E C O N O M I C S B U L L E T I N

An Expository Note on Alchian-Allen Theorem When Sub-Utility Functions are Homogeneous of Degree n > 0 with Two-Stage Budgeting

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Abstract

This expository note shows a proof of Alchian and Allen's conjecture broadly known as a phrase `shipping the good apples out' meaning that consumers purchase fine quality relatively more than coarse one if a fixed charge is imposed. Their statement is often referred as the Alchian-Allen theorem (or effect). The proof requires conditions about homogeneity, inner solution and substitutability and they also justify two-stage budgeting. In order this work to be an exposition, I emphasize graphical representations, however, a comparison with the proof of Borcherding and Silberberg (JPE; 1978) is also considered. This comparison clarifies difference between proofs using Hicksian demand functions and using Marshallian demand functions (with some specific conditions). Extending sequential budgeting procedures, we also discuss perspectives toward multiple-quality analysis. That turns out a definition based on sequential budgeting may open a way of experimental study about the effect.

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1. Introduction

"Ship the good apples out." Alchian and Allen (1967, pp.63-64) propose consumers purchase superior quality (higher price) relatively more than inferior one (lower price) when a fixed transaction (transportation) fee is uniformly imposed. Even now it is viable and applications of the theorem still have some interests — for example, recent studies such as Smith and Chang (2002) apply to a monetary model and Hummels and Skiba (2004) reconcile with iceberg cost hypothesis with an empirical investigation. Theoretically, using Hicksian demand functions in order to eliminate income effects and introducing "all other goods" (or a Hicksian composite good), Borcherding and Silberberg (1978) have proved the conjecture under a certain condition and Bauman (2004) extended the proof with multiple composite goods to justify the condition of Borcherding and Silberberg. Yet, it is ambiguous when there is some income effects as suggested by Gould and Segall (1969) and, in addition, Umbeck (1980) criticizes the specification of Borcherding and Silberberg using Hicksian demand functions with this regard.[1]

On the other hand of theoretical debates, the Alchian-Allen effect has attracted numerous empirical studies. These empirical works are classified into two groups. The one simply examines the influence of fixed transportation costs — for example, Staten and Umbeck (1989), Cowen and Tabarrok (1995), and Pritchett and Chambarlain (1993)[2] — and the other verifies the existence of the effect — see, for example, Bertonazzi et al. (1993) as well as Hummels and Skiba (2004). In applications for historical experiences, we also find the Alchian-Allen effect as such discussed by Thornton and Ekelund (2004, Chapter 2) in terms of North-South trade during the Civil War, and Temin (1989) and Irwin and Temin (2001) in terms of implementing tariffs on cotton imports and cotton industry in the antebellum United States. These historical observations also tell the importance of the argument about the Alchian-Allen effect in contemporary trade policy and further theoretical investigations.

This expository note demonstrates Alchian and Allen's conjecture can be explained without Hicksian demand functions but utility functions of homogeneous of degree $n > 0$ and weakly separable to justify two-stage budgeting procedures. The two-stage budgeting procedure is well studied by William M. Gorman (for example, see Gorman 1959 and 1995a) and it is also well explained by economists such as Bliss (1975, Chapter 7). Currently, some researchers are interested in this procedure as a computational method of large models. As an exposition, I emphasize graphical representations of the proof as well as algebraic aspects.

Discussions develop as follows. In Section 2, the proof of the Alchian-Allen theorem applying homogeneous utility function is provided with emphasis on two-stage budgeting and graphical representations. Then, in Section 3, the difference of proofs of this note and of Borcherding and Silberberg is compared with an argument about cross elasticity by Appendix A. For further studies, Section 4 discusses some possible extensions and Section 5 concludes the article.

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[1] About applicability of the theorem, Razzolini et al. (2003) consider all possible market structures and suggest only monopoly applies to the case.

[2] About the discussion of applying the Alchian-Allen theorem by Pritchett and Chamberlain has some controversies — see a series of debate by Komlos and Alecke (1996) and Pritchett (1997). Their arguments are potentially relevant to the argument of market structures given by Razzolini et al. (2003). My on-going paper (Saito 2008) also verifies this argument to propose another source of price differences.

2. Proof of the Theorem

Suppose there are two goods x and y, and x has two alternative qualities, fine and coarse, respectively denoted as x_1 and x_2. I assume y is the numéraire of this model. The difference in the quality of x is characterized by prices. Let p_1 and p_2 be respective prices of x_1 and x_2; thence, $p_1 > p_2 > 0$ applies. Let $V = V(x_1, x_2, y)$ be the utility function of a consumer. Let x-good and y-good are substitutes of each other, and assume weak separability to modify the definition of the utility function as $V[U(x_1, x_2), y]$, where $X = U(x_1, x_2)$ is the sub-utility function (index) and, by construction of X, x_1 and x_2 are also substitutes of each other. Required properties of V and U to have well-behaving demand functions are then summarized as follows.

Assumption 1 *Utility function V is strictly quasi-concave in X and y; and sub-utility function U is also strictly quasi-concave in x_1 and x_2.*

Consider the following two-stage budgeting problem of the consumer. At the first stage, regarding the sub-utility index X as a good we solve:

$$\text{Maximize} \quad V(X, y) \quad \text{subject to} \quad PX + y \leq m, \tag{1}$$

where $P > 0$ is the price index of x and $m > 0$ is the income respectively in terms of the price of y.[3] Assume no corner solution. Then, this problem gives an optimum value of X and y as functions of P and m, as such, $X^* = X^*(P, m)$ and $y^* = y^*(P, m)$. Let X^* be given by Problem (1) to consider the second stage problem such that:

$$\text{Minimize} \quad p_1x_1 + p_2x_2 \quad \text{subject to} \quad U(x_1, x_2) \geq X^*, \tag{2}$$

which determines consumptions of x_1 and x_2 with respect to X^* and prices.

Remark 1 *Suppose U is homogeneous of degree $n > 0$ and the constraint of Problem (2) is not slack. Then, at the optimum, Problem (1) is identical to the next problem if it has an inner solution:*

$$\text{Maximize} \quad V(x_1, x_2, y) \quad \text{subject to} \quad p_1x_1 + p_2x_2 + y \leq m. \tag{3}$$

Proof. Let $\lambda \geq 0$ be the Lagrange multiplier for Problem (3). From the first order condition for an inner solution, at the optimum, we have

$$p_1x_1 + p_2x_2 = \frac{1}{\lambda} \cdot \left(x_1 \cdot \frac{\partial U}{\partial x_1} + x_2 \cdot \frac{\partial U}{\partial x_2} \right) \equiv \frac{n}{\lambda} \cdot U(x_1, x_2),$$

where the last equivalence follows from Euler's Homogeneous Function Theorem because of the assumption such that U is homogeneous of degree $n > 0$. Applying $P = n/\lambda$ and rewriting V with the sub-utility function, we can see Problem (1) and Problem (3) are identical at the optimum. Note, in order to guarantee $P \in (0, \infty)$, we require $n \in (0, \infty)$ and $\lambda \in (0, \infty)$. ■

[3] Gorman (1995b) provides a detailed argument about price indices of aggregated goods.

Theorem 1 (Alchian-Allen Theorem) *Assume Assumption 1. Suppose there is no corner solution. Then the consumer raises the consumption on the fine quality relative to that of the coarse one if the sub-utility function is homogeneous of degree $n > 0$.*

Proof. Applying Remark 1, we can work on the two-step optimization process represented by Problem 1 and following Problem 2 if the sub-utility function is homogeneous of degree $n > 0$. Let $T > 0$ be a fixed transaction cost. Then, by the assumption on prices, we have

$$\frac{p_1}{p_2} > \frac{p_1 + T}{p_2 + T}. \tag{4}$$

Thus, the relative price of the fine quality declines in terms of the coarse one when the fixed transaction cost is uniformly imposed. In addition, I also note the slope of the iso-utility locus of V on y-X plane is represented by

$$\frac{dX}{dy} = -\frac{\partial V/\partial y}{\partial V/\partial X},$$

which follows from

$$dX = \frac{\partial U}{\partial x_1}dx_1 + \frac{\partial U}{\partial x_2}dx_2,$$

and

$$dV = \frac{\partial V}{\partial X}\frac{\partial U}{\partial x_1}dx_1 + \frac{\partial V}{\partial X}\frac{\partial U}{\partial x_2}dx_2 + \frac{\partial V}{\partial y}dy = 0.$$

Then, we can also see the slope of iso-utility locus of V on y-X plane increases as y increases by Assumption 1 for V.

Based on the above arguments, we can analyze the influence of the fixed transportation cost using Figure 1 and Figure 2 (Assumption 1 for U gives the shape of the iso-utility locus of U on x_1-x_2 plane). Suppose A in Figure 1 is the initial consumption point. If a fixed transaction cost is imposed, the income in terms of P declines because λ decreases as consumer prices of x_1 and x_2 rises—in particular, λ is not altered by marginal utilities and that is determined after the change in consumer prices; hence, only consumer prices matter at this point. Then this change is depicted by (i) in the figure. Accordingly the consumer brings the utility level down in accordance with (ii) in the figure; thence, the optimum point moves from A to B.

On x_1-x_2 plane (Figure 2), the decline in X is depicted by the downward movement of the sub-utility function denoted by (iii) in the figure because $n > 0$. Suppose C is the initial consumption point corresponding to A. If the fixed transaction cost is imposed, the iso-utility locus of the sub-utility function shift downward and the consumer minimizes the expenditure at E because the slope of the sub-utility function must be the relative price of the fine and the coarse qualities. In this sense, the decline in the relative price of the fine quality in terms of the coarse one is depicted by (iv) in the figure. Accordingly the consumption ratio of the fine quality to the coarse one increases as depicted by (v) because the homogeneity assumption on the sub-utility function guarantees the slope of OD to be larger than that of OE, where D represents the consumption point under the same price level with the utility level with the fixed transportation cost. Hence, we can see the fixed transaction cost raises the relative consumption of the fine quality to the coarse one under Assumption 1and additional assumptions, such that, there is no corner solution and the sub-utility function is homogeneous of degree $n > 0$. ∎

3. Proofs Applying Hicksian v. Marshallian

In the previous section, applying diagrams, we could see the Alchian-Allen effect to hold under conditions of utility functions to justify two-budgeting procedures. In the next, see algebraic characters to consider making comparisons with another proof. Algebraically, similar to Borcherding and Silberberg (1978), the influence of fixed transportation costs stated in the Alchian-Allen theorem shown by Theorem 1 (or Figure 2) is represented by the inequality such that

$$\frac{\partial\left(x_1/x_2\right)}{\partial T} > 0,$$

where the difference is that the relative demand function x_1/x_2 is Marshallian in this note while it is Hicksian in Borcherding and Silberberg. The following assessment clarifies the difference between two proofs, this note and of Borcherding and Silberberg.

Let x_i^H and x_i^M respectively be Hicksian and Marshallian demand functions of x-good of respective qualities. For Hicksian demand functions, following Borcherding and Silberberg, we find

$$\frac{\partial\left(x_1^H/x_2^H\right)}{\partial T} = \frac{x_1^H}{x_2^H}\cdot\left[\left(\frac{\varepsilon_{11}}{p_1}+\frac{\varepsilon_{12}}{p_2}\right)-\left(\frac{\varepsilon_{21}}{p_1}+\frac{\varepsilon_{22}}{p_2}\right)\right], \tag{5}$$

where ε_{ij} is the elasticity of **Hicksian** demand of x_i in terms of the price of x_j. Let subscript $j = 3$ denote the index for the composite good y. Using the Third Law of Hicks[4] as such

$$\sum_{j=1}^{3}\varepsilon_{ij} = 0 \quad (i = 1, 2, 3),$$

we find

$$\varepsilon_{12} = -\varepsilon_{11} - \varepsilon_{13} \quad \text{and} \quad \varepsilon_{22} = -\varepsilon_{21} - \varepsilon_{23}.$$

Thus, (5) can be arranged to get

$$\frac{\partial\left(x_1^H/x_2^H\right)}{\partial T} = \frac{x_1^H}{x_2^H}\cdot\left[\left(\varepsilon_{11}-\varepsilon_{21}\right)\cdot\left(\frac{1}{p_1}-\frac{1}{p_2}\right)+\left(\varepsilon_{13}-\varepsilon_{23}\right)\cdot\frac{1}{p_2}\right], \tag{6}$$

which also corresponds to equation (5) in Borcherding and Silberberg as well. The first term within the braces is positive because $\varepsilon_{11} < 0$, $\varepsilon_{21} > 0$, and $p_1 > p_2$. However, the second term cannot be determined in the model. In the proof of Borcherding and Silberberg, they rely on close substitutability between x_1 and x_2 or empirical results.[5]

Next, consider the proof using Marshallian demand functions demonstrated by this paper. Let $x_i^H\left(p_1, p_2, \bar{U}\right) = x_i^M\left(p_1, p_2, m - y\right)$ at the equilibrium, where $m-y$ is the residual income

[4] See Hicks (1946, pp. 309-11).

[5] See Borcherding and Silberberg (1978, p.135). With this regard, Bauman (2004) suggests the condition of Borcherding and Silberberg gets likely satisfied as the number of other goods increase.

$\bar{U}$ is the corresponding utility level to $m-y$. Then, the corresponding equation to (5) for this case is arranged as

$$\frac{\partial\left(x_1^M/x_2^M\right)}{\partial T}=\frac{\partial\left(x_1^H/x_2^H\right)}{\partial T}+\frac{x_1^M}{x_2^M}\cdot(\mu_1-\mu_2)\cdot\left(\frac{\eta_1}{p_1}+\frac{\eta_2}{p_2}\right), \tag{7}$$

where μ_i and η_i are the income elasticity of demand of x_i and the elasticity of the residual income in terms of p_i defined respectively by

$$\mu_i=\frac{\partial x_i^M}{\partial(m-y)}\frac{m-y}{x_i^M}\quad\text{and}\quad\eta_i=\frac{\partial(m-y)}{\partial p_i}\frac{p_i}{m-y}.$$

If we consider a two-good model using x_1 and x_2, then, we can easily find the Alchian-Allen effect because $p_1>p_2$ as claimed in the proof of the theorem using equation (4). The two-stage budgeting procedure under the n-th degree homogeneity allows us to treat the model as if it is the two-good model of x-good on the second stage; hence, we can find $\partial\left(x_1^H/x_2^H\right)/\partial T>0$.[6] In addition, expansion paths are upward linear for any degrees of homogeneity and that suggests rates of changes in consuming x_1 and x_2 in response to the unit change in the residual income shall be the same; hence, we must have $\mu_1\equiv\mu_2$. Subsequently, we can see the income effect, which is represented by the second term of (7), vanishes if we apply homogeneous utility functions. Thus, applying the same algebraic expression to Borcherding and Silberberg, we can give a proof for the Alchian-Allen effect as such

$$\frac{\partial\left(x_1^M/x_2^M\right)}{\partial T}>0.$$

4. Toward Multiple-Quality-Multiple-Good Analysis

The simplest extension is to introduce multiple "all other goods" as it is examined by Bauman (2004). Suppose all goods mentioned are separable in utility. Then, the two-stage budgeting procedure is justified (Gorman 1995a)[7] and it immediately gives the proof of the theorem because we just need to regard y as another index instead of regarding it as a composite good. In this sense, we can also extend our idea to include some other quality goods in the composite-good index to perform the Alchian-Allen effect in these other goods.

As another possibility, we can introduce multiple qualities more than three. Suppose the separability condition of Gorman is satisfied. If all qualities are substitutes of each others, then, the Alchian-Allen effect in such a model seems to be immediately proved applying accordingly many budgeting stages to x-good — in particular, if there are N such qualities classified by prices, then, we need $N-1$ budgeting stages.[8] Notice, however, in such a model, we just look at the relation between a quality and the neighboring index — for example, if the problem is solved in an upward manner, which means solve the model from the lowest

[6] See also Appendix A with relevance to (6).

[7] See also Brown and Chang (1976) for more detailed arguments about aggregations of commodities under general equilibrium frameworks with production.

[8] See, for example, Norman et al. (2001) for such sequential budgeting procedures.

quality to the highest, the relation between quality $N - k + \ell$ and quality $N - k$ is not considered but $N - k + \ell$ and the index containing qualities larger than $N - k + \ell + 1$. If this notion is admitted, based on the two-stage budgeting, the Alchian-Allen theorem is alternatively rewritten as follows.

Definition 1 (Multiple-Quality Alchian-Allen Effect) *Suppose there are a differentiated good by quality classes. Let qualities are indexed by $i = 1, 2, \ldots, N$. Denote the price of quality i by p_i and assume $p_1 > p_2 > \cdots > p_N$. Similarly, denote the consumption of quality i by x_i. Define X_i to be the index of consumption containing all better than i or equivalent qualities such that $X_1 \equiv x_1$. Let $T \geq 0$ be a fixed transaction cost. Then, the Alchian-Allen theorem is redefined by*

$$\frac{d\left(X_{i-1}/x_i\right)}{dT} > 0, \ \forall i = 2, 3, \ldots, N.$$

Relative relations among commodities in response to price shocks are usually determined by respective elasticities. In this case, even if relative consumptions are determined bilaterally, general descriptions of orders are a sort of puzzle. We can avoid this puzzling situation in multiple-quality models if Definition 1 is applied. To get some intuition, consider a three-quality model as such there is a standard quality ($i = 2$) as well as the fine quality ($i = 1$) and the coarse one ($i = 3$), which may gain much more interests in empirical applications. Assume the two-stage budgeting is justified and it holds that

$$\frac{d\left(X_2/x_3\right)}{dT} > 0 \quad \text{and} \quad \frac{d\left(x_1/x_2\right)}{dT} > 0. \tag{8}$$

The above setting indicates the demand for the coarse one declines in terms of the finer ones in an aggregated sense and the demand for standard one declines in terms of the fine one. Then, we can say there is the Alchian-Allen effect by Definition 1. However, it is easily verified that relative demands for each good cannot be determined because

$$\frac{d\left(x_1/x_3\right)}{dT} = \frac{x_1}{X_2}\underset{(+)}{\frac{d\left(X_2/x_3\right)}{dT}} + \frac{X_2}{x_3}\underset{(+/-)}{\frac{d\left(x_1/X_2\right)}{dT}}.$$

In particular, for $d\left(x_1/x_3\right)/dT > 0$, the fixed cost elasticity of demand for the fine quality must be smaller than that for the coarse one, so that

$$-\frac{dx_1}{dT}\frac{T}{x_1} < -\frac{dx_3}{dT}\frac{T}{x_3},$$

which is a very strong requirement and a cause of puzzling (see below).

In the example, suppose we have (8) and

$$-\frac{dx_1}{dT}\frac{T}{x_1} > -\frac{dx_3}{dT}\frac{T}{x_3},$$

which suggests the Alchian-Allen effect holds between the standard and the finest ones but it does not hold between the finest and the coarse one in the original sense. Yet, it is still possible to have (8) because the inequality about the coarse quality is expressed in terms of

X_2. Then, should we say there is no Alchian-Allen effect? If Definition 1 is applied, it exists but we still have no consensus about it. These arguments show that some difficulties exist to consider the Alchian-Allen effect bilaterally for all combinations of goods. However, the Alchian-Allen effect applying Definition 1, which can be proved by two-budgeting, gives a logical-but-easy way to deal with multiple-quality cases of the Alchian-Allen effect.

Finally, consider plausibility and viability of the Alchian-Allen effect with multiple-quality based on Definition 1. The philosophy of the Alchian-Allen theorem is to compare two classes of commodities. In applications, however, we need to classify quality standards to verify the effect. Then, it is not difficult to set the basis but it is difficult to select upper classes to compare with; one polar is just the neighboring class and the other polar is all the upper classes as suggested by Definition 1. Hence, Definition 1 suggests an alternative toward multiple-quality models but it is a detached criterion based on a logical procedure. In addition, Norman et al. (2003 and 2004) run experiments to verify the sequential budgeting problem. Applying Definition 1 and similar procedures with Norman et al., it opens a way for the Alchian-Allen contexts to execute experiments.

5. Concluding Remarks

This expository note has shown the existence of the Alchian-Allen effect under a specific condition justifying two-stage budgeting procedures in a three-good model (a composite good y and another good x of two qualities x_1 and x_2) using Marshallian demand functions and sequential budgeting. The idea of the proof is to apply intuitions in two-good models — only of x-good. Technically it has been done by using homogeneous functions that corresponds to justifying sequential budgeting. Then, we found graphical representations of the proof of the theorem. In addition, we also found an algebraic relationship to another proof presented by Borcherding and Silberberg.

The graphical representation developed in this paper assuredly reduces efforts of applied researchers working on the Alchian-Allen theorem with general equilibrium frameworks (that contains at least three good). For example, in partial equilibrium models, it is much easier than general equilibrium contexts to deal with the Alchian-Allen effect because these models are supposed to have minimum of two goods. However, calculations in algebraic approaches impose larger efforts on applied general equilibrium works. Under the justification conditions of utility functions for two-stage budgeting discussed by Gorman and this study, graphical treatments ease these costs and researchers can concentrate on their own interests within general equilibrium frameworks.

In addition, an alternative definition of the Alchian-Allen effect for multiple-quality models is introduced. The definition in this study is based as a relationship between a certain quality and aggregated finer qualities. Although there is no common consensus about the definition of the Alchian-Allen effect in multiple-quality contexts, it provides a direction in terms of two-stage budgeting. This alternative definition also gives a way of executing experiments about the effect as well.

Appendix

Appendix A: Price Index and Cross Elasticity

This appendix section verifies how cross elasticity is defined using price index of aggregated goods. In terms of income effect, it clarifies a technical difference between proofs using Hicksian demand functions and Marshallian demand functions with the homogeneous uility assumption.

Let the sub-utility function for x-good is homogeneous of degree $n > 0$. Then, by linear homogeneity of demand in the residual income, the derived indirect utility function for x-good is homogeneous of degree n in the residual income, so that, without loss of further generality, we can define a function v that satisfies

$$X = V(p_1, p_2, m - y) \equiv v(p_1, p_2) \times (m - y)^n .$$

Therefore, the price index for the first stage must satisfy

$$PX + y = m \quad \Longrightarrow \quad P = \frac{(m - y)^{n-1}}{v(p_1, p_2)}.$$

Next, consider cross elasticities ε_{i3}. According to (6), the proof becomes much simpler if $\varepsilon_{31} \equiv \varepsilon_{32}$. Applying the envelope theorem and Young's theorem to the expenditure function, $\varepsilon_{3i} = \varepsilon_{i3}$ holds. However, we will find

$$\varepsilon_{3i} = \left(\frac{\partial y}{\partial P} \frac{P}{y} \right) \cdot \left(\frac{\partial P}{\partial p_i} \frac{p_i}{P} \right),$$

which suggests $\varepsilon_{31} \not\equiv \varepsilon_{32}$ because

$$\frac{\partial P}{\partial p_1} \frac{p_1}{P} = \frac{\partial (1/v)}{\partial p_1} \frac{p_1}{P} \times (m - y)^{n-1} \not\equiv \frac{\partial (1/v)}{\partial p_2} \frac{p_2}{P} \times (m - y)^{n-1} = \frac{\partial P}{\partial p_2} \frac{p_2}{P}.$$

In particular, if $\varepsilon_{31} \equiv \varepsilon_{32}$, then, using Roy's identity, we must have

$$\frac{p_1}{p_2} \equiv \frac{\partial (1/v) / \partial p_2}{\partial (1/v) / \partial p_1} = \frac{-\partial v / \partial p_2}{-\partial v / \partial p_1} = \frac{-\dfrac{\partial V / \partial p_2}{\partial V / \partial (m - y)}}{-\dfrac{\partial V / \partial p_1}{\partial V / \partial (m - y)}} = \frac{x_2^H}{x_1^H},$$

which imposes $p_1 x_1 \equiv p_2 x_2$ at the equilibrium; hence, consumption shares of each quality must be the same. In the model, there is no such restriction of consumption share. Therefore, we can say the first term of (6) always dominates the second term, which is supposed to be sufficiently small in the proof of Borcherding and Silberberg, to have $\partial \left(x_1^H / x_2^H \right) / \partial T > 0$ under the homogeneity assumption of this study.

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Figures

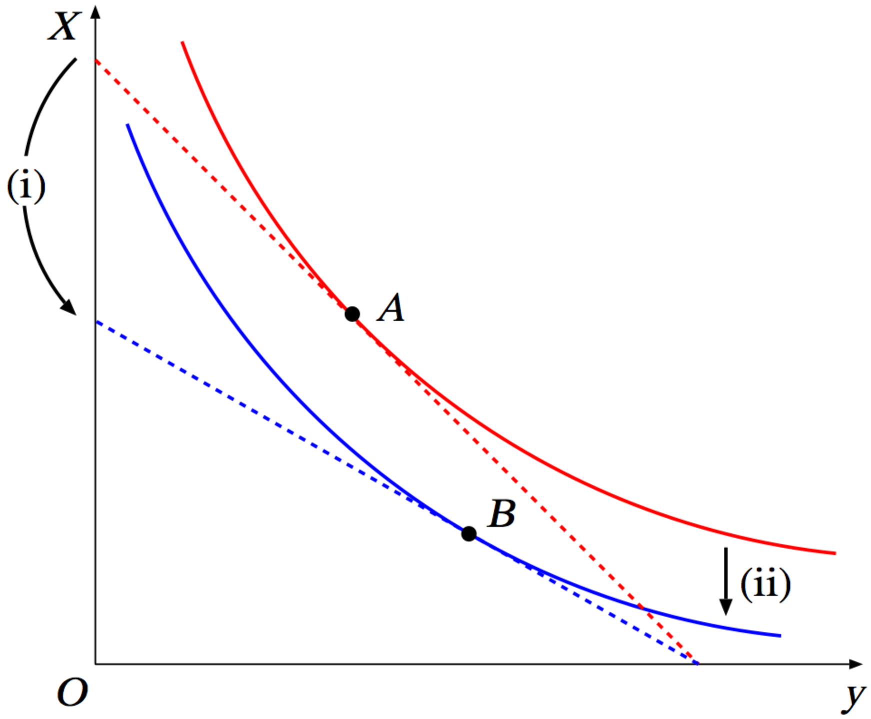

Figure 1: First Stage

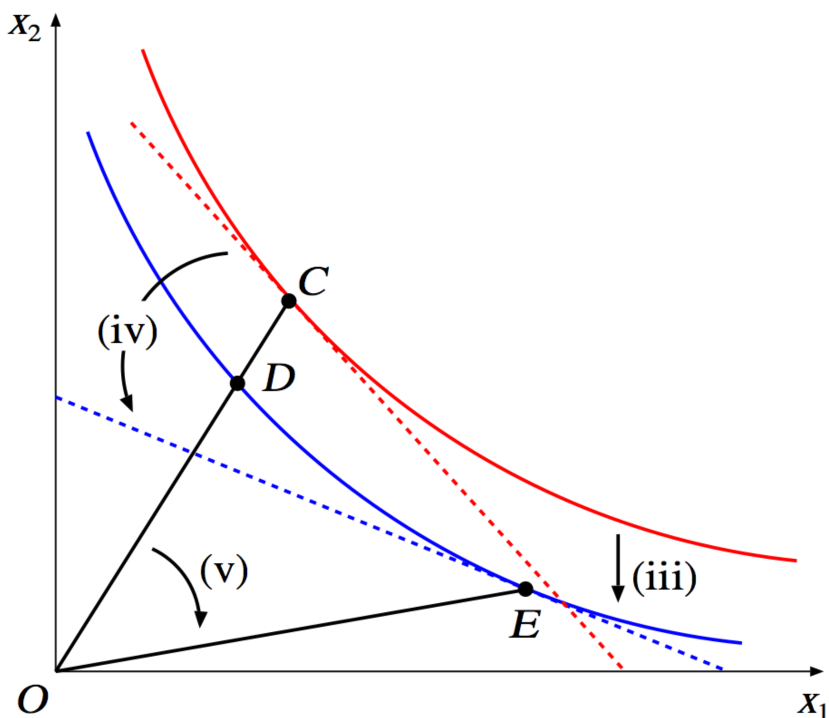

Figure 2: Second Stage