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Endogenous participation in imperfect labor and capital markets

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### Abstract

We introduce endogenous participation in an economy with labor and financial market frictions. Agents can choose to be workers or entrepreneurs or not to participate in any market. We examine how the transition rates between these three options are affected by productivity shocks (business cycle conditions) and by changes in the level of market frictions (cross-country institutional quality variations).

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#### 1. Introduction

Each year many workers decide to become self-employed, while many self-employed become employees and the levels of these transitions depend on macroeconomic conditions. For example, panel (a) of Figure 1 suggests that each year in Great Britain between 1.5% and 2% of those in employment move into self-employment. There is some suggestion that this rate may be pro-cyclical. Meanwhile, about 2% to 3% of those out of employment enter self-employment each year. Panel (b) shows that each year about 10% of the self-employed become employed while about 6% of the self-employed exit into non-employment.<sup>1</sup>

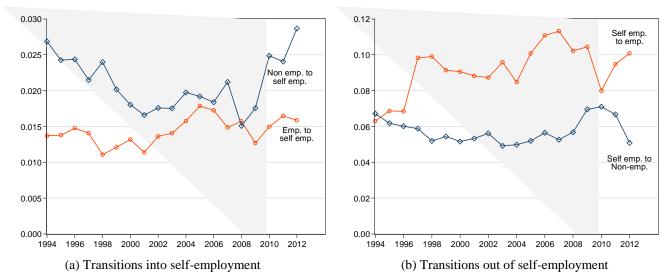


Figure 1. Annual transition rates into and out of self-employment. Shaded area indicates consecutive quarters of negative GDP growth. Sample includes all individuals aged 16-60. Weighted to UK population. Source: Quarterly Labour Force Survey 5-quarter longitudinal panel.

Clearly, the choice between self-employment and employment is influenced not only by the state of the economy but also by conditions in the financial and labor markets. The level of labor market frictions affects availability of jobs while the level of financial market frictions affects availability of funding for prospective entrepreneurs.

We analyze a simple model with labor and financial market frictions where agents can stay away from the market, or enter the labor market, or become entrepreneurs.<sup>2</sup> We do so by introducing a labor market in the Holmström and Tirole (1997) fixed investment model. Then we examine how the transition rates between these three options are affected by productivity

<sup>&</sup>lt;sup>1</sup> The fact that transition rates out of self-employment are so much higher is related to the fact that the stock of self-employed is much smaller than the stocks of employed and non-employed (unemployed plus those people between 16 and 60 who do not participate in the labor market).

<sup>&</sup>lt;sup>2</sup> There is an extensive literature that investigates the interactions between financial and labor market frictions (see, for example, Acemoglu, 2011; Arnold, 2002; Farmer, 1985; Greenwald and Stiglitz, 1993; Hall, 2011; Wasmer and Weil, 2004). However, in all these papers agents can be either workers or self-employed without being allowed to move between the two forms of employment.

shocks (business cycle conditions) and by changes in the level of market frictions (crosscountry institutional quality variations).

#### 2. The Model

The one-period single-good economy is populated by a continuum of risk-neutral agents of measure 1. Agents are endowed with one indivisible unit of labor and *z* units of the good that is distributed across the population according to the function *G* (with density *g*) on the interval  $[\underline{z}, \overline{z}] \in \mathbb{R}^+$ . All agents have three options. They can become entrepreneurs and run a project, they can enter the labor market, or they can abstain from any economic activity.

Entering the labor market entails a utility cost  $\gamma$  distributed across the population according to the function *F* (with density *f*) on the interval  $[\underline{\gamma}, \overline{\gamma}] \in R^+$ . The utility of an agent is equal to  $C - \gamma I$  where *C* denotes consumption and  $I \in \{0,1\}$  equals 1 if the agent enters the labor market and 0 otherwise. The distributions *F* and *G* are independent.

There is a risky technology that requires an entrepreneur's labor to manage it, an investment of *K* units of the good and one unit of labor. With probability  $\in \{p^L, p^H\}$ ,  $(0 < p^L < p^H < 1)$  the technology yields *X* units of the single good and with probability 1 - p yields nothing. Success depends on the entrepreneur's behavior. Working hard increases the likelihood of success while shirking offers a private benefit *B*.

#### Assumption 1: $K > \overline{z}$ .

The inequality implies that no agent can self-finance a project. Those agents who are successful in securing external finance must hire a worker. Let w and r denote the equilibrium wage (paid at the completion of the project) and the equilibrium gross interest rate. We restrict our attention to equilibrium outcomes where projects have a positive expected value only when entrepreneurs exert effort.

Assumption 2: 
$$p^{H}(X - w) - rK \ge 0 > p^{L}(X - w) - rK + B$$
.

Put differently, if there was an entrepreneur wealthy enough to self-finance the project, he would choose to exert effort.

#### 2.1. The Financial Market

Agents who do not become entrepreneurs either invest their endowments in the financial market or they store them. We follow Holmström and Tirole (1997) in assuming that financial markets are competitive. Agents will choose to invest their endowments in the

financial market rather than store them if: (a) entrepreneurs, who are protected by limited liability, exert effort,<sup>3</sup> and (b) the interest rate r is not less than 1 (the return to storage).

Let R denote the loan repayment. For an entrepreneur who obtained a loan equal to l the repayment must satisfy

$$p^H R = lr$$

so that lenders are indifferent between the loan contract and the market rate of return r. Limited liability and the fact that wages are only paid when projects are successful imply that the payoff to entrepreneurs when projects fail is equal to zero when they do exert effort and equal to B when they do not exert effort. Then, entrepreneurs exert effort if the following incentive compatibility constraint

$$p^{H}(X - R - w) \ge p^{L}(X - R - w) + B$$

is satisfied. The left-hand side equals the expected payoff of the project when the entrepreneur exerts effort while the right-hand side shows the expected payoff when she does not exert effort. Notice that wages and loan repayments are only paid out when projects succeed. Clearly, for sufficiently high borrowing the above incentive constraint will be violated even if Assumption 2 still holds. Rearranging yields

$$X - w - \varphi \ge R$$

where  $\varphi \equiv \frac{B}{p^H - p^L}$  measures agency costs. The constraint sets an upper bound on the repayment. Combining the above two conditions we find that only those agents with endowments greater than

$$z^* = K - \frac{p^H}{r} (X - w - \varphi) \tag{1}$$

obtain external funds. We refer to the rest of the agents as 'financially constrained'.<sup>4</sup>

#### 2.2. The Labor Market

Those agents that have secured a loan can become entrepreneurs and run a project by hiring a worker. Agreed wages are paid at the completion of the project and thus workers earn income only if their corresponding project succeeds. The division of surplus between entrepreneurs, who have secured a loan, and workers is determined by a generalized Nash

<sup>&</sup>lt;sup>3</sup> Given that projects are not profitable when entrepreneurs do not exert effort.

<sup>&</sup>lt;sup>4</sup> Giannetti (2011) provides evidence showing that liquidity constraints negatively affect the probability of being self-employed.

bargaining rule. When wages are set above the market clearing wage some of those agents who enter the labor market are not matched with firms and become involuntary unemployed; a possibility that agents anticipate when they decide whether or not to enter the labor market. Let  $\pi$  denote the employment rate, i.e. the proportion of agents that enters the labor market and are matched.

Consider the wage determination process. Let  $z^{I} \leq z$  measure the level of internal finance. The entrepreneur's expected payoff conditional on a successful bargaining outcome is equal to the project's expected profits plus the return from the funds invested in the financial market  $p^{H}(X - w - R) + (z - z^{I})r$ . Given that  $p^{H}R = lr = (K - z^{I})r$  the last expression can be written as  $p^{H}(X - w) - (K - z^{I})r + (z - z^{I})r = p^{H}(X - w) - (K - z)r$ . When bargaining fails, her payoff equals  $(z - z^{I})r$  given that (a) the loan agreement precedes the attempt to hire a worker and (b) that she is protected by limited liability. The worker's expected payoff conditional on a successful bargaining outcome equals  $p^{H}w + zr - \gamma$ , which is equal to expected wages plus return on savings minus labor market entry cost. When bargaining fails her, payoff equals  $zr - \gamma$ .<sup>5</sup> Let  $\alpha$  measure the bargaining power of workers. Then,

$$w = argmax(p^Hw)^{\alpha}(p^H(X-w) - (K-z^I)r)^{1-\alpha}.$$

#### Lemma 1: The optimal level of internal finance is independent of the endowment level.

**Proof:** Setting the first-order-condition of the optimization problem equal to zero and solving for the wage yields  $w = \frac{\alpha(p^H x - (K - z^I)r)}{p^H}$ . The proposition follows from the observations (a) the wage is increasing in the level of internal finance,  $z^I$ , and (b) the entrepreneur's payoff depends on the level of internal finance only through the determination of the wage. The lemma implies that entrepreneurs contribute the lowest possible amount of internal

**Lemma 2:** The common wage is given by

$$w = \frac{\alpha}{1-\alpha} \varphi \equiv \theta \varphi$$

finance. Thus we have  $z^{I} = z^{*}$ .

<sup>&</sup>lt;sup>5</sup> Given that the incentive compatibility constraint must be satisfied in equilibrium (otherwise entrepreneurs would not receive external finance) we restrict our attention to those payoffs associated with the entrepreneur exerting effort.

**Proof:** Setting  $z^{I} = z^{*}$ , implies that  $w = \frac{\alpha(p^{H}X - (K - z^{*})r)}{p^{H}}$ . By substituting (1) for  $z^{*}$  and solving for the wage rate we complete the proof.

The equilibrium wage depends on the degree of imperfections in both markets.<sup>6</sup> The higher the bargaining power of workers (higher  $\alpha$ ), the higher the wage. The wage also increases with  $\varphi$ , a measure of financial market agency costs. As the level of internal finance increases the entrepreneur's obligation to her creditors decreases, thus increasing the surplus whose division is negotiated between the two parties.

Above, we assumed that the allocation of bargaining power is independent of the employment rate. In what follows, we consider the more realistic case where the bargaining power of workers is increasing in the employment rate. As the employment rate increases the influence of the outsiders, i.e. the involuntary unemployed, declines. Thus, we consider the new wage function

$$w = \frac{\alpha(\pi,\delta)}{1-\alpha(\pi,\delta)}\varphi = \delta\theta(\pi)\varphi; \ \theta'(\pi) > 0; \ \delta > 0,$$
(2)

where  $\delta$  is a shift parameter.

#### 2.2. Occupational Decisions

The expected utility derived from entering the labor market is

$$U^L = \pi p^H w + rz - \gamma.$$

The first term denotes expected labor income and the second term denotes gross financial income. The expected utility of entrepreneurs is

$$U^E = p^H (X - w) - r(K - z).$$

When the agent decides neither to become an entrepreneur, nor to participate in the labor market, her utility  $U^N$  is given by her financial income

$$U^N = rz$$

Assumption 2 implies that  $U^E - U^N > 0$ . All agents with  $z \ge z^*$  either become entrepreneurs or enter the labor market. Next, comparing  $U^L$  and  $U^N$  we find that among the agents with endowments less than  $z^*$ , those with  $\gamma > \pi p^H w$  do not enter the labor market. Let

<sup>&</sup>lt;sup>6</sup> This dependence was originally shown by Peroti and Spier (1993) while its macroeconomic implications have also been analyzed by Wasmer and Weil (2004).

$$\gamma^H = \pi p^H w. \tag{3}$$

Comparing  $U^E$  and  $U^L$  we find that among those agents with endowments greater than  $z^*$ , those with  $\gamma > \pi p^H w - (p^H (X - w) - rK)$  become entrepreneurs. Let

$$\gamma^{L} = \pi p^{H} w - (p^{H} (X - w) - rK) \ge 0.$$
<sup>(4)</sup>

Assumption 2 implies that  $\gamma^H > \gamma^L$ . Clearly if  $\gamma^L = 0$  then all agents with  $z \ge z^*$  will become entrepreneurs.

#### 2.3. Equilibrium

The availability of storage implies that there are two types of equilibria. There is one equilibrium where the financial market clears at a gross interest rate greater than unity and nobody stores. The mass of financially unconstrained agents is sufficiently high to allow the market to clear. Small perturbations only change the interest rate without affecting output and employment.

We focus on the other equilibrium where the gross interest rate is equal to one (return to storage) and some endowments are stored. Labor market clearing implies

$$\pi \{ F(\gamma^H) G(z^*) + F(\gamma^L) (1 - G(z^*)) \} = (1 - F(\gamma^L)) (1 - G(z^*)).$$
(5)

The left-hand side equals the supply of labor. The first term in the brackets equals the mass of financially constrained agents who enter the labor market, while the second term equals the corresponding mass of unconstrained agents. The right hand-side equals the demand for labor (unconstrained agents who become entrepreneurs).

Equations (1) and (5) solve for the parameters  $z^*$ ,  $\gamma^H$ ,  $\gamma^L$ , w and  $\pi$ . The amount invested in storage *V* is

$$V = \hat{z} - K (1 - F(\gamma^L)) (1 - G(z^*)).$$
(6)

The second term of the right-hand side equals aggregate investment. The following proposition summarizes the comparative static results of the model when  $\gamma^L = 0$ .

**Proposition 1:** (Equilibrium with Storage) Suppose that  $\gamma^L = 0$ . Then,

(a) 
$$\frac{d\pi}{d\varphi} < 0$$
, (b)  $\frac{dV}{d\phi} > 0$ , (c)  $\frac{d\pi}{d\delta} < 0$ , (d)  $\frac{dV}{d\delta} > 0$ , (e)  $\frac{d\pi}{dx} > 0$ , and (f)  $\frac{dV}{dx} < 0$ .

After an increase in either workers' bargaining power or in agency costs the wage rate increases, causing a drop in output (increase in storage) and thus a fall in the employment rate. In contrast, a positive productivity shock has the opposite effects. The results related to the

employment rate still hold when  $\gamma^L > 0$ . A sufficient condition, but by no means necessary, for the results related to storage to be still valid is that the direct effects on output dominate the indirect employment effects on the wage rate.

#### 3. Macroeconomic Implications

We first analyze the impact of productivity shocks on employment and participation rates to clarify the implications of our model for the behavior of workers over the business cycle. Then we introduce variations in market frictions to assess how cross-country institutional differences affect cross-country variations in participation and employment rates.

#### **3.1. Productivity Shocks**

Consider an increase in *X*. Proposition 1 implies that the employment rate will rise. When  $\gamma^L = 0$ , the participation rate is equal to  $1 - (1 - F(\gamma^H))G(z^*)$ . Differentiating with respect to *X* yields

$$f(\gamma^{H})p^{H}\frac{d\pi}{dx}\delta\left(1+\theta'(\pi)\right)\theta G(z^{*})+g(z^{*})\frac{p^{H}}{r}\left(1-F(\gamma^{H})\right)>0.$$

The participation rate increases after a positive productivity shock. Both terms are positive as the positive shock encourages (a) entrepreneurship, and (b) entry in the labor market.

#### **3.2. Institutional Variations**

A higher value of  $\delta$  signifies a stronger union and thus a less flexible labor market. Similarly, a higher  $\varphi$  captures a less efficient financial market. Proposition 1 suggests that an increase in any of these parameters has a negative effect on the employment rate. What happens to the participation rate is more complicated and for simplicity we restrict our attention to the case when  $\gamma^L = 0$ .

Differentiating the participation rate with respect to  $\varphi$  yields

$$f(\gamma^{H})p^{H}\delta\left(\frac{d\pi}{d\varphi}\varphi(\theta(\pi)+\pi\theta'(\pi))+\pi\theta(\pi)\right)G(z^{*})-g(z^{*})\frac{p^{H}}{r}\left(1-F(\gamma^{H})\right)\geq 0.$$

The term in the brackets is positive which implies, if  $\left|\frac{d\pi}{d\varphi}\varphi(\theta(\pi)+\pi\theta'(\pi))\right|$  < 1, the whole expression will be positive. The expression equals the elasticity of the expected wage function  $\pi w$  (see (2)) with respect to the level of financial frictions. When it is less than 1 (the effects of an increase in frictions on the wage dominate its effects on the employment rate) the rise in frictions has a positive effect on the participation rate. This, for example, will

be the case for countries with, *ceteris paribus*, higher employment rates. The second term captures the negative effect of higher financial market inefficiency on entrepreneurship.

We draw two implications about the relationship between cross-country institutional variations and corresponding variations in labor market outcomes. The symmetry of the wage function with respect to  $\delta$  and  $\varphi$  implies that the two variations have the same qualitative effects.<sup>7</sup> Furthermore, more flexible markets imply higher employment rates. However, cross-country variations in participation rates depend on the relative responses of wages and employment rates to institutional variations. Among countries with high frictions those with higher employment rates have higher participation rates.

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<sup>&</sup>lt;sup>7</sup> Financial market inefficiency has a stronger effect due to its impact on entrepreneurship ( $\delta$  does not affect  $z^*$ ).

## Appendix

## **Proof of Proposition 1:**

Setting r = 1 and totally differentiating (5) we get

$$\begin{cases} F(\gamma^{H})G(z^{*}) + (1 - G(z^{*}))F(\gamma^{L}) + \\ \pi(p^{H}\delta\theta(\pi)\varphi + \pi p^{H}\delta\theta'(\pi)\varphi)f(\gamma^{H})G(z^{*}) + \\ (1 + \pi)f(\gamma^{L})(1 - G(z^{*}))(p^{H}\delta\theta(\pi)\varphi + \pi p^{H}\delta\theta'(\pi)\varphi + p^{H}\delta\theta'(\pi)\varphi) + \\ (\pi(F(\gamma^{H}) - F(\gamma^{L})) + (1 - F(\gamma^{L})))g(z^{*})p^{H}\delta\theta'(\pi) \\ + \begin{cases} \pi^{2}f(\gamma^{H})p^{H}G(z^{*})\delta\theta(\pi) + \\ (\pi(F(\gamma^{H}) - F(\gamma^{L})) + (1 - F(\gamma^{L})))(1 + \delta\theta(\pi))g(z^{*})p^{H} + \\ (1 + \pi)f(\gamma^{L})(1 - G(z^{*}))(\pi p^{H}\delta\theta(\pi) + p^{H}\delta\theta(\pi)) \\ \end{cases} \\ + \begin{cases} \pi^{2}f(\gamma^{H})G(z^{*})p^{H}\theta(\pi)\varphi + (\pi(F(\gamma^{H}) - F(\gamma^{L})) + (1 - F(\gamma^{L})))g(z^{*})p^{H}\theta(\pi)\varphi + \\ (1 + \pi)f(\gamma^{L})(1 - G(z^{*}))(\pi p^{H}\theta(\pi)\varphi + p^{H}\theta(\pi)\varphi) \\ \end{cases} \\ d\delta \\ - \{(\pi(F(\gamma^{H}) - F(\gamma^{L})) + (1 - F(\gamma^{L})))g(z^{*})p^{H} + (1 + \pi)(1 - G(z^{*}))f(\gamma^{L})p^{H}\}dX = 0 \end{cases}$$

From which the proofs of parts (a), (c) and (e) directly follow.

Totally differentiating (6) we get

$$\begin{split} \frac{dV}{d\varphi} &= \frac{\partial V}{\partial \pi} \frac{d\pi}{d\varphi} + \frac{\partial V}{\partial \varphi} \\ &= \begin{cases} f(\gamma^L) \big( 1 - G(z^*) \big) K \big( (p^H \delta \theta(\pi) \varphi + \pi p^H \delta \theta'(\pi) \varphi + p^H \delta \theta'(\pi) \varphi) \big) + \\ K \big( 1 - F(\gamma^L) \big) g(z^*) p^H \delta \theta'(\pi) \varphi \end{cases} \\ &+ f(\gamma^L) \big( 1 - G(z^*) \big) K \big( \pi p^H \delta \theta(\pi) + p^H \delta \theta(\pi) \big) \\ &+ K \big( 1 - F(\gamma^L) \big) g(z^*) (1 + \delta \theta(\pi)) p^H \end{split}$$

$$\begin{aligned} \frac{dV}{d\delta} &= \frac{\partial V}{\partial \pi} \frac{d\pi}{d\delta} + \frac{\partial V}{\partial \delta} \\ &= \begin{cases} f(\gamma^L) (1 - G(z^*)) K(p^H \delta \theta(\pi) \varphi + \pi p^H \delta \theta'(\pi) \varphi + p^H \delta \theta'(\pi) \varphi) + \\ K(1 - F(\gamma^L)) g(z^*) p^H \delta \theta'(\pi) \varphi \end{cases} \frac{d\pi}{d\delta} \\ &+ \left[ K f(\gamma^L) (1 - G(z^*)) p^H \theta(\pi) \varphi + K (1 - F(\gamma^L)) g(z^*) p^H \theta(\pi) \varphi \right] \end{aligned}$$

$$\begin{aligned} \frac{dV}{dX} &= \frac{\partial V}{\partial \pi} \frac{d\pi}{dX} + \frac{\partial V}{\partial X} \\ &= \begin{cases} f(\gamma^L) (1 - G(z^*)) K(p^H \delta \theta(\pi) \varphi + \pi p^H \delta \theta'(\pi) \varphi + p^H \delta \theta'(\pi) \varphi) + \\ K(1 - F(\gamma^L)) g(z^*) p^H \delta \theta'(\pi) \varphi \end{cases} \frac{d\pi}{dX} \\ &- [Kf(\gamma^L) (1 - G(z^*)) p^H + K(1 - F(\gamma^L)) g(z^*) p^H] \end{aligned}$$

Parts (b), (d) and (f) follow directly from parts (a), (b) and (c) for the case when  $\gamma^L = 0$ . For  $\gamma^L > 0$ , we need the additional assumption that the direct effects on output dominate the indirect employment effects on the wage rate and thus on the participation rate.