



Submission Number: PED11-11-00051

A note on tax base, public debt, and investors' beliefs

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## *Abstract*

This paper provides a new evidence and theoretical support for the role of market expectation in the public debt markets. Dispersion of investors' beliefs over the default probability is empirically associated with the size of tax base during the crisis of 2008-10. That small differences in the fundamentals may result in multiple equilibria for several Eurozone countries can be captured in a model of debt crisis with self-fulfilling nature, investors' beliefs, and relevant controls.

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May 2011

### Abstract

This paper provides a new evidence and theoretical support for the role of market expectation in the public debt markets. Dispersion of investors' beliefs over the default probability is empirically associated with the size of tax base during the crisis of 2008-10. That small differences in the fundamentals may result in multiple equilibria for several Eurozone countries can be captured in a model of debt crisis with self-fulfilling nature, investors' beliefs, and relevant controls.

Keywords: fiscal space, sovereign default, market expectations

JEL codes: E6, F4, H6, O1

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## 1. Introduction

This paper studies the relative importance of economic fundamentals and investors' beliefs in the public debt markets. Driven by the fiscal challenge facing many countries in the aftermath of the 2008-09 global crisis, the ongoing uncertainty focuses on the southwest periphery countries in the Euro area -- Greece, Ireland, Italy, Portugal, and Spain. Specifically, investors are reluctant to roll over and restructure the public debt of these Euro economies, with the significant increase in risk premia attached to the sovereigns during 2009-11. Consequently, while the cost of sovereign borrowing increased across countries during the global crisis, the market pessimism about fiscal adjustment in these Euro countries are prolonged.

Given their economic fundamentals relative to other industrial and comparable emerging market countries, the experiences of Greece, Ireland, Italy, Portugal, and Spain are quite unique. Table 1 provides the 2006 average of public debt/GDP, the 2000-06 average of tax revenue/GDP, and the 2008-10 average of sovereign CDS spreads across countries. The tax bases are averaged across several years to smooth for business cycle fluctuations. The relative size of debt and tax can be interpreted as the tax years it would take to repay the public debt.<sup>1</sup> These numbers indicate that the size of public debt and tax base are not remarkably worse in the troubled Euro countries, yet they continue to have greater difficulty in managing the public debt.

<Table 1>

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<sup>1</sup> For detailed definition and assessment of the debt/tax ratio, see Aizenman and Jinjark (2010).

The next section reports some stylized facts about recent development in the public debt markets. The news is that investors' positions in the debt markets may not be based entirely on the economic fundamentals. We posit that the investors may believe that fiscal conditions in some countries will deteriorate markedly and price the sovereign risks based on their beliefs (and their beliefs about others' beliefs) about the future fundamentals. Section 3 presents a simple model that corroborates our conjecture.

## 2. Stylized facts

To gauge the role of investor's beliefs about economic fundamentals, as well as their beliefs about others' beliefs on the default probability, Figure 1 provides the notional amount outstanding (billion USD) of sovereign CDS positions as of February 25, 2011 based on the Depository Trust & Clearing Corporation (DTCC) data. The net amount is the position once offsetting contracts have been netted out the gross amount.<sup>2</sup>

Interestingly, the gross positions of sovereign CDS have no clear relationship with country size or government debt (e.g. the position on South Korea is more than twice the U.S.; the position on Italy is the largest.). It is also evident that the gross positions are many times larger than the net positions, implying there are significant trade offsettings and dispersion of the market expectations in the sovereign debt markets.

<Figure 1>

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<sup>2</sup> See also DTCC (2011) and The Economist (2010), October 14.

Table 2 reports regressions of the sovereign CDS gross/net positions as a dependent variable on the public debt/GDP, tax revenue/GDP, prediction errors on CDS (10-year) spreads. The spread prediction errors are obtained by regressing CDS spreads on lags and controls for economic fundamentals.<sup>3</sup> While the data on the sovereign CDS positions and relevant variables are limited to the last two years (2008-10) for 47 countries, the estimation results indicate that the public debt/GDP and the prediction errors on sovereign spreads are positively associated with the relative gross/net positions for the Euro countries, but not for the non-Euro countries. Larger tax base/GDP is, however, associated with lower gross/net positions in all countries. The evidence suggests that small differences in fundamentals, particularly the tax base, may result in large differences in investors' beliefs and outcomes.

<Table 2>

### 3. A model

Consider a government that starts a period with a public debt overhang  $b\bar{Y}$  and wants to roll it over, subject to the demand of investors.<sup>4</sup> Tax revenue is collected during the period and inflation can be engineered to reduce the debt service. The government offers investors an interest factor of  $R$ ; the real interest rate is  $R-1$ . The investors are risk neutral and require the expected return equals to the risk-free rate,  $R^f$ .

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<sup>3</sup> The control variables are public debt/tax revenue, US interest rate, external debt/GDP, trade/GDP, output growth, GDP per capita, currency depreciation, and inflation rate. See Aizenman, Hutchison and Jinjark (2011) for detailed estimation on the mispricing of sovereign spreads in the context of the 2008-10 crisis.

<sup>4</sup> This section owes to the richer models of Calvo (1988), Cole and Kehoe (2000), and Aizenman and Marion (2010).

An inflation surprise  $\pi$  reduces the real outstanding debt to  $\frac{b\bar{Y}}{1+\pi}$ . Inflation reduces output  $Y = \bar{Y}(1 - \alpha\pi^\theta)$ ;  $\theta \geq 1$ . The government imposes a tax on output at the rate  $t$ , and pays the tax collection cost  $\tau$ , obtaining the net tax revenue  $Y(t - 0.5\tau t^2)$ . The exogenous government spending is  $g\bar{Y}$ . The intertemporal budget constraint is

$$D_2 = R \left[ \frac{b\bar{Y}}{1+\pi} - \bar{Y}(1 - \alpha\pi^\theta)(t - 0.5\tau t^2) + g\bar{Y} \right]$$

$$D_{s+1} = R \left[ D_s - \bar{Y}(t - 0.5\tau t^2) + g\bar{Y} \right]$$

where  $D_s$  is the real public debt at the start of period  $s$ . Forward iterations,

$$\bar{Y} \left( -\alpha\pi^\theta + \frac{R}{R-1} \right) (t - 0.5\tau t^2) = \frac{b\bar{Y}}{1+\pi} + g\bar{Y} \left( \frac{R}{R-1} \right)$$

$$t - 0.5\tau t^2 = \frac{\frac{b}{1+\pi} + g \left( \frac{R}{R-1} \right)}{1 - \alpha\pi^\theta + \frac{1}{R-1}} \equiv \Omega$$

The net tax revenue (the left-hand side) should be sufficient to payoff the long-run fiscal expense share of output,  $\Omega$ . If  $b = 0$  and  $\pi = 0$ , the net tax revenue covers the government expenditure,  $g\bar{Y}$ . If  $g = 0$  and  $\pi = 0$ , the net tax revenue covers the annuity value of the debt overhang as a fraction of permanent output,  $\frac{b(R-1)}{R}$ . The tax Laffer

$$\text{curve is } t(b, \pi, \alpha, \theta, g, \tau, R) = \frac{1 - (1 - 2\tau\Omega)^{\frac{1}{2}}}{\tau}$$

Two conditions determine the equilibrium. First, the government defaults if the net tax revenue is smaller than the long-run fiscal expense as a share of output:

$t - 0.5\tau t^2 < \Omega$ . Denote  $R_{\max}$  the interest factor that corresponds to the maximum

possible value of  $R$ , and  $R_{\min}$  the minimum value (i.e.  $t - 0.5\tau t^2 > \Omega$ ). The default probability density  $f[ ]$ , defined over the values of  $R$  as the implicit function  $q( )$  of

$$\frac{\Omega}{t - 0.5\tau t^2}, \text{ is}$$

$$p = f \left[ q \left( \frac{\Omega}{t - 0.5\tau t^2} \right) \right]$$

In the present model, uncertainty over the public debt servicing stems from the factors  $\{b, \pi, \alpha, \theta, g, \tau\}$  that affect the tax base and the long-run fiscal expense. For example, the default probability increases with initial debt overhang  $b$ ; e.g. due to political uncertainty and a strategic debt accumulation by current policymaker in order to restrain the fiscal resource available to future policymakers.<sup>5</sup> Alternatively, if the inflation rate is distributed uniformly on some interval  $[\bar{\pi} - \varepsilon, \bar{\pi} + \varepsilon]$ ;  $\varepsilon > 0, \bar{\pi} - \varepsilon \geq 0$ , the default is more likely with  $\pi = \bar{\pi} + \varepsilon$ ; on the other hand, the default is less likely if the government has some flexibility to generate a small inflation surprise  $\pi = \bar{\pi} + \varepsilon$ . The default probability becomes 1 for  $R > R_{\max}$ , and 0 for  $R < R_{\min}$ . Any factors that lower the tax base or increase the long-run fiscal expense will increase the default probability. Figure 2 illustrates this condition where  $p$  has a uniform distribution.

<Figure 2>

The second condition comes from the requirement that the expected return demanded by the investors must equal to the risk-free rate. To the investors, a payoff from holding the public debt is 0 with probability  $p$  and  $R$  with probability  $1-p$ . It follows that

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<sup>5</sup> See Alesina and Tabellini (1990).

$$R^f = (1 - p)R$$

that is,

$$p = \frac{R - R^f}{R}$$

Figure 2 shows that the government must offer higher interest factor as the default probability increases.  $R$  is infinite if the default is certain.

Possible equilibria include (i) points where both conditions are met: given the default probability, the investors are willing to hold the debt and the expected tax base covers the long-run fiscal expense at a given interest factor – i.e. points  $x$  and  $z$ ; and (ii) a point where the investors refuses to purchase the public debt at any interest factor because they are certain that the government will not be able to service the debt – i.e.

$$p = 1; R = \infty.$$

At point  $z$ , if the majority of investors believe that the default probability is slightly below  $p_z$ , then they may revise downward the default probability all the way to  $p_x$  (default is effectively a rare event). Similarly, if the majority believe that the default probability is slightly above  $p_z$ , the default probability may rise gradually to the point where no investor is willing to hold the debt at any interest factor. The investor belief may be influenced by the concern whether the government has sufficient tax revenue to service the public debt and long-run fiscal expense.

Consider a possible implication of that a small increase in the risk-free rate may have on two economies with the same fundamentals. In one economy the government is able to generate a small inflation surprise (though not necessarily desirable to the investors), thereby reducing the required debt service. In the other there is an



institutional arrangement that restricts the government from using the inflation surprise. It is plausible that the former converges to  $p_x$ , whereas in the latter the investors are not willing to hold the debt due to inflexibility of the government and prospective default.

Any shift in the investor belief over the relevant factors, including output cost of inflation, government spending, tax evasion and collection cost, and the risk-free rate, will have similar implications to the above example. Default can be self-filling and the investor's beliefs about others' beliefs are relevant to the multiple equilibria.

#### 4. Conclusion

The fiscal challenge may be due to fundamentals and, in the case of the Eurozone countries during 2009-11, the policy inflexibility and market expectations. Managing the public debt can prove to be difficult if the markets price on the investors' beliefs about future fundamentals. We provide empirical support for a link between dispersion in market expectation and the fundamentals. For a majority of countries during the global crisis, the size of tax base is negatively associated with the dispersion of market expectation of default probability.

Uniquely, the negative effects of the amount of debt outstanding and the market mispricing of sovereign risk are specific to the Euro countries. A model of debt crisis with self-fulfilling feature and multiple equilibria can explain this new evidence. Any shift in investors' beliefs about future fundamentals (e.g. induced by the associated cost of financial system bailouts; the resultant benefit of fiscal stimulus) can inadvertently affect the debt-servicing ability, even that of the government with ample tax base and fundamentals.

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Table 1. Public debt in 2006, tax base 2000-06, and sovereign spreads 2008-10.

<u>Country</u>	<u>Public Debt/GDP</u>	<u>Tax/GDP</u>	<u>Sovereign Spread</u>	<u>Country</u>	<u>Public Debt/GDP</u>	<u>Tax/GDP</u>	<u>Sovereign Spread</u>
ARG	0.59	0.25	1701.22	KOR	0.31	0.27	181.50
AUS	0.15	0.32	63.24	LTU	0.23	0.20	326.32
AUT	0.65	0.45	72.38	LVA	0.25	0.19	485.62
BEL	0.93	0.48	52.16	MEX	0.44	0.20	217.95
BGR	0.16	0.28	287.78	MYS	0.43	0.18	125.27
BRA	0.66	0.40	222.89	NLD	0.60	0.40	41.42
CHL	0.06	0.25	130.25	NOR	0.55	0.46	23.75
CHN	0.18	0.10	107.06	NZL	0.23	0.38	64.69
CZE	0.33	0.41	108.42	PER	0.27	0.15	230.81
DEU	0.70	0.37	28.41	PHL	0.49	0.16	292.99
DNK	0.42	0.52	47.51	POL	0.49	0.36	142.12
ESP	0.46	0.39	73.84	PRT	0.71	0.37	67.71
EST	0.06	0.19	281.78	QAT	0.24	0.31	151.00
FRA	0.73	0.46	33.26	RUS	0.09	0.20	313.55
GBR	0.60	0.38	59.76	SVK	0.32	0.35	88.92
GRC	1.07	0.35	124.05	SVN	0.26	0.42	79.50
HRV	0.32	0.23	236.80	SWE	0.40	0.50	52.32
HUN	0.76	0.45	258.96	THA	0.41	0.18	154.82
IDN	0.31	0.18	379.08	TUN	0.43	0.23	188.62
IRL	0.55	0.34	127.47	TUR	0.43	0.31	318.85
ISL	0.86	0.47	471.27	UKR	0.27	0.22	1349.78
ISR	0.76	0.37	135.08	VEN	0.31	0.25	1239.94
ITA	1.11	0.45	87.35	VNM	0.46	0.24	333.85
JPN	2.06	0.28	47.38	ZAF	0.29	0.31	245.65
KAZ	0.09	0.19	442.12				

Table 2. Empirical relationship between fundamentals and gross/net positions on sovereign CDS, 2008-10. This table provides regression analysis of the gross/net sovereign CDS position as a dependent variable (y). The net amount is the position once offsetting contracts have been netted out the gross amount (see also Figure 1). The lagged independent variables (x) are the public debt/GDP, tax revenue/GDP, and prediction errors on sovereign CDS spreads. A constant term is included (not reported). Standard errors are in parentheses; ( ) denotes unadjusted standard errors; < > clustered standard errors; [ ] Newey-West standard errors. \*\*\* (\*\*, \*) denotes statistical significance at 1 (5, 10) percent level.

$y = \frac{Gross}{Net}$ CDS positions 2008-10			
$x =$	Euro area countries	Non-euro countries	All countries
	coeff./s.e.	coeff./s.e.	coeff./s.e.
<u>Public Debt</u> GDP	4.22 (1.41) *** <1.62> ** [1.70] **	-1.72 (2.66) <2.31> [2.75]	-1.28 (2.15) <1.88> [2.22]
<u>Tax Revenue</u> GDP	-34.68 (7.80) *** <7.67> *** [7.48] ***	-23.92 (8.54) *** <7.27> *** [8.64] ***	-30.88 (6.82) *** <6.17> *** [7.32] ***
<u>Actual CDS Spreads</u> Predicted CDS Spreads	1.11 (0.25) *** <0.22> *** [0.23] ***	-0.30 (0.33) <0.26> [0.29]	-0.14 (0.28) <0.24> [0.26]
R-squared	0.71	0.15	0.24
Countries	12	35	47
Observations	24	70	94

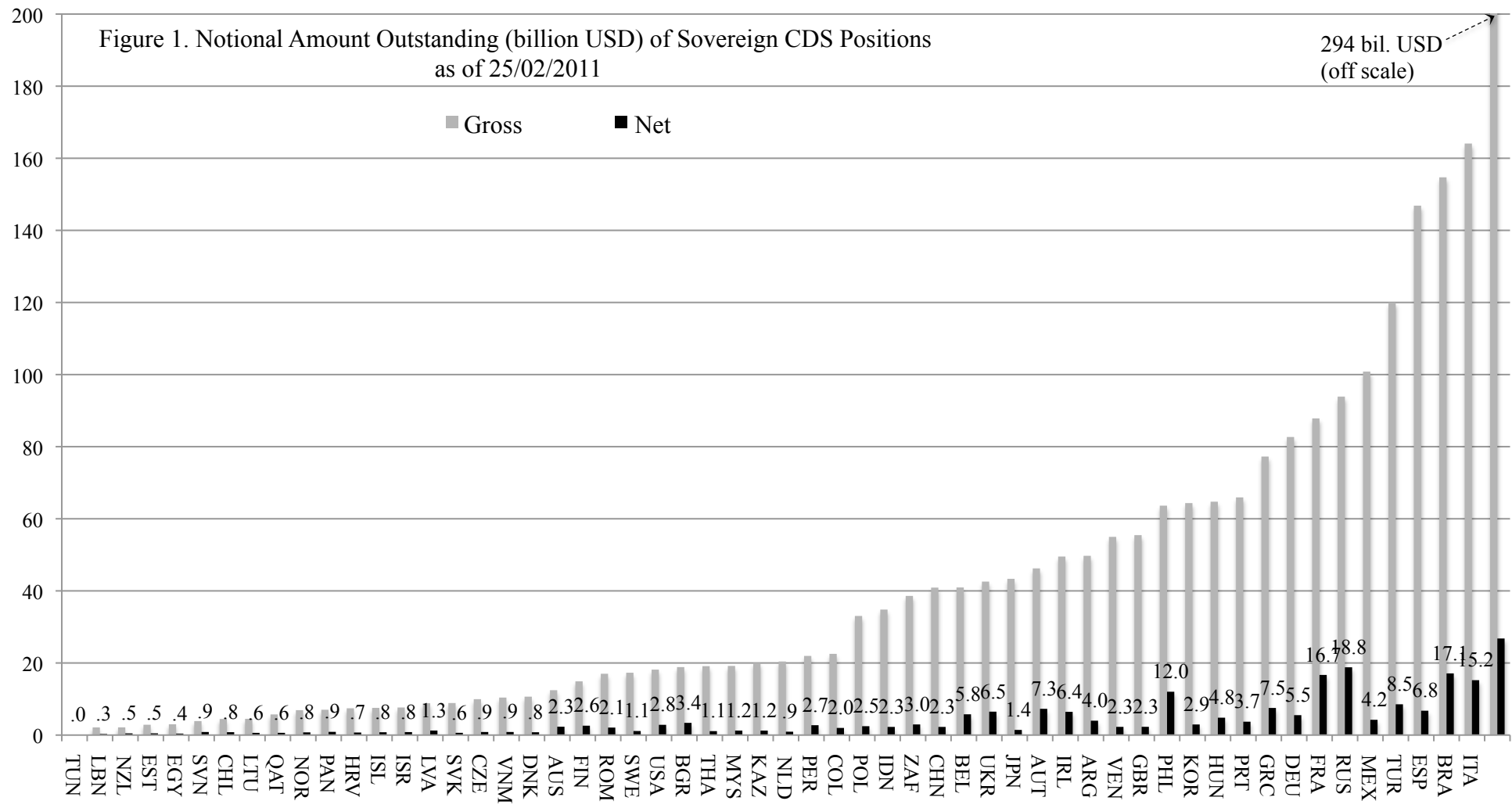


Figure 2. Default probability and interest factor.

