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Intergenerational Child Care Support and the Fluctuating Fertility: A Note

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Abstract

This paper describes the relation between fertility and intergenerational child-care support provided by grandparents in a model based on the overlapping generations model with endogenous fertility. Results show that intergenerational child-care support causes fertility to fluctuate. This paper presents an alternative model of fluctuating fertility shown by the `Easterlin Hypothesis' described by Benhabib and Nishimura (1989), and others. Moreover, this paper shows that an increase in rewards for child care supplied by older people does not always increase child care. Consequently, younger people can not depend on sufficient intergenerational child-care support.

1   Introduction

Intergenerational cooperation is often observed in families. Parents (or grandparents) assist in child care and household tasks. The children (parents) provide financial support to their parents (grandparents), as described by Schultz (1988) and others. Such intergenerational cooperation can ease the burden of child care and promote the upbringing of a child. In fact, the fertility of nuclear families is less than that of three-generation families in Japan[1], which implies that the family type is one factor determining the fertility rate in a family. Even if parents have their children, the parents can work sufficiently thanks to the grandparents' efforts. They (older parents) do housekeeping or care for children in place of the parents (younger parents).

This paper establishes an overlapping generations model with intergenerational cooperation and presents the relation between fertility and intergenerational child-care support. Moreover, this paper considers gifts from children (younger people) to parents (older people). Zhang and Zhang (1998) consider an endogenous fertility model with introduced gifts. The gift incorporated by Zhang and Zhang (1998) is included with the stipulation that parents must do something to receive a gift. However, the gift considered herein is that the parents must do housekeeping or child care to receive the gift.

The 'Easterlin Hypothesis' suggests the possibility of self-generating fluctuations in population growth: baby booms and baby busts are repeated. A large population will face stiffer economic competition, lower incomes, congestion, and crowding if other means of production as well as the social infrastructure do not expand simultaneously. The result might be a decline in fertility levels as parents attempt to maintain an adequate standard of living for themselves. Moreover, using the framework developed by Barro and Becker (1989), Benhabib and Nishimura (1989) demonstrate that the fertility rate might fluctuate according to capital per capita. Our paper presents an alternative mechanism of fertility rate fluctuation by incorporating intergenerational child-care support. This paper shows that intergenerational cooperation produces the fluctuation of the fertility rate and that whether fluctuation converges or not is determined intergenerationally by

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[1]This fact was uncovered using a survey conducted by the Ministry of Land, Infrastructure and Transport (2002).

relative altruism. Moreover, even if younger people increase the gift for their parents to induce the housekeeping or child-care time supplied by their parents, the parents reduce the housekeeping or child-care time in a steady state. Consequently, fertility might also decrease. Intergenerational child-care support is fundamentally equal to the child-care service supplied by nursery schools and other facilities. Apps and Rees (2004) and Martínez and Iza (2004) consider the child-care sector and show that fertility increases thanks to the child-care service because the opportunity cost for child bearing and subsequent care decreases. Galor and Weil (1996) present a model in which an increase in the wage rate increases the opportunity cost for child care. Thereby, fertility decreases.[2] Then, child-care services can stop decreasing fertility. On the other hand, this paper describes that even if younger people increase the gift as the reward for child care to increase intergenerational child care, the child-care support supplied by older people does not increase as younger people might expect. Consequently, younger people must not and do not depend on sufficient intergenerational child care.

The structure of the paper is the following. In the next section, we describe intergenerational cooperation and develop an overlapping generation model with cooperation. In section 3, we show that fluctuation of the fertility rate can be a result of relative altruism. In section 4, we prove a relation between intergenerational child-care support and altruism. Finally, we conclude the paper.

2 The Model

We consider the most popular overlapping generations (OLG) framework, which subsumes that each agent lives for only two periods. We regard the cohort that is born at $t-1$ as generation t; agents of this generation are young in period t and older in period $t+1$. At each point in time, two generations exist: young and old generations. The lifetime of

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[2] Galor and Weil (1996) present a model in which the relation between the fertility and the income per capita (wage rate) is negative. However, Groezen, Leers, and Meijdam (2003) and Fanti and Gori (2009) present a model for which this relation is positive. In these models, the opportunity cost for child care is not considered explicitly. Fertility is characterized as increasing because of an income effect. Groezen, Leers, and Meijdam (2003) consider an overlapping generations model based on the small open economy hypothesis by which individuals draw utility from the number of children raised and consumption at both younger and older times. On the other hand, Fanti and Gori (2009) consider an overlapping generations model of a closed economy.

agents of a generation is t. All agents are assumed to have an identical utility function, which depends on the number of children in the young period and which depends on consumption and leisure during the older period. Therefore, we assume that the utility function u_t is the following.[3]

$$u_t = \alpha \ln n_t + \beta \ln c_{t+1} + (1 - \alpha - \beta) \ln l_{t+1}, \ 0 < \alpha < 1, \ 0 < \beta < 1 \tag{1}$$

Therein, n_t is the number of children of generation t in the young period, and c_{t+1}, l_{t+1} represents the consumption and leisure of this generation in the old period.

Next, we consider the economic behavior of each agent of generation t. They are endowed with a unit of time for both their young and old periods. In the young period t, they decide on labor times and the number of children they have n_t. That is to say, they divide a unit of time between the labor supply and their child rearing. Child rearing requires ϕ units of time per child. Therefore, the time allocated for child rearing and labor income are denoted, respectively, as ϕn_t and $(1 - \phi n_t)w_t$. The wage rate is denoted as w_t. Consequently, we can obtain the following budget constraint when each agent is young.

$$s_t = (1 - \phi n_t)w_t + z_t(w_t - h) \tag{2}$$

In that equation, s_t denotes the savings of the young in period t. In addition, z_t signifies the times of child rearing assistance from the previous generation (old when the period is t), which is the child-care assistance from their parents. It is defined as $\frac{1-l_t}{n_{t-1}}$. In addition, h stands for the reward per unit of support time for the older generation in period t.[4] We assume that the child-care time supplied by younger people and the time supplied by older people are perfect substitutes. Actually, h is not determined through the market; it depends on mutual negotiation that takes place between the younger and the older generations. In other words, h denotes the degree of relative altruism in the economy. Large h implies that the young generation is more (less) altruistic (selfish) than the old

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[3]This assumption is conventional in modeling of endogenous fertility. (Galor and Weil (1996), Groezen, Leers, and Meijdam (2003), and others.)

[4]z_t is defined by $\frac{1-l_t}{n_{t-1}}$. Furthermore, n_{t-1} denotes the number of children the older person has. We assume that the older people supplies child care equally among all of their children.

generation. We consider h as a gift from the children (younger people) to parents (older people). The older people must take care of care children to receive the gift.

Moreover, the magnitude of z_t can be interpreted as the time of intergenerational support in the society. Younger people ask the older people to care for their children if $w_t > h$. Otherwise, the younger people care for their children by themselves, and the intergenerational child-care support vanishes. For that reason, we consider the case in which the intergenerational child care exists: $w_t > h$. To a considerable degree, z_t increases the labor time decreased by the child care. The labor time is shown as $1 - \phi n_t + z_t$. However, the condition $1 - \phi n_t + z_t \leq 1$ must hold.

During the older period $t + 1$, they use their time for leisure and to support their children's care. Assuming that l_{t+1} denotes the leisure time of an agent of generation t in their old period $t+1$, the reward from child-care assistance for their children is $(1-l_{t+1})h$. Therefore, the budget constraints for each t-generation agent in the old period $t + 1$ are

$$c_{t+1} = (1 + r_{t+1})s_t + (1 - l_{t+1})h, \tag{3}$$

where r_{t+1} is the interest rate.

Each agent treats w_t, r_{t+1} and $z_t (= \frac{1-l_t}{n_{t-1}})$ as given and chooses n_t, c_{t+1} and l_{t+1} to maximize utility from eq. (1), subject to eqs. (2) and (3). The optimal allocations are shown as follows.[5]

$$n_t = \frac{\alpha}{\phi w_t} I_t, \tag{4}$$

$$c_{t+1} = (1 + r_{t+1})\beta I_t, \tag{5}$$

$$l_{t+1} = \frac{(1 + r_{t+1})(1 - \alpha - \beta)}{h} I_t \tag{6}$$

Therein, $I_t \equiv w_t + (w_t - h)z_t + \frac{h}{1+r_{t+1}}$.

Finally, the production function of final goods is given as a neoclassical constant-returns-to-scale function $Y_t = F(K_t, L_t)$, where Y_t denotes the aggregate final goods, K_t stands for the aggregate capital stock, and L_t represents the labor input. Assuming perfect competition and defining $f(k_t) \equiv \frac{Y_t}{L_t}$ and $k_t \equiv \frac{K_t}{L_t}$, the wage rate w_t and the interest rate r_t are shown, respectively, as $w_t = f(k_t) - f'(k_t)k_t$ and $r_t = f'(k_t)$.

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[5]See Appendix A for a detailed proof.

3 Equilibrium

The analyses described in this paper subsume a small open economy. Therefore, the interest rate, the wage rate, and the capital–labor ratio are constant. Substituting the optimal allocation of n_t into $z_{t+1} = \frac{1 - l_{t+1}}{n_t}$, we obtain the following equation:

$$z_{t+1} = \frac{\phi w}{\alpha} \left(\frac{1}{w + (w - h)z_t + \frac{h}{1+r}} - \frac{(1+r)(1-\alpha-\beta)}{h} \right). \tag{7}$$

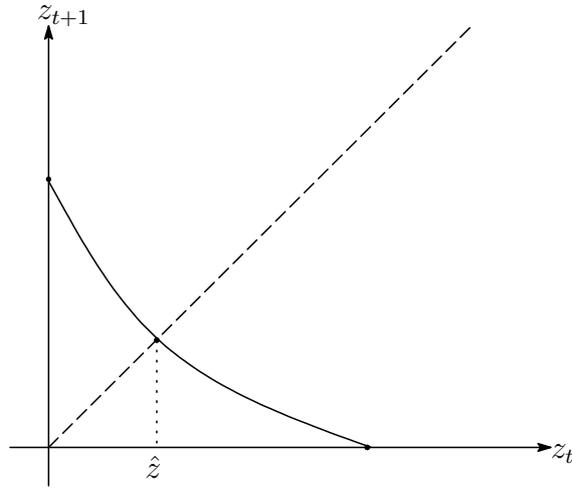The locus of this equation is depicted in Fig. 1.[6]

Fig. 1 z in a steady state

This equation shows the dynamic change in the degree of intergenerational child-care support provided by grandparents. If z_t is low, then z_{t+1} is high, and vice versa. A higher z_t produces a high fertility rate. However, in the next period, the support that households receive declines. Therefore, the fertility rate also decreases. Consequently, in the subsequent period, this support increases because of the decrease in households, which increases the fertility rate. This process continues. As a result, z_t fluctuates over time, as does the fertility rate. Then, the following proposition is established.

Proposition 1 An intergenerational child-care support in a steady state $\hat{z}$ is unique. The child-care time per child ϕ is small, z_t can converge to $\hat{z}$ with fluctuation. On the

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[6]See Appendix B for a detailed proof.

other hand, if ϕ is large, then z_t can diverge from $\hat{z}$ with fluctuation.

Proof Considering the steady state $z_{t+1} = z_t = \hat{z}$, Fig. 1 shows that $\hat{z}$ is unique. Then, z in the steady state, as defined by $\hat{z}$, is shown as follows.[7]

$$\hat{z} = \frac{-\left(w + \frac{h}{1+r} + \frac{\phi w(1+r)(1-\alpha-\beta)(w-h)}{\alpha h}\right) + \sqrt{D}}{2(w-h)} \tag{8}$$

In that equation, $D \equiv \left(w + \frac{h}{1+r} + \frac{\phi w(1+r)(1-\alpha-\beta)(w-h)}{\alpha h}\right)^2 - \frac{4\phi w(w-h)}{\alpha}\left(1 - \frac{(1+r)(1-\alpha-\beta)}{h}\left(w + \frac{h}{1+r}\right)\right)$.
Actually, $\hat{z}$ is locally stable if $-1 < \frac{dz_{t+1}}{dz_t}|_{z_{t+1}=z_t=\hat{z}} < 0$.[8] Actually, $\frac{dz_{t+1}}{dz_t}|_{z_{t+1}=z_t=\hat{z}}$ is shown as

$$\frac{dz_{t+1}}{dz_t}|_{z_{t+1}=z_t=\hat{z}} = -\frac{\phi w(w-h)}{\alpha\left(w + (w-h)\hat{z} + \frac{h}{1+r}\right)^2}. \tag{9}$$

This stable condition can not be satisfied if the child-care time per child ϕ is large.[9] Then, z_t diverges with the fluctuation. Figure 2 presents the dynamic path. Q.E.D.
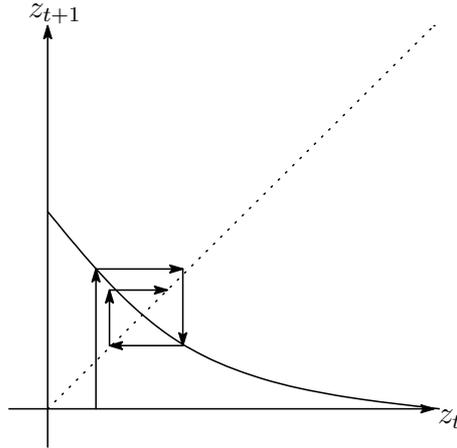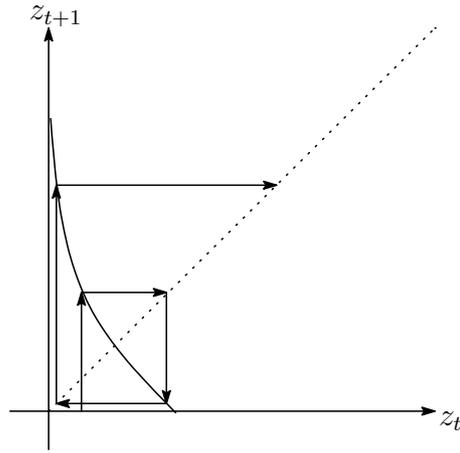

Fig. 2-1 Convergence
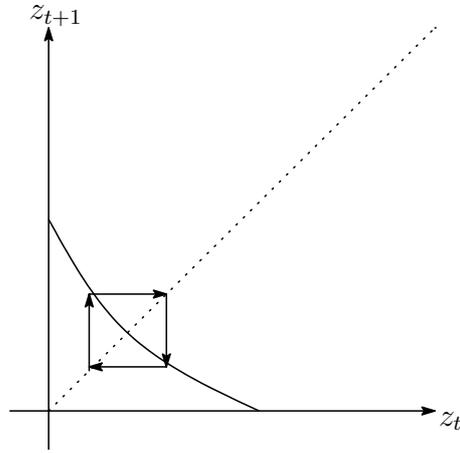

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Fig. 2-2 Divergence

Fig. 2-3 Cycle

This proposition proves that this paper presents an alternative model with fluctuating fertility shown by the 'Easterlin Hypothesis', as described by Benhabib and Nishimura (1989), and others.

If the z_t that agents in generation t receive is small, then z_{t+1} increases because of the agents' supply of additional child-care support to gain more income. However, if z_t is large, then z_{t+1} decreases because agents spend more time in leisure as a result of their sufficient income. Consequently, z_t fluctuates over time, as does fertility rate n_t. If the amount of change in z_t is equal to that in z_{t+1}, then z_t and n_t fluctuate to infinity. Intergenerational child-care support z_t can diverge over time if ϕ is large. The higher ϕ lowers the fertility rate. Therefore, the child-care support provided by grandparents to their children is large. This effect increases over time, so divergence occurs.

From the description provided above, we can explain two interpretations. First, the people of the older generation, who have insufficient income in the younger period are willing to provide child care to secure resources for their old age. On the other hand, with sufficient income, the older generation enjoys leisure. In Japan, considering that the old generation—after rapid economic growth—has sufficient income, we can infer that the older generation views provision of child care negatively; for that reason, the fertility rate is decreasing.

Second, the results presented in this paper show that if an increase in the number of children n_{t-1} lowers the intergenerational child-care support per household z_t under constant l_t, then it is brought lower to z_t after a baby boom (high n_{t-1}). Moreover, we can advance the following interpretation. After the baby boom (n_{t-1}), too few parents existed for the number of households. For that reason, the average intergenerational child care across households is low (that is, low z_t).

4   Analysis

This section presents a description of the relation between an increase in the reward for the child-care support offered by the older generation, denoted as h, and that support per household in the steady state, denoted as z. Thereby, the following proposition is established.

Proposition 2   If $\hat{z} < \frac{1}{1+r}$, then the rise in h pulls up $\hat{z}$ when the price effect in higher than the income effect. However, if $\hat{z} > \frac{1}{1+r}$, then the rise in h always pulls up $\hat{z}$.

Proof   Differentiating z with respect to h, we obtain

$$\frac{d\hat{z}}{dh} = \frac{\frac{\phi w}{\alpha}\left(\frac{\hat{z}-\frac{1}{1+r}}{\left(w+(w-h)\hat{z}+\frac{h}{1+r}\right)^2} + \frac{(1+r)(1-\alpha-\beta)}{h^2}\right)}{1+\frac{\phi w(w-h)}{\alpha\left(w+(w-h)\hat{z}+\frac{h}{1+r}\right)^2}}. \tag{10}$$

The sign of this equation is ambiguous. The term of $\frac{(1+r)(1-\alpha-\beta)}{h^2}$ in the numerator represents the price effect. The rise in h indicates an increase in the opportunity cost for leisure among older people. Therefore, the rise in h, i.e. the price effect, increases $\hat{z}$. The

term $\frac{\hat{z}-\frac{1}{1+r}}{\left(w+(w-h)\hat{z}+\frac{h}{1+r}\right)^2}$ in the numerator represents the income effect. If $\hat{z} < \frac{1}{1+r}$, then the rise in h increases households' incomes. Therefore, the leisure and the fertility rate also rise. Consequently, the income effect in the case of $\hat{z} < \frac{1}{1+r}$ decreases $\hat{z}$. If the price effect is higher than the income effect, then the rise in h increases $\hat{z}$. However, if $\hat{z} > \frac{1}{1+r}$, then the sign of $\hat{z} - \frac{1}{1+r}$ is positive. Therefore, the rise in h always increases $\hat{z}$. Q.E.D.

When $\hat{z} < \frac{1}{1+r}$—that is, when the intergenerational child-care support $\hat{z}$ is small—the rise in h does not always increase $\hat{z}$. We can interpret the reward of child care h as security in old age or as a gift. This proposition shows that an increase in security in old age does not always pull up intergenerational child-care support.

Then, the fertility in the steady state $\hat{n}$ is altered by an increase in gift h, as shown by $\frac{d\hat{n}}{dh} = \frac{\alpha}{\phi w}\left(\frac{1}{1+r} - \hat{z} + (w-h)\frac{d\hat{z}}{dh}\right)$. We obtain $\frac{d\hat{n}}{dh} > 0$ if $\hat{z} < \frac{1}{1+r}$ and $\frac{d\hat{z}}{dh} > 0$ (the income effect is small). Therefore, we note that an increase in the gift h does not always increase fertility: even if younger people consider increasing the gift given to older people for more care, the older people reduce their care for children. Consequently, fertility decreases also.

5   Concluding Remarks

This paper has presented analyses of the relation between fertility and intergenerational child-care support (child-care support per household offered by older people) based on the overlapping generations model in which households determine the number of children (the fertility rate) and child-care support by older people. Among the results that have been advanced in this paper, the following two are especially noteworthy.

First, if intergenerational support is introduced into the endogenous fertility model, then the fertility rate fluctuates. Furthermore, the breadth of that fluctuation is determined by the relevant parameter (e.g. care time per child).

Second, a rise in the degree of relative altruism (or security or gift in old age) does not always increase the fertility rate and child care supplied by older people. Therefore, older people do not supply child care to the degree that younger people expect.

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Appendix A

The Lagrange function is set as shown below:

$$L \equiv \alpha \ln n_t + \beta \ln c_{t+1} + (1 - \alpha - \beta) \ln l_{t+1},$$
$$+ \lambda \left(w_t(1 + z_t - \phi n_t) - hz_t + \frac{h}{1 + r_{t+1}}(1 - l_{t+1}) - \frac{c_{t+1}}{1 + r_{t+1}} \right).$$

Therein, λ is the Lagrange multiplier. The first-order conditions are expressed as follows.

$$\frac{\partial L}{\partial n_t} = \frac{\alpha}{n_t} - \lambda \phi w_t = 0, \tag{A.1}$$

$$\frac{\partial L}{\partial c_{t+1}} = \frac{\beta}{c_{t+1}} - \frac{\lambda}{1 + r_{t+1}} = 0 \tag{A.2}$$

$$\frac{\partial L}{\partial l_{t+1}} = \frac{1 - \alpha - \beta}{l_{t+1}} - \frac{\lambda h}{1 + r_{t+1}} \tag{A.3}$$

$$\frac{\partial L}{\partial \lambda} = w_t(1 + z_t - \phi n_t) - hz_t + \frac{h}{1 + r_{t+1}}(1 - l_{t+1}) - \frac{c_{t+1}}{1 + r_{t+1}} = 0 \tag{A.4}$$

Substituting (A.1)–(A.3) into (A.4), we obtain $\lambda = \frac{1}{w_t + (w_t - h)z_t + \frac{h}{1 + r_{t+1}}}$. By substituting λ into (A.1)–(A.3), optimal allocations (4)–(6) are shown.

Appendix B

We can depict Fig. 1 if $\frac{dz_{t+1}}{dz_t} < 0$ and $\frac{\partial^2 z_{t+1}}{\partial z_t^2} > 0$. With (7), we can derive the following.

$$\frac{dz_{t+1}}{dz_t} = -\frac{\phi w(w - h)}{\alpha \left(w + (w - h)z_t + \frac{h}{1+r} \right)^2} < 0$$

Moreover, we derive $\frac{\partial^2 z_{t+1}}{\partial z_t^2}$ as follows.

$$\frac{\partial^2 z_{t+1}}{\partial z_t^2} = \frac{2\phi w(w - h)^2}{\alpha \left(w + (w - h)z_t + \frac{h}{1+r} \right)^3} > 0$$

Therefore, z_{t+1} must be positive at $z_t = 0$ to hold $\hat{z} > 0$. This condition is $\frac{1}{w + \frac{h}{1+r}} - \frac{(1+r)(1-\alpha-\beta)}{h} > 0$; that is,

$$h > \frac{(1 - \alpha - \beta)(1 + r)w}{\alpha + \beta}.$$

Consequently, h must be in $\frac{(1-\alpha-\beta)(1+r)w}{\alpha+\beta} < h < w$. Unless this condition holds, z_{t+1} becomes negative. (Then, no steady state of $\hat{z} > 0$ exists.) The grandparents take their

time and use it as leisure time if z_{t+1} or z_t or $\hat{z}$ is negative. Our paper is intended to present analyses of intergenerational child care by which the grandparents give their time. Therefore, we do not consider such a case.

Appendix C

Considering (7), we obtain the following equation in the steady state.

$$\hat{z} = \frac{\phi w}{\alpha} \left(\frac{1}{w + (w - h)\hat{z} + \frac{h}{1+r}} - \frac{(1+r)(1-\alpha-\beta)}{h} \right)$$

With some calculation, we obtain the following equation.

$$(w-h)\hat{z}^2 + \left(w + \frac{h}{1+r} + \frac{\phi w(1+r)(1-\alpha-\beta)(w-h)}{\alpha h} \right) \hat{z}$$
$$- \frac{\phi w}{\alpha} \left(1 - \frac{(1+r)(1-\alpha-\beta)}{h} \left(w + \frac{h}{1+r} \right) \right) = 0 \qquad \text{(C.1)}$$

Therein, $1 - \frac{(1+r)(1-\alpha-\beta)}{h} \left(w + \frac{h}{1+r} \right) > 0$ if $\hat{z} > 0$ or $h > \frac{(1-\alpha-\beta)(1+r)w}{\alpha+\beta}$. The locus of the left side in this equation is depicted in Fig. 3.
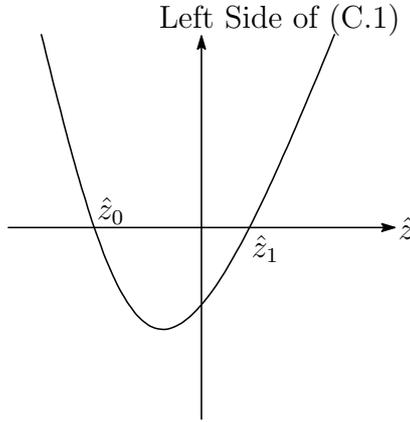

Fig. 3 Solution of (C.1)

This equation has two solutions as $\hat{z}$: one for negative value $\hat{z}_0$ and the other for positive value $\hat{z}_1$. However, because $\hat{z}$ must be positive, $\hat{z}$ is determined uniquely as $\hat{z}_1$. Actually, $\hat{z}_1$ is shown as (8).

Appendix D

If $\hat{z}$ is locally stable, then the following is true.

$$-\frac{\phi w(w-h)}{\alpha \left(w + (w-h)\hat{z} + \frac{h}{1+r}\right)^2} > -1$$

$$\frac{\phi w}{\alpha \left(w + (w-h)\hat{z} + \frac{h}{1+r}\right)} < \frac{w + (w-h)\hat{z} + \frac{h}{1+r}}{w - h}$$

Considering that $\frac{\phi w}{\alpha \left(w+(w-h)\hat{z}+\frac{h}{1+r}\right)} = \hat{z} + \frac{\phi w(1+r)(1-\alpha-\beta)}{\alpha h}$, then

$$\hat{z} + \frac{\phi w(1+r)(1-\alpha-\beta)}{\alpha h} < \frac{w + \frac{h}{1+r}}{w - h} + \hat{z}$$

$$\phi < \frac{\alpha h \left(w + \frac{h}{1+r}\right)}{w(w-h)(1+r)(1-\alpha-\beta)}.$$

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