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# Optimal population and education

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## *Abstract*

If raising and educating children is a private cost to households, while the increase in savings returns implied by the resulting future increase in skilled labor is a public good, fertility and educations choices made by households at competitive equilibria typically deliver a suboptimal mix of size and skills of the population. In particular, households underinvest in their children education compared to the optimal level. Imposing a tax-financed compulsory education is shown to be unlikely to implement the optimal steady state, even if the mandatory level of education is the optimal one. Nevertheless, a social security scheme that makes pension payments to a household contingent to its fertility and investment in its children's education allows to implement the first-best steady state. The social security budget is balanced period by period financing pensions through a payroll tax on the increase in children's labor income that is due to the human capital investment effort of their parents.

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Abstract. If raising and educating children is a private cost to households, while the increase in savings returns implied by the resulting future increase in skilled labor is a public good, fertility and educations choices made by households at competitive equilibria typically deliver a suboptimal mix of size and skills of the population. In particular, households underinvest in their children education compared to the optimal level. Imposing a tax-financed compulsory education is shown to be unlikely to implement the optimal steady state, even if the mandatory level of education is the optimal one. Nevertheless, a social security scheme that makes pension payments to a household contingent to its fertility and investment in its children's education allows to implement the first-best steady state. The social security budget is balanced period by period financing pensions through a payroll tax on the increase in children's labor income that is due to the human capital investment effort of their parents.

## 1. Introduction

The most obvious economic decisions that households make routinely are how much to work and whether to save or borrow and how (i.e. in what assets) in order to smooth consumption over time. But households decide also whether to reproduce, to what extent, and how many resources (time and income) to invest in their children. These decisions have huge economic consequences in the aggregate. For instance,

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everything else remaining constant, changes in the overall fertility rate propagate across the population pyramid producing variations in the dependency ratio that may reduce the output per capita for generations as a result. On the other hand, the impact on output per capita of a fall in the fertility rate may be offset by a higher investment in children's education that increases their productivity, so that a quantity-quality trade-off is faced in the choice of population.

Nevertheless, households make typically reproductive and educational decisions (possibly only beyond some compulsory elementary schooling in the case of the latter) independently of each other and disregarding their impact on the aggregate, given the negligible size of each individual household compared to the entire economy. As a consequence, the resulting fertility and allocation of resources (including those devoted to educate children) will typically be suboptimal. In the case of reproductive and education decisions this will certainly be so if the cost of raising children is a private cost to the household, while its benefit is a public good (through an increase in the amount and skills of future labor supply, which raises both the return to savings and the possible pension transfers to the current generation), since under such conditions households will try to free-ride on other households fertility and education efforts.

In this paper, I characterize in an overlapping generations setup the optimal steady state fertility and human capital investment (along with the optimal savings), and show that they cannot be a laissez-faire competitive equilibrium outcome. I show nonetheless, that a pay-as-you-go social security that makes pensions contingent to the household fertility and investment in their children's human capital —and financed by a payroll tax *on the returns of the human capital investment*, and not on the entire labor income— implements the optimal steady state as a competitive equilibrium steady state. Surprisingly, the historically most common policy used to address the problem of households underinvestment in education, namely tax-financed compulsory education, only delivers the optimal steady state under conditions unlikely to be met.

Research addressing the issue of optimal population size goes back to at least Phelps (1967), followed by the characterization in Samuelson (1975) of the optimal (exogenous) growth rate of the population[1] and a subsequent extensive literature. Most of the literature addresses the issue of population size from the viewpoint of the sustainability of pay-as-you-go pension systems, and the need to tie pension payments

[1]Deardoff (1976) and Michel and Pestieau (1993) qualified the results showing the that a solution to first-order conditions used in Samuelson (1975) could be a minimum instead of a maximum.

to individual fertility in order to make social security sustainable and implement the optimal population size has been repeatedly been put forward. In that literature fertility has often been made endogenous by introducing an explicit utility from having children (see Eckstein and Wolpin (1985) and, more recently, Abio, Mahieu and Patxot (2004), Michel and Wigniolle (2007)) as a consequence of the fact that otherwise households would not reproduce as soon as reproduction itself is costly for them.

Although many papers on the optimal population size for the sustainability of PAYG pension schemes have addressed the issue separately from that of parental investments in their children's education, there are nonetheless papers in which the two decisions have been analyzed jointly. Galor and Weil (2002) consider for instance a household quantity-quality choice of children following the model of household fertility behavior in Becker (1960). Nevertheless households are supposed to derive utility from the total income earned by its children, again to offset the fact that children (both their quantity and quality) are supposed to be costly to parents (in terms of time here, and hence of lost labor income).[2]

Schoonbroodt and Tertilt (2010) consider also parents deriving utility from both the number of children and their utility, but they address directly the problem of the misalignment of parents' incentives because of their inability appropriate the returns of the cost in making children. The authors explore the consequences of granting to parents property rights over some of theirs children income, but there is no human capital dimension in their analysis, so that the quantity-quality trade-off is overlooked.

In Cremer et al. (2006) individuals' utility depend, as in this paper, only on their own consumption. As a consequence, parents do not invest in increasing the probability of having children, which still they somehow arbitrarily have at the lowest of two exogenously given rates. Moreover, no quantity-quality trade-off is faced by the households and the only technology available is a storage technology allowing to transfer the endowment from young to old at an exogenously given fixed return. Thus any link between reproductive (and educational) choices and savings returns is again missing.

[2]As a matter of fact, the goal of Galor and Weil (2002) is rather to provide a framework with endogenous fertility and technological change able to account for the observed pattern of demographic and technological transition. As a consequence, the paper makes modeling choices leading to an economy where households do not face a savings problem, voiding of meaning any social security concern.

In this paper, I choose to make the households utility to depend only on their consumption, and not to derive utility from having children or their well-being. I do so in order to uncover more clearly (and crudely) the interest generations have in the reproduction and formation of new generations as factors of production needed for their own future well-being. I also disconnect the cost of child rearing from that of reproduction: what is costly is not reproduction per se, but taking care of kids to the point of making of them skilled labor. This hopefully brings to the forefront that households do not necessarily (and unfortunately) commit to take care of the children they produce so much as to commit to make of them educated highly productive individuals by the mere fact of having them; that is a choice, as well as that of neglecting them (as sad as it may be). Making such modeling choices I hope to make stand out, albeit in an admittedly oversimplified manner, the relevant mechanisms to take into account in designing the optimal population and education policies.

## 2. The economy

Consider an economy of 2-period lived overlapping generations of agents (households) that, when young, can supply labor and reproduce at the rate of their choice. Consumption can be produced out of labor and the amount previously produced but not consumed. Returns to scale are constant, both factors are needed for production, and capital is supposed to depreciate completely in one period. Households derive utility only from consumption[3] so that they supply labor inelastically.[4] Reproduction per se is not costly, but taking care of children (i.e. "educating" them) is.[5] On the other hand, educating children increases the effective units of labor they will supply.

In principle, households have an interest in a high supply of effective labor when they will be old, in order to get the most of the capital savings. Indeed, the more households reproduce —costlessly as long as they do not invest in educating their children—, the more labor (albeit unskilled) will be available for production next period, increasing output, but at the same time there will be more mouths to

[3]In particular, agents neither derive utility from having children nor from their children's utility.
[4]This is inessential for the main point of the paper, and will be relaxed to include a choice of labor supply in later versions of the paper.
[5]Note that "education" here encompasses everything from diapers to PhD. Without this "education" investment, (surviving) children become "plain vanilla" young agents with no special skills whatsoever.

feed, putting pressure on resources tomorrow. Alternatively, effective labor can be increased tomorrow reproducing less today but increasing the investment in education. The problem is that while reproduction is costless, education is not, so that doing so puts pressure on resources today.

Clearly, as long as the education costs are born by households while its returns cannot be appropriated by them, households will underinvest in it, hoping to free-ride on the others. But if the returns to education exceed its cost this is inefficient. What is then the optimal mix of quantity and quality of labor for the society? Can that optimal combination be the result of decentralized choices of individuals in a competitive setup? If not, is there some some policy intervention that makes of the optimal quantity and quality of population a competitive outcome? These are the questions addressed in the following sections.

The rest of the paper is organized as follows. Section 3 characterizes the steady state that a planner maximizing the representative agent's utility would choose. It turns out that such a steady state is not a competitive equilibrium steady state unless in the latter the demand for money happens to be zero (and moreover the return to capital exceeds that of labor) which is a knife-edge case (Proposition 1). Section 4 addresses the problem form the viewpoint of a representative agent operating under competitive conditions. Section 5 characterizes the resulting competitive equilibria and shows that, under the assumption of costly children only educated, the fertility rate remains undetermined while no education is provided by parents in any such equilibrium (Proposition 2). Section 6 provides the characterization of the competitive equilibrium steady state that underpins the result in Proposition 1. Section 7 shows that the planner's steady state can nonetheless be implemented as a competitive outcome with a social security paying pensions contingent to both individual fertility *and parental education effort*. The social security needs to be financed through a payroll tax *on the excess income resulting form education* (Proposition 3). At the competitive equilibrium the demand for money is zero. Section 8 shows that nonetheless money is essential, even if in zero demand at equilibrium, since removing it prevents the planner steady state to be implementable this way. Finally, Section 6 concludes.

## 3. The planner's problem

Consider a planner seeking to maximize the steady state utility of the representative household.[6] The planner chooses a steady state profile of consumptions $c_0, c_1$, per worker capital savings $k$, a population growth rate $n$,[7] and an investment in its children education or human capital $h$, solution to

$$\max_{0 \leq c_0, c_1, k, 1+n, h} u_0(c_0) + u_1(c_1)$$
$$(1+n)\big[c_0 + k + e((1+n)h)\big] + c_1 \leq F\big(k, (1+n)(1+h)\big)$$

On the right-hand side of the feasibility constraint, $F$ being a neoclassical production function, no output is produced as soon as reproduction stops and therefore labor supply evaporates. Also the term $e((1+n)h)$ in the constraint is the cost of producing $1+n$ children and transforming, for each of them, the unit of labor they are endowed with into $1+h$ efficiency units of labor. This cost is supposed to satisfy $e(0) = 0 = e'(0)$ and $e' > 0, e'' > 0$ for strictly positive levels of education investment.[8]

Since both labor and capital are assumed to be necessary for positive production, as soon as it is assumed that $u_i'(0) = +\infty$ for $i =, 1, 2$, it is guaranteed then that, at the solution, it must hold $k, 1+n > 0$, as well as $c_0, c_1 > 0$. Also, for any given $1+n, k > 0$, the output net of resources invested in education is maximized by an $h \geq 0$ such that

$$F_L(\frac{k}{1+n}, 1+h) - e'((1+n)h)(1+n) \leq 0$$

and

$$h[F_L(\frac{k}{1+n}, 1+h) - e'((1+n)h)(1+n)] = 0$$

[6] Heterogeneous households within each generation will be addressed in later versions of the paper.

[7] So that population grows between periods by a factor of $1+n$ , meaning that a household has $1+n$ households as descendants (the basic economic agent here is the household, and the mating process of individuals is thus overlooked). Therefore $1+n$ does not correspond to the commonly used Total Fertility Rate defined as the average number of children born to a woman, but to roughly half of it. Specifically, the replacement rate, which in terms of TFR is (slightly over) 2, in this setup corresponds to $1+n = 1$, i.e. $n = 0$ (in this simple setup abstraction is made of the slight excess of boys over girls in births and of factors like child mortality).

[8] Note that the cost $e((1+n)h)$ of producing any amount $1+n$ of unskilled labor, i.e. with $h = 0$, is zero, since $e((1+n)0) = 0$. Moreover, with such a cost function the solution to the first-order conditions of the household problem is not a minimum. This is not the case for the planner's problem, as usual, and the use of only the first-order conditions in this case needs to be further justified. This will be addressed later.

so that $e'(0) = 0$ implies $h > 0$ as well. Therefore, the solution to the planner is interior and, since its feasibility constraint can also be written (dividing both sides by $1 + n > 0$) as

$$c_0 + \frac{c_1}{1+n} + k + e((1+n)h) \leq F\Big(\frac{k}{1+n}, 1+h\Big)$$

a solution to the planner's problem is necessarily characterized by

(1) the FOCs

$$\begin{pmatrix} u_0'(c_0) \\ u_1'(c_1) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \frac{1}{1+n} \\ 1 - F_K\left(\frac{k}{1+n}, 1+h\right)\frac{1}{1+n} \\ -\frac{c_1}{(1+n)^2} + he'((1+n)h) + F_K\left(\frac{k}{1+n}, 1+h\right)\frac{k}{(1+n)^2} \\ (1+n)e'((1+n)h) - F_L\left(\frac{k}{1+n}, 1+h\right) \end{pmatrix}$$

for some $\lambda \geq 0$

(2) and the feasibility constraint

$$c_0 + \frac{c_1}{1+n} + k + e((1+n)h) = F\Big(\frac{k}{1+n}, 1+h\Big)$$

(since $\lambda > 0$ necessarily from any of the first two coordinates in the vectors above),

**Proposition.** *At the optimal steady state consumptions, capital savings, fertility, and education investment are all strictly positive, i.e.* $c_0, c_1, k, 1+n, h > 0$ *and satisfy*

$$\begin{aligned} \frac{u_0'(c_0)}{u_1'(c_1)} &= 1 + n = F_K\Big(\frac{k}{1+n}, 1+h\Big) = F_L\Big(\frac{k}{1+n}, 1+h\Big)e'((1+n)h)^{-1} \\ c_0 + \frac{c_1}{1+n} + k &+ e((1+n)h) = F\Big(\frac{k}{1+n}, 1+h\Big) \\ \frac{c_1}{1+n} &= F_k\Big(\frac{k}{1+n}, 1+h\Big)\frac{k}{1+n} + (1+n)he'((1+n)h) \end{aligned} \tag{P}$$

Since, as it will be shown in the next section, at a competitive equilibrium steady state $h = 0$, the next property follows straightforwardly.

**Proposition 1.** *In the standard overlapping generations*[9] *extended to include a fertility choice and an education investment in the descendants labor productivity, no competitive equilibrium steady state is the optimal steady state*

## 4. The market economy

Consider a representative agent born at $t$ choosing a profile of consumptions $c_0^t, c_1^t$, capital savings $k^t$, monetary savings $M^t$, fertility $n^t$, and children's education $h^t$ such that it solves[10]

$$\max_{0 \leq c_0, c_1, k, M, 1+n, h} u_0(c_0) + u_1(c_1)$$
$$c_0 + k + \frac{M}{p_t} + e((1+n)h) \leq w_t(1 + h^{t-1})$$
$$c_1 \leq r_{t+1}k + \frac{M}{p_{t+1}}$$

given monetary prices for the consumption good $p_t, p_{t+1}$, the real wage $w_t$, the return to capital savings $r_{t+1}$, and the increase of his own endowment in effective units of labor chosen by his parents $h^{t-1}$.

The solution to the representative agent's problem is necessarily characterized by[11]

$$h^t = 0$$
$$n^t > -1$$

[9] Populated by 2-period lived agents with only first-period inelastic labor supply.

[10] More precisely, the second period budget constraint is

$$c_1 \leq \begin{cases} r_{t+1}k + \frac{M}{p_{t+1}} & \text{if } -1 < n \\ 0 & \text{if } -1 = n \end{cases}$$

with the representative agent internalizing the impact of the fertility choice on the real value of savings: capital and monetary savings become worthless should the representative agent (i.e. everyone) choose not to have any descendants. A marginal utility of second period consumption $u_1'(c_1)$ going to infinity as $c_1$ vanishes will ensure that the representative agent chooses some $n > -1$. In effect, doing so guarantees that his second period income is not zero without necessarily costing him anything since he has always the choice of setting $h^t = 0$.

[11] Since $n = -1$ implies $c_1 = 0$ while $u_1'(0) = +\infty$ (see footnote 10).

along with the first-order conditions

$$\begin{pmatrix} u_0'(c_0^t) \\ u_1'(c_1^t) \\ 0 \\ 0 \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ \frac{1}{p_t} \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{1}{p_{t+1}} \end{pmatrix}$$

for some $\lambda^t, \mu^t > 0$, and the budget constraints

$$c_0^t + k^t + \frac{M^t}{p_t} + e((1+n^t)h^t) = w_t(1+h^{t-1})$$

$$c_1^t = r_{t+1}k^t + \frac{M^t}{p_{t+1}}$$

That is to say,[12]

$$\frac{u_0'(c_0^t)}{u_1'(c_1^t)} = \frac{p_t}{p_{t+1}} = r_{t+1}$$
$$c_0^t + k^t + \frac{M^t}{p_t} + e((1+n^t)h^t) = w_t(1+h^{t-1})$$
$$c_1^t = r_{t+1}k^t + \frac{M^t}{p_{t+1}}$$
$$h^t = 0$$
$$n^t > -1$$

[12] A solution to these conditions cannot be a minimum since the associated Lagrangian

$$\begin{aligned}\mathcal{L}(\lambda_0^t, \lambda_1^t, c_0^t, c_1^t, k^t, M^t) = u_0(c_0^t) + u_1(c_1^t) \\ - \lambda_0^t[c_0^t + k^t + \frac{M^t}{p_t} + e((1+n^t)h^t) - w_t(1+h^{t-1})] \\ - \lambda_1^t[c_1^t - r_{t+1}k^t - \frac{M^t}{p_{t+1}}]\end{aligned}$$

has a Hessian

$$\begin{pmatrix} 0 & 0 & -1 & 0 & -1 & -\frac{1}{p_t} \\ 0 & 0 & 0 & -1 & r_{t+1} & \frac{1}{p_{t+1}} \\ -1 & 0 & u_0''(c_0^t) & 0 & 0 & 0 \\ 0 & -1 & 0 & u_1''(c_1^t) & 0 & 0 \\ -1 & r_{t+1} & 0 & 0 & 0 & 0 \\ -\frac{1}{p_t} & \frac{1}{p_{t+1}} & 0 & 0 & 0 & 0 \end{pmatrix}$$

whose principal minors of order 5 and 6 satisfy respectively

$$\begin{aligned}(-1)^5 H_5(-6) &= -(u_0'' + r_{t+1}u_1'') > 0 \\ (-1)^5 H_5(-5) &= -(\frac{1}{p_t^2}u_0'' + \frac{1}{p_{t+1}^2}u_1'') > 0 \\ (-1)^5 H_5(-4) &= 0 \geq 0 \\ (-1)^5 H_5(-3) &= 0 \geq 0 \\ (-1)^6 H_6 &= 0 \geq 0\end{aligned}$$

(where $H_5(-i)$ stands for the principal minor of order 5 without the $i$-th row and column) which implies that a solution to the first-order conditions is not a minimum since the second order necessary condition for a minimum is not satisfied. Moreover, the second order necessary conditions for a maximum are satisfied and, although the second order sufficient conditions are not, the existence itself of a maximum is guaranteed by the compactness of the budget set and the continuity of preferences.

The output per worker at $t$ is given by

$$y_t = F\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1+h^{t-1}\Big)$$

where $n^{t-1}$ is the rate of growth of the population chosen by generation $t-1$, so that

$$\begin{aligned} w_t &= F_L\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1+h^{t-1}\Big) \\ r_{t+1} &= F_K\Big(\frac{k^t}{1+n^t}, 1+h^t\Big) \end{aligned}$$

## 5. Competitive equilibria

A competitve equilibrium is characterized by a sequence $\{c_0^t, c_1^t, k^t, M^t, h^t, p_t\}_{t\in\mathbb{Z}}$ such that, for all $t$,

$$\begin{aligned} \frac{u_0'(c_0^t)}{u_1'(c_1^t)} = \frac{p_t}{p_{t+1}} &= F_K\Big(\frac{k^t}{1+n^t}, 1+h^t\Big) \\ c_0^t + k^t + \frac{M^t}{p_t} + e((1+n^t)h^t) &= F_L\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1+h^{t-1}\Big)(1+h^{t-1}) \\ c_1^t &= F_K\Big(\frac{k^t}{1+n^t}, 1+h^t\Big)k^t + \frac{M^t}{p_{t+1}} \\ \frac{M^t}{M^{t+1}} &= 1+n^t \end{aligned}$$

(where the last condition is equivalent to the feasibility of the allocation of resources) along with

$$\begin{aligned} h^t &= 0 \\ n^t &> -1 \end{aligned}$$

that is to say

$$\begin{aligned}
\frac{u_0'(c_0^t)}{u_1'(c_1^t)} &= \frac{p_t}{p_{t+1}} = F_K\Big(\frac{k^t}{1+n^t},1\Big) \\
c_0^t + k^t + \frac{M^t}{p_t} &= F_L\Big(\frac{k^{t-1}}{1+n^{t-1}},1\Big) \\
c_1^t &= F_K\Big(\frac{k^t}{1+n^t},1\Big)k^t + \frac{M^t}{p_{t+1}} \\
\frac{M^t}{M^{t+1}} &= 1+n^t \\
h^t &= 0
\end{aligned}$$

with

$$n^t > -1$$

but undetermined.

**Proposition 2.** *In the standard overlapping generations*[13] *extended to include a fertility choice and an education investment in the descendants labor productivity, at a competitive equilibrium the fertility rate is undetermined and the investment in children's skills is zero.*

The result stated in the previous proposition about parents not investing at all on children's education is not surprising from a theoretical viewpoint, once one notices that educating children is a private cost while its returns is a public good. But in case it seems nonetheless counterfactual, let us remind that the model leaves out of the picture the compulsory basic education mandated and financed through taxes by the state. Not that long ago in historical terms in the developed world, and still today in some underdeveloped countries, that imposition (or the ability of enforce it) did not exist, and in that context the claim does not look counterfactual.

Approaching the model further to reality will require to include non-compulsory tax-financed secondary and European-style tax-financed higher education. In this case the result is likely to leave the investment beyond the compulsory level undetermined, since while the returns to education do not go to parents, neither do the costs (at least directly), so that the misalignment of incentives is not there anymore. American-style private higher education, insofar makes the children bear both the returns and the costs to their education investment (through loans to that end)

[13]Populated by 2-period lived agents with only first-period inelastic labor supply.

solves the incentives problem as well, albeit in the opposite direction (i.e. roughly making the agent receiving the returns pay for the costs, instead of freeing from the (direct) costs the agent that is not receiving the returns, as European-style higher education does), and including it changes qualitatively the model. These extensions are addressed below.

## 6. Competitive equilibrium steady state

A competitive equilibrium steady state is characterized by the following conditions.

**Definition.** *Competitive equilibrium steady state:* $\{c_0, c_1, k, m, n, h\}$ *such that*

$$\begin{aligned}
\frac{u_0'(c_0)}{u_1'(c_1)} &= 1 + n = F_K\Big(\frac{k}{1+n}, 1+h\Big) \\
c_0 + k + m + e((1+n)h) &= F_L\Big(\frac{k}{1+n}, 1+h\Big)(1+h) \\
\frac{c_1}{1+n} &= F_K\Big(\frac{k}{1+n}, 1+h\Big)\frac{k}{1+n} + m \\
h &= 0 \\
n &> -1
\end{aligned}$$

That is to say, since at a competittive equilibrium steady state $h = 0$, it is characterized by $\{c_0, c_1, k, m, n\}$ such that

$$\begin{aligned}
\frac{u_0'(c_0)}{u_1'(c_1)} &= 1 + n = F_K\Big(\frac{k}{1+n}, 1\Big) \\
c_0 + k + m &= F_L\Big(\frac{k}{1+n}, 1\Big) \\
\frac{c_1}{1+n} &= F_K\Big(\frac{k}{1+n}, 1\Big)\frac{k}{1+n} + m \\
n &> -1
\end{aligned} \tag{4}$$

and implementing the optimal steady state therefore requires some intervention.

Historically, compulsory basic education financed through taxes has been the most usual policy to address the problem of underinvestment in education by individual households. Alternatively, I consider also a policy that makes households pensions contingent to their choices on fertility and children's education. It turns out that the conditions under which such a policy implements the optimal steady state are much less stringent than those required for a tax-financed compulsory education to attain the same goal. In what follows we consider first what does the introduction of public education change, and then what fertility and education contingent pensions change.

## 7. The market economy with tax-funded compulsory education

Consider a representative agent born at $t$ choosing a profile of consumptions $c_0^t, c_1^t$, capital savings $k^t$, monetary savings $M^t$, fertility $n^t$, and children's education $h^t$ such that it solves

$$\max_{0 \leq c_0, c_1, k, M, 1+n, h} u_0(c_0) + u_1(c_1)$$

$$c_0 + k + \frac{M}{p_t} + e((1+n)h) \leq w_t(1 + h^{t-1} + h_t^p) - T_t$$

$$c_1 \leq r_{t+1}k + \frac{M}{p_{t+1}}$$

given monetary prices for the consumption good $p_t, p_{t+1}$, the real wage $w_t$, the return to capital savings $r_{t+1}$, the increase of his own endowment in effective units of labor chosen by his parents $h^{t-1}$ and that imposed by the government and financed through a lump-sum tax $T_t$.[14]

The solution to the representative agent's problem is characterized by

$$h^t = 0$$

$$n^t > -1$$

[14] Again in the second budget constraint, which is more precisely

$$c_1 \leq \begin{cases} r_{t+1}k + \frac{M}{p_{t+1}} & \text{if } -1 < n \\ 0 & \text{if } -1 = n \end{cases}$$

the representative agent internalizes the impact of the fertility choice on the real value of savings. A marginal utility of second period consumption $u_1'(c_1)$ going to infinity as $c_1$ vanishes ensures again that the representative agent chooses some $n > -1$.

(since $n = -1$ implies $c_1 = 0$ while $u_1'(0) = +\infty$) along with the first-order conditions

$$\begin{pmatrix} u_0'(c_0^t) \\ u_1'(c_1^t) \\ 0 \\ 0 \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ \frac{1}{p_t} \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{1}{p_{t+1}} \end{pmatrix}$$

for some $\lambda^t, \mu^t > 0$, and

$$c_0^t + k^t + \frac{M^t}{p_t} + e((1+n^t)h^t) = w_t(1 + h^{t-1} + h_t^p) - T_t$$
$$c_1^t = r_{t+1}k^t + \frac{M^t}{p_{t+1}}$$

That is to say,

$$\frac{u_0'(c_0^t)}{u_1'(c_1^t)} = \frac{p_t}{p_{t+1}} = r_{t+1}$$
$$c_0^t + k^t + \frac{M^t}{p_t} = w_t(1 + h^{t-1} + h_t^p) - T_t$$
$$c_1^t = r_{t+1}k^t + \frac{M^t}{p_{t+1}}$$
$$h^t = 0$$
$$n^t > -1$$

The government budget will be balanced period-by-period iff

$$T_t = e((1+n^t)h_{t+1}^p)$$

The output per worker at $t$ is given by

$$y_t = F\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1 + h_t^p\Big)$$

where $n^{t-1}$ is the rate of growth of the population chosen by generation $t-1$, so that

$$w_t = F_L\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1 + h_t^p\Big)$$
$$r_{t+1} = F_K\Big(\frac{k^t}{1+n^t}, 1 + h_{t+1}^p\Big)$$

## 8. Competitive equilibria with tax-financed compulsory education

Given a compulsory education policy $\{h_t^p\}_t$ and the taxes allowing to finance it $\{T_t\}_t$, a competitive equilibrium is characterized by a sequence $\{c_0^t, c_1^t, k^t, M^t, h^t, p_t\}_{t\in\mathbb{Z}}$ such that, for all $t$,

$$h^t = 0$$

$$n^t > -1$$

and

$$\frac{u_0'(c_0^t)}{u_1'(c_1^t)} = \frac{p_t}{p_{t+1}} = F_K\Big(\frac{k^t}{1+n^t}, 1+h_{t+1}^p\Big)$$

$$c_0^t + k^t + \frac{M^t}{p_t} = F_L\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1+h_t^p\Big)(1+h_t^p) - T_t$$

$$c_1^t = F_K\Big(\frac{k^t}{1+n^t}, 1+h_t^p\Big)k^t + \frac{M^t}{p_{t+1}}$$

$$\frac{M^t}{M^{t+1}} = 1+n^t$$

where the last condition is equivalent to the feasibility of the allocation of resources. if

$$T_t = e((1+n^{t-1})h_{t+1}^p)$$

A competitive equilibrium steady state under an education policy $h^p$ and the taxes $T = e((1+n)h^p)$ paying for it, is then characterized by $h = 0$ and $\{c_0, c_1, k, m, n\}$ such that

$$\frac{u_0'(c_0)}{u_1'(c_1)} = 1+n = F_K\Big(\frac{k}{1+n}, 1+h^p\Big)$$

$$c_0 + k + m = F_L\Big(\frac{k}{1+n}, 1+h^p\Big)(1+h^p) - e((1+n)h^p)$$

$$\frac{c_1}{1+n} = F_K\Big(\frac{k}{1+n}, 1+h^p\Big)\frac{k}{1+n} + m$$

with

$$n > -1$$

but undetermined.

Since the planner's steady state is necessarily characterized by

$$\begin{aligned}\frac{u_0'(c_0)}{u_1'(c_1)} &= 1+n = F_K\Big(\frac{k}{1+n}, 1+h\Big) = F_L\Big(\frac{k}{1+n}, 1+h\Big)e'((1+n)h)^{-1} \\ c_0 + \frac{c_1}{1+n} &+ k + e((1+n)h) = F\Big(\frac{k}{1+n}, 1+h\Big) \\ \frac{c_1}{1+n} &= F_k\Big(\frac{k}{1+n}, 1+h\Big)\frac{k}{1+n} + (1+n)he'((1+n)h)\end{aligned} \tag{P}$$

then the planner's steady state can be decentralized as a competitive equilibrium steady state if under the policy $h^p = h$ the choice $c_0, c_1, k, m, 1+n$ is such that

$$F_K\Big(\frac{k}{1+n}, 1+h\Big) = F_L\Big(\frac{k}{1+n}, 1+h\Big)e'((1+n)h)^{-1}$$

$$m = (1+n)he'((1+n)h)$$

**Proposition.** *In the standard overlapping generations*[15] *extended to include a fertility choice and an education investment in the descendants labor productivity, a competitive equilibrium steady state* $\{c_0, c_1, k, m, n\}$ *under a compulsory education* $h^p$ *financed by lump-sum tax on labor income delivers the optimal steady state savings, fertility, and educational investment if, and only if, it satisfies*

$$F_K\Big(\frac{k}{1+n}, 1+h^p\Big) = F_L\Big(\frac{k}{1+n}, 1+h^p\Big)e'((1+n)h)^{-1}$$

$$m = (1+n)h^p e((1+n)h^p)$$

Note that, while the characterization of a competitive equilibrium steady state has one degree of freedom (it leaves the fertility undetermined), the conditions for it to implement the optimal steady state impose on it two additional equations, with the risk of overdeterminacy.

## 9. Fertility-education contingent pensions

Consider instead an overlapping generations economy with a representative agent

[15] Populated by 2-period lived agents with only first-period inelastic labor supply.

born at $t$ choosing a solution $(c_0^t, c_1^t, k^t, M^t, n^t, h^t)$ to the problem[16]

$$\max_{0\le c_0,c_1,k,M,1+n,h} u_0(c_0)+u_1(c_1)$$

$$c_0+k+\frac{M}{p_t}+e((1+n)h)\le w_t(1+(1-\tau)h^{t-1})$$
$$c_1\le r_{t+1}k+\frac{M}{p_{t+1}}+\tau w_{t+1}(1+n)h$$

given $p_t, p_{t+1}, w_t, r_{t+1}$,$h^{t-1}$ and $\tau$.

The solution to the problem of agent $t$ is interior under the assumptions made,[17] characterized by the first-order conditions

$$\begin{pmatrix} u_0'(c_0^t) \\ u_1'(c_1^t) \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ \frac{1}{p_t} \\ h^t e'((1+n^t)h^t) \\ (1+n^t)e'((1+n^t)h^t) \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{1}{p_{t+1}} \\ -\tau w_{t+1}h^t \\ -\tau w_{t+1}(1+n^t) \end{pmatrix}$$

for some $\lambda^t, \mu^t > 0$, along with

$$c_0^t+k^t+\frac{M^t}{p_t}+e((1+n^t)h^t)=w_t(1+(1-\tau)h^{t-1})$$
$$c_1^t=r_{t+1}k^t+\frac{M^t}{p_{t+1}}+\tau w_{t+1}(1+n^t)h^t$$

[16]Once more the second-period budget constraint is, more precisely,

$$c_1\le\begin{cases} r_{t+1}k+\frac{M}{p_{t+1}}+\tau w_{t+1}(1+n)h & \text{if } -1<n \\ 0 & \text{if } -1=n \end{cases}$$

[17]As with the planner, $e'(0)=0$ prevents $h^t$ from being 0, since the the maximum present value of the pension net of education investment, for any $n$, is characterized by

$$\frac{p_{t+1}}{p_t}\tau w_{t+1}(1+n)-e'((1+n)h)(1+n)\le 0$$

and

$$h\Big[\frac{p_{t+1}}{p_t}\tau w_{t+1}(1+n)-e'((1+n)h)(1+n)\Big]=0.$$

That is to say, the agent would choose $c_0^t, c_1^t, k^t, M^t, n^t, h^t$ such that

$$\frac{u_0'(c_0^t)}{u_1'(c_1^t)} = \frac{p_t}{p_{t+1}} = r_{t+1} = \tau w_{t+1} e'((1+n^t)h^t)^{-1}$$

$$c_0^t + k^t + \frac{M^t}{p_t} + e((1+n^t)h^t) = w_t(1+(1-\tau)h^{t-1})$$

$$c_1^t = r_{t+1}k^t + \frac{M^t}{p_{t+1}} + \tau w_{t+1}(1+n^t)h^t$$

The output per worker at $t$ is given by

$$y_t = F\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1+h^{t-1}\Big)$$

where $n^{t-1}$ is the rate of growth of the population chosen by generation $t-1$, so that

$$w_t = F_L\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1+h^{t-1}\Big)$$

$$r_{t+1} = F_K\Big(\frac{k^t}{1+n^t}, 1+h^t\Big)$$

For a given $\tau$, a competitive equilibrium is characterized by $\{c_0^t, c_1^t, k^t, M^t, n^t, h^t, p_t\}_{t\in\mathbb{Z}}$ such that, for all $t$,

$$\frac{u_0'(c_0^t)}{u_1'(c_1^t)} = \frac{p_t}{p_{t+1}} = F_K\Big(\frac{k^t}{1+n^t}, 1+h^t\Big) = \tau F_L\Big(\frac{k^t}{1+n^t}, 1+h^t\Big)e'((1+n^t)h^t)^{-1}$$

$$c_0^t + k^t + \frac{M^t}{p_t} + e((1+n^t)h^t) = F_L\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1+h^{t-1}\Big)(1+(1-\tau)h^{t-1})$$

$$c_1^t = F_K\Big(\frac{k^t}{1+n^t}, 1+h^t\Big)k^t + \frac{M^t}{p_{t+1}} + \tau F_L\Big(\frac{k^t}{1+n^t}, 1+h^t\Big)(1+n^t)h^t$$

$$\frac{M^t}{M^{t+1}} = 1+n^t$$

—where the last condition is equivalent to the feasibility of the allocation of resources—

For a given $\tau$, a competitive equilibrium steady state is characterized by $\{c_0, c_1, k, m, n, h\}$ such that

$$\begin{aligned}\frac{u_0'(c_0)}{u_1'(c_1)} &= 1+n = F_K\Big(\frac{k}{1+n}, 1+h\Big) = \tau F_L\Big(\frac{k}{1+n}, 1+h\Big)e'((1+n)h)^{-1} \\ c_0 + k + m + e((1+n)h) &= F_L\Big(\frac{k}{1+n}, 1+h\Big)(1+(1-\tau)h) \\ \frac{c_1}{1+n} &= F_K\Big(\frac{k}{1+n}, 1+h\Big)\frac{k}{1+n} + m + \tau F_L\Big(\frac{k}{1+n}, 1+h\Big)h\end{aligned}$$

Note that it coincides with the planner's if $\tau = 1$ and $m = 0$, since then the system becomes

$$\begin{aligned}\frac{u_0'(c_0)}{u_1'(c_1)} &= 1+n = F_K\Big(\frac{k}{1+n}, 1+h\Big) = F_L\Big(\frac{k}{1+n}, 1+h\Big)e'((1+n)h)^{-1} \\ c_0 + k + e((1+n)h) &= F_L\Big(\frac{k}{1+n}, 1+h\Big) \\ \frac{c_1}{1+n} &= F_K\Big(\frac{k}{1+n}, 1+h\Big)\frac{k}{1+n} + F_L\Big(\frac{k}{1+n}, 1+h\Big)h\end{aligned}$$

which is equivalent to the planner's steady steady state system

$$\begin{aligned}\frac{u_0'(c_0)}{u_1'(c_1)} &= 1+n = F_K\Big(\frac{k}{1+n}, 1+h\Big) = F_L\Big(\frac{k}{1+n}, 1+h\Big)e'((1+n)h)^{-1} \\ c_0 + \frac{c_1}{1+n} + k + e((1+n)h) &= F\Big(\frac{k}{1+n}, 1+h\Big) \\ \frac{c_1}{1+n} &= F_k\Big(\frac{k}{1+n}, 1+h\Big)\frac{k}{1+n} + (1+n)he'((1+n)h)\end{aligned}$$

if

$$F_L\Big(\frac{k}{1+n}, 1+h\Big)h = (1+n)he'((1+n)h).$$

**Proposition 3.** *In the standard overlapping generations*[18] *extended to include a fertility choice and an education investment in the descendants labor productivity, the planner's steady state is a monetary competitive equilibrium steady state with zero demand for money, if the increase in labor income resulting from parental investment in children and their education is completely taxed away and transferred to the parents.*

[18]Populated by 2-period lived agents with only first-period inelastic labor supply.

Note that all the intergenerational transfers needed to implement the planner's steady state are realized transferring as fertility-education contingent pension to agent $t$ at $t+1$ the amount $\tau w_{t+1}(1+n^t)h^t$ raised at $t+1$ by the payroll tax paid by agent $t+1$ on the increase in labor income coming from the education investment made by parents $t$. As a consequence, there is no need to use money to complement such transfers, from which the demand for money is zero.

Nevertheless, the presence of money is essential, even if it is not demanded at equilibrium, to guarantee that the rate at which agents can transfer wealth across periods —either saving in capital or through the fertility-education contingent pension scheme— coincides with the population growth factor implied by their fertility choice. This becomes apparent computing the competitive equilibrium steady state of the same economy without money. This is done in the next section.

## 8. Money is needed, even if not demanded

Consider an overlapping generations economy like the previous one, with the only difference that agents can only save in terms of capital. The same policy of fertility-education contingent pensions financed by a payroll tax on the increase of labor income due to educations investments is in place.

An interior solution to the problem of agent $t$ is characterized by the first-order conditions

$$\begin{pmatrix} u_0'(c_0^t) \\ u_1'(c_1^t) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ h^t e'((1+n^t)h^t) \\ (1+n^t)e'((1+n^t)h^t) \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\tau w_{t+1} h^t \\ -\tau w_{t+1}(1+n^t) \end{pmatrix}$$

for some $\lambda^t, \mu^t > 0$, and

$$c_0^t + k^t + e((1+n^t)h^t) = w_t(1+(1-\tau)h^{t-1})$$
$$c_1^t = r_{t+1}k^t + \tau w_{t+1}(1+n^t)h^t$$

That is to say, the agent would choose $c_0^t, c_1^t, k^t, n^t, h^t$ such that

$$\frac{u_0'(c_0^t)}{u_1'(c_1^t)} = r_{t+1} = \tau w_{t+1} e'((1+n^t)h^t)^{-1}$$
$$c_0^t + k^t + e((1+n^t)h^t) = w_t(1+(1-\tau)h^{t-1})$$
$$c_1^t = r_{t+1}k^t + \tau w_{t+1}(1+n^t)h^t$$

The output per worker at $t$ is given by

$$y_t = F\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1+h^{t-1}\Big)$$

where $n^{t-1}$ is the rate of growth of the population chosen by generation $t-1$, so that

$$\begin{aligned} w_t =& F_L\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1+h^{t-1}\Big) \\ r_{t+1} =& F_K\Big(\frac{k^t}{1+n^t}, 1+h^t\Big) \end{aligned}$$

For a given $\tau$, a competitive equilibrium is characterized by $\{c_0^t, c_1^t, k^t, n^t, h^t, p_t\}_{t\in\mathbb{Z}}$ such that, for all $t$,

$$\begin{aligned} \frac{u_0'(c_0^t)}{u_1'(c_1^t)} &= F_K\Big(\frac{k^t}{1+n^t}, 1+h^t\Big) = \tau F_L\Big(\frac{k^t}{1+n^t}, 1+h^t\Big)e'((1+n^t)h^t)^{-1} \\ c_0^t + k^t + e((1+n^t)h^t) &= F_L\Big(\frac{k^{t-1}}{1+n^{t-1}}, 1+h^{t-1}\Big)(1+(1-\tau)h^{t-1}) \\ c_1^t &= F_K\Big(\frac{k^t}{1+n^t}, 1+h^t\Big)k^t + \tau F_L\Big(\frac{k^t}{1+n^t}, 1+h^t\Big)(1+n^t)h^t \end{aligned}$$

—the feasibility of the allocation of resources is guaranteed by the budget constraints.

For a given $\tau$, a competitive equilibrium steady state is characterized by $\{c_0, c_1, k, n, h\}$ such that

$$\begin{aligned} \frac{u_0'(c_0)}{u_1'(c_1)} &= F_K\Big(\frac{k}{1+n}, 1+h\Big) = \tau F_L\Big(\frac{k}{1+n}, 1+h\Big)e'((1+n)h)^{-1} \\ c_0 + k + e((1+n)h) &= F_L\Big(\frac{k}{1+n}, 1+h\Big)(1+(1-\tau)h) \\ \frac{c_1}{1+n} &= F_K\Big(\frac{k}{1+n}, 1+h\Big)\frac{k}{1+n} + \tau F_L\Big(\frac{k}{1+n}, 1+h\Big)h \end{aligned}$$

which is *not* equivalent, even if $\tau = 1$, to

$$\begin{aligned} \frac{u_0'(c_0)}{u_1'(c_1)} &= 1+n = F_K\Big(\frac{k}{1+n}, 1+h\Big) = F_L\Big(\frac{k}{1+n}, 1+h\Big)e'((1+n)h)^{-1} \\ c_0 + \frac{c_1}{1+n} + k + e((1+n)h) &= F\Big(\frac{k}{1+n}, 1+h\Big) \\ \frac{c_1}{1+n} &= F_k\Big(\frac{k}{1+n}, 1+h\Big)\frac{k}{1+n} + (1+n)he((1+n)h) \end{aligned}$$

i.e. the planner's steady steady state system since, in the absence of money, nothing guarantees that the productivity of capital is the growth factor of the population implied by the agents' fertility choice.

## 9. Concluding remarks

The model above shows that the decisions on fertility and education taken by households in a decentralized way typically lead to a suboptimal steady state. The reason for that is that producing future skilled labor is a private cost on the returns of which other households can free-ride.

While the problem has been recognized in the literature, two main innovations are introduced in the approach followed in this paper. Firstly, rather than wondering what is the optimal population size households want to produce, I draw the attention to the fact that it is not just the quantity but also the quality of the population that matters for the future returns to capital savings. Thus I let the agents choose both their fertility and how much they educate their children. Secondly, having the previous literature unnecessarily intertwined the (low) costs of producing kids with the (high) costs of producing skilled labor out of kids, I disentangle the two and as a consequence need not relay on altruism or children in the utility function to avoid the population collapsing.

The main results in the paper are, on the one hand, that the competitive equilibria steady state are typically suboptimal, and on the other hand, that the optimal steady state can nonetheless be implemented as a competitive equilibrium outcome if it is put in place a social security whose pension payments are made to depend on the households' choices on both fertility and education, and that is financed by a payroll tax on the increase in labor income of the children. Moreover it is shown that the common policy of a tax-financed compulsory education is unlikely to implement the optimal steady state, even if the mandatory education is set to be the optimal one.

The analysis can and should be extended in many directions, some of which have been mentioned throughout the paper, but the message stemming from the simple setup considered here should not change much as a result.

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