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Voting on Retirement Age: When does Postponing Retirement Find Political Support?

Berthold Ulrich Wigger
Karlsruhe Institute of Technology

Abstract

The present paper asks whether and how postponing retirement will find political support in an aging society. The paper considers a model in which the legal retirement age is the outcome of majority voting. The paper first establishes conditions that allow for a consistent application of the median voter theorem. It then demonstrates that an increase in life expectancy in fact leads to a higher legal retirement age in the political equilibrium. In contrast, an increase in the level of intergenerational redistribution leads to a lower legal retirement age.

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Berthold U. Wigger*

Karlsruhe Institute of Technology and CESifo Munich

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Preliminary and Incomplete

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The present paper asks whether and how postponing retirement will find political support in an aging society. The paper considers a model in which the legal retirement age is the outcome of majority voting. The paper first establishes conditions that allow for a consistent application of the median voter theorem. It then demonstrates that an increase in life expectancy in fact leads to a higher legal retirement age in the political equilibrium. In contrast, an increase in the level of intergenerational redistribution leads to a lower legal retirement age.

JEL classification: D72, H55, J26

Key words: Retirement age, Majority voting, Aging, Intergenerational redistribution

*Berthold U. Wigger, Karlsruhe Institute of Technology, Department of Economics, D-76128 Karlsruhe, Germany, phone: ++49-721-608-43731, wigger@kit.edu.

1 Introduction

Population aging due to an increase in life expectancy is a trend that virtually all developed economies have in common. While the trend as such is hardly alarming, its consequences for old-age security give reason to substantial personal and social disquiet. This particularly holds for societies that have established an unfunded public pension scheme in the past as population aging jeopardizes the financial sustainability of such schemes.

Basically, three strategies are available to stabilize the financing of unfunded public pensions when the population is aging. First, increasing revenue by either raising contribution rates, general taxes or debt. Second, cutting benefits. And third, postponing retirement. The first strategy has already been widely used, if not to say largely exhausted. The second strategy becomes the more unpopular, the older the population grows. Therefore, most societies are left with the third strategy.

The present paper asks whether and how postponing retirement will find political support in an aging society. The paper considers a model in which the legal retirement age is the outcome of majority voting. The paper first establishes conditions that allow for a consistent application of the median voter theorem. It then demonstrates that an increase in life expectancy in fact leads to a higher legal retirement age in the political equilibrium. In contrast, an increase in the level of intergenerational redistribution leads to a lower legal retirement age.

Related literature: Sheshinsky (1978), Crettez and Patricia Le Maitre (2002), Cremer and Pestieau (2003), Fehr, Sterkeby and Thøgersen (2003), Conde-Ruiz and Galasso (2004), De La Croix, Mahieu and Rillaers (2004), Casamatta, Cremer

and Pestieau (2005), Lacomba and Lagos (2006, 2007).

2 The Model

2.1 The Macroeconomy

The population consists of overlapping generations. Time is continuous. At each time t the oldest generation dies and a new generation is born. The population grows at the constant rate n , the interest rate on capital, r , is time-invariant, and labor productivity evolves at the constant rate g .

2.2 The Individuals

Individuals live for T periods, inelastically supply one unit of labor in the first R periods, and are retired in the remaining $T - R$ periods.

Consider an individual of age $A < T$ at time t , i.e., an individual born at time $t - A$. The remaining lifetime utility of an individual of age A at time t is

$$U_{A,t} = \int_0^{T-A} u[c_{A,t}(\theta)]e^{-\rho\theta}d\theta - \int_0^{\max\{0,R-A\}} z(A+\theta)e^{-\rho\theta}d\theta, \quad (1)$$

where u denotes instantaneous utility from consumption, $c_{A,t}(\theta)$ denotes consumption at time $t + \theta$ of an individual born at time $t - A$, $z(A + \theta)$ denotes the disutility the individual derives from labor at the age of $A + \theta$, and $\rho \geq 0$ is the individual discount rate. Instantaneous utility from consumption is assumed to be logarithmic, that is, $u(c) = \ln c$.

The budget constraint at time t of an individual born at time $t - A$ is given by

$$B_{A,t} = S_{A,t} + \int_0^{\max\{0, R-A\}} (1 - \tau)w(t + \theta)e^{-r\theta} d\theta + \int_{\max\{0, R-A\}}^{T-A} \pi(t + \theta)e^{-r\theta} d\theta, \quad (2)$$

where $w(t + \theta)$ is the wage rate at time $t + \theta$, so that $w(t + \theta) = w(t)e^{g\theta}$. Further, τ is the contribution rate of the public pension scheme which is assumed to be time-invariant, $\pi(t + \theta)$ is the public pension benefit at time $t + \theta$, and $S_{A,t}$ is the cumulated amount of savings or debt that an individual born at time $t - A$ has accumulated until time t . It is assumed that individuals are not endowed with any inherited wealth or debt at the beginning of their (economic) life, so that $S_{0,t} = 0$ for all t .

An individual of age A at time t chooses a flow of instantaneous consumption $c_{A,t}$ that maximizes her (remaining) lifetime utility $U_{A,t}$ taking the budget constraint $B_{A,t}$ into account. This leads to the following consumption function:

$$c_{A,t}(\theta) = \frac{\rho B_{A,t}}{1 - e^{-\rho(T-A)}} e^{(r-\rho)\theta}, \quad \text{for } \theta \in [0, T - A]. \quad (3)$$

2.3 The Public Pension Scheme

The public pension scheme balances at each time t , that is,

$$\int_0^R \tau w(t)e^{-n\theta} d\theta = \int_R^T \pi(t)e^{-n\theta} d\theta, \quad (4)$$

which is equivalent to

$$\pi(t) = \tau \frac{1 - e^{-nR}}{e^{-nR} - e^{-nT}} w(t). \quad (5)$$

For further reference define the contribution rate that leads to an equalization of net labor income and public pension benefits at each time t . This contribution rate, denoted as $\bar{\tau}$ in what follows, is implicitly defined by $(1 - \bar{\tau})w(t) = \pi(t)$. Substituting for $\pi(t)$ by means of (5), it follows that

$$\bar{\tau} = \frac{e^{-nR} - e^{-nT}}{1 - e^{-nT}}. \quad (6)$$

For $\tau < \bar{\tau}$, net labor income exceeds the pension benefit at each time t , whereas for $\tau > \bar{\tau}$ the opposite holds true.

3 Majority Voting

Substituting optimal consumption defined by (3) into lifetime utility defined by (1), yields indirect (remaining) lifetime utility at time t of an individual born at time $t - A$ as a function of the contribution rate τ , the demographic parameters T and n , and the cumulated amount of savings $S_{A,t}$, that is, $V_{A,t} = V_{A,t}[R, T, n, \tau, S_{A,t}]$, which is defined as

$$V_{A,t} = \int_0^{T-A} \ln \left[\frac{\rho B_{A,t}}{1 - e^{-\rho(T-A)}} + (r - \rho)\theta \right] e^{-\rho\theta} d\theta - \int_0^{\max\{0, R-A\}} z(A + \theta) e^{-\rho\theta} d\theta. \quad (7)$$

Herein, $B_{A,t}$ is determined by

$$B_{A,t} = S_{A,t} + \frac{w(t)}{r - g} \left[(1 - \tau) \left(1 - e^{-(r-g)\max\{0, R-A\}} \right) \right. \quad (8)$$

$$\left. + \tau \frac{1 - e^{-nR}}{e^{-nR} - e^{-nT}} \left(e^{-(r-g)\max\{0, R-A\}} - e^{-(r-g)(T-A)} \right) \right], \quad (9)$$

which follows from (2) and (5). In what follows, the subscript t is omitted for notational simplicity as long as confusion can be ruled out. Thus, V_A denotes

remaining indirect utility of an individual of age A at time t , B_A the remaining lifetime budget, and S_A the accumulated wealth or debt.

3.1 Single-Peakedness

In order to determine the legal retirement age that results as the outcome of majority voting, it is essential that V_A is single-peaked in R for all $A \in [0, T]$. The following results provide conditions that guarantee single-peakedness of V_A .

Lemma 1

- i. B_A is strictly increasing in R for all $R \in [A, T]$ if $\tau \leq \bar{\tau}$.*
- ii. B_A is strictly concave in R for all $R \in [A, T]$ if $\tau \leq \bar{\tau}$ and $r \geq n + g$.*

Proof: See the Appendix.

Lemma 2 *B_A is strictly increasing in R for all $R \in [0, A]$.*

Proof: See the Appendix.

Lemma 3 *V_A is strictly concave in R for all $R \in [A, T]$ if $\tau \leq \bar{\tau}$, $r \geq n + g$, $z' \geq 0$, and if ρ is sufficiently small.*

Proof: See the Appendix.

Lemma 4 *V_A is strictly increasing in R for all $R \in [0, A]$.*

Proof: See the Appendix.

Lemmata 1, 2, 3 and 4 imply the following result.

Proposition 1 *Individuals of all ages $A \in [0, T]$ have single-peaked preferences with respect to R if $\tau \leq \bar{\tau}$, $r \geq n + g$, $z' \geq 0$, and if ρ is sufficiently small.*

Example 1 *Consider the following example of the model. Let $A = 0$, $T = 50$, $w(t) = 1$, $r = .03$, $n = .01$, $g = 0$, $\tau = .2$, and $z(A + \theta) = 3$. Figure 1 plots V_A as a function of R for $\rho = .01$ and for $\rho = .1$, respectively.*

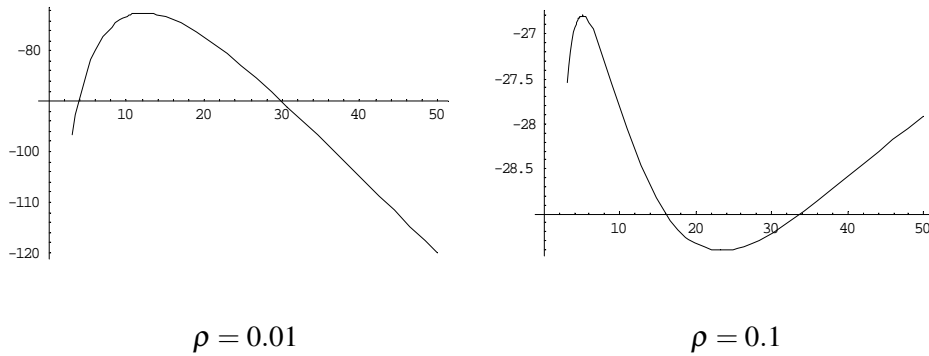


Figure 1: Example

3.2 Voting Preferences

The voting preference of an individual of age A solves

$$\max_R V_A[R, \tau, T, n, S_A]$$

which leads to

$$R_A = R[\tau, T, n, S_A].$$

The wealth or debt that an individual of age A at time t has accumulated, S_A , depends, among other variables, on the legal retirement age that has prevailed until time t . Let the history of legal retirement age be denoted by \tilde{R} , the accumulated wealth or debt of an individual of age A at time t can be written as a function $S_A = S_A(\tilde{R})$.

Definition 1 *A self-perpetuating voting preference of an individual of age A is implicitly defined by*

$$R_A = R[\tau, T, S_A(R_A)].$$

Proposition 2 *A self-perpetuating voting preference exists for individuals of all ages $A \in [0, T]$ and is uniquely determined.*

Proof: See the Appendix.

Proposition 3 *Self-perpetuating voting preferences satisfy*

$$\frac{\partial R_A}{\partial A} = \begin{cases} 0, & \text{if } A < \hat{A}, \\ 1, & \text{if } A > \hat{A}, \end{cases}$$

where $\hat{A} = R[\tau, T, S_{\hat{A}}(\hat{A})]$.

Proof: See the Appendix.

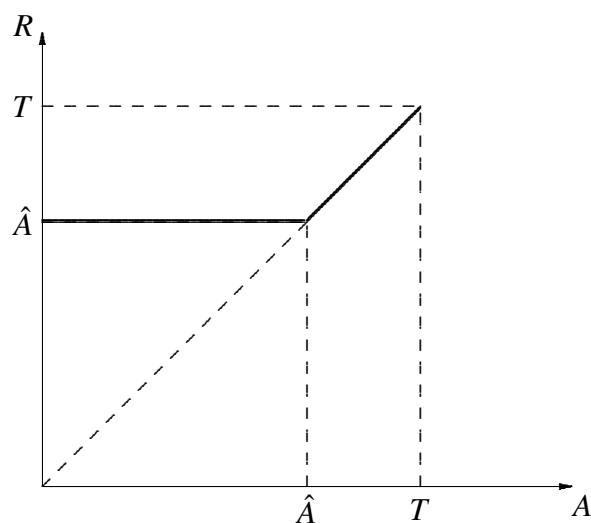


Figure 2: Self-perpetuating Voting Preferences

3.3 Voting Equilibrium

Proposition 4 *The self-perpetuating voting equilibrium retirement age is given by*

$$R_M = R[\tau, T, S_M(R_M)],$$

where M denotes the median of the age distribution.

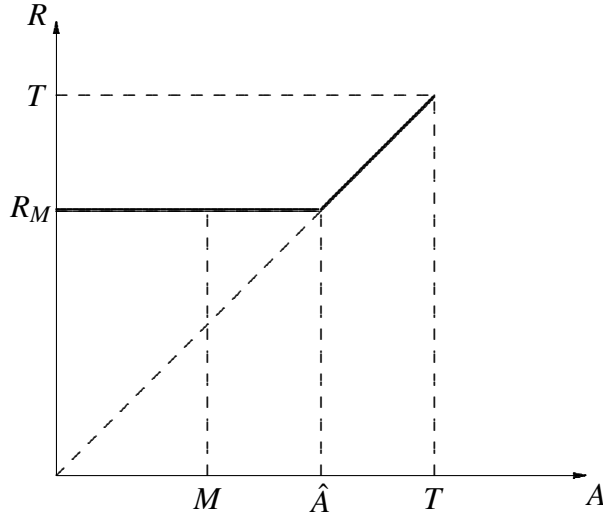


Figure 3: Voting Equilibrium

4 Aging

Proposition 5 *The higher is the lifetime T , the higher is the self-perpetuating voting equilibrium retirement age R_M .*

Proof: See the Appendix.

5 Intergenerational Redistribution

Proposition 6

- i. *Let $r > n + g$. Then, the higher is the contribution rate τ , the higher is the self-perpetuating retirement age R_M if $M < \hat{A}$, and leaves R_M unaffected if $M \geq \hat{A}$.*
- ii. *Let $r = n + g$. Then, the self-perpetuating retirement age R_M does not vary with the contribution rate τ for all $M \in [0, T]$.*

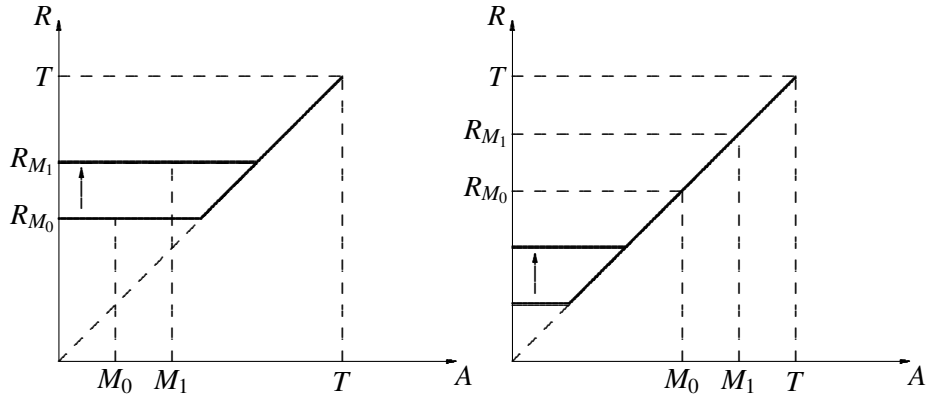


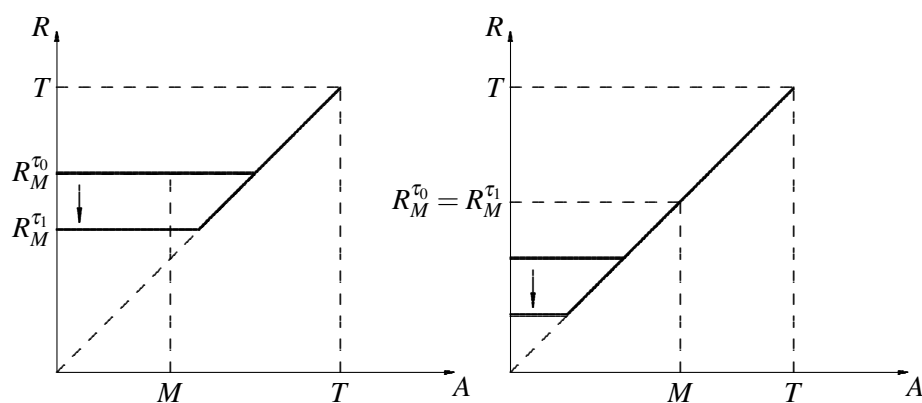
Figure 4: Increase in Lifetime T

Proof: See the Appendix.

6 Conclusion

The paper can convey both, good news and bad news. The good news is that aging in the form of an increase in life expectancy can be expected to strengthen political support for an increase in the legal retirement age. The bad news is that the higher the level of intergenerational redistribution embodied in the public pension scheme is, the lower the political support for an increase in the legal retirement age will be.

The following conclusion that can be drawn from the present analysis. If policy manages to stabilize the contribution rate, that is, limits the degree of intergenerational redistribution embodied in an unfunded public pension scheme, an increase in the legal retirement age as a response to aging will find political support. Policy should thus refrain from compensating older individuals for an increase in the legal retirement age by granting more generous pension benefits.

Figure 5: Increase in the Contribution Rate τ

Such a compensation would not only jeopardize public pension financing as such. It would also weaken the political support for an increase in the legal retirement age.

According to the OECD (2007): "...recent changes to the pension system [in Germany] have lowered target pensions and Germany is one of the few OECD countries that have moved the retirement age beyond age 65." The German reform progress lends support to the present analysis. The results of this paper in fact suggest that limiting the degree of intergenerational redistribution and increasing the legal retirement age are not just two different strategies to secure future public pension financing when the population is aging. Rather, limiting the degree of intergenerational redistribution strengthens society's willingness to accept an increase in the legal retirement age.

Appendix

Proof of Lemma 1

Proof of i. Differentiate B_A as determined by (9) with respect to R for $R \in [A, T]$.

This leads to

$$\begin{aligned}
\frac{\partial B_A}{\partial R} \frac{r-g}{w(t)} e^{-(r-g)A} = & \\
(1-\tau)(r-g)e^{-(r-g)R} + \tau n e^{-nR} \frac{e^{-(r-g)R} - e^{-(r-g)T}}{e^{-nR} - e^{-nT}} & \\
+ \tau(1-e^{-nR}) n e^{-nR} \frac{e^{-(r-g)R} - e^{-(r-g)T}}{(e^{-nR} - e^{-nT})^2} & \\
- \tau(1-e^{-nR})(r-g)e^{-(r-g)R} \frac{1}{e^{-nR} - e^{-nT}} & \tag{A1}
\end{aligned}$$

For $\tau \leq \bar{\tau}$ it follows that

$$\begin{aligned}
\frac{\partial B_A}{\partial R} \frac{r-g}{w(t)} e^{-(r-g)A} \geq & \\
(1-\bar{\tau})(r-g)e^{-(r-g)R} + \tau n e^{-nR} \frac{e^{-(r-g)R} - e^{-(r-g)T}}{e^{-nR} - e^{-nT}} & \\
+ \tau(1-e^{-nR}) n e^{-nR} \frac{e^{-(r-g)R} - e^{-(r-g)T}}{(e^{-nR} - e^{-nT})^2} & \\
- \bar{\tau}(1-e^{-nR})(r-g)e^{-(r-g)R} \frac{1}{e^{-nR} - e^{-nT}} &
\end{aligned}$$

Substituting $\bar{\tau}$ by (6), it follows that

$$\begin{aligned} \frac{\partial B_A}{\partial R} \frac{r-g}{w(t)} e^{-(r-g)A} \geq & \\ \tau n e^{-nR} \left[\frac{e^{-(r-g)R} - e^{-(r-g)T}}{e^{-nR} - e^{-nT}} \right. & \\ \left. + (1 - e^{-nR}) \frac{e^{-(r-g)R} - e^{-(r-g)T}}{(e^{-nR} - e^{-nT})^2} \right] > 0. & \end{aligned}$$

This proves i.

Proof of ii. Since B_A is C^2 in R for $R \in [A, T]$, it will be shown that $\partial^2 B_A / \partial R^2 < 0$ for all $R \in [A, T]$ if $\tau \leq \bar{\tau}$ and $r \geq n + g$. Differentiation of (A1) with respect to R yields

$$\frac{\partial^2 B_A}{\partial R^2} \frac{r-g}{w(t)} e^{-(r-g)A} = \alpha(\tau) + \beta + \gamma + \delta + \phi + \psi + \sigma + \zeta(\tau) + \mu,$$

where

$$\begin{aligned} \alpha(\tau) &= -(r-g)^2(1-\tau)e^{-(r-g)R}, \\ \beta &= -\tau n^2 e^{-nR} \frac{e^{-(r-g)R} - e^{-(r-g)T}}{e^{-nR} - e^{-nT}}, \\ \gamma &= \tau n e^{-nR} \frac{ne^{-nR}(e^{-(r-g)R} - e^{-(r-g)T}) - (r-g)e^{-(r-g)R}(e^{-nR} - e^{-nT})}{(e^{-nR} - e^{-nT})^2}, \\ \delta &= -\tau n^2 e^{-2nR} \frac{e^{-(r-g)R} - e^{-(r-g)T}}{(e^{-nR} - e^{-nT})^2}, \\ \phi &= -\tau(1 - e^{-nR})n^2 e^{-nR} \frac{e^{-(r-g)R} - e^{-(r-g)T}}{(e^{-nR} - e^{-nT})^2}, \\ \psi &= \tau(1 - e^{-nR})ne^{-nR} \times \\ &\quad \frac{2ne^{-nR}(e^{-(r-g)R} - e^{-(r-g)T}) - (r-g)e^{-(r-g)R}(e^{-nR} - e^{-nT})}{(e^{-nR} - e^{-nT})^3}, \\ \sigma &= \tau n(r-g)e^{-nR} e^{-(r-g)R} \frac{1}{e^{-nR} - e^{-nT}}, \\ \zeta(\tau) &= \tau(1 - e^{-nR})(r-g)^2 e^{-(r-g)R} \frac{1}{e^{-nR} - e^{-nT}}, \\ \mu &= \tau(1 - e^{-nR})n(r-g)e^{-nR} e^{-(r-g)R} \frac{1}{(e^{-nR} - e^{-nT})^2}. \end{aligned}$$

Now observe that $\alpha(\tau) + \zeta(\tau) \leq \alpha(\bar{\tau}) + \zeta(\bar{\tau})$ for $\tau \leq \bar{\tau}$. Substituting $\bar{\tau}$ by (6), it follows that $\alpha(\bar{\tau}) + \zeta(\bar{\tau}) = 0$, so that

$$\frac{\partial^2 B_A}{\partial R^2} \frac{r-g}{w(t)} e^{-(r-g)A} \leq \beta + \gamma + \delta + \phi + \psi + \sigma + \mu.$$

Manipulation of γ leads to

$$\gamma = \frac{\tau n^2 (r-g) e^{[-2n-(r-g)]R}}{(e^{-nR} - e^{-nT})^2} \left[\frac{1 - e^{-(r-g)(T-R)}}{r-g} - \frac{1 - e^{-n(T-R)}}{n} \right].$$

Since

$$\frac{1 - e^{-(r-g)(T-R)}}{r-g} \leq \frac{1 - e^{-n(T-R)}}{n}$$

for $r-g \geq n$ with = if $r-g = n$, it follows that $\gamma \leq 0$ for $r-g \geq n$. Adding ψ and μ leads to

$$\phi + \mu = \frac{2\tau(1-\tau)n^2(r-g)e^{[-2n-(r-g)]R}}{(e^{-nR} - e^{-nT})^3} \left[\frac{1 - e^{-(r-g)(T-R)}}{r-g} - \frac{1 - e^{-n(T-R)}}{n} \right],$$

so that, by the same argument, $\phi + \mu \leq 0$ for $r-g \geq n$. Adding δ and σ leads to

$$\delta + \sigma = \frac{\tau n^2 (r-g) e^{[-2n-(r-g)]R}}{(e^{-nR} - e^{-nT})^2} \left[\frac{1 - e^{-(r-g)(T-R)}}{r-g} - \frac{1 - e^{-n(T-R)}}{n} \right],$$

so that $\delta + \sigma \leq 0$ for $r-g \geq n$. It follows that

$$\frac{\partial^2 B_A}{\partial R^2} \frac{r-g}{w(t)} e^{-(r-g)A} \leq \beta + \phi,$$

if $\tau \leq \bar{\tau}$ and $r \geq n+g$. Since

$$\begin{aligned} \beta + \phi &= -\tau n^2 e^{-nR} \frac{e^{-(r-g)R} - e^{-(r-g)T}}{e^{-nR} - e^{-nT}} \\ &\quad - \tau(1 - e^{-nR}) n^2 e^{-nR} \frac{e^{-(r-g)R} - e^{-(r-g)T}}{(e^{-nR} - e^{-nT})^2} < 0, \end{aligned}$$

it follows that $\partial^2 B_A / \partial R^2 < 0$ if $\tau \leq \bar{\tau}$ and $r \geq n+g$. This proves ii.

Proof of Lemma 2

Differentiate B_A as determined by (9) with respect to R for $R \in [0, A)$. This leads to

$$\frac{\partial B_A}{\partial R} \frac{r-g}{w(t)} = \tau n e^{-nR} \frac{1 - e^{-(r-g)(T-A)}}{e^{-nR} - e^{-nT}} \left(1 + \frac{1 - e^{-nR}}{e^{-nR} - e^{-nT}} \right) > 0.$$

Proof of Lemma 3

Since V_A is C^2 in R for $R \in [A, T]$, it suffices to show that $\partial^2 V_A / \partial R^2 < 0$ if $\tau \leq \bar{\tau}$, $r \geq n + g$ and ρ sufficiently small. Differentiation of (7) with respect to R for $R \in [A, T]$ leads to

$$\frac{\partial V_A}{\partial R} = \frac{1}{B_A} \frac{\partial B_A}{\partial R} \frac{1}{\rho} (1 - e^{-\rho(T-A)}) - z(R) e^{-\rho(R-A)},$$

and

$$\begin{aligned} \frac{\partial^2 V_A}{\partial R^2} = & \left[-\frac{1}{B_A^2} \left(\frac{\partial B_A}{\partial R} \right)^2 + \frac{1}{B_A} \frac{\partial^2 B_A}{\partial R^2} \right] \frac{1}{\rho} (1 - e^{-\rho(T-A)}) \\ & + [\rho z(R) - z'(R)] e^{-\rho(R-A)}. \end{aligned}$$

Since $B_A > 0$ and finite, $\lim_{\rho \rightarrow 0} \frac{1}{\rho} (1 - e^{-\rho(T-A)}) = T - A$ by L'Hospital's Rule, and $\partial B_A / \partial R$ and $\partial^2 B_A / \partial R^2$ independent of ρ , it follows that

$$\lim_{\rho \rightarrow 0} \frac{\partial^2 V_A}{\partial R^2} = -(T - A) \left[-\frac{1}{B_A^2} \left(\frac{\partial B_A}{\partial R} \right)^2 - \frac{1}{B_A} \frac{\partial^2 B_A}{\partial R^2} \right] - z'(R).$$

Since $\partial^2 B_A / \partial R^2 < 0$ if $\tau \leq \bar{\tau}$ and $r \leq n + g$ by Lemma 1, it follows that

$$\lim_{\rho \rightarrow 0} \frac{\partial^2 V_A}{\partial R^2} < 0,$$

and Lemma 3 follows by a standard continuity argument.

Proof of Lemma 4

Differentiation of (7) with respect to R for $R \in [0, A)$ leads to

$$\frac{\partial V_A}{\partial R} = \frac{1}{B_A} \frac{\partial B_A}{\partial R} \frac{1}{\rho} (1 - e^{-\rho(T-A)}).$$

Since $\partial B_A / \partial R > 0$ for $R \in [0, A)$ by Lemma 2, it follows that $\partial V_A / \partial R > 0$ for $R \in [0, A)$.

A References

- Casamatta, Georges, Helmuth Cremer, and Pierre Pestieau (2005), Voting on Pensions with Endogenous Retirement Age, *International Tax and Public Finance*, 12, 7-28.
- Conde-Ruiz, J. Ignacio and Vincenzo Galasso (2004), The Macroeconomics of Early Retirement, *Journal of Public Economics*, 88, 1849-1869.
- Cremer, Helmuth and Pierre Pestieau (2003), The Double Dividend of Postponing Retirement, *International Tax and Public Finance*, 10, 419-434.
- Crettez, Bertrand and Patricia Le Maitre (2002), Optimal Age of Retirement and Population Growth, *Journal of Population Economics*, 15, 737-755.
- De La Croix, David, Géraldine Mahieu and Alexandra Rillaers (2004), How Should the Allocation of Resources Adjust to the Baby Bust?, *Journal of Public Economic Theory*, 6, 607-636.
- Fehr, Hans, Wenche Irén Sterkeby and Øystein Thøgersen (2003), Social Security Reforms and Early Retirement, *Journal of Population Economics*, 16, 345-361.
- Lacomba Juan A. and Francisco Lagos (2006), Population Aging and Legal Retirement Age, *Journal of Population Economics*, 19, 507-519.
- Lacomba Juan A. and Francisco Lagos (2007), Political Election on Legal Retirement Age, *Social Choice and Welfare*, 29, 1-17.
- OECD (2007), *Pensions at a Glance - Public Policies across OECD Countries*, 2007 Edition.
- Sheshinski, Eytan (1978), A Model of Social Security and Retirement Decisions, *Journal of Public Economics*, 10, 337-360.