

## The Harberger-Laursen-Metzler effect with with Marshallian preferences

Constantine Angyridis  
*Ryerson University*

Arman Mansoorian  
*York University*

### *Abstract*

The effects of a terms of trade deterioration on the current account are studied when the representative agent has Marshallian preferences, with which the rate of time preference is a decreasing function of savings. A terms of trade deterioration reduces the permanent income of the representative agent. With Marshallian preferences, savings fall and the country runs a current account deficit. The numerical evaluations of the model suggest that with standard functional forms and reasonable parameter values the Harberger-Laursen-Metzler effect is recovered in an infinite horizon model with an endogenous rate of time preference.

---

We would like to thank the Social Sciences and Humanities Research Council of Canada for financial support. All the remaining errors are our own.

**Citation:** Angyridis, Constantine and Arman Mansoorian, (2008) "The Harberger-Laursen-Metzler effect with with Marshallian preferences." *Economics Bulletin*, Vol. 6, No. 11 pp. 1-11

**Submitted:** August 22, 2007. **Accepted:** March 6, 2008.

**URL:** <http://economicsbulletin.vanderbilt.edu/2008/volume6/EB-07F40020A.pdf>

# 1 Introduction

A long-standing debate in open economy macroeconomics is with regards to the effects of a terms of trade deterioration on the current account. This debate originated in the works of Habereger (1950) and Laursen and Metzler (1950), who predicted that a terms of trade deterioration should lead to a fall in savings and a current account deficit; the Harberger-Laursen-Metzler (H-L-M) effect.

Obstfeld (1982) was the first to examine the H-L-M effect in an optimizing framework. He used an endowment economy in which the infinitely lived representative agent had Uzawa (1968) preferences, where the rate of time preference is an increasing function of instantaneous utility. With Uzawa preferences, the equality of the rate of time preference to the world interest rate, which the small open economy takes as given, dictates a unique level of utility that must be maintained in the steady state. By reducing his permanent income, a terms of trade deterioration requires an increase in the steady state net foreign asset holdings of the representative agent. Hence, aggregate expenditure falls and the country must run a current account surplus.

Svensson and Razin (1983) show that the main reason for Obstfeld's result is that for the stability of the steady state equilibrium he has to assume that the rate of time preference is increasing in instantaneous utility. Persson and Svensson (1985) argue that the assumption that the rate of time preference is increasing in instantaneous utility is "arbitrary and even counterintuitive" (p. 45).<sup>1</sup> As a result, they decide to use the overlapping generations model.

Sen and Turnovsky (1989) use an infinite horizon model with a fixed rate of time preference, but allow for labour-leisure choice and capital accumulation. In their model, the effects of a terms of trade deterioration on the current account depend primarily on the behaviour of investment.

Finally, Mansoorian (1993) recovers the H-L-M effect in an infinite horizon model using the habit persistence model of Ryder and Heal (1973), where habits develop over past levels of consumption. With the habit persistence model, if preferences exhibit adjacent complementarity, then after a terms of trade deterioration savings will fall, giving rise to a current account deficit.

In this paper, we propose another alternative to recover the H-L-M effect in an infi-

---

<sup>1</sup>Similar remarks are also made by Blanchard and Fischer (1989, pp. 74-75), who state that "the Uzawa function, with its assumption (that the rate of time preference is increasing in instantaneous utility), is not particularly attractive as a description of preferences and is not recommended for general use."

nite horizon model; Marshallian preferences, recently formalized by Gootzeit, Schneider and Smith (2002). According to Marshall, agents derive direct utility from the act of saving. Hence, for example, an empire-builder would take pride in his achievement; and would not exert great efforts in building an empire simply to smooth his consumption. Marshall (1920, IV.VII.8, pp. 230-234) gives an example of a physician who gives financial support to a factory; and derives great satisfaction in doing so. Indeed, in modern societies, where wealth is an important measure of social status, people do undergo forced savings to purchase, for example, luxurious dwellings.

Marshall's view of savings, therefore, stands in contrast to the classical economists' view, who regarded savings as arising purely to smooth consumption over time, with no direct utility involved. Gootzeit *et. al.* formulate the Marshallian view of savings by making the rate of time preference a decreasing function of savings; higher savings reduce the rate of time preference and increase lifetime utility. With such preferences, any shock that increases the permanent income of the representative agent would lead him to save more than with time separable preferences, because he would see an opportunity for long term improvement in his egoistic achievements. On the other hand, a shock that reduces his permanent income would hurt his long term opportunities; he would save less than with time separable preferences, because his ego is hurt.

Hence, with Marshallian preferences, a terms of trade deterioration, by reducing the permanent income of the representative agent, reduces savings, leading to a current account deficit. We further show that with reasonable functional forms and parameter values the model exhibits saddlepoint stability. Hence, our results stand in contrast to the view in the literature that with an endogenous rate of time preference the stability requirements preclude the H-L-M effect in an infinite horizon model.

The paper is organized as follows. Section II presents the model. Section III works out the effects of a terms of trade deterioration. Section IV evaluates the model numerically. Section IV concludes.

## 2 The Model

The model is that of a small open economy, with perfect capital mobility, flexible prices and no uncertainty. The setting is similar to that of Obstfeld (1982), and Sen and Turnovsky (1989).

## 2.1 The Problem of the Representative Agent

The preferences of the representative household are given by

$$\int_0^{\infty} e^{-\Theta_t} U(\omega(c_t^h, c_t^f)) dt, \quad (1)$$

where  $c_t^h$  and  $c_t^f$  denote the consumptions of the domestic and foreign goods, respectively.  $\omega(\cdot)$  is a homothetic aggregator function measuring total consumption, while  $U(\cdot)$  measures total utility from  $\omega(\cdot)$  at time  $t$ . Finally,  $\Theta_t$  is the discount factor from time  $t$  to 0; that is,

$$\Theta_t = \int_0^t \theta_s ds. \quad (2)$$

Hence,

$$\dot{\Theta}_t = \theta_t, \quad (3)$$

where  $\theta_t$  is the rate of time preference at time  $t$ . Following Gootzeit *et. al.*, we assume that  $\theta_t$  is a decreasing and concave function of net asset accumulation  $\dot{b}_t$ :

$$\theta_t = \theta(\dot{b}_t) \quad (4)$$

with  $\theta'(\dot{b}_t) < 0$  and  $\theta''(\dot{b}_t) < 0$ . When there is no asset accumulation,  $\theta$  has a fixed value  $\bar{\theta}$ .

As our focus is on the behaviour of savings, we keep the production side at its simplest form. The representative agent is endowed with  $y$  units of the home good at any time  $t$  and nothing of the foreign good. The home produced good is the numeraire, while the price of the foreign good is at  $p$ , which is taken as given.

The internationally traded bonds are the only available assets and they have a fixed rate of return  $r$ . Let  $b_t$  be the real asset holdings of the representative agent. His flow budget constraint is

$$\dot{b}_t = rb_t + y - c_t^h - pc_t^f, \quad (5)$$

The problem of the representative agent is to choose a sequence of consumption levels to maximize (1), subject to (3), (5), the initial conditions  $b_0$  and  $\Theta_0$ , and the standard intertemporal solvency condition. This maximization problem can be solved in two stages. In the first stage, for a given level of expenditures  $Z_t$  at any  $t$ , maximize  $\omega(c_t^h, c_t^f)$  subject to  $Z_t = c_t^h + pc_t^f$  giving us the indirect utility function  $Z_t V(p)$ . In the second stage, choose the values of  $Z_t$  that maximize lifetime utility.

Writing the present value Hamiltonian for the second stage, one can derive the optimality conditions as

$$H_Z = U'Ve^{-\Theta_t} - \Lambda_t + \Phi_t \theta' = 0, \quad (6)$$

$$-H_b = \dot{\Lambda}_t \Rightarrow -r\Lambda_t + r\Phi_t\theta' = \dot{\Lambda}_t, \quad (7)$$

$$H_\Theta = \dot{\Phi}_t \Rightarrow -e^{-\Theta_t}U = \dot{\Phi}_t, \quad (8)$$

where  $\Lambda_t$  and  $-\Phi_t$  are, respectively, the shadow prices of assets and time preference.

To simplify these optimality conditions, define the current value shadow prices  $\lambda_t$  and  $\phi_t$  as  $\lambda_t = \Lambda_t e^{\Theta_t}$  and  $\phi_t = \Phi_t e^{\Theta_t}$ . Hence,  $\dot{\lambda}_t = \dot{\Lambda}_t e^{\Theta_t} + \lambda_t \theta_t$ , and  $\dot{\phi}_t = \dot{\Phi}_t e^{\Theta_t} + \phi_t \theta_t$ . Then eliminate  $\Lambda_t$ ,  $\Phi_t$ ,  $\dot{\Lambda}_t$  and  $\dot{\Phi}_t$  from (6) to (8), and rewrite these conditions as

$$U'V - \lambda_t + \phi_t\theta' = 0, \quad (9)$$

$$-r\lambda_t + r\phi_t\theta' + \lambda_t\theta_t = \dot{\lambda}_t, \quad (10)$$

$$-U + \phi_t\theta_t = \dot{\phi}_t, \quad (11)$$

which have the standard interpretations.

## 2.2 The Perfect Foresight Path

To derive the perfect foresight path, first take the time derivative of (9), and use (10) and (11) to eliminate  $\lambda$  and derive the differential equation for  $Z$  (the Euler equation):

$$\dot{Z} = \frac{U'V + \phi_t\theta'}{U''V^2 - \phi_t\theta''} \left[ \theta_t - \frac{\theta'(\phi_t\theta_t - U) + \phi_t\theta''r(rb_t + y - Z)}{U'V + \phi_t\theta'} - \frac{U'V}{U'V + \phi_t\theta'}r \right]. \quad (12)$$

The dynamics of the model are then described by equations (5), (11) and (12). Linearizing these equations around the steady state, we obtain

$$\begin{bmatrix} \dot{b} \\ \dot{Z} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} r & -1 & 0 \\ \Psi & \Omega & 0 \\ \bar{\phi}\theta'r & -(U'V + \bar{\phi}\theta') & \bar{\theta} \end{bmatrix} \begin{bmatrix} b_t - \bar{b} \\ Z_t - \bar{Z} \\ \phi_t - \bar{\phi} \end{bmatrix}, \quad (13)$$

where overbars denote steady state values,  $\Omega = \frac{-U''V^2r + \bar{\theta}U''V^2 - \theta''U + \bar{\phi}\theta''r}{\Delta}$ ,  $\Psi = \frac{r[\theta'U'V + \theta''U - \bar{\phi}\theta''r]}{\Delta}$ , and  $\Delta = U''V^2 - \bar{\phi}\theta''$ . Some important properties of system (13) are highlighted in the following two propositions.

**Proposition 1:** The system (13) is saddlepoint stable if<sup>2</sup>

$$U''U - (U')^2 > 0. \quad (14)$$

**Proof:** Notice the Jacobian matrix  $J$  of the system (13) has determinant

$$\det(J) = -\left(\frac{\bar{\theta}r}{\Delta}\right)\theta'U'V \left[ \frac{U''U}{(U')^2} - 1 \right] \quad (15)$$

---

<sup>2</sup>Condition (14) is identical to condition (5) in Obstfeld (1990, p. 49), which he needs for stability in his two period model with Uzawa type of time preference.

and trace

$$\text{tr}(J) = 2\bar{\theta}. \quad (16)$$

Since system (13) has one predetermined ( $b$ ) and two jump variables ( $Z$  and  $\phi$ ), it exhibits saddlepoint stability if  $\det(J) < 0$  and  $\text{tr}(J) > 0$ . Notice,  $\Delta < 0$ ,  $\bar{\theta} > 0$ ,  $r > 0$ ,  $\theta' < 0$ ,  $V > 0$ ,  $U' > 0$  and  $U'' < 0$ . Thus, for  $\det(J)$  to be negative we need  $U''U - (U')^2 > 0$ , while  $\text{tr}(J) > 0$  always.<sup>3</sup> **Q.E.D.**

**Proposition 2.** Let  $\omega = ZV$ . For the CRRA class of preferences

$$U(\omega) = \begin{cases} \frac{\omega^{1-\sigma} - 1}{1-\sigma}, & \text{for } \sigma > 0, \sigma \neq 1 \\ \ln(\omega), & \text{for } \sigma = 1, \end{cases}$$

condition (14) will be satisfied for (i)  $0 < \sigma < 1$ , if  $\omega < \sigma^{1/(1-\sigma)}$ , (ii)  $\sigma > 1$ , if  $\omega < (1/\sigma)^{1/(\sigma-1)}$ , and (iii)  $\sigma = 1$ , if  $\omega < e^{-1}$ .

**Proof.** For  $\sigma \neq 1$ , multiply both sides of (14) by  $-(\omega/U')$ , and use  $U = (\omega^{1-\sigma} - 1)/(1-\sigma)$  and  $U' = \omega^{-\sigma}$  to rewrite this condition as  $(\omega^{1-\sigma} - 1)/(1-\sigma) < -1$ . Next, for  $0 < \sigma < 1$ , this inequality yields  $\omega < \sigma^{1/(1-\sigma)}$ , while for  $\sigma > 1$  it yields  $\omega < (1/\sigma)^{1/(\sigma-1)}$ . Finally, when  $\sigma = 1$ , condition (14) reduces to  $(-1/\omega^2)[\ln(\omega) + 1] > 0$ , which holds if  $\omega < e^{-1}$ . **Q.E.D.**

### 3 The Effects of a Terms of Trade Deterioration

Now consider the effects of a terms of trade deterioration on the current account. The steady state is given by (5), (11) and (12), with  $\dot{b} = \dot{Z} = \dot{\phi} = 0$ , which yield the following comparative statics results:

$$\frac{d\bar{Z}}{dp} = \left( \frac{V'}{V^2(U''U - (U')^2)} \right) [-\bar{Z}V(U''U - (U')^2) - U'U], \quad (17)$$

$$\frac{d\bar{b}}{dp} = \left( \frac{1}{r} \right) \frac{d\bar{Z}}{dp}. \quad (18)$$

**Proposition 3.** If preferences are in the CRRA class and satisfy (14), then  $d\bar{Z}/dp < 0$  and  $d\bar{b}/dp < 0$ .

**Proof.** From (14) and (17), for  $d\bar{Z}/dp < 0$  we require that  $-\omega[U''U - (U')^2] > U'U$ . Given that for CRRA preferences  $\sigma = -U''(\omega/U')$ , this condition can be rewritten as  $\sigma U +$

---

<sup>3</sup>A necessary, but not sufficient, condition for (14) to hold is  $U < 0$ . Notice that an increase in the rate of time preference  $\theta$  reduces the absolute value of lifetime utility. Hence, with  $U < 0$  an increase in  $\theta$  will increase lifetime utility; that is, with  $U < 0$  the shadow price of time preference  $-\phi$  is positive ( $\phi < 0$ ).

$U'\omega > U$ . Rearranging and substituting for  $U$  and  $U'$ , the above inequality reduces to  $\omega^{1-\sigma} > \omega^{1-\sigma} - 1$ , which is always true. Thus, for the CRRA preferences that satisfy (14)  $d\bar{Z}/dp < 0$  and  $d\bar{b}/dp < 0$ . **Q.E.D.**

Hence, for a wide set of reasonable preference specifications, the steady state net foreign asset position falls, and the H-L-M effect is recovered. Intuitively, the terms of trade deterioration reduces the permanent income of the representative agent. Faced with a blow to his ego, from the deterioration in his long term prospects, the representative agent reduces his savings, leading to a current account deficit.

## 4 A Numerical Evaluation of the Model

Here we evaluate the model numerically. We assume that the aggregator function  $\omega(\cdot)$  is Cobb-Douglas; that is,  $\omega(c_t^h, c_t^f) = (c_t^h)^\alpha (c_t^f)^{1-\alpha}$ , with the corresponding indirect utility function  $\omega = Z_t V(p)$ , where  $V(p) = (\alpha^\alpha (1-\alpha)^{1-\alpha})/p^{1-\alpha}$ .

The discount rate  $\theta$  is assumed to take the following form

$$\theta_t = 1 + \bar{\theta} - e^{\eta b_t}, \text{ with } \eta \geq 0. \quad (19)$$

This specification for  $\theta$  is consistent with the assumptions made by Gootzeit *et al.*; that is,  $\theta(0) = \bar{\theta}$ ,  $\theta' = -e^{\eta b_t} \eta < 0$ ,  $\theta'' = -e^{\eta b_t} \eta^2 < 0$ .

Following the RBC literature (see, e.g., Cooley and Prescott, 1995), we set  $\sigma = 2$  and  $r = 0.02$ . We also set  $\bar{\theta} = 0.01$ .<sup>4</sup> Our results are not sensitive to the choice of  $0 < \alpha < 1$ ; we set  $\alpha = 0.7$ . The size of  $y$  depends on our choice of units; we set  $y = 0.6$ .

As the literature does not provide us with an appropriate value for  $\eta$ , we derive a theoretical range of admissible values for it.

**Proposition 4.** For  $\sigma = 2$ , the admissible values of  $\eta$  fall in the range  $(4(r - \bar{\theta})V, +\infty)$ .

**Proof.** Evaluate (12) in the steady state to obtain:

$$\bar{Z}^\sigma - V^{1-\sigma} \bar{Z} - \frac{(1-\sigma)(r - \bar{\theta})V^{1-\sigma}}{\eta} = 0. \quad (20)$$

For  $\sigma = 2$ , this is a quadratic equation in  $\bar{Z}$ . Let  $d = (1/V^2) - 4(r - \bar{\theta})/\eta V$ . Then for (20) to have real roots, we need  $d > 0$ , which yields a lower bound for  $\eta$ :  $\eta > 4(r - \bar{\theta})V$ . Based on Proposition 2, the stability condition in the present case is:  $\bar{Z} < 1/(2V)$ . It

---

<sup>4</sup>In this model, we should have  $\bar{\theta} < r$ . To see why, notice from footnote 3 that  $\phi < 0$ . Next, notice, from the Euler equation (12) that in the steady state  $\bar{\theta} = \frac{U'V}{U'V + \phi\theta'}$ , which, with  $\phi < 0$ , implies that  $\bar{\theta} < r$ .

is easy to show that from the two real roots, only one satisfies the stability condition;  $\bar{Z} = (1/2) [(1/V) - \sqrt{d}]$ . Furthermore,  $\bar{Z}$  is strictly decreasing in  $\eta$ : as  $\eta$  tends to infinity,  $\bar{Z}$  goes to zero. Hence,  $\eta \in (4(r - \bar{\theta})V, +\infty)$  is the admissible range. **Q.E.D.**

With  $p = 1$ , the choice of  $\eta = 0.023$  gives us the initial steady state with  $Z_0 = 0.7$  and  $b_0 = 5$ .<sup>5</sup> Consider the effects of an increase in  $p$  to 1.2. The new steady state will have  $\bar{Z} = 0.6559$  and  $\bar{b} = 2.7952$ . The adjustment paths for aggregate expenditures  $Z$  and net foreign assets  $b$  are shown in Figure 1. On impact, there is a sharp increase in expenditures, which results in a current account deficit. Over time, the country's net foreign asset position deteriorates until it reaches its new steady state equilibrium.

Finally, we have re-done our analysis for  $\sigma = 0.5$  and  $\sigma = 1$ . Our results indicate that the adjustments of  $Z_t$  and  $b_t$  will be true qualitatively as in the case with  $\sigma = 2$ . With these three values of  $\sigma$  (0.5, 1, 2) we cover all cases in Proposition 2.

## 5 Conclusion

According to Marshall, agents derive direct utility from the act of savings. Gootzeit *et. al.* formulate this by making the rate of time preference a decreasing function of savings; higher savings reduce the rate of time preference and increase lifetime utility. We demonstrated that with these preferences, a terms of trade deterioration, by lowering the permanent income of the representative agent, reduces savings, leading to a current account deficit. We also showed that with reasonable functional forms and parameter values the model exhibits saddlepoint stability. Hence, our results stand in contrast to the view in the literature that with an endogenous rate of time preference the stability requirements preclude the H-L-M effect in an infinite horizon model.

---

<sup>5</sup>For the parameter values  $r = 0.02$ ,  $\bar{\theta} = 0.01$ ,  $\alpha = 0.07$  and initial  $p$  equal to 1, the lower bound for  $\eta$  is  $4(r - \bar{\theta})V = 0.0217$ .



## References

- [1] Blanchard, Olivier J., and Stanley Fischer, 1989, *Lectures on Macroeconomics*. Cambridge, Mass.: MIT Press.
- [2] Cooley, Thomas, and Edward Prescott, 1995, *Economic growth and business cycles*, in Thomas Cooley, ed., *Frontiers of Business Cycle Research*. Princeton University Press.
- [3] Gootzeit, Michael, Johannes Schneider, and William Smith, 2002, Marshallian recursive preferences and growth, *Journal of Economic Behaviour and Organization* 49, 381-404.
- [4] Harberger, A., 1950, Currency depreciation, income, and the balance of trade, *Journal of Political Economy* 58, 47-56.
- [5] Laursen, S. and L.A. Metzler, 1950, Flexible exchange rates and the theory of employment, *Review of Economics and Statistics* 32, 281-299.
- [6] Mansoorian, Arman, 1993, Habit Persistence and the Harberger-Laursen-Metzler Effect in an Infinite Horizon Model. *Journal of International Economics*, 34, 153-166.
- [7] Marshall, Alfred, 1920, *Principles of Economics*, 8-th edition, Macmillan: London.
- [8] Obstfeld, Maurice, 1982, Aggregate spending and the terms of trade: is there a Harberger-Laursen-Metzler effect? *Quarterly Journal of Economics*, 97, 251-270.
- [9] Obstfeld, Maurice, 1990, Intertemporal dependence, impatience, and dynamics, *Journal of Monetary Economics*, 26, 45-75.
- [10] Persson, T., and L.E.O. Svensson, 1985, Current account dynamics and the terms of trade: Harberger-Laursen-Metzler effect two generations later, *Journal of Political Economy* 93, 43-65.
- [11] Ryder, Harl E., and Geoffrey M. Heal, Optimal Growth with Intertemporally Dependent Preferences, *Review of Economic Studies*, January 1973, 40(1), pp. 1-31.
- [12] Sen, Partha, and Stephen J. Turnovsky, 1989, Deterioration of the Terms of Trade and Capital Accumulation: A Re-examination of the Laursen-Metzler Effect. *Journal of International Economics*, 26: 227-250.

- [13] Svensson, L.E.O., and A. Razin, 1983, The terms of trade and the current account: the Harberger-Laursen-Metzler effect, *Journal of Political Economy* 91: 97-125.
- [14] Uzawa, Hirofumi, 1968, Time Preference, the Consumption Function, and Optimal Asset Holdings, in J.N. Wolfe (ed.), *Value, Capital and Growth: Papers in Honour of Sir John Hicks*, Aldine Publishing Company, Chicago, Illinois.

Figure 1: Transition Path to the New Steady State for  $b(t)$  and  $Z(t)$

