
Submitted: May 26, 2008. **Accepted:** May 29, 2008.
URL: http://economicsbulletin.vanderbilt.edu/2008/volume4/EB-08D70025A.pdf

1 Introduction

Expectations about future redistribution policy affect the current individuals' decision making on educational investment, which, in turn, has an effect on the future distribution of income and, thus, on the future voting behavior over redistribution policy. Hassler et al. (2003) and Hassler et al. (2007) have provided politico-economic frameworks that incorporate this feedback mechanism and have shown that the mechanism results in multiple political equilibria: the poor-majority state, and the rich-majority state. In the poor-majority state, expectations of higher future redistribution lead to lower educational investments and, thus, to a lower proportion of highly educated individuals. This implies a larger size of poor individuals, which, in turn, increases future demand for redistribution, resulting in a higher level of redistribution. In the rich-majority state, expectations of lower future redistribution lead to higher educational investments and, thus, to a higher proportion of highly educated individuals. This implies a larger size of rich individuals, which, in turn, decreases future demand for redistribution, resulting in a lower level of redistribution.

Based on this feedback mechanism, Hassler et al. (2003) and Hassler et al. (2007) show that, depending on parameter values, the economy attains one of the following two types of equilibria: a unique poor-majority equilibrium, or multiple equilibria. Hassler et al. (2003) assume age-independent taxation that creates dynamic motion of redistribution, and they provide dynamic predictions of redistribution policies. The case of a unique poor-majority equilibrium provides an explanation for continental European countries after the oil shock, whereas the case of multiple equilibria provides an explanation for Anglo-Saxon countries that experienced a switch from a poor-majority equilibrium to a rich-majority equilibrium after the 1980s. In contrast, Hassler et al. (2007) assume age-dependent taxation that produces stationary redistribution policy. Focusing on the case of multiple equilibria, they provide an explanation for the cross-country differences in redistribution policies among western countries that share similar political backgrounds.

The theory of Hassler et al. (2003) and Hassler et al. (2007) contributes to the understanding of the differences with respect to redistribution policies between some OECD countries. However, their theory does not fully explain the empirical observation in Nordic countries (Denmark, Finland, Norway, and Sweden) that have experienced high graduation rates of tertiary education (OECD, 2005) and high levels of redistribution (OECD, 2007). Furthermore, empirical observations indicate that these features in the Nordic countries have been persistent for the last few decades. The aim of this paper is to develop a framework that provides a prediction of a persistent state featured by a high proportion of educated people and a high level of redistribution.

For the purpose of analysis, we focus on education costs in the model of Hassler et al. (2007) and extend their model by generalizing the education cost function. Hassler et al. (2007) assumed that the cost of investment in education e is given by $(e)^2$, whereas we assume that it is given by $\eta \cdot (e)^2$, where $\eta > 0$. For cases of high values of η, with Hassler et al. (2007) ($\eta = 1$) as a special case, individuals have a weak incentive to invest in education, which results in a similar equilibrium characterization to Hassler et al. (2007); the economy attains the poor-majority equilibrium or the rich-majority equilibrium depending on expectations by individuals. However, for cases of low values of η, individuals have a strong incentive to invest in education even if they expect a high future tax rate and, thus, a low return from the investment. Therefore, for cases of low education costs, the economy attains a rich-majority equilibrium that is featured by a large proportion of highly educated individuals and a high level of redistribution. The generalization of the education cost function creates a new prediction, which was not presented by Hassler et al. (2003) and Hassler et al. (2007).

The organization of this paper is as follows. Section 2 presents the model. Section 3 characterizes the political equilibrium. Section 4 explores empirical implications of our results.

2 The Model

The model is based on that developed by Hassler et al. (2007). Time is discrete and is denoted by $t = 1, 2, \cdots$. The economy consists of a continuum of agents living for two periods. Each generation has a unit mass. Agents are, at birth, identical.

Agents can affect their prospects in life by an educational investment. In particular, they become either rich or poor and, by undertaking a costly investment, can increase the probability e of becoming rich. The cost of investment is given by $\eta \cdot (e)^2$, $\eta > 0$; Hassler et al. (2007) considered the special case of $\eta = 1$. Rich agents earn a high wage, normalized to unity, in both periods, whereas poor agents earn a low wage, normalized to zero.

The government provides transfers (s) financed by taxes levied on the rich. The tax rates are age dependent, τ^o for the old and τ^y for the young. The tax rates are determined before the young agents decide on their investment. The government budget balances in every period.

There is no storage technology in this economy. Each individual uses up his/her endowments within a period. Therefore, the expected utility of agents alive at time t is given as follows: $V_t^{os} = 1 - \tau_t^o + s_t$, $V_t^{ou} = s_t$, and $V_t^y = e_t \cdot \{(1 - \tau_t^y) + \beta(1 - \tau_{t+1}^o)\} + (s_t + \beta s_{t+1}) - \eta(e_t)^2$, where V_t^{os} and V_t^{ou} denote the utility of the old who were successful in youth and the old who were unsuccessful in youth, and V_t^y denotes the expected utility of young agents born in period t. Note that V_t^y is computed prior to an individual's success or failure. The parameter $\beta \in (0, 1)$ is the discount factor.

Given these preferences, the optimal investment of young agents is:

$$e^*(\tau_t^y, \tau_{t+1}^o) = \frac{1}{2\eta}(1 + \beta - \tau_t^y - \beta\tau_{t+1}^o). \tag{1}$$

Because young agents are identical ex ante, agents of the same cohort choose the same investment, implying that the proportion of old poor agents in period $t + 1$ is given by:

$$u_{t+1} \equiv 1 - e^*(\tau_t^y, \tau_{t+1}^o) = \frac{1}{2\eta}(2\eta - (1 + \beta) + \tau_t^y + \beta\tau_{t+1}^o).$$

Thus, the proportion of the old poor agents in period $t + 1$, u_{t+1}, depends on the income tax rate levied on the young rich in period t, τ_t^y, and the tax rate levied on the old rich in period $t + 1$, τ_{t+1}^o.

The government runs a balanced budget in every period, implying that the budget can be expressed as:

$$2s_t = (1 - u_t)\tau_t^o + e^*(\tau_t^y, \tau_{t+1}^o)\tau_t^y = W(u_t, \tau_t^o) + Z(\tau_t^y, \tau_{t+1}^o), \tag{2}$$

where $W(u_t, \tau_t^o) \equiv (1 - u_t)\tau_t^o$ is the tax revenue financed by the old, and $Z(\tau_t^y, \tau_{t+1}^o) \equiv e^*(\tau_t^y, \tau_{t+1}^o)\tau_t^y$ is the tax revenue financed by the young.

3 Political Equilibrium

This section characterizes a political equilibrium where agents vote on taxation, period by period. Following Hassler et al. (2007), it is assumed that agents vote on current taxes at the beginning of each period but that only the old vote. The respective utility of the old rich and the old poor can be written as:

$$\begin{aligned} V_t^{os} &= 1 - \tau_t^o + \frac{1}{2} \cdot \left(W(u_t, \tau_t^o) + Z(\tau_t^y, \tau_{t+1}^o)\right), \\ V_t^{ou} &= \frac{1}{2} \cdot \left(W(u_t, \tau_t^o) + Z(\tau_t^y, \tau_{t+1}^o)\right). \end{aligned}$$

The current paper focuses on stationary Markov-perfect equilibria, where the state of the economy is summarized by the proportion of the current old poor agents (u_t). A (*stationary Markov-perfect*) *political equilibrium* is defined as a triplet of functions $\{T^o, T^y, U\}$, where $T^o : [0,1] \to [0,1]$ and $T^y : [0,1] \times [0,1] \to [0,1]$ are two public policy rules, $\tau_t^o = T^o(u_t)$ and $\tau_t^y = T^y$, and $U : [0,1] \to [0,1]$ is a private decision rule, $u_{t+1} = U(\tau_t^y)$, such that the following functional equations hold.

1. $T^o(u_t) = \arg\max_{\tau_t^o \in [0,1]} W^{dec}(\tau_t^o, u_t)$, where:

$$W^{dec}(\tau_t^o, u_t) = \begin{cases} 1 - \tau_t^o + \frac{1}{2}W(u_t, \tau_t^o) & \text{if } u_t \leq 1/2, \\ \frac{1}{2}W(u_t, \tau_t^o) & \text{if } u_t > 1/2. \end{cases}$$

2. $U(\tau_t^y) = 1 - e^*(\tau_t^y, \tau_{t+1}^o)$, with $\tau_{t+1}^o = T^o(U(\tau_t^y))$.

3. $T^y = \arg\max_{\tau_t^y \in [0,1]} Z(\tau_t^y, \tau_{t+1}^o)$ subject to $\tau_{t+1}^o = T^o(U(\tau_t^y))$.

The first equilibrium condition requires that τ_t^o maximizes the utility of the decisive old voter who is either the old rich (if $u_t \leq 1/2$) or the old poor (if $u_t > 1/2$). We assume that the rich decide τ_t^o in the case of an equal number of rich and poor voters ($u_t = 1/2$). The second equilibrium condition implies that all young agents choose their investments optimally, given τ_t^y and τ_{t+1}^o, and that agents hold rational expectations about future taxes and distributions of types. The third equilibrium condition requires that τ_t^y is chosen to maximize the revenue from the young.

3.1 The Determination of T^o and U

We now solve the three equilibrium conditions recursively. The mapping $T^o(\cdot)$ that satisfies equilibrium condition 1 is given by:

$$T^o(u_t) = \begin{cases} 0 & \text{if } u_t \leq 1/2, \\ 1 & \text{if } u_t > 1/2. \end{cases} \tag{3}$$

The decisive voter will set $\tau_t^o = 0$ if the rich are the majority: $u_t \leq 1/2$, whereas it will set $\tau_t^o = 1$ if the poor are the majority: $u_t > 1/2$.

Next, we rewrite equilibrium condition 2 by substituting in the optimal investment $e^*(\tau_t^y, \tau_{t+1}^o)$. This yields the following functional equation:

$$U(\tau_t^y) = \frac{1}{2\eta}\left(2\eta - (1+\beta) + \tau_t^y + \beta T^o(U(\tau_t^y))\right). \tag{4}$$

Because $T^o(\cdot) \in \{0, 1\}$ is given by (3), any solution of (4) must be a combination of the two linear functions: $U(\tau_t^y) = (2\eta - (1+\beta) + \tau_t^y)/2\eta$ if $T^o = 0$ and $U(\tau_t^y) = (2\eta - 1 + \tau_t^y)/2\eta$ if $T^o = 1$.

Under the assumption of rational expectations about future taxes and distributions of types, the solution to the functional equation (4) is given by:

$$U(\tau_t^y) = \begin{cases} (2\eta - (1+\beta) + \tau_t^y)/2\eta & \text{if } \tau_t^y \le 1 - \eta \\ \{(2\eta - (1+\beta) + \tau_t^y)/2\eta, (2\eta - 1 + \tau_t^y)/2\eta\} & \text{if } 1 - \eta < \tau_t^y \le 1 - \eta + \beta \\ (2\eta - 1 + \tau_t^y)/2\eta & \text{if } 1 - \eta + \beta < \tau_t^y. \end{cases} \tag{5}$$

Suppose that young agents in period t expect $\tau_{t+1}^o = 0$ ($\tau_{t+1}^o = 1$); the rich (poor) old are the majority of voters in period $t+1$. Under this expectation, young agents choose their investments as $e^*(\tau_t^y, 0)(e^*(\tau_t^y, 1))$. This expectation is rational if $e^*(\tau_t^y, 0) \ge 1/2$ (if $e^*(\tau_t^y, 1) < 1/2$), i.e., if $\tau_t^y \le 1 - \eta + \beta$ (if $1 - \eta < \tau_t^y$). Figure 1 illustrates possible cases of U that satisfy the second equilibrium condition (5). Since our focus is on the educational costs, these cases are classified according to the size of η.

Suppose that the costs of education are high such that $\eta > 1$ (see Panels (a) and (b) of Figure 1). A high η implies that educational investment is costly, thereby giving individuals a disincentive to invest in education. Thus, a high η results in a low probability of being successful and the majority in the next period can be poor $\forall \tau_t^y \in [0, 1]$. Alternatively, suppose that the costs of education are low such that $\eta \in (0, 1]$ (see Panels (c) and (d) of Figure 1). The individuals have a strong incentive to invest in education, which results in a high probability of being successful. Thus, the majority in the next period can be rich for $\tau_t^y \in [0, \min(1 - \eta + \beta, 1)]$. In particular, if $\tau_t^y \le 1 - \eta$, the majority in the next period is always the rich.[1]

As depicted in Figure 1, there are multiple, self-fulfilling expectations of U for a certain set of τ_t^y. Which U arises in equilibrium depends on the expectations of agents. To illustrate U in equilibrium, we define the critical rate of τ_t^y: $\theta \in (\max(0, 1-\eta), \min(1 - \eta + \beta, 1)]$. The rate θ, which depends on the expectations of agents, is the highest tax rate that yields a majority of old rich in the next period. For $\tau_t^y > \theta$, the majority is the poor. However, for $\tau_t^y \le \theta$, the majority in the next period is either the rich or the poor depending on the expectations by agents. Thus, the equilibrium features a monotonic or a nonmonotonic function U. Without loss of generality, we hereafter focus our attention on equilibria that feature a monotonic function U (see Figure 2 for an example). The function U is, thus, given by:

$$U(\tau_t^y) = \begin{cases} (2\eta - (1+\beta) + \tau_t^y)/2\eta & \text{if } \tau_t^y \le \theta, \\ (2\eta - 1 + \tau_t^y)/2\eta & \text{if } \tau_t^y > \theta. \end{cases} \tag{6}$$

[1] Hassler et al. (2007) ignored the cases displayed in Panels (c) and (d) by assuming a specific form of the education cost function with $\eta = 1$. If $\eta = 1$, there is no $\tau_t^y \in [0, 1]$ that satisfies $\tau_t^y \le 1 - \eta$; there is no case in which the majority, in the next period, is always the rich. Their analysis was, therefore, limited to the cases illustrated in Panels (a) and (b). The current paper extends the model of Hassler et al. (2007) by generalizing the cost function of education, and shows that, in the cases presented in Panels (c) and (d), there is a political equilibrium whose outcome is qualitatively different from that of Hassler et al. (2007).

3.2 The Determination of T^y and the Characterization of Political Equilibria

Given the characterization of T^o and U satisfying equilibrium conditions 1 and 2, respectively, we now consider the political determination of τ_t^y that satisfies equilibrium condition 3. Because there are two possible cases of the majority, we introduce the corresponding definitions of political equilibria: a *poor-majority equilibrium* featured by $u > 1/2$, and a *rich-majority equilibrium* featured by $u \leq 1/2$.

We first show the two types of equilibria that correspond to those shown in Hassler et al. (2007). The first (second) part of the following proposition includes the result of Proposition 2 (Proposition 3) in Hassler et al. (2007) as a special case. Therefore, we leave out the interpretation of the following proposition.[2]

Proposition 1.

(i) *Suppose that* $\beta < (\eta - 1/2)^2/(1-\eta)$ *holds. There exists a set of poor-majority equilibria such that* $\forall t$, $T^o(u_t)$ *is given by* (3), $T^y = 1/2$, *and* U *is given by* (6). *The equilibrium outcome is unique and such that* $\forall t$, $\tau_t^y = 1/2, \tau_t^o = 1$, $u_t = 1 - 1/4\eta$, and $2s_t = 3/8\eta$.

(ii) *Suppose that* $\beta > 1 - 2\eta$ *and* $\beta \geq (1/\eta)(\eta - 1/2)^2$ *hold. There exists a set of rich-majority equilibria such that* $\forall t$, $T^o(u_t)$ *is given by* (3), $T^y = \theta \in \left(\max(1-\eta, \tilde{\theta}(\beta)), \min((1+\beta)/2, 1+\beta-\eta)\right]$, *and* U *is given by* (6), where $\tilde{\theta}(\beta) \equiv \left(1 + \beta - \sqrt{\beta(\beta+2)}\right)/2$. *The equilibrium outcome is indeterminate and such that* $\forall t$, $\tau_t^y = \theta, \tau_t^o = 0$, $u_t = (2\eta - (1+\beta) + \theta)/2\eta$, *and* $2s_t = (1-\theta+\beta)^2/2\eta$.

Proof. See the appendix.

The second part of Proposition 1 suggests that, in the rich-majority equilibrium: (i) the decisive voters fail to attain the top of the Laffer curve $Z(\tau_t^y, 0)$; and (ii) the outcome is indeterminate. We find that these properties depend on the specification of the education cost function. Indeed, the following proposition shows that when, costs are low, there is a rich-majority equilibrium where: (i) the decisive voters attain the top of the Laffer curve; and (ii) the outcome is determinate.

Proposition 2. *Suppose that* $\beta \leq 1 - 2\eta$ *holds. There exists a set of rich-majority equilibria such that* $\forall t$, $T^o(u_t)$ *is given by* (3), $T^y = (1+\beta)/2$, *and* U *is given by* (6). *The equilibrium outcome is unique and such that* $\forall t$, $\tau_t^y = (1+\beta)/2, \tau_t^o = 0$, $u_t = (2\eta - (1+\beta)/2)/2\eta$, *and* $2s_t = (1+\beta)^2/8\eta$.

Proof. See the appendix.

In order to understand the result, consider the cases illustrated in Panels (c) and (d) of Figure 1. If $1 - \eta < (1+\beta)/2$, i.e., if $\beta > 1 - 2\eta$, agents have an option that induces a future majority of rich by setting $\tau_t^y = \theta$. The tax rate on the young depends on the expectations by agents (see Panel (a) of Figure 3). However, if $(1+\beta)/2 \leq 1 - \eta$, i.e., if $\beta \leq 1 - 2\eta$, expectations no longer affect the determination of τ_t^y. Voters can choose

[2] The full version of this paper (Arawatari and Ono, 2007) provides the interpretation in detail.

$\tau_t^y = (1+\beta)/2$ and maximize the revenue from the young by taxing on the top of the Laffer curve (see Panel (b) of Figure 3).

Figure 4 illustrates the parameter conditions derived in Propositions 1 and 2. The next proposition summarizes the results established so far. The proof of the proposition is immediate from Figure 4.

Proposition 3

(i) *If* $\beta < (1/\eta)(\eta - 1/2)^2$, *there exists a unique poor-majority equilibrium as in Proposition 1(i).*

(ii) *If* $(1/\eta)(\eta - 1/2)^2 \leq \beta < (\eta - 1/2)^2/(1-\eta)$, *the equilibrium is indeterminate. There exist both a poor-majority equilibrium, as in Proposition 1(i), and a set of rich-majority equilibria, as in Proposition 1(ii).*

(iii) *If* $\beta > 1-2\eta$ *and* $\beta \geq (\eta-1/2)^2/(1-\eta)$, *there exists a set of rich-majority equilibria, as in Proposition 1(ii).*

(iv) *If* $\beta \leq 1-2\eta$, *there exists a unique rich-majority equilibrium, as in Proposition 2.*

By assuming the cost function of education with $\eta = 1$, Hassler et al. (2007) focused on a special case and showed only two patterns of political equilibria, as in Propositions 3(i) and 3(ii). Contrary to their analysis, however, we focus on a wider range of η by assuming a generalized cost function. Under this extended framework, we can derive an additional two patterns of political equilibria, as in Propositions 3(iii) and 3(iv), which were not shown by Hassler et al. (2007).

The mechanism behind our main finding is intuitive and can be understood as follows. Suppose that the investment cost is low such that $\eta \leq 1$. In this case, it is more attractive for agents to invest in education. In particular, when the cost is sufficiently low such that $\beta \geq (\eta-1/2)^2/(1-\eta)$, agents have an incentive to invest in education even if they expect a high future tax rate and, thus, a low return from the investment. Then, it becomes possible to tax heavily the young without compromising the future political equilibrium. Therefore, in this case, there is a future majority of rich irrespective of the policy involved.

4 Empirical Implications

The equilibrium level of redistribution differs across political equilibria. Consider first the case of a high η, say η_{high}, that leads to multiple equilibria, as in Proposition 3(ii). As already shown by Hassler et al. (2007), this case provides an explanation for the cross-country differences in the size of the government and the distributions of the rich and the poor. To provide an empirical viewpoint, we take the graduation rate of tertiary education as a proxy variable of the size of the rich e^*. Then, the poor-majority equilibrium corresponds to some continental European countries such as Austria, France, Germany, and Italy, featured by large government sizes and low graduation rates; the rich-majority equilibrium corresponds to Australia, Iceland, Ireland, the United Kingdom, and the United States, featured by small government sizes and high graduation rates (OECD, 2005, 2007).

Although this case can describe the differences between some OECD countries, it does not fit the empirical fact of the Nordic countries; which have large government sizes and high graduation rates (OECD, 2005, 2007). However, the case of a low η, say $\eta_{low}(< \eta_{high})$, that leads to only a rich-majority equilibrium, as in Propositions 3(iii) and 3(iv), can provide an explanation for the Nordic countries. In this case, the majority is rich, and the level of redistribution is given by $2s_t = (1-\theta+\beta)^2/2\eta_{low}$ or $2s_t = (1+\beta)^2/8\eta_{low}$, which is generally larger than that in the poor-majority equilibrium, $2s_t = 3/8\eta_{high}$. In addition, this equilibrium is persistent in the sense that there is no possibility of shifting toward a poor-majority equilibrium. Therefore, by focusing on educational costs, we can show the equilibrium that fits the empirical fact of the Nordic countries, which was not shown by Hassler et al. (2003) and Hassler et al. (2007).

We have shown the existence of the equilibrium featured by a population with a highly educated majority and a high level of redistribution by considering the cases of low education costs. The existence of a qualitatively similar equilibrium may be shown by allowing the wage of the successful agents, w, to differ from unity (Hassler et al., 2003). A higher wage inequality results in a unique equilibrium where the majority of rich prefer a large size of government and imposes a high tax rate on the young. However, this case does not fit the empirical fact of the Nordic countries that are featured by low inequality (Björklund et al., 2002). Therefore, focusing on education costs rather than wage inequality is a key to explaining the empirical fact of the Nordic countries in the framework of Hassler et al. (2003) and Hassler et al. (2007). Low costs of education can be interpreted as easy access to tertiary education, which is featured by low university tuition fees in the Nordic countries.

5 Appendix

5.1 Proof of Proposition 1

(i) Suppose that, at time t, agents know that $\tau_t^y = 1/2$ and expect that $\tau_{t+1}^o = 1$. Then, $1 - e^*(1/2, 1) = u_{t+1} = 1 - 1/4\eta = 1 - (1/4)(1/\eta)$. Because the condition $\beta < (\eta - 1/2)^2/(1-\eta)$ ensures that $\eta > 1/2$ (see Figure 4), $u_{t+1} = 1 - (1/4)(1/\eta) > 1/2$ holds for all t. By (3), this implies that $\tau_{t+1}^o = 1$, fulfilling initial expectations. Therefore, there exists a high-tax equilibrium if the decisive voter finds it optimal to set $\tau_t^y = 1/2$.

To establish that setting $\tau_t^y = 1/2$ is optimal for the decisive voter, we note the following properties of the function Z: (i) $Z(\tau_t^y, 0)$ is concave in τ_t^y and is maximized at $\tau_t^y = (1+\beta)/2$; (ii) $Z(\tau_t^y, 0) > Z(\tau_t^y, 1)$. Given these properties, we derive the conditions that (i) setting $\tau_t^y = (1+\beta)/2$ is not available under the expectation $\tau_{t+1}^o = 0$; (ii) $Z(\tau_t^y, 0)$ is maximized at $\tau_t^y = \theta$; and (iii) $Z(\theta, 0) < Z(1/2, 1)$ hold. These conditions are given by (i) $(1+\beta)/2 \leq 1-\eta$, (ii) $\theta \in (\max(0, 1-\eta), (1+\beta)/2)$, and (iii) $\theta < \tilde{\theta}(\beta)$, respectively. The second and third conditions require that θ is set within the range $(\max(0, 1-\eta), \tilde{\theta}(\beta))$. The condition $\beta < (\eta - 1/2)^2/(1-\eta)$, which guarantees that the set $(\max(0, 1-\eta), \tilde{\theta}(\beta))$ is not empty, includes the first condition, as illustrated in Figure 4. ■

(ii) Under the assumption that U is monotonic, $T^o(U(\theta)) = 0$ implies that $\forall \tau_t^y \leq \theta$, $T^o(U(\tau_t^y)) = 0$, and $U(\tau_t^y) = (2\eta - (1+\beta) + \tau_t^y)/2\eta$. Consequently, the relevant payoff function in the range $\tau_t^y \leq \theta$ is $Z(\tau_t^y, 0)$. The function $Z(\tau_t^y, 0)$ is a hump-shaped function of τ_t^y, with a maximum at $(1+\beta)/2$.

Given the first assumption $\beta > 1 - 2\eta$, i.e., $1 - \eta < (1+\beta)/2$, we can consider the following two cases: (a) the case of $1 - \eta < (1+\beta)/2 < 1 + \beta - \eta$; and (b) the case of $1 + \beta - \eta \leq (1+\beta)/2$. In case (a), $Z(\tau_t^y, 0)$ is increasing in the range $\tau_t^y < (1+\beta)/2$ and decreasing in the range $\tau_t^y \geq (1+\beta)/2$. In case (b), $Z(\tau_t^y, 0)$ is increasing in the range $\tau_t^y < 1+\beta-\eta$. Therefore, the decisive voters prefer $\tau_t^y = \theta \in (1-\eta, \min(1+\beta-\eta, (1+\beta)/2)]$ to any $\tau_t^y < \theta$ under the assumption of $\beta > 1 - 2\eta$.

An alternative option for the decisive voter is to set $\tau_t^y = 1/2$ under the expectation of $\tau_{t+1}^o = 1$. The choice of $\tau_t^y = \theta$ under the expectation of $\tau_{t+1}^o = 0$ is sustained as an equilibrium if $Z(\theta, 0) \geq Z(1/2, 1)$, i.e., if $\theta \geq \tilde{\theta}(\beta)$, where:

$$\tilde{\theta}(\beta) \equiv \frac{1 + \beta - \sqrt{\beta(\beta+2)}}{2} < \frac{1+\beta}{2}.$$

Therefore, there exists a set of low-tax equilibria featured by $T^o(u_t)$ is given by (3), $T^y = \theta$ and U is given by (6) if θ is set within the range:

$$\left(\max(1-\eta, \tilde{\theta}(\beta)), \min((1+\beta)/2, 1+\beta-\eta)\right].$$

The condition $\beta \geq (1/\eta)(\eta - 1/2)^2$, which is rewritten as $\tilde{\theta}(\beta) \leq 1 + \beta - \eta$, implies that the above range is not empty. ■

5.2 Proof of Proposition 2

Suppose that, in period t, agents know that $\tau_t^y = (1+\beta)/2$ and expect that $\tau_{t+1}^o = 0$. Then $u_{t+1} = 1 - e^*((1+\beta)/2, 0) = (2\eta - (1+\beta)/2) \leq 1/2$ if and only if $\beta \geq -1 + 2\eta$.

The condition $\beta \geq -1 + 2\eta$ holds under the assumption of $\beta \leq 1 - 2\eta$ (see Figure 4). By (3), the condition $u_{t+1} \leq 1/2$ implies that $\tau^o_{t+1} = 0$, fulfilling initial expectations. Given that $Z(\tau^y_t, 0) > Z(\tau^y_t, 1) \forall \tau^y_t \in [0, 1]$ and that $\arg\max_{\tau^y_t \in [0,1]} Z(\tau^y_t, 0) = (1 + \beta)/2$, setting $\tau^y_t = (1 + \beta)/2$ is optimal for the decisive voter.

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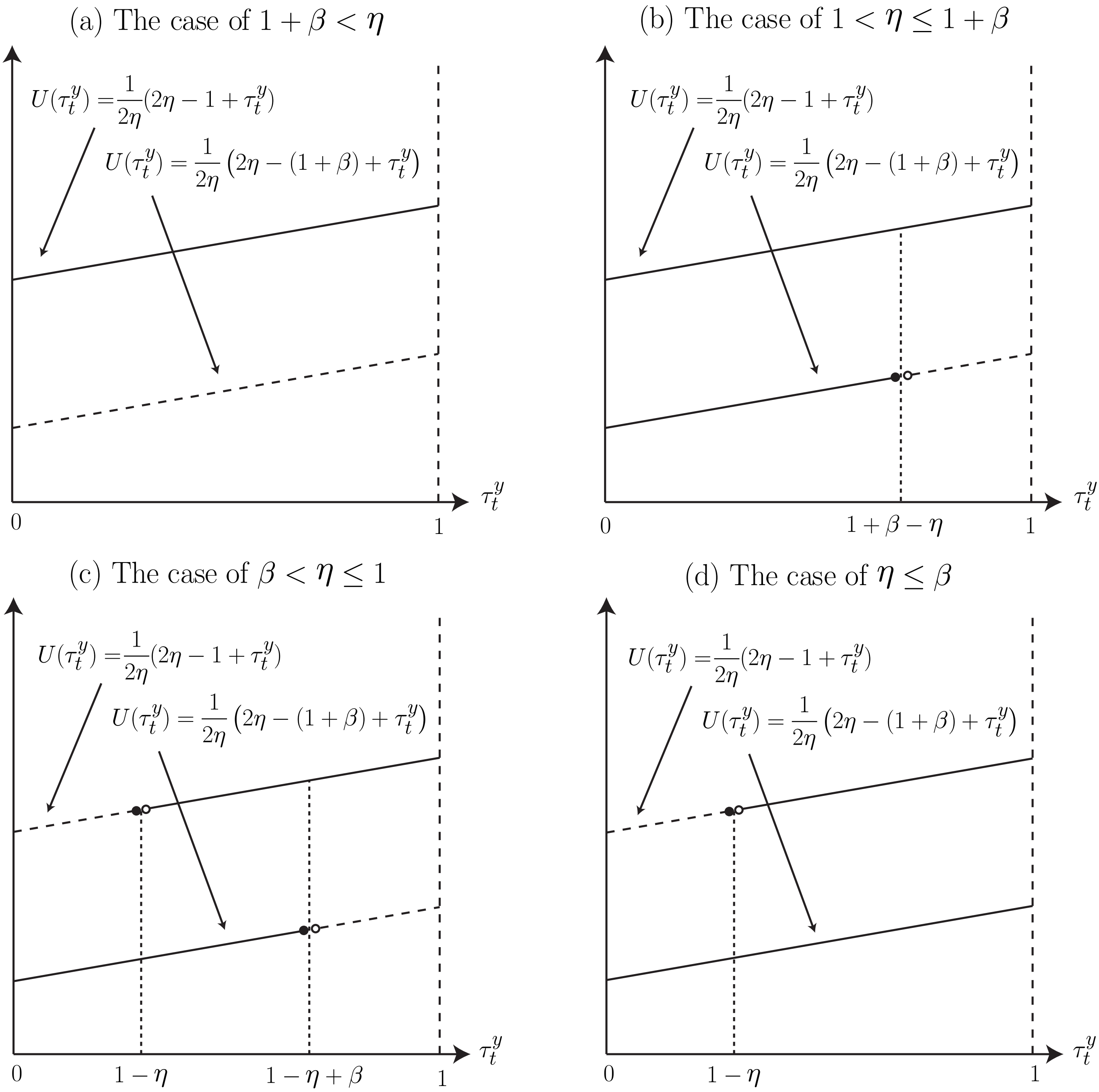

Figure 1: The figures represent the equilibrium decision rule $u_{t+1} = U(\tau_t^y)$. The solid lines show the graphs of U satisfying equilibrium condition 2. The panel (a) illustrates the case of $1+\beta<\eta$; the panel (b) illustrates the case of $1<\eta\leq 1+\beta$; the panel (c) illustrates the case of $\beta<\eta\leq 1$; the panel (d) illustrates the case of $\eta\leq\beta$.

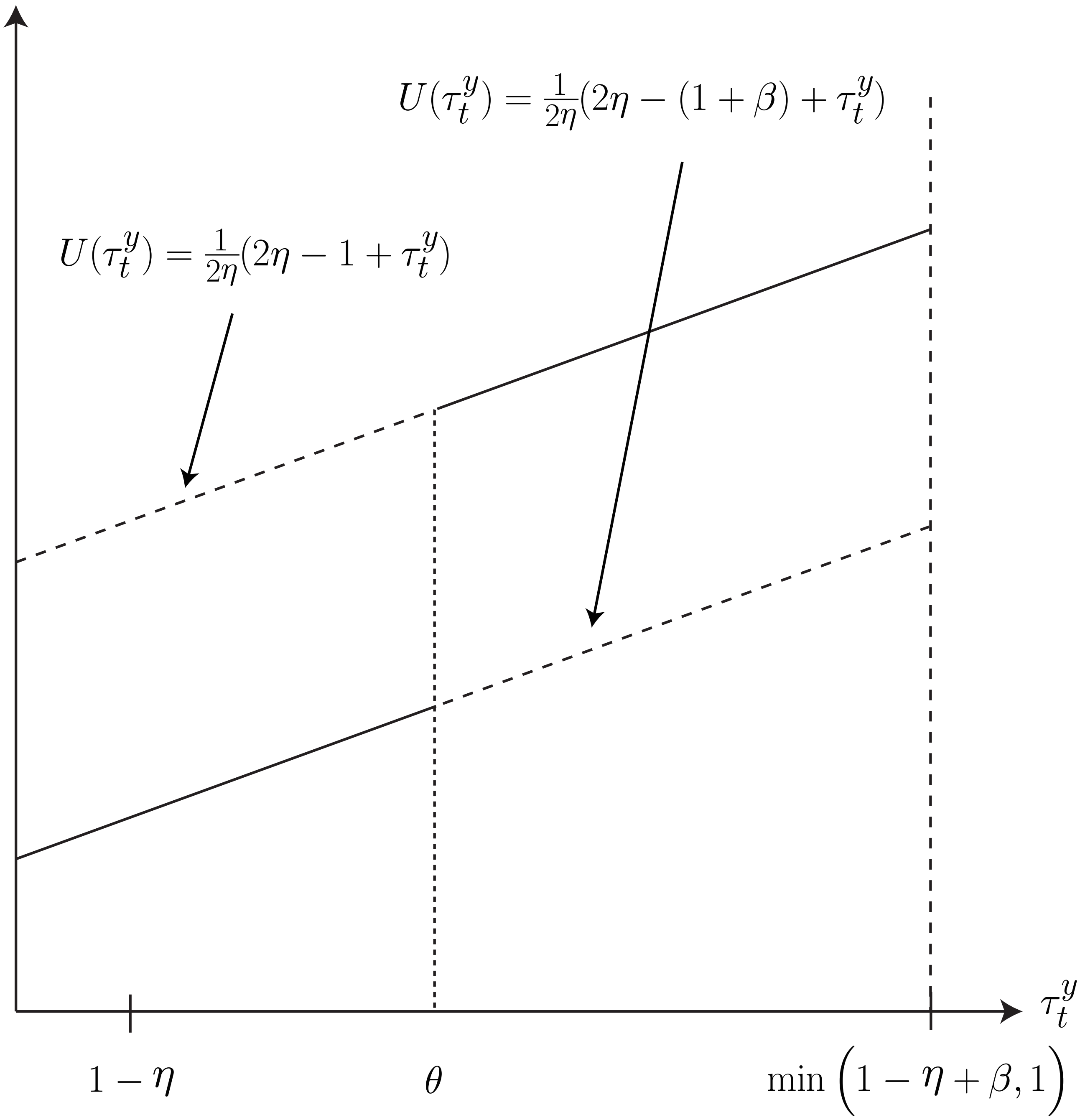

Figure 2: The figure illustrates an example of a monotonic function U for the case of $\eta \leq 1$.

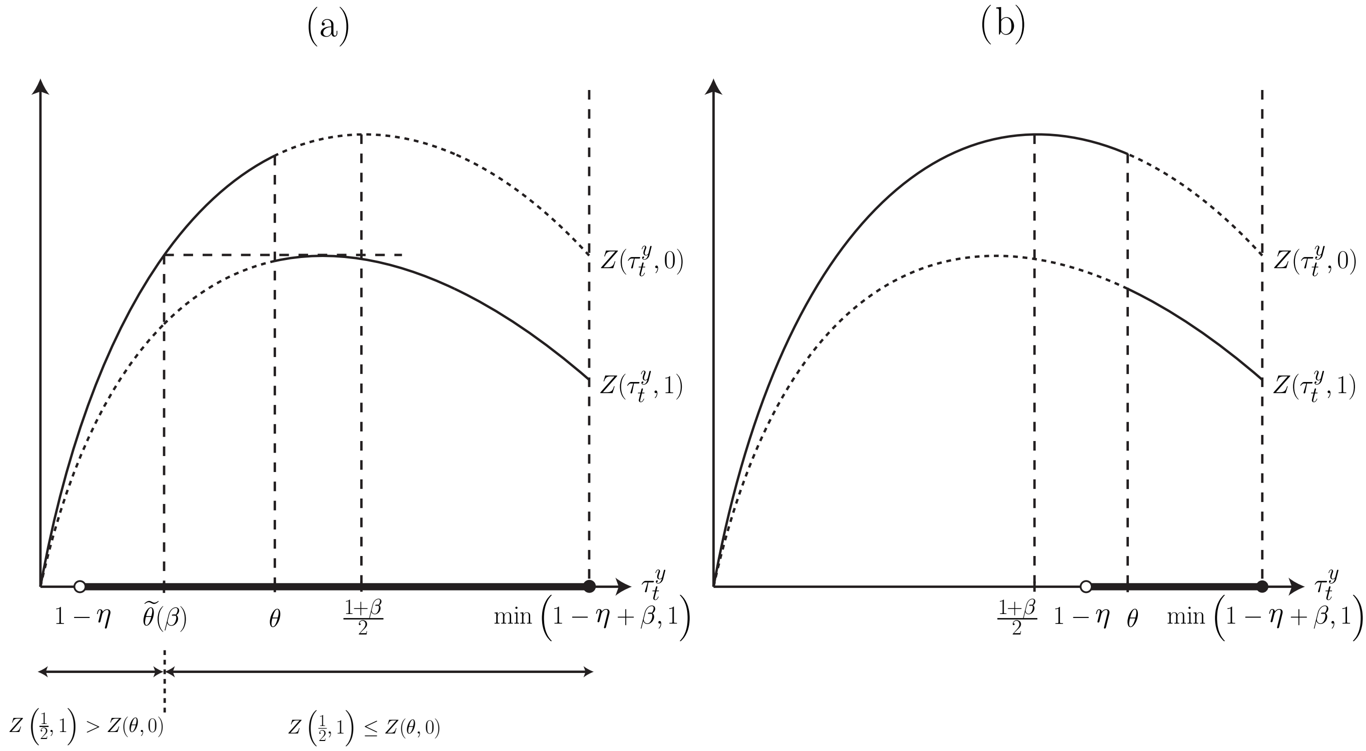

Figure 3: The solid curves illustrate the available tax revenue from the young, Z. The panel (a) represents the case of $\beta > 1-2\eta$, i.e., $1-\eta < (1+\beta)/2$. The panel (b) represents the case of $\beta \leq 1-2\eta$, i.e., $1-\eta \geq (1+\beta)/2$.

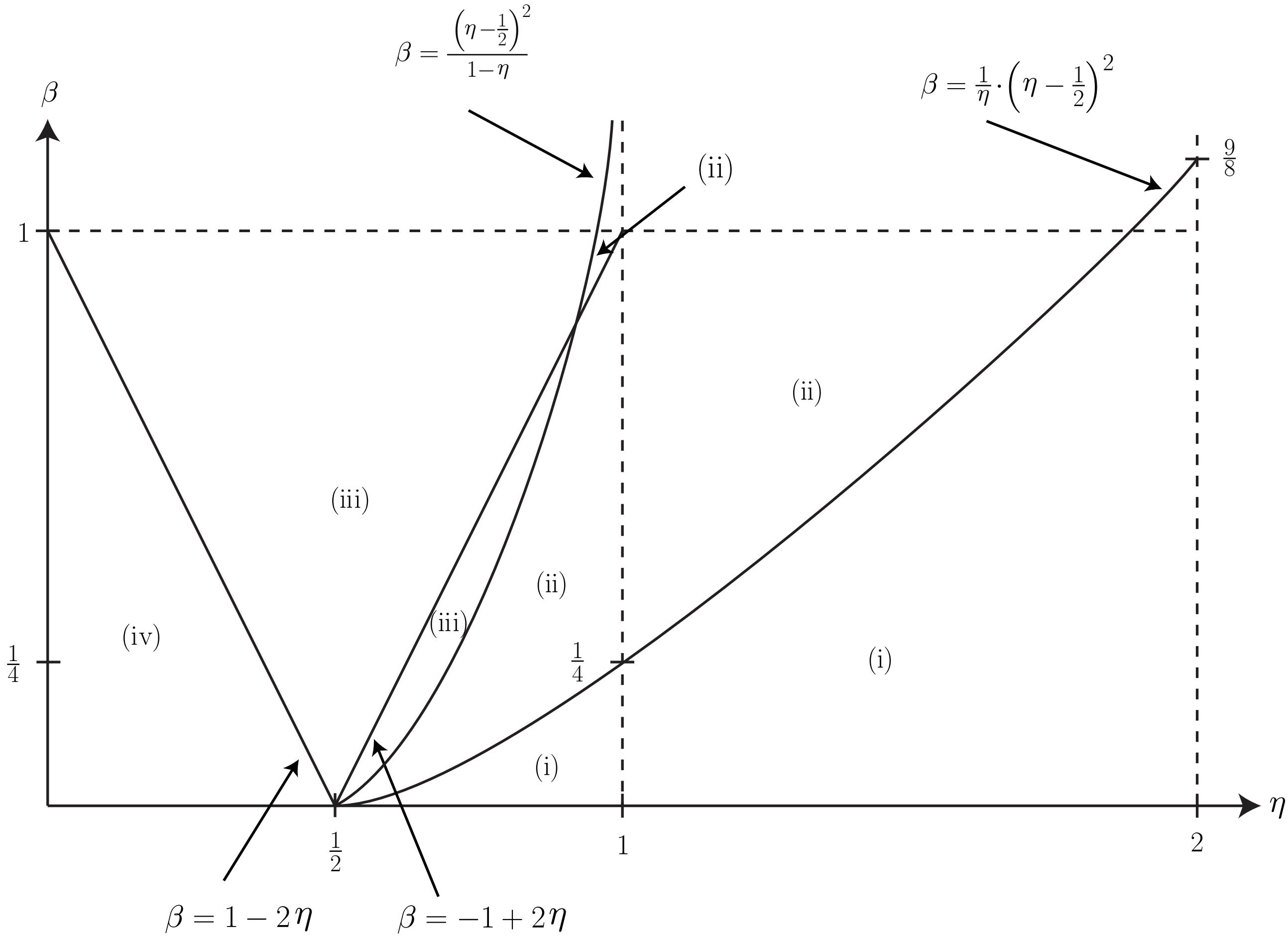

Figure 4: The figure displays the set of parameters (β, η) classified according to the characterization of political equilibrium in Propositions 1 and 2. The areas (i), (ii), (iii) and (iv) satisfy the parameter conditions given in Proposition 3 (i), (ii), (iii) and (iv), respectively.