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Department of Economics 415 Calhoun Hall Vanderbilt University Nashville, TN 37235 USA (615) 343-8495 (fax)

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A theoretical analysis of the Fairtrade certification program

Andrea S Podhorsky
York University

Abstract

This paper develops a model to study how the Fairtrade certification program can be best utilized to transfer income to producers of primary commodities in developing countries. Prices received by farmers are often low due to their disadvantaged position vis-a-vis large intermediary buyers between local and world markets. The Fairtrade program establishes a minimum price and alternate distribution channels that bypass intermediaries in the supply chain. In the model, it is assumed that consumers are willing to pay a premium for Fairtrade products. Firms in the final goods market voluntarily choose to certify their products according to whether this premium exceeds their certification cost. In the primary market, farmers sell their commodity to an intermediary that has monopsony power and the intermediary sells it to final goods producers. Farmers that are selected into the Fairtrade program, however, sell their product to final goods producers through a nonprofit Fairtrade intermediary. The model shows that the price of the conventional commodity for final goods producers is decreasing in the Fairtrade price floor. Hence the program works to squeeze the intermediary's monopsony profits and consumers benefit from the lower cost of uncertified goods in addition to the greater ethical quality of certified goods. The model is used to characterize the Pareto optimal price floor and the price floor that maximizes the total producer surplus of selected and unselected farmers.

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A Theoretical Analysis of the Fairtrade Certification Program (preliminary and incomplete)

Andrea Podhorsky
York University

April 2011

Abstract

In this paper I develop a model to study how the Fairtrade certification program can be best utilized to transfer income to producers of primary commodities in developing countries. Low world commodity prices in tandem with a disadvantaged position vis-a-vis large intermediary buyers between local and world markets often result in prices received by farmers that are insufficient to cover their costs of production and living. The fair-trade labeling program establishes a minimum guaranteed price (a price floor) and alternate distribution channels that bypass intermediaries in the supply chain. In the model, it is assumed that consumers value the ethical quality of products that have been certified under the Fairtrade program and are willing to pay a premium for Fairtrade products. Firms in the final goods market voluntarily choose to certify their products according to whether the premium for Fairtrade products exceeds their certification cost. In the primary market, farmers sell their commodity to an intermediary that has monopsony power, and the intermediary processes the commodity and sells it to final goods producers. Farmers that are selected into the Fairtrade program, however, sell their product to final goods producers through a nonprofit Fairtrade intermediary and receive the minimum price established by the Fairtrade program. I use the model to show that the price of the conventional commodity for final goods producers is decreasing in the Fairtrade price floor. Consequently, the Fairtrade program works to squeeze the intermediary's monopsony profits and consumers benefit from the lower cost of uncertified conventional goods, in addition to the greater ethical quality of certified goods. Also, for initial increases in the Fairtrade price floor, the wage received by farmers that have not been selected into the Fairtrade program is increasing. For further increases in the Fairtrade price floor, however, there is eventually a trade-off between the returns to selected farmers and those that are not selected into the program. The model is used to characterize the Pareto optimal Fairtrade price floor as well as the Fairtrade price floor that maximizes the total producer surplus of selected and unselected farmers.

1 Introduction

2 The Model

2.1 Final Goods Market

Maximizing

$$U_i = \left[\int_{\upsilon\in V} \lambda(r(\upsilon))^{\rho} c_i(\upsilon)^{\rho} d\upsilon\right]^{\frac{1}{\rho}} \tag{1}$$

where, for a given r_f,

$$r(\upsilon) = \begin{cases} r_f, & \text{if variety } \upsilon \text{ is labeled} \\ \underline{r}, & \text{if variety } \upsilon \text{ is unlabeled} \end{cases}$$

subject to

$$\int_{\upsilon\in V} p\left(\upsilon\right) c_i\left(\upsilon\right) d\upsilon \leq Y_i$$

yields

$$c_i\left(p(\upsilon)\right) = \frac{p\left(\upsilon\right)^{-1} \lambda_p(r(\upsilon)) Y_i}{P^{1-\sigma}} \tag{2}$$

where $\sigma = \frac{1}{1-\rho}$,

$$P = \left[\int_{\upsilon\in V} \lambda_p(r(\upsilon) d\upsilon\right]^{\frac{1}{1-\sigma}} \tag{3}$$

is the real price index and

$$\lambda_p(r(\upsilon) = \left[\frac{\lambda(r(\upsilon))}{p\left(\upsilon\right)}\right]^{\sigma-1}.$$

For each variety, consumer i weights the utility he receives from consuming the quantity $c_i(\upsilon)$ with the subjective function $\lambda(r(\upsilon))$ that serves to characterize how he privately values the ethical quality of variety υ. I assume that λ is continuously differentiable and increasing at a rate that is diminishing in ethical quality so that $\lambda'(r_f) > 0$ and $\frac{\lambda'(r_f)(1+r_f)}{\lambda(r_f)} \to 0$ as $r_f \to \infty$ (ie. $\lambda'(r_f) \to 0$ for finite but large r_f.)[1] Also, consumers value varieties that possess a minimal level of ethical quality so that $\lambda\left(\underline{r}\right) > 0$.

[1] As we'll see, these assumptions ensure that consumption of labeled goods is concave in the Fairtrade price floor r_f.

Final goods producers (firms) purchase the processed commodity from an intermediary in the commodity market, package and brand it, and then sell to final consumers. Firms incur variable costs that include labor and raw materials. One unit of labor, at cost w, and one unit of raw materials, at cost p_m, produces one unit of output. Hence variable costs are given by

$$VC = (w + p_m)\, c\,(p)$$

where, from (2),

$$c\,(p(\upsilon)) = \frac{p\,(\upsilon)^{-1}\,\lambda_p(r(\upsilon))Y}{P^{1-\sigma}} \tag{4}$$

is aggregate consumption of a single variety. Firms that purchase the Fairtrade commodity incur a cost of r_f per unit of raw materials while firms that purchase the conventional commodity incur a cost of p_c per unit of raw materials so that

$$p_m = \begin{cases} r_f, & \text{if variety } \upsilon \text{ is labeled} \\ p_c, & \text{if variety } \upsilon \text{ is unlabeled} \end{cases}.$$

Each firm maximizes its profits by charging a markup over marginal cost equal to $\frac{\sigma}{\sigma-1}$ and hence the price of a final good is $p\,(\upsilon) = \frac{w+p_m}{\rho}$, and a firm's profits are

$$\begin{aligned} \pi &= [p\,(\upsilon) - (w + p_m)]\, c\,(p\,(\upsilon)) \\ &= \frac{\lambda_p(r(\upsilon))Y}{\sigma P^{1-\sigma}}. \end{aligned}$$

Firms must pay an entry fee F (in labor units) to enter the industry. Upon entry to the industry each firm learns its type φ which is drawn independently according to the common distribution H. Each firm's type determines its certification fee $\frac{f_c}{\varphi}$ (measured in units of labor). Hence firms that draw a high type φ, have a lower certification cost. Since firms do not know their certification cost with certainty prior to entry, this models a firm's initial uncertainty about its cost to label its product. (Analogous to Melitz (2003).) According to rules established by the Fairtrade Labeling Orgaizations International (FLO), cases may arise where the Fairtrade minimum prices are set at relevant levels, but where the payer bears the costs of certain activities which are not reflected in the price. For instance, if a producer takes responsibility for an activity (e.g. transport to the harbour, specific packing or processing), the cost of this additional activity will be added to the Fairtrade minimum price paid by the Fairtrade payer to the producer.[2] I assume that each firm's type φ is drawn independently from a Pareto distribution with shape parameter $h > 1$ and scale parameter 1 so that

$$H(\varphi) = 1 - \varphi^{-h}.$$

With this distribution, there are relatively few high type firms in the population since the probability that a firm draws $\varphi > \varphi'$, for some φ', is decreasing in φ'.

The firm with type φ^* that is indifferent to labeling its product is defined by

$$\pi(r_f) - \pi(\underline{r}) = \frac{wf_c}{\varphi^*}. \tag{5}$$

Knowing φ^*, it follows from (5) that

$$P^{1-\sigma} = QM \tag{6}$$

where

$$Q = \left(1 - \varphi^{*-h}\right)\lambda_p(\underline{r}) + \varphi^{*-h}\lambda_p(r_f) \tag{7}$$

is the average ethical quality of available product varieties and M is the mass of entrants to the industry. Also, we can express the aggregate consumption of labeled and unlabeled varieties as

$$\begin{aligned} C_L &= n\frac{\left(\frac{w+r_f}{\rho}\right)^{-1}\lambda_p(r_f)Y}{QM} \\ &= \frac{\alpha\rho Y}{w+r_f} \end{aligned} \tag{8}$$

[2] See "Additional requirements for purchasing from certified producers (direct or via an exporter), Section 3.4.05" located at http://www.fairtrade.org.uk.

$$C_{UL} = m\frac{\left(\frac{w+p_c}{\rho}\right)^{-1}\lambda_p(\underline{r})Y}{QM} = \frac{(1-\alpha)\,\rho Y}{w+p_c} \tag{9}$$

respectively, where $n = \varphi^{*-h}M$ is the mass of labeled varieties, $m = \left(1-\varphi^{*-h}\right)M$ is the mass of unlabeled varieties and $\alpha = \frac{\varphi^{*-h}\lambda_p(r_f)}{Q}$ is the share of income Y that is spent on labeled varieties. Since $\lambda_p(r_f) > 0$ and $\lambda_p(\underline{r}) > 0$, it follows from (7) that $0 \leq \alpha < 1$.[3]

Prior to entering the industry, expected profits are given by

$$E[\Pi] = \int_1^\infty \frac{\lambda_p(r)\,Y}{\sigma P^{1-\sigma}} h(\varphi)\,d\varphi - \int_{\varphi^*}^\infty w f_c(\varphi)\,h(\varphi)\,d\varphi = \frac{Y}{\sigma M} - \frac{h}{h+1} w f_c \varphi^{*-(h+1)}.$$

Free entry ensures zero expected profits net of the entry fee F so that

$$\frac{L}{\sigma M} - \frac{h}{h+1} f_c \varphi^{*-(h+1)} = F. \tag{10}$$

The following lemma shows that there is a unique equilibrium if the certification cost f_c is sufficiently large.

Lemma 1 *A unique equilibrium exists in the final goods market if* $\frac{1}{h+1} > \frac{F}{f_c}$.

Proof. See the Appendix. ■

2.2 Intermediaries

I assume that there is a single intermediary firm in the commodity market that purchases the raw commodity input from farmers at a price of r_c and then sells the processed commodity to final goods producers at a price of p_c. The intermediary has market power as a buyer of the commodity but for simplicity, and to isolate the intermediary's monopsony power, I assume that the intermediary is a perfectly competitive seller to final goods producers that sell to consumers in the final goods market.

[3] We'll see from the proof of Lemma 1 that $\varphi^{*-h} < 1$.

Also, the intermediary purchases x units of the commodity from farmers but after processing the input, sells ξx units to final goods producers, where $\xi < 1$. While, for simplicity, no explicit labor costs are incurred by the intermediary, ξ units of the commodity 'melt' away after processing.

The introduction of the Fairtrade program provides a different channel through which farmers can sell their input to final goods producers. The Fairtrade program establishes a price floor r_f for the commodity input so that final goods producers must pay farmers at least r_f per unit of output. Equivalently, we can think of a non-profit Fairtrade intermediary that purchases the commodity input from farmers at a price of r_f per unit of output and sells directly to participating final goods producers at a price of r_f per unit of output. For simplicity, costs incurred by Fairtrade intermediaries are wholly funded by the certification fees paid by final goods producers that participate in the program.[4] The Fairtrade intermediaries earn zero profits so that the intermediary in the conventional market has no incentive to mimic or join the Fairtrade program. The number of Fairtrade contracts x_f awarded to raw commodity producers is determined by the demand for Fairtrade products in the final goods market, according to consumer preferences for labeled products. At a price of r_f, x_f units of the labeled product is demanded by consumers.

Fairtrade and conventional products have different supply and demand curves, since the Fairtrade label differentiates products according to their ethical quality and Fairtrade commodities are sourced from zero-profit Fairtrade intermediaries. Both markets clear, so that the program does not result in overproduction of the raw commodity or introduce new market distortions, but resolves an information problem in the final goods market by revealing to consumers the wage paid to producers of the raw commodity. Since the demand for Fairtrade labeled products is assumed to be sufficiently small relative to the aggregate market, the conventional market continues to operate and both markets interact.

The timing of the game is as follows. First, the Fairtrade labeling authority chooses the Fairtrade price floor r_f and allocates Fairtrade contracts to farmers of the raw commodity. Second, the intermediary in the conventional commodity market decides how much of the raw commodity to

[4] Note that in actuality, 75% of the Fairtrade program's income is funded by certification fees. See: "Applicable Costs" in the section "Responsibilities as a Licensee" at http://www.fairtrade.org.uk/business_services/product_certification.aspx

purchase, and hence the price it offers to raw commodity producers. Third, firms decide whether to enter the final goods market. There are an infinite number of potential entrants, and nature draws each entrant's type independently from the common distribution H. Each firm pays an identical fixed cost F, thereafter sunk, to enter the industry and each firm learns its type upon entry. A firm's type determines its cost to certify its product. For simplicity, consumers also learn each firm's type. Fourth, upon learning its type, each firm decides whether to label its product. Fifth, firms produce. Finally, consumers observe final goods prices and which product varieties are labeled, and decide how much to consume of each.

2.3 Commodity Market

The supply of the raw commodity is given by

$$x = \frac{r_c - a}{b}$$

where r_c is the wage received by farmers (the commodity's price). Under the Fairtrade program, Fairtrade contracts $x_f\left(r_f\right)$ are accepted by farmers when the market price $r_c < r_f$. Hence, under the Fairtrade program, supply of the raw commodity in the conventional market (the residual supply curve S_R) is given by

$$x = \begin{cases} \frac{r_c - a}{b} - x_f, \text{ if } r_c < r_f \\ \frac{r_c - a}{b}, \text{ if } r_c \geq r_f \end{cases}$$

or, equivalently,

$$r_c = \begin{cases} a + b\left(x + x_f\right), \text{ if } r_c < r_f \\ a + bx, \text{ if } r_c \geq r_f \end{cases} . \tag{11}$$

The intermediary's profits are

$$\pi_m = \left(\xi p_c - r_c\right) x\left(r_c\right) - f_m \tag{12}$$

where f_m are fixed costs incurred by the intermediary. The intermediary treats x_f as given so that, for a given p_c, it is optimal for the intermediary to purchase

$$x^* = \frac{\xi p_c - a - b x_f}{2b} \tag{13}$$

units of the commodity.[5] It follows from (11) and (13) that the price received by farmers is given by

$$r_c^* = \frac{\xi p_c + a + b x_f}{2}.$$

Although the Fairtrade price r_f is chosen exogenously by a Fairtrade program authority, the Fairtrade premium $r_f - r_c$ is endogenous and depends on conditions in the conventional market that determine p_c and the demand for Fairtrade products, which determines x_f. For a given p_c, the spread received by the intermediary, per unit of output, is

$$\xi p_c - r_c = \frac{\xi p_c - a - b x_f}{2}. \tag{14}$$

It follows that Fairtrade contracts x_f result in a smaller spread since, for a given r_c, the elasticity of supply

$$E_s = \frac{r_c}{r_c - a - b x_f}$$

is increasing in x_f, which reduces the intermediary's monopsony power. The intermediary will remain in the industry so long as

$$\pi_m^* = \frac{(\xi p_c - a - b x_f)^2}{4b} > f_m.$$

[5] I assume throughout that $r_f > r_c$. It follows that r_f is such that

$$\begin{aligned} x &< \frac{r_f - a}{b} - x_f \\ \iff & \frac{\xi p_c - a - b x_f}{2b} < \frac{r_f - a}{b} - x_f \\ \iff & p_c < p_u \end{aligned}$$

where

$$p_u = \frac{2 r_f - a - b x_f}{\xi}$$

and is shown in Figure 1.

The intermediary's profits are decreasing in Fairtrade contracts x_f, processing costs (which are decreasing in ξ) and, from (13), are increasing in the output of the conventional commodity x^*.[6]

2.4 Market Clearing

Market clearing in the conventional commodity market requires that aggregate demand for unlabeled products by final goods producers (derived from the aggregate demand for unlabeled products by consumers C_{UL}) is equal to the supply of the conventional input by the intermediary so that the price paid to the intermediary for the processed input p_c is implicitly defined by

$$C_{UL} = \xi \frac{\xi p_c - a - bx_f}{2b}. \tag{15}$$

Also, market clearing in the Fairtrade market requires that Fairtrade contracts are equal to the aggregate demand for labeled products by final goods producers (derived from the aggregate demand for labeled products by consumers C_L) so that for a given r_f, x_f is determined by

$$x_f = C_L. \tag{16}$$

Figure 1 depicts the effects of the introduction of the fairtrade program in the conventional commodity market for the case where the intermediary incurs no processing costs, so that $\xi = 1$. Prior to the introduction of the program, the intermediary sells the commodity to final goods producers according to it's marginal cost MC_o. For a given quantity of the raw commodity, the intermediary's marginal costs exceed the price of the raw commodity since it is the sole buyer, and hence it confronts the entire upward sloping supply curve (it has to increase its price on all units of the commodity purchased whenever it purchases an additional unit.) Since it is a perfectly competitive seller, however, it maximizes its profits by selling a quantity such that the commodity's price p_c is equal to its marginal cost, and hence optimal supply to final goods producers is given by the intermediary's marginal cost of purchasing the raw commodity. Also, prior to the

[6] Note that while Fairtrade contracts reduce output, which exacerbates the market failure in the conventional market, as we'll see, aggregate output of the joint market $x + x_f$ increases.

program, demand for the commodity by final goods producers is given by C_o, so that a quantity of x_o is purchased from commodity producers by the intermediary and a price of p_{co} prevails in the commodity market, and raw commodity producers receive a wage of r_{co}. Once the program establishes a given Fairtrade price floor r_f, Fairtrade contracts x_f are determined according to the demand for labeled goods and S_R depicts the residual supply for the conventional raw commodity given by (11). Consequently, the intermediary's marginal costs increase to MC_1 and since products are now differentiated according to their ethical quality, demand for the conventional commodity decreases to C_{UL}.[7] Due to program, then, and as shown formally in Proposition 2, the price of the commodity falls to p_{c1} and, since aggregate output at p_{c1} has increased ($x_1 + x_f > x_o$), the wage received by farmers increases to r_{c1}. Hence the spread received by the intermediary decreases and since output x in the conventional commodity market decreases, the intermediary's profits π_m fall.

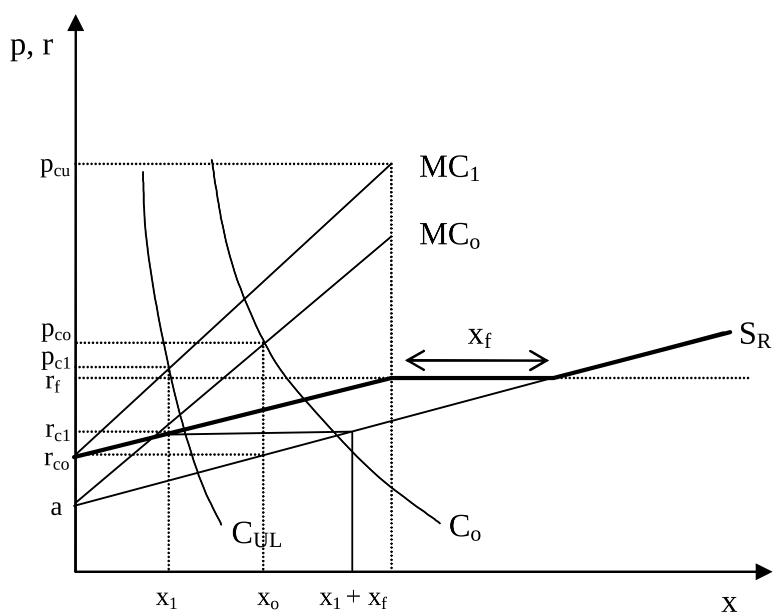

Figure 1. The Commodity Market; case $\xi = 1$.

To summarize, the model is defined by the equations (5), (6), (7), (8), (9), (10) and (13), which,

[7]Note that $C_o = \frac{\rho L}{1+p_c}$ while $C_{UL} = \frac{(1-\alpha)\rho L}{1+p_c}$ so that for a given p_c, $C_{UL} < C_o$ since $\alpha > 0$.

after normalizing the wage $w = 1$, can be reduced to the following system of five equations in the unknowns φ^*, Q, M, p_c and α.

$$\frac{(\lambda_p(r_f) - \lambda_p(\underline{r}))L}{\sigma QM} = \frac{f_c}{\varphi^*} \tag{17}$$

$$Q = \left(1 - \varphi^{*-h}\right)\lambda_p(\underline{r}) + \varphi^{*-h}\lambda_p(r_f) \tag{18}$$

$$\frac{L}{\sigma M} - \frac{h}{h+1} f_c \varphi^{*-(h+1)} = F \tag{19}$$

$$\frac{(1-\alpha)\rho L}{\xi(1+p_c)} + \frac{\alpha\rho L}{2(1+r_f)} = \frac{\xi p_c - a}{2b} \tag{20}$$

$$\alpha = \frac{\varphi^{*-h}\lambda_p(r_f)}{Q} \tag{21}$$

where

$$\begin{aligned} \lambda_p(r_f) &= \rho^{\sigma-1}\left[\frac{\lambda(r_f)}{1+r_f}\right]^{\sigma-1} \\ \lambda_p(\underline{r}) &= \rho^{\sigma-1}\left[\frac{\lambda(\underline{r})}{1+p_c}\right]^{\sigma-1} \end{aligned} \tag{22}$$

and $r_f \in [r_c, \overline{r})$.[8]

2.4.1 Comparative Statics

Proposition 2 *(i) Fairtrade contracts x_f, the proportion of firms that label φ^{*-h}, the proportion of income spent on labeled goods α and the wage to farmers in the conventional market r_c are quasi-concave in r_f, while output in the conventional market x and the mass of entrants M is quasi-convex in r_f. (ii) The commodity price p_c is decreasing and consumer welfare W is increasing in r_f.*

Proof. See the Appendix. ■

As the Fairtrade price floor r_f increases from r_c, I assume that the demand for labeled products and hence Fairtrade contracts x_f increase. From (4), this necessitates that a consumer's private valuation for ethical quality $\lambda(r_f)$ is increasing rapidly in r_f and consequently, from (5), a proportion of firms will optimally choose to certify their products. Hence the threshold firm type φ^*

[8] Note that $\overline{r}$ is implicitly defined by $\lambda_p(r_f) - \lambda_p(\underline{r}) = 0$, so that, from (17), $r_f \in [r_c, \overline{r}]$ ensures that $\varphi^* > 0$.

decreases, since a greater proportion of firms label their products, and the proportion of income spent on labeled products α increases. From part (ii) of the proposition, however, the price of the conventional product p_c is decreasing in r_f so that the relative consumption of Fairtrade products is falling. Hence as r_f continues to increase, the greater relative cost of purchasing Fairtrade products becomes overwhelming, and x_f begins to fall. Also, the threshold firm type φ^* begins to increase, since profits to final goods producers that certify their products are decreasing relative to those that sell their products unlabeled, and the proportion of income spent on labeled products α begins to fall.

As Fairtrade contracts increase, output in the conventional market x begins to fall. From (9) and (13), and as shown in Figure 1, the supply of the conventional product by the intermediary decreases (to MC_1) and demand for the conventional product by final goods producers also decreases (to C_{UL}) since the share of income spent of labeled goods α is increasing. Once Fairtrade contracts x_f begin to fall, output in the conventional market x begins to rise once the decline in the share of income spent on labeled goods α outweighs the continually decreasing commodity price p_c. Hence output of the conventional commodity x is quasi-convex in the Fairtrade price floor r_f and, since p_c is decreasing in r_f, x reaches a minimum while Fairtrade contracts x_f are decreasing in r_f. As shown in the appendix, it follows that aggregate output $x + x_f$ is quasi-concave in r_f, and exceeds x_o if and only if $r_f < p_{co}$. From (11), we have that the wage received by farmers that are not selected into the program r_c is increasing in aggregate output $x + x_f$. This is also clear from Figure 1, since the horizontal distance between the original supply curve and the residual supply curve at r_{co} is equal to x_f. It follows that r_c is quasi-concave in r_f. Finally, from (19) it follows that the mass of entrants M is increasing in φ^* since expected certification costs are decreasing in φ^*. Since φ^* is quasi-convex in r_f, it follows that the mass of entrants M is quasi-convex in r_f.

The commodity price p_c is decreasing in the Fair-trade price floor. We can express (20) as

$$\frac{(1-\alpha)\,\rho L}{\xi\,(1+p_c)} = \frac{\xi p_c - a}{2b} - \frac{\alpha \rho L}{2\,(1+r_f)} \tag{23}$$

so that demand for the conventional commodity (LHS of (23)) is equal to its supply (RHS of (23)).

For a given p_c, an increase in r_f from r_c results in a greater proportion of income spent on labeled goods α. Also, since $x_f = \frac{\alpha \rho L}{1+r_f}$ increases, we have that α is increasing rapidly and supply in the conventional commodity market (the RHS of (23)) decreases. Since the decrease in $1-\alpha$ is at the same rate, the decrease in demand for the conventional commodity must be greater (the increase in α on the RHS of (23) is moderated by the increase in r_f.) Hence p_c must fall in response to a greater Fairtrade price-floor r_f whenever α is increasing. If α is decreasing in response to a greater r_f, then for a given p_c, the supply of the conventional commodity must increase. The demand for the conventional commodity must increase by less (the decrease in supply is exacerbated by a greater r_f) so that p_c also falls in response to a greater r_f whenever α is decreasing.[9]

From (1) it follows that consumer welfare is given by

$$\begin{aligned} W &= \frac{Y}{P} \\ &= (QM)^{\frac{1}{\sigma-1}} L. \end{aligned}$$

As shown in the appendix, W is increasing in both the private valuation for labeled goods per unit price $\lambda_p(r_f)$ and the private valuation for unlabeled goods per unit price $\lambda_p(\underline{r})$. As the Fairtrade price floor r_f increases, $\lambda_p(r_f)$ is increasing by assumption. Also, since the price of the conventional commodity p_c is decreasing, $\lambda_p(\underline{r})$ is also increasing. Hence consumer welfare W is increasing in r_f. [Note that this is due to the fact that $\lambda(\underline{r})$ is independent of r_c, so that once r_c begins to fall, $\lambda_p(\underline{r})$ continues to rise... But if we had $\lambda_p(r_c)$, then preferences would be endogenous and the model circular?]

2.5 Monopsony Profits

Recall that the intermediary's profits are increasing in output x. Hence the Fairtrade program can only reduce the intermediary's profits by decreasing the size of the conventional market, and the intermediary will earn positive profits (gross of fixed costs f_m) so long as $x > 0$. As r_f is first

[9] Note that (23) is independent of the market structure for final goods ie. the argument holds for $\rho = 1$. Moreover, the previous argument is strengthened when the intermediary has market power (as a consequence, $1+r_f$ is multiplied by 2), but would still hold if the intermediary were perfectly competitive. Hence p_c is decreasing faster when the intermediary has market power.

increased from r_c, the spread that the intermediary receives for each unit of output is decreasing. As shown in Proposition 2, the price that the intermediary receives in the commodity market p_c is decreasing in r_f. Also, the wage received by farmers that are not selected into the program r_c is first increasing in r_f. Once r_c begins to fall, however, the spread received by the intermediary continues to decrease since p_c is decreasing faster than r_c, so long as output in the conventional market x is decreasing. Moreover, since the spread is decreasing if and only if output x is decreasing, it follows that the intermediary's profits π_m are decreasing in r_f whenever x is decreasing.

2.6 Producer Surplus

The total producer surplus of all farmers includes those that have been selected into the fairtrade program as well as those that have not been selected. As shown in Figure 2, we can express total producer surplus as

$$\begin{aligned} PS &= \int_a^{r_c} \frac{p-a}{b} dp + T \\ &= \frac{1}{2}(r_c - a)(x + x_f) + T \end{aligned} \tag{24}$$

where transfers to selected farmers are given by

$$T = (r_f - r_c)\, x_f.$$

From (11) and (24) it's clear that baseline producer surplus (PS less T) is increasing in aggregate output $x + x_f$, while transfers to selected producers depend on the magnitude of the price floor r_f and Fairtrade contracts x_f. As r_f is first increased from r_c, while aggregate output $x + x_f$ is increasing, baseline producer surplus is increasing and transfers to selected farmers T are also increasing. Once aggregate output begins to fall, however, the producer surplus of farmers that are not selected into the program begins to fall while transfers to selected farmers T continues to rise (selected farmers are compensated by the fall in r_c by the transfer T). Hence while aggregate output is decreasing, there is a trade-off between the objectives of maximizing the welfare of farmers that have been selected into the program and those that have not been selected. Total producer surplus is depicted in Figure 2 by the area with horizontal stripes (baseline producer surplus) and the area with vertical stripes (the transfer to selected producers T).

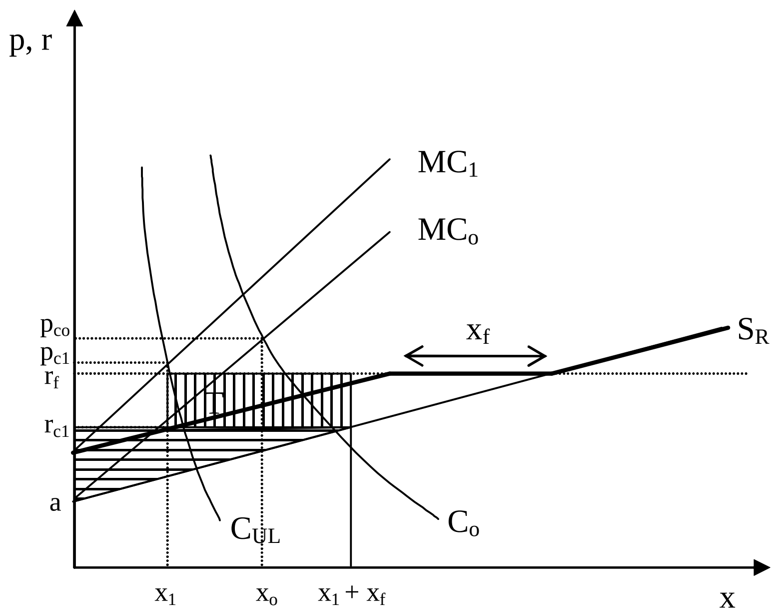

Figure 2. Producer Surplus; case $\xi = 1$.

2.7 Efficiency

It's clear from the above analysis that the Pareto optimal Fairtrade price floor r_f maximizes aggregate output. As we've seen, while aggregate output is increasing, both selected and unselected farmers are better off. Also, consumer welfare is increasing in r_f. The following proposition further characterizes the Pareto optimal r_f. The price floor that maximizes aggregate output must be less than the commodity price p_c [this is in contrast to the optimal (non-voluntary) price floor, which would be set equal to the world price p_c. See Deardorff and Rajaraman (2009)], and it is less that the price floor that would maximize Fairtrade contracts.

Proposition 3 *(i) If* $r_f' = \arg\max\left(x + x_f\right)$, *then* $p_c > r_f'$. *(ii)* $\arg\max\left(x + x_f\right) < \arg\max x_f <$

$\arg\max \alpha < \arg\max PS < \arg\max T$.

Proof. See the appendix. ■

Since the Pareto optimal Fairtrade price floor is lower than the standard that would maximize Fairtrade contracts x_f, it is also lower than the price floor that minimizes output in the conventional market x. Interestingly, efficiency does not require that the size of the conventional market is as small as possible, despite that the existence of the intermediary in the conventional market introduces the only market failure. Rather, since labeled and unlabeled products are differentiated goods (and not perfect substitutes), Pareto optimality requires that the joint output of both the conventional and Fairtrade markets is as large as possible. Since the intermediary maximizes profits by restricting output, the Pareto optimal price floor ensures that its monopsony power is best mitigated, subject to the extent of consumer demand for labeled products. Furthermore, as shown in part (ii) of the proposition, the Pareto optimal price floor is also less than the standard that would maximize the proportion of income spent on Fairtrade products α (which, as shown in the appendix, is equivalent to the standard that would maximize the proportion of firms that choose to label their products φ^{*-h}). The price floor that maximizes producer surplus PS, however, exceeds the price floor that would maximize α, so that the share of income spent on labeled products and the proportion of firms that label their products is not fully maximal. The transfer to producers T is also not fully maximal, due to the trade-off between greater baseline producer surplus and greater transfers T. Note that the standard that maximizes PS represents a potential Pareto improvement (according to the Kaldor-Hicks criterion), however, since producer surplus is increasing over $[r_c, \arg\max PS]$. Producers that have been selected into the Fairtrade program could, in principle, compensate those that have not been selected and lose from increases in the price floor beyond $\arg\max(x + x_f)$. [Am still working to understand whether r_f is greater/less than p_c at $r_f = \arg\max PS$] Also, as shown in the appendix, we have that $x + x_f > x_o$ if and only if $r_f < p_{co}$ so that, in comparison with the status quo of no labeling program, producers of the raw commodity that have not been selected into the program are better off so long as $r_f < p_{co}$. Hence, since p_c is decreasing in r_f, unselected producers are better off than if there is no labeling program for some values of r_f that exceed $r'_f = \arg\max(x + x_f)$ (which makes unselected producers best off).

Work in progress:

(1) generalize results for any continuous distribution function H.

(2) Consider effects of the Fairtrade program when the intermediary has no market power and compare with above analysis. (It seems that, in this instance, the commodity price p_c is also decreasing in the Fairtrade price floor r_f however, since there is no spread received by the intermediary, there would be a strict trade-off between the return to farmers that are not selected and those that are selected into the program for all r_f.

(3) Generalize the model to include a finite number of intermediaries N with market power (oligopsony), and consider the effects on the overall analysis as N changes.

(4) Compare the Fairtrade program with an export tax, as in Deardorff and Rajaraman (1990), and with a direct transfer to producers from consumers.

(5) Add a consumption externality to the model.

(6) Consider the consequences of the intermediary also having market power as a seller of the processed commodity.

Appendix

Preliminaries

Assumptions for λ :

$$\begin{aligned}
\frac{\partial x_f}{\partial r_f} &= \frac{\lambda_p'(r_f)}{\lambda_p(r_f)} - \frac{1}{1+r_f} \\
&= (\sigma-1)\frac{\lambda'(r_f)}{\lambda(r_f)} - \frac{\sigma}{(1+r_f)} > 0 \\
&\Longleftrightarrow \frac{\lambda'(r_f)(1+r_f)}{\lambda(r_f)} > \frac{\sigma}{\sigma-1} \\
&\quad \text{which is true iff } r_f \text{ small} \\
\text{so assume } \frac{\lambda'(r_f)(1+r_f)}{\lambda(r_f)} &\to 0 \text{ as } r_f \to \infty. \\
\text{ie. } \lambda'(r_f) &\to 0 \text{ for finite but large } r_f. \\
&\quad \text{consequence is that } x_f \text{ is concave in } r_f
\end{aligned}$$

Proof of Lemma 1

Normalizing $w = 1$, equilibrium in final goods market requires

$$\begin{aligned}
\frac{[\lambda_p(r_f) - \lambda_p(\underline{r})]L}{\sigma Q M} &= \frac{f_c}{\varphi^*} \\
\frac{[\lambda_p(r_f) - \lambda_p(\underline{r})]\left[F + \frac{h}{h+1} f_c \varphi^{*-(h+1)}\right]}{[(1-\varphi^{*-h})\lambda_p(\underline{r}) + \varphi^{*-h}\lambda_p(r_f)]} &= \frac{f_c}{\varphi^*} \\
[\lambda_p(r_f) - \lambda_p(\underline{r})]\left[\frac{F}{f_c}\varphi^* + \frac{h}{h+1}\varphi^{*-h}\right] &= \left(1-\varphi^{*-h}\right)\lambda_p(\underline{r}) + \varphi^{*-h}\lambda_p(r_f) \\
[\lambda_p(r_f) - \lambda_p(\underline{r})]\frac{F}{f_c}\varphi^* - \frac{1}{h+1}[\lambda_p(r_f) - \lambda_p(\underline{r})]\varphi^{*-h} &= \lambda_p(\underline{r}) \\
\frac{F}{f_c}\varphi^* - \frac{1}{h+1}\varphi^{*-h} &= \frac{\lambda_p(\underline{r})}{\lambda_p(r_f) - \lambda_p(\underline{r})} \\
\frac{1}{h+1}\varphi^{*-h-1} + \frac{\lambda_p(\underline{r})}{(\lambda_p(r_f) - \lambda_p(\underline{r}))}\varphi^{*-1} &= \frac{F}{f_c}
\end{aligned}$$

The LHS of above equation is decreasing in φ^* and $LHS \to 0$ as $\varphi^* \to \infty$. Also, if $\varphi^* = 1$

$$LHS = \frac{1}{h+1} + \frac{\lambda_p(\underline{r})}{(\lambda_p(r_f) - \lambda_p(\underline{r}))}.$$

Hence an equilibrium exists and is unique if

$$\frac{1}{h+1} + \frac{\lambda_p(\underline{r})}{(\lambda_p(r_f) - \lambda_p(\underline{r}))} > \frac{F}{f_c}$$

and it is sufficient to require $\frac{1}{h+1} > \frac{F}{f_c}$. Note that, as a consequence, $\varphi^* > 1$ or, equivalently,

$\varphi^{*-h} < 1$.

Proof of Proposition 2

Normalizing $w = 1$ and differentiating the system with respect to r_f yields:

$$\frac{\lambda_p(r_f) - \lambda_p(\underline{r})}{Q}\frac{L}{\sigma M} = \frac{f_c}{\varphi^*}$$

$$\frac{\lambda'_p(r_f) - \frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f}}{\lambda_p(r_f) - \lambda_p(\underline{r})} - \widehat{QM} = -\widehat{\varphi^*}$$

$$Q = \left(1 - \varphi^{*-h}\right)\lambda_p(\underline{r}) + \varphi^{*-h}\lambda_p(r_f)$$

$$\widehat{Q} = -\frac{h\varphi^{*-h}(\lambda_p(r_f) - \lambda_p(\underline{r}))}{Q}\widehat{\varphi^*} + \frac{\varphi^{*-h}\lambda'_p(r_f)}{Q} + \frac{\left(1 - \varphi^{*-h}\right)}{Q}\frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f}$$

$$\frac{L}{\sigma M} - I = F$$

$$\widehat{M} = -\frac{I}{I+F}\widehat{I}$$

$$I = \frac{h}{h+1}f_c\varphi^{*-(h+1)}$$

$$\widehat{I} = -(h+1)\widehat{\varphi^*}.$$

Solving, we have

$$\widehat{M} = \frac{I}{I+F}(h+1)\widehat{\varphi^*}$$

and hence

$$\begin{aligned}
\widehat{QM} &= -\frac{h\varphi^{*-h}(\lambda_p(r_f) - \lambda_p(\underline{r}))}{Q}\widehat{\varphi^*} + \frac{\varphi^{*-h}\lambda'_p(r_f)}{Q} + \frac{\left(1 - \varphi^{*-h}\right)}{Q}\frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f} + \frac{I}{I+F}(h+1)\widehat{\varphi^*} \\
&= \left[-\frac{h\varphi^{*-h}(\lambda_p(r_f) - \lambda_p(\underline{r}))}{Q} + \frac{I}{I+F}(h+1)\right]\widehat{\varphi^*} + \frac{\varphi^{*-h}\lambda'_p(r_f)}{Q} + \frac{\left(1 - \varphi^{*-h}\right)}{Q}\frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f} \\
&= \frac{1}{Q}\left[\varphi^{*-h}\lambda'_p(r_f) + \left(1 - \varphi^{*-h}\right)\frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f}\right] \\
&= \alpha\frac{\lambda'_p(r_f)}{\lambda_p(r_f)} + (1-\alpha)\frac{1}{\lambda_p(\underline{r})}\frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f}
\end{aligned}$$

since $-\frac{h\varphi^{*-h}\left(\lambda_p(r_f) - \lambda_p(\underline{r})\right)}{Q} + \frac{I}{I+F}(h+1) = 0$. Hence $\widehat{QM}$ is increasing in r_f, since we'll see that

$\frac{dp_c}{dr_f} < 0$ for all r_f. Also,

$$\begin{aligned}
\widehat{\varphi^*} &= \widehat{QM} - \frac{\lambda_p'(r_f) - \frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f}}{\lambda_p(r_f) - \lambda_p(\underline{r})} \\
&= -\frac{\lambda_p(r_f)\,\lambda_p(\underline{r})}{Q\left(\lambda_p(r_f) - \lambda_p(\underline{r})\right)}\left[\frac{\lambda_p'(r_f)}{\lambda_p(r_f)} - \frac{1}{\lambda_p(\underline{r})}\frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f}\right]
\end{aligned}$$

so that φ^* is convex, since $\frac{\lambda_p'(r_f)}{\lambda_p(r_f)} > \frac{1}{\lambda_p(\underline{r})}\frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f}$ iff r_f is sufficiently small.

$$\begin{aligned}
\widehat{Q} &= -\frac{h\varphi^{*-h}\left(\lambda_p(r_f) - \lambda_p(\underline{r})\right)}{Q}\widehat{\varphi^*} + \left[\frac{\varphi^{*-h}\lambda_p'(r_f)}{Q} + \frac{\left(1-\varphi^{*-h}\right)}{Q}\frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f}\right] \\
&= \frac{h\varphi^{*-h}}{Q}\frac{\lambda_p(r_f)\,\lambda_p(\underline{r})}{Q}\left[\frac{\lambda_p'(r_f)}{\lambda_p(r_f)} - \frac{1}{\lambda_p(\underline{r})}\frac{d\lambda_p(\underline{r})}{dp_c}\frac{dp_c}{dr_f}\right] + \widehat{QM}
\end{aligned}$$

$\widehat{Q} > 0$ when $\widehat{\varphi^*} = 0$ so that $\arg\max Q > \arg\max \alpha = \arg\min \varphi^*$.

$$\begin{aligned}
\widehat{\alpha} &= -h\widehat{\varphi^*} + \frac{\lambda_p'(r_f)}{\lambda_p(r_f)} - \widehat{Q} \\
&= -\frac{h\lambda_p(\underline{r})}{Q}\widehat{\varphi^*} + \frac{\lambda_p'(r_f)}{\lambda_p(r_f)} - \widehat{QM} \\
&= \lambda_p(\underline{r})\left[\frac{h\lambda_p(\underline{r})\,\lambda_p(r_f) + Q\left(\lambda_p(r_f) - Q\right)}{Q^2\left(\lambda_p(r_f) - \lambda_p(\underline{r})\right)}\right]\left\{\frac{\lambda_p'(r_f)}{\lambda_p(r_f)} - \frac{\frac{d\lambda_p(\underline{r})}{dp_c}}{\lambda_p(\underline{r})}\frac{dp_c}{dr_f}\right\}
\end{aligned}$$

$$\begin{aligned}
\widehat{\alpha} &> 0 \iff \frac{\lambda_p'(r_f)}{\lambda_p(r_f)} > \frac{\frac{d\lambda_p(\underline{r})}{dp_c}}{\lambda_p(\underline{r})}\frac{dp_c}{dr_f} \\
&\iff \widehat{\varphi^*} < 0
\end{aligned}$$

Hence α is concave and shares the same extremum as φ^*. Also, we have

$$\widehat{x_f} = \widehat{\alpha} - \frac{1}{1+r_f}$$

so that $\arg\max x_f < \arg\max \alpha$.

From market clearing we have

$$\alpha = \frac{(\xi p_c - a)\,\xi\,(w+p_c)\,(w+r_f) - 2\,(w+r_f)\,b\rho L}{\left[\xi\,(w+p_c) - 2\,(w+r_f)\right]b\rho L}$$

so that $0 < \alpha < 1$ implies

$$\xi\,(w+p_c) - 2\,(w+r_f) < 0$$

$$(\xi p_c - a)\,\xi\,(w + p_c) < 2b\rho L$$

and

$$(\xi p_c - a)\,(w + r_f) > b\rho L.$$

Differentiating the market clearing condition yields

$$\frac{dp_c}{dr_f} = \frac{[(\xi p_c - a)\,\xi\,(1 + p_c) - 2b\rho L]\,(1 + p_c)}{\xi^2\,(1 + p_c)^2 + (1 - \alpha)\,2b\rho L}\left[\begin{array}{c}\widehat{\alpha} \\ -\frac{\xi(1+p_c)}{[\xi(1+p_c)-2(1+r_f)](1+r_f)}\end{array}\right]$$

so as $\alpha \to 0$, $(\xi p_c - a)\,\xi\,(1 + p_c) - 2b\rho L \to 0$ so that $\frac{dp_c}{dr_f} \to 0$.

Solving for $\frac{dp_c}{dr_f}$ and $\widehat{\alpha}$:

$$\widehat{\alpha} = \Psi\left\{\frac{\lambda_p'(r_f)}{\lambda_p(r_f)} - \frac{\frac{d\lambda_p(\underline{r})}{dp_c}}{\lambda_p(\underline{r})}\frac{dp_c}{dr_f}\right\}$$

where

$$\Psi = \lambda_p(\underline{r})\left[\frac{h\lambda_p(\underline{r})\,\lambda_p(r_f) + Q\,(\lambda_p(r_f) - Q)}{Q^2\,(\lambda_p(r_f) - \lambda_p(\underline{r}))}\right] > 0.$$

Hence

$$\begin{aligned}
\widehat{\alpha} &= \Psi\frac{\lambda_p'(r_f)}{\lambda_p(r_f)} - \Psi\frac{\frac{d\lambda_p(\underline{r})}{dp_c}}{\lambda_p(\underline{r})}\frac{dp_c}{dr_f} \\
&\iff \widehat{\alpha} = \frac{\Psi\left[\xi^2\,(1 + p_c)^2 + (1 - \alpha)\,2b\rho L\right]}{\left[\xi^2\,(1 + p_c)^2 + (1 - \alpha)\,2b\rho L\right] + \Psi\frac{\frac{d\lambda_p(\underline{r})}{dp_c}}{\lambda_p(\underline{r})}\,[(\xi p_c - a)\,\xi\,(1 + p_c) - 2b\rho L]\,(1 + p_c)} \\
&\quad \left\{\frac{\lambda_p'(r_f)}{\lambda_p(r_f)} + \frac{\frac{d\lambda_p(\underline{r})}{dp_c}}{\lambda_p(\underline{r})}\frac{\alpha\xi\,(1 + p_c)^2\,b\rho L}{\left[\xi^2\,(1 + p_c)^2 + (1 - \alpha)\,2b\rho L\right](1 + r_f)^2}\right\}
\end{aligned}$$

where $$\frac{\Psi\left[\xi^2\,(1 + p_c)^2 + (1 - \alpha)\,2b\rho L\right]}{\left[\xi^2\,(1 + p_c)^2 + (1 - \alpha)\,2b\rho L\right] + \Psi\frac{\frac{d\lambda_p(\underline{r})}{dp_c}}{\lambda_p(\underline{r})}\,[(\xi p_c - a)\,\xi\,(1 + p_c) - 2b\rho L]\,(1 + p_c)} > 0$$

and

$$\frac{\alpha\xi\,(1 + p_c)^2\,b\rho L}{\left[\xi^2\,(1 + p_c)^2 + (1 - \alpha)\,2b\rho L\right](1 + r_f)^2} > 0.$$

So when α is small (r_f is small), α is increasing, but as r_f increases, $\frac{\lambda_p'(r_f)}{\lambda_p(r_f)} \to 0$ and since $\frac{\frac{d\lambda_p(\underline{r})}{dp_c}}{\lambda_p(\underline{r})} < 0$, α begins to fall. Hence α is concave in r_f.

Also, it follows that

$$\frac{dp_c}{dr_f} = \frac{\left[(\xi p_c - a)\,\xi\,(1+p_c) - 2b\rho L\right](1+p_c)}{\xi^2 (1+p_c)^2 + (1-\alpha)\,2b\rho L}\widehat{\alpha} - \frac{\left[(\xi p_c - a)\,\xi\,(1+p_c) - 2b\rho L\right](1+p_c)}{\xi^2 (1+p_c)^2 + (1-\alpha)\,2b\rho L}\frac{\xi\,(1+p_c)}{\left[\xi\,(1+p_c) - 2\,(1+r_f)\right](1+r_f)}$$

which is negative for all $r_f < \overline{r}_f$ since from the market clearing condition, $0 < \alpha < 1$ implies

$$\xi\,(1+p_c) - 2\,(1+r_f) < 0$$

$$(\xi p_c - a)\,\xi\,(1+p_c) < 2b\rho L$$

and

$$(\xi p_c - a)\,(1+r_f) > b\rho L$$

and we have that $\frac{\frac{d\lambda_p(r)}{dp_c}}{\lambda_p(r)} < 0$.

Aggregate output z can be expressed as

$$\begin{aligned} z &= x + x_f \\ &= \frac{\xi p_c - a - bx_f}{2b} + x_f \\ &= \frac{\xi p_c - a}{2b} + \frac{x_f}{2} \end{aligned}$$

so that

$$\frac{dz}{dr_f} = \frac{1}{2b}\left(\xi\frac{dp_c}{dr_f}\right) + \frac{1}{2}\frac{dx_f}{dr_f}.$$

Since $\frac{dp_c}{dr_f} < 0$ for all r_f, $\frac{dx_f}{dr_f} = 0 \Longrightarrow \frac{dz}{dr_f} < 0$ so that $\arg\max z < \arg\max x_f$, since x is decreasing in x_f. Hence z is decreasing on $[\arg\max x_f,\ \overline{r}]$.

Also, we can express

$$\begin{aligned} z &= \frac{\alpha\rho L}{1+r_f} + \frac{(1-\alpha)\,\rho L}{1+p_c} \\ &= \alpha\left[\frac{p_c - r_f}{(1+r_f)\,(1+p_c)}\right]\rho L + \frac{\rho L}{1+p_c} \end{aligned}$$

so that

$$\frac{dz}{dr_f} = \frac{\alpha\rho L}{(1+r_f)}\left\{\left[\frac{p_c - r_f}{(1+p_c)}\right]\widehat{\alpha} - \frac{1}{(1+r_f)}\right\} - \frac{1}{1+p_c}\frac{(1-\alpha)\,\rho L}{(1+p_c)}\frac{dp_c}{dr_f}.$$

Hence, when r_f is small so that α is small, since the second term dominates and $\frac{dp_c}{dr_f} < 0$, we have that $\frac{dz}{dr_f} > 0$. Hence z is quasi-concave in r_f. Also, $z > x_o = \frac{\rho L}{1+p_{co}} \Longleftrightarrow p_{co} - r_f > 0$.

Parameter restrictions:

(i) From market clearing we have

$$\alpha = \frac{(\xi p_c - a)\,\xi\,(w + p_c)\,(w + r_f) - 2\,(w + r_f)\,b\rho L}{[\xi\,(w + p_c) - 2\,(w + r_f)]\,b\rho L}.$$

Since $r_c < r_f$, from the residual supply curve we have that

$$\xi p_c < 2r_f - a - bx_f$$

$\Longrightarrow 2r_f > \xi p_c$. Hence

$$\xi\,(w + p_c) - 2\,(w + r_f) < 0$$

and since $\alpha > 0$, we have that

$$(\xi p_c - a)\,\xi\,(w + p_c) < 2b\rho L.$$

Also, $\alpha < 1$ implies

$$\begin{aligned}\frac{(\xi p_c - a)\,\xi\,(w + p_c)\,(w + r_f) - 2\,(w + r_f)\,b\rho L}{[\xi\,(w + p_c) - 2\,(w + r_f)]\,b\rho L} &< 1 \\ (\xi p_c - a)\,(w + r_f) &> b\rho L.\end{aligned}$$

So it's required that

$$\xi\,(w + p_c) - 2\,(w + r_f) < 0$$

$$(\xi p_c - a)\,\xi\,(w + p_c) < 2b\rho L$$

and

$$(\xi p_c - a)\,(w + r_f) > b\rho L.$$

(ii) Recall that

$$\begin{aligned}\lambda_p\,(r_f) &= \rho^{\sigma-1}\left[\frac{\lambda\,(r_f)}{w + r_f}\right]^{\sigma-1} \\ \lambda_p\,(\underline{r}) &= \rho^{\sigma-1}\left[\frac{\lambda\,(\underline{r})}{w + p_c}\right]^{\sigma-1}.\end{aligned}$$

Hence we need to set an upper bound on r_f, $\overline{r}$ to ensure that $\varphi^* > 1$. From (17), it is necessary that

$$\begin{aligned}\lambda_p\,(r_f) - \lambda_p\,(\underline{r}) &> 0 \\ &\Longleftrightarrow \frac{\lambda\,(r_f)}{w + r_f} > \frac{\lambda\,(\underline{r})}{w + p_c}.\end{aligned}$$

Note that if $r_f = r_c \Longrightarrow \lambda\,(r_f) = \lambda\,(\underline{r})$ but $\lambda_p\,(r_f) > \lambda_p\,(\underline{r})$ since $r_c < p_c$. Since p_c is decreasing in r_f, however, we need to require $r_f < \overline{r}$, where $\overline{r}$ is implicitly defined by

$$\frac{\lambda\,(\overline{r})}{\lambda\,(\underline{r})}\,(w + p_c\,(\overline{r})) = w + \overline{r}$$

This permits $r_f > p_c$ since $\frac{\lambda(\bar{r})}{\lambda(\underline{r})} > 1$.

Proof of Proposition 3

(i) We can express

$$\begin{aligned} z &= x + x_f \\ &= \frac{\rho L - (1 + r_f)\, x_f}{(1 + p_c)} + x_f \\ &= \frac{\rho L + (p_c - r_f)\, x_f}{(1 + p_c)}. \end{aligned}$$

Since p_c is decreasing in r_f, there exists a $\widetilde{p}_c = \widetilde{r}_f$ such that $\widetilde{z} = \frac{\rho L}{(1+\widetilde{p}_c)}$ and $z > \widetilde{z}$ iff $p_c > \widetilde{p}_c$ iff $r_f < \widetilde{r}_f$. Hence z must be decreasing at $r_f = \widetilde{r}_f$ so that when $r_f = \arg\max z$, $p_c > r_f$.

(ii) We can express

$$\begin{aligned} PS &= \frac{1}{2}(r_c - a)(x + x_f) + ((r_f - a) - (r_c - a))\, x_f \\ &= \frac{1}{2} b\,(x + x_f)(x + x_f) + ((r_f - a) - b\,(x + x_f))\, x_f \\ &= \frac{1}{2} b\,(x + x_f)^2 - b\,(x + x_f)\, x_f + (r_f - a)\, x_f \end{aligned}$$

which is always positive, and is increasing in aggregate output $z = x + x_f$. We have that $r_c = a + b\,(x + x_f)$ so that $\frac{dr_c}{dr_f} > 0 \Longleftrightarrow \frac{dz}{dr_f} > 0$.

$$\begin{aligned} \frac{dPS}{dr_f} &= bz\frac{dz}{dr_f} + \frac{dT}{dr_f} \\ &= bz\frac{dz}{dr_f} - b\left(\frac{dz}{dr_f}x_f + z\frac{dx_f}{dr_f}\right) + x_f + (r_f - a)\frac{dx_f}{dr_f} \\ &= [z - x_f]\, b\frac{dz}{dr_f} + x_f + [(r_f - a) - bz]\frac{dx_f}{dr_f} \\ &= xb\frac{dz}{dr_f} + x_f + [r_f - a - bz]\frac{dx_f}{dr_f} \\ &= xb\frac{dz}{dr_f} + x_f + (r_f - r_c)\frac{dx_f}{dr_f} \end{aligned}$$

so when z is maxed, PS continues to increase since $r_f > r_c$ and $\frac{dx_f}{dr_f} > 0$ ie. transfers T are increasing. We have that

$$\begin{aligned} T &= (r_f - r_c)\, x_f \\ &= ((r_f - a) - (r_c - a))\, x_f \\ &= ((r_f - a) - bz)\, x_f \end{aligned}$$

$$\frac{dT}{dr_f} = -bx_f\frac{dz}{dr_f} + x_f + [(r_f - a) - bz]\frac{dx_f}{dr_f}.$$

Since $\arg\max z < \arg\max x_f$, when $\frac{dx_f}{dr_f} = 0$, we have that $\frac{dz}{dr_f} < 0$ and it follows that $\frac{dT}{dr_f} > 0$. Hence $\arg\max z < \arg\max x_f < \arg\max T$. Also, at r_f such that $\frac{dPS}{dr_f} = 0$, since $\arg\max z < \arg\max T$, it follows that $\frac{dz}{dr_f} < 0$ and $\frac{dT}{dr_f} > 0$ and hence $\arg\max z < \arg\max PS < \arg\max T$. We can also express

$$\begin{aligned}
\frac{dPS}{dr_f} &= bz\frac{dz}{dr_f} - b\left(\frac{dz}{dr_f}x_f + z\frac{dx_f}{dr_f}\right) + x_f + (r_f - a)\frac{dx_f}{dr_f} \\
&= x_f + xb\frac{dx}{dr_f} + [r_f - (a + bx_f)]\frac{dx_f}{dr_f} \\
&= x_f + x^2 b\left[-\frac{\alpha}{1-\alpha}\widehat{\alpha} - \frac{p_c}{1+p_c}\widehat{p_c}\right] + x_f[r_f - (a + bx_f)]\left[\widehat{\alpha} - \frac{1}{1+r_f}\right] \\
\widehat{\alpha} &= 0 \Longrightarrow \\
\frac{dPS}{dr_f} &= x_f\left[\frac{1 + (a + bx_f)}{1 + r_f}\right] - x^2 b\left[\frac{p_c}{1+p_c}\widehat{p_c}\right] > 0 \\
\text{since } \widehat{p_c} &< 0 \text{ for all } r_f
\end{aligned}$$

and hence $\arg\max PS > \arg\max \alpha$. Finally, since $\arg\max x_f < \arg\max \alpha$, it follows that $\arg\max z < \arg\max x_f < \arg\max \alpha < \arg\max PS < \arg\max T$.