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Dynamic Global Game with Multiple Signals

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Based on the framework of dynamic global game with positive network effects and one type of signal on the optimal timing of irreversible investment, this paper extends the analysis further to cover the case of dynamic global game with multiple signals.

The new results from this paper indicate that when economic agents are facing with multiple types of irreversible investment alternatives and multiple independent signals, dynamic increasing differences are more likely to be violated than the case of single signal. The absence of dynamic increasing differences in this case will be more likely to induce economic agents to invest in the same time others do than in the case of single signal. Therefore, the new results reinforce the tendency towards multiple equilibria.

The policy implication found by this paper suggests that the more imprecise about multiple independent public signals on past investment activities, the more likely investors from different sectors will tend to invest at the same time others do and/or to choose the same type of investment project others do due to the effects of self-fulfilling beliefs.

1. Introduction

Multiple equilibria prevail whenever economic agents' actions are complementary or capable of creating sufficient positive network effect. Policy makers then have to resolve an equilibrium selection problem first before any relevant public policies can be addressed. Carlsson and van Damme (1993) developed an equilibrium selection theory for the case of two-player static coordination games of which being referred to as a global game. The framework was later extended to the case of dynamic global game with only one type of signal by Heidhues and Melissas (2006). The main contribution of dynamic global

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game is to identify the market outcome out of multiple equilibria by iteratively eliminating all the other dominated strategies.

This paper extends the one-type signal dynamic global game model into a dynamic global game with two types of independent signals[2]. This is to complicate the matter further since players must not only choose their optimal investment period but also the type of investment project. One of the examples is the investment problem of choosing an appropriate housing type to be constructed on a piece of vacant land. The remainder of this paper is organized as follows. The model is introduced in the section 2. In section 3, conditions that dynamic increasing differences will be satisfied or violated are identified for the case of multiple independent signals. Conclusion is given in section 4. Proof is provided in the Appendix.

2. The model

There are many risk neutral players who are denoted by i, where $i \in [0,1]$. Each player has the opportunity to choose among two risky irreversible investment projects denoted by A and B. A player can invest at time one, at time two, or not to invest at all. If she invests in project A (B), she has to pay for fixed cost equal to $F_A \geq 0 \left(F_B \geq 0\right)$ and a per-period operating cost $c_A \geq 0 \left(c_B \geq 0\right)$. If a player invests in project A in period one, she gets utility from the whole lifetime of project A (B) equal to

$$\begin{aligned} V_{1j}^i &= \left(\theta_j + n_{1j} - F_j - c_j\right) + \beta\left(\theta_j + n_{1j} + n_{2j} - c_j\right) + \beta^2\left(\theta_j + n_{1j} + n_{2j} - c_j\right) + \dots, \\ &= \frac{1}{1-\beta}\left(\theta_j + n_{1j} + \beta n_{2j} - c_j\right) - F_j, \qquad j = A, B. \end{aligned} \tag{1}$$

Where $\theta_A \left(\theta_B\right)$ is per-period return from project A (B). $\theta = \left(\theta_A, \theta_B\right)$ is a two dimensional vector of state of the world. Each of its elements is independently and randomly drawn from a uniform distribution along the entire real line. $n_{1A}\left(n_{1B}\right)$ and $n_{2A}\left(n_{2B}\right)$ denotes the mass of players who invest in project A (B) at time one and two respectively, where $0 \leq n_{1A} + n_{1B} + n_{2A} + n_{2B} \leq 1$. δ is an interest rate, where $\delta \in (0,1)$. β is a discount factor, where $\beta = 1/(1+\delta)$. If a player invests in project A (B) in period two instead, she gets

[2] See Broadie,M. and J. Detemple (1997) for alternative analysis of the problem of multiple assets with stochastic prices in the absence of network effect.

$$V_{2j}^{i} = \frac{\beta}{1-\beta}\left(\theta_j + n_{1j} + n_{2j} - c_j\right) - \beta F_j, \qquad j = A, B. \qquad (2)$$

If player i decides not to invest in both periods, she gets zero.

All players have two private and imperfect signals concerning the two-dimensional state of the world denoted by $\theta = (\theta_A, \theta_B)$. Player i's first-period signal is $s_{1j}^{i} = \theta_j + \varepsilon_{2j}^{i} + \varepsilon_{1j}^{i}$, where $j = A$ or B, and her second-period signal is $s_{2j}^{i} = \theta_j + \varepsilon_{2j}^{i}$ where $j = A$ or B. The error s $\varepsilon_{2A}^{i}\left(\varepsilon_{2B}^{i}\right)$ are uniformly distributed in the population over the interval $\left[-\varepsilon_A(-\varepsilon_B), \varepsilon_A(\varepsilon_B)\right]$. Half of the population receives an error $\varepsilon_{1A}^{i}\left(\varepsilon_{1B}^{i}\right) = \varepsilon_A(\varepsilon_B)$, and half of the population receives an error $\varepsilon_{1A}^{i}\left(\varepsilon_{1B}^{i}\right) = -\varepsilon_A(-\varepsilon_B)$. Errors $\varepsilon_{1A}^{i}\left(\varepsilon_{1B}^{i}\right)$ and $\varepsilon_{2A}^{i}\left(\varepsilon_{2B}^{i}\right)$ are independently distributed in the population. In addition, errors $\varepsilon_{1A}^{i}\left(\varepsilon_{2A}^{i}\right)$ and $\varepsilon_{1B}^{i}\left(\varepsilon_{2B}^{i}\right)$ are independently distributed in the population. Hence, a quarter of the population receives an error $\varepsilon_{1A}^{i} = \varepsilon_A$ and $\varepsilon_{1B}^{i} = \varepsilon_B$. A quarter of the population receives an error $\varepsilon_{1A}^{i} = \varepsilon_A$ and $\varepsilon_{1B}^{i} = -\varepsilon_B$. A quarter of the population receives an error $\varepsilon_{1A}^{i} = -\varepsilon_A$ and $\varepsilon_{1B}^{i} = \varepsilon_B$. Finally, the last quarter of the population receives an error $\varepsilon_{1A}^{i} = -\varepsilon_A$ and $\varepsilon_{1B}^{i} = -\varepsilon_B$.

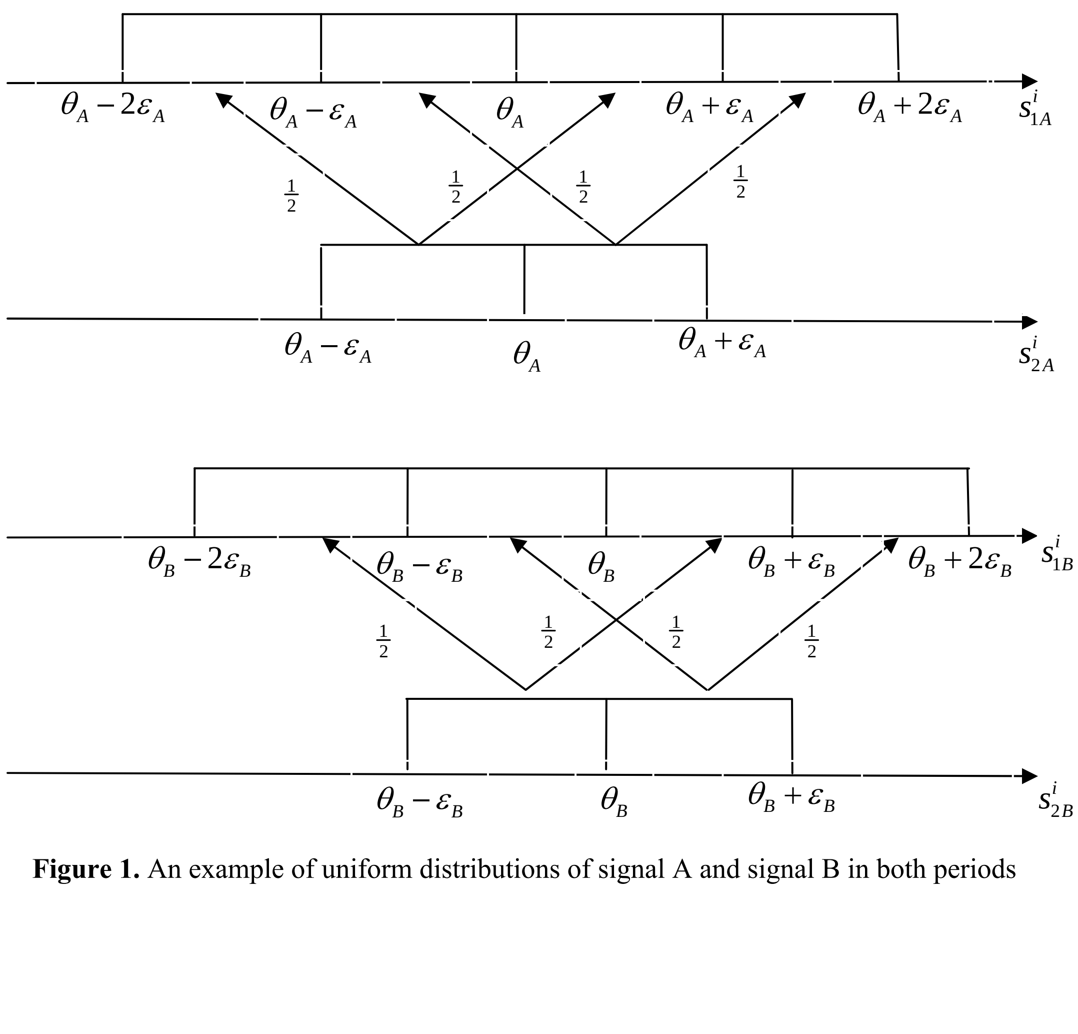

Figure 1. An example of uniform distributions of signal A and signal B in both periods

The first-period signal $s_{1A}^i\left(s_{1B}^i\right)$ is constructed by adding noise to $s_{2A}^i\left(s_{2B}^i\right)$ as proposed in Heidhues and Melissas (2006). In addition, since signals s_{tA}^i and s_{tB}^i are also independently distributed for t = 1 and 2, this implies that

(i) $E\left(\theta_j \mid s_{2j}^i, s_{2l}^i, s_{1j}^i, s_{1l}^i\right) = E\left(\theta_j \mid s_{2j}^i\right), \qquad for \;\; j,l = A,B.,\; j \neq l$,

(ii) $\theta_j \mid s_{2j}^i \sim U\left[s_{2j}^i - \varepsilon_j, s_{2j}^i + \varepsilon_j\right],\; and \;\; E\left(\theta_j \mid s_{2j}^i, s_{2l}^i\right) = s_{2j}^i, \qquad for \;\; j,l = A,B.,\; j \neq l$,

(iii) The first-period signals are also uniformly distributed around $\theta_j, \quad for \;\; j = A,B$.

The timing of the game is as follows:

At time t = 0, nature chooses θ_A and θ_B. All players receive their first-period signals.

At time t = 1, all players simultaneously decide whether to invest or to wait.

At time t = 2, player i observes whether $\varepsilon_{1A}^i\left(\varepsilon_{1B}^i\right) = \varepsilon_A\left(\varepsilon_B\right)$ or $\varepsilon_{1A}^i\left(\varepsilon_{1B}^i\right) = -\varepsilon_A\left(-\varepsilon_B\right)$. Player i neither observes n_{1A} nor n_{1B}. If she did not invest at time one, she decides whether or not to invest at time two.

At time t = 3, All players receive their payoffs.

Each player's time-one action space is denoted by D^1, where D^1 = {invest in A, invest in B, not invest}. Player i's time-two action space is denoted by D^2. Player i's time-two action space is D^2 = {invest in A, invest in B, not invest} if her time-one action equals {not invest}. On the other hand, player i's time-two action space is D^2 = {not invest} if her time-one action either equals {invest in A} or {invest in B}. Player i's time-one observable history is $H_1^i = \left\{s_{1A}^i, s_{1B}^i \mid s_{1A}^i \in \Re, s_{1B}^i \in \Re\right\}$. Her observable history at time two is

$H_2^i = \left\{\left(s_{1A}^i, s_{2A}^i\right),\left(s_{1B}^i, s_{2B}^i\right) \mid s_{1A}^i, s_{1B}^i \in \Re, s_{2A}^i \in \left\{s_{1A}^i - \varepsilon_A, s_{1A}^i + \varepsilon_A\right\}, s_{2B}^i \in \left\{s_{1B}^i - \varepsilon_B, s_{1B}^i + \varepsilon_B\right\}\right\} \times D^1$.

Let's also denote player i's strategy by $\sigma^i = \left(\sigma_{1A}^i, \sigma_{1B}^i, \sigma_{2A}^i, \sigma_{2B}^i\right)$ and denote strategy profile by σ.

Next, define the expected payoff of a player who invests in the second period after getting signals s_{2A}^i, s_{2B}^i, given the strategy profile σ, as

$$h\left(s_{2A}^i, s_{2B}^i, \sigma\right) \equiv E\left\{\max\left\{\left(\frac{\beta}{1-\beta}\left(s_{2A}^i + n_{1A} + n_{2A} - c_A\right) - \beta F_A\right), \left(\frac{\beta}{1-\beta}\left(s_{2B}^i + n_{1B} + n_{2B} - c_B\right) - \beta F_B\right)\right\} \middle| s_{2A}^i, s_{2B}^i, \sigma\right\}. \tag{3}$$

The expected payoff of a player who invests in the first period after getting signals s_{1A}^i, s_{1B}^i, given the strategy profile σ, is defined as

$$h\left(s_{1A}^i, s_{1B}^i, \sigma\right) \equiv \max E\left[\left\{\left(\frac{1}{1-\beta}\left(s_{1A}^i + n_{1A} + \beta n_{2A} - c_A\right) - F_A\right), \left(\frac{1}{1-\beta}\left(s_{1B}^i + n_{1B} + \beta n_{2B} - c_B\right) - F_B\right)\right\} \middle| s_{1A}^i, s_{1B}^i, \sigma\right]. \tag{4}$$

Player i's gain of waiting, given her first signals s_{1A}^i, s_{1B}^i and σ, is defined as

$$\begin{aligned} W\left(s_{1A}^i, s_{1B}^i, \sigma\right) \equiv & \frac{\beta}{4}\max\left\{0, h\left(s_{1A}^i+\varepsilon_A, s_{1B}^i+\varepsilon_B, \sigma\right)\right\}+\frac{\beta}{4}\max\left\{0, h\left(s_{1A}^i-\varepsilon_A, s_{1B}^i-\varepsilon_B, \sigma\right)\right\} \\ & +\frac{\beta}{4}\max\left\{0, h\left(s_{1A}^i+\varepsilon_A, s_{1B}^i-\varepsilon_B, \sigma\right)\right\}+\frac{\beta}{4}\max\left\{0, h\left(s_{1A}^i-\varepsilon_A, s_{1B}^i+\varepsilon_B, \sigma\right)\right\}. \end{aligned} \tag{5}$$

Hence, the optimal condition to invest in the first period for player i with signals s_{1A}^i, s_{1B}^i is as follow

$$g\left(s_{1A}^i, s_{1B}^i, \sigma\right) \equiv h\left(s_{1A}^i, s_{1B}^i, \sigma\right)-W\left(s_{1A}^i, s_{1B}^i, \sigma\right) \geq 0. \tag{6}$$

3. Dynamic increasing differences and uniqueness

The concept of dynamic increasing differences introduced by Heidhues and Melissas (2006) is applied to the case of multiple signals in this section.

The following analysis illustrates that under a very special case that dynamic increasing differences are satisfied for the case of dynamic global games with multiple signals.

Denote the difference in ex-post payoffs between investing in A in the first period and not investing (or keeping the option to invest alive) is defined as

$$\Delta V^i\left(a_1, \Delta\right) \equiv \left\{\frac{1}{1-\beta}\left(\theta_A+n_{1A}+\beta n_{2A}-c_A\right)-F_A\right\}-\Delta, \tag{7}$$

where

$$\begin{aligned} \Delta &= \max\left\{\left(\frac{\beta}{1-\beta}\left(\theta_A+n_{1A}+n_{2A}-c_A\right)-\beta F_A\right),\left(\frac{\beta}{1-\beta}\left(\theta_B+n_{1B}+n_{2B}-c_B\right)-\beta F_B\right)\right\} \\ &= \max\left\{\Delta_A, \Delta_B\right\}. \end{aligned}$$

Hence, Δ is the ex-post value of the option to wait. Suppose that

$$\Delta=\Delta_A=\left(\frac{\beta}{1-\beta}\left(\theta_A+n_{1A}+n_{2A}-c_A\right)-\beta F_A\right), \text{ where } \Delta_A>\Delta_B>0.$$

Then, the difference in ex-post payoffs between investing in A in the first period and not investing can be written as

$$\begin{aligned} \Delta V^i\left(a_1, \Delta_A\right) \equiv \left(\frac{1}{1-\beta}\right)\Big[&\left\{\left(\theta_A+n_{1A}+\beta n_{2A}-c_A\right)-(1-\beta)F_A\right\} \\ &-\beta\left\{\left(\theta_A+n_{1A}+n_{2A}-c_A\right)-(1-\beta)F_A\right\}\Big], \end{aligned} \tag{8}$$

and denote the difference in ex-post payoffs between investing in type A in the second period and not investing by

$$\Delta V^i\left(\Delta_A, 0\right) \equiv \left(\frac{\beta}{1-\beta}\right)\left\{\left(\theta_A+n_{1A}+n_{2A}-c_A\right)-(1-\beta)F_A\right\}. \tag{9}$$

Under the situation that (a) $\Delta_A > \Delta_B > 0$, and (b) $0 < \beta < 1$, then the ex-post payoff functions shown in equations (7) to (9) exhibit dynamic increasing differences if and only if:

(i) $\frac{\partial \Delta V^i\left(\Delta_A, 0\right)}{\partial n_{2A}} = \frac{\beta}{1-\beta} > 0$,

(ii) $\frac{\partial \Delta V^i\left(a_1, \Delta_A\right)}{\partial n_{2A}} = \frac{\beta}{1-\beta} - \frac{\beta}{1-\beta} = 0$,

(iii) $\frac{\partial \Delta V^i\left(\Delta_A, 0\right)}{\partial n_{1A}} - \frac{\partial \Delta V^i\left(\Delta_A, 0\right)}{\partial n_{2A}} = \frac{\beta}{1-\beta} - \frac{\beta}{1-\beta} = 0$,

(iv) $\frac{\partial \Delta V^i\left(a_1, \Delta_A\right)}{\partial n_{1A}} - \frac{\partial \Delta V^i\left(a_1, \Delta_A\right)}{\partial n_{2A}} = 1 - 0 = 1 > 0$.

Condition (i) states that as more players invest in A in the second period, investing in the second period is preferred to not investing. Condition (ii) states that if more people invest in period 2, it does not make any difference either invest early or invest late. Condition (iii) states that if more people move from investing late to investing early, it does not make any difference between invest late and not invest at all. Condition (iv) states that investing early becomes more profitable.

In this special case it turns out that all the conditions of dynamic increasing differences are satisfied. Proposition 1 indicates that in this case there exists an essentially unique rationalizable outcome.

Proposition 1. Given that (a) $\Delta_A > \Delta_B > 0$, (b) $0 < \beta < 1$, and (c) the ex-post payoff function of investing in A satisfies dynamic increasing differences.

Then there exists a unique symmetric switching equilibrium and hence *an essentially unique rational outcome*. (See the proof in Appendix 1)

A strategy profile $k_A^* = \left(k_{1A}^*, k_{2A}^*\right)$ in which $k_{t,A}^* < \infty$ (for t = 1, 2) is an equilibrium (strategy profile) in symmetric switching strategies if and only if it satisfies the following necessary equations

(i) $$\left[\frac{1}{1-\beta}\left(k_{1A}^* + E\left\{n_{1A} + \beta n_{2A} \middle| k_A^*, s_{1A}^i = k_{1A}^*\right\} - c_A\right) - F_A\right] = W\left(s_{1A}^i = k_{1A}^*, k_A^*\right), \qquad (10)$$

where

$$W\left(s_{1A}^i = k_{1A}^*, k_A^*\right) = \frac{\beta}{2}\max\left\{0, \left(\frac{1}{1-\beta}\left(k_{1A}^* + \varepsilon_A + E\left\{n_{1A} + n_{2A} \middle| s_{1A}^i = k_{1A}^*, k_A^*\right\} - c_A\right) - F_A\right)\right\}$$
$$+\frac{\beta}{2}\max\left\{0, \left(\frac{1}{1-\beta}\left(k_{1A}^* - \varepsilon_A + E\left\{n_{1A} + n_{2A} \middle| s_{1A}^i = k_{1A}^*, k_A^*\right\} - c_A\right) - F_A\right)\right\},$$

(ii) $$\left[\frac{1}{1-\beta}\left(k_{1B}^{*}+E\left\{n_{1B}+\beta n_{2B}\middle|k_{B}^{*},s_{1B}^{i}\le k_{1B}^{*}\right\}-c_{B}\right)-F_{B}\right]\le W\left(s_{1B}^{i}\le k_{1B}^{*},k_{B}^{*}\right), \qquad (11)$$

where

$$W\left(s_{1B}^{i}\le k_{1B}^{*},k_{B}^{*}\right)=\frac{\beta}{2}\max\left\{0,\left(\frac{1}{1-\beta}\left(s_{1B}^{i}+\varepsilon_{B}+E\left\{n_{1B}+n_{2B}\middle|s_{1B}^{i}\le k_{1B}^{*},k_{B}^{*}\right\}-c_{B}\right)-F_{B}\right)\right\}$$
$$+\frac{\beta}{2}\max\left\{0,\left(\frac{1}{1-\beta}\left(s_{1B}^{i}-\varepsilon_{B}+E\left\{n_{1B}+n_{2B}\middle|s_{1B}^{i}\le k_{1B}^{*},k_{B}^{*}\right\}-c_{B}\right)-F_{B}\right)\right\},$$

(iii) $$W\left(s_{1A}^{i}=k_{1A}^{*},k_{A}^{*}\right)>W\left(s_{1B}^{i}\le k_{1B}^{*},k_{B}^{*}\right). \qquad (12)$$

Condition (i) states that in equilibrium, returns from investing early in A equals the option value of A. Condition (ii) states that the returns from investing early in B must be lower to or equal the option value of B. Condition (iii) states that the option value of A is larger than the option value of B. Under situation, player i's optimal investment decision is to invest in A in period one.

However, in general investors who have many different risky investment alternatives may realize that there are many more promising projects which will be feasible in the future than those projects in which they are going to invest now. For example, it might be worthwhile for a firm to wait for a brand-new production technology which will be fully commercialized next year than to commit itself to the existing old technology by investing now.

Under such a situation, it is equivalent to say that equation (12) is no longer true, hence one has

$$W\left(s_{1A}^{i}=k_{1A}^{*},k_{A}^{*}\right)<W\left(s_{1B}^{i}\le k_{1B}^{*},k_{B}^{*}\right) \qquad (13)$$

Therefore, the results from Proposition 1 are not applied to this case. This is because the difference in ex-post payoffs between investing in A in the first period and not investing as shown in equation (8) is no longer true. The new equation for the difference in ex-post payoffs between investing in A in the first period and not investing must be such that

$$\Delta V^{i}\left(a_{1},\Delta_{B}\right)\equiv\left(\frac{1}{1-\beta}\right)\Big[\left\{\left(\theta_{A}+n_{1A}+\beta n_{2A}-c_{A}\right)-\left(1-\beta\right)F_{A}\right\}$$
$$-\beta\left\{\left(\theta_{B}+n_{1B}+n_{2B}-c_{B}\right)-\left(1-\beta\right)F_{B}\right\}\Big], \qquad (14)$$

where $$\Delta_{B}=\left(\frac{\beta}{1-\beta}\left(\theta_{B}+n_{1B}+n_{2B}-c_{B}\right)-\beta F_{B}\right)>\Delta_{A}>0,$$

is the highest ex-post value of the option to wait.

Consequently, this inevitably leads to the situation in which the previous condition (ii) of dynamic increasing differences is no longer applied. The new applicable condition must be

(ii) $\frac{\partial \Delta V^i\left(a_1, \Delta_B\right)}{\partial n_{2B}} = -\left(\frac{\beta}{1-\beta}\right) < 0$.

It can be seen that this new condition (ii) states that if more people invest in period 2, it turns out to be not profitable to invest early. Hence, it clearly violates the dynamic increasing differences.

In sum, in this case the absence of dynamic increasing differences implies that investors have more incentives to coordinate in such a way that they attempt to invest at the same time others do and invest in the same type of project chosen by others. Therefore, this can lead to multiple equilibria.[3]

4. Conclusion

This paper extends the existing dynamic global game with single signal introduced by Heidhues and Melissas (2006) into the case in which investors can receive multiple independent signals. This extension is justified on the ground that investors often encounter many different risky and irreversible investment options.

Because of multiple signals, dynamic increasing differences tend to be violated more easily than the case of single signal. This is because investors may realize that there are many more promising projects which will be feasible in the near future than those projects in which they are going to invest now. Therefore, dynamic increasing differences can be violated and can lead to multiple equilibria.

The policy implication found by this paper suggests that the more imprecise about multiple independent public signals on past investment activities, the more likely investors from different sectors will tend to invest at the same time others do and to choose the same type of investment project others do due to the effects of self-fulfilling beliefs.

Future research may shed some light on a more general problem in which all the signals are, to some degree, correlated.

[3] The main idea of the proof should be directly followed from the proof, for the case of single signal, in Heidhues and Melissas (2006)

Appendix 1

The Proof of Proposition 1

First note that since signal A and signal B are independently distribute, then one can replace the optimal condition to invest, says in A, listed in equation (6),

$$g\left(s_{1A}^i, s_{1B}^i, \sigma\right) \equiv h\left(s_{1A}^i, s_{1B}^i, \sigma\right) - W\left(s_{1A}^i, s_{1B}^i, \sigma\right) \geq 0,$$

by the following conditions

(i) $g\left(s_{1A}^i, \sigma_A\right) \equiv h\left(s_{1A}^i, \sigma_A\right) - W\left(s_{1A}^i, \sigma_A\right) \geq 0$,

(ii) $g\left(s_{1B}^i, \sigma_B\right) \equiv h\left(s_{1B}^i, \sigma_B\right) - W\left(s_{1B}^i, \sigma_B\right) \leq 0$,

(iii) $W\left(s_{1A}^i, \sigma_A\right) > W\left(s_{1B}^i, \sigma_B\right)$.

where $W\left(s_{1A}^i, \sigma_A\right) \equiv \dfrac{\beta}{2}\max\left\{0, h\left(s_{1A}^i + \varepsilon_A, \sigma_A\right)\right\} + \dfrac{\beta}{2}\max\left\{0, h\left(s_{1A}^i - \varepsilon_A, \sigma_A\right)\right\}$,

and $W\left(s_{1B}^i, \sigma_B\right) \equiv \dfrac{\beta}{2}\max\left\{0, h\left(s_{1B}^i + \varepsilon_B, \sigma_B\right)\right\} + \dfrac{\beta}{2}\max\left\{0, h\left(s_{1B}^i - \varepsilon_B, \sigma_B\right)\right\}$.

Next, the rest of the proof proceeds as follows. For $0 < \beta < 1$, the difference in ex-post payoffs (of investing in A) between investing in the first period and investing in the second period must be positive,

$$\Delta V^i\left(a_1, \Delta_A\right) \equiv \left\{\frac{1}{1-\beta}\left(\theta_A + n_{1A} + \beta n_{2A} - c_A\right) - F_A\right\} - \left\{\frac{\beta}{1-\beta}\left(\theta_A + n_{1A} + n_{2A} - c_A\right) - \beta F_A\right\} > 0.$$

Next, given also that the difference in ex post payoffs between investing in A and B in the first period is defined as,

$$\Delta V^i\left(a_1, b_1\right) \equiv \left\{\frac{1}{1-\beta}\left(\theta_A + n_{1A} + \beta n_{2A} - c_A\right) - F_A\right\} - \left\{\frac{1}{1-\beta}\left(\theta_B + n_{1B} + \beta n_{2B} - c_B\right) - F_B\right\} > 0.$$

And given that the difference in ex post payoffs between investing in A and B in the second period is such that $\Delta_A > \Delta_B > 0$, or

$$\Delta_A - \Delta_B \equiv \left\{\frac{\beta}{1-\beta}\left(\theta_A + n_{1A} + n_{2A} - c_A\right) - \beta F_A\right\} - \left\{\frac{\beta}{1-\beta}\left(\theta_B + n_{1B} + n_{2B} - c_B\right) - \beta F_B\right\} > 0.$$

Hence, under the assumption that no other player invests in either period, or $\left(n_{1A} = n_{2A} = n_{1B} = n_{2B} = 0\right)$, player i who did not invest in the first period and has signal, in the second period, which satisfied $\left(s_{2A}^i - c_A - (1-\beta)F_A\right) > \left(s_{2B}^i - c_B - (1-\beta)F_B\right)$ would want to invest (in A).

Suppose player i has signal $\left(s_{1A}^i - c_A - (1-\beta)F_A - \varepsilon_A\right) > \left(s_{1B}^i - c_B - (1-\beta)F_B - \varepsilon_B\right)$ in period one. From the assumption of the distribution of signals used by this study, player i can foresee that she would also want to invest in time two. Then she should decide to invest immediately in the first period to save the waiting cost.

Therefore, under the assumption that no other player invests in either period, player i will invest immediately once her first period signal is higher than some constant thresholds $\overset{-1}{s_{1A}}$.

This means that, in time one, dynamic increasing differences ensure that all players who receive signals $s_{1A}^{i} > \bar{s}_{1A}$ will prefer investing early to investing late.

One the other hand, under the assumption that no other player invests in the first period, dynamic increasing differences ensure that all players, in time two, who receive signal $s_{2A}^{i} > c_{A} + (1-\beta) F_{A}$ will prefer investing (in A) to not investing. So player i should invest once her second period signal, s_{2A}^{i}, is higher than some constant thresholds $\overset{-1}{s_{2A}}$, where $\overset{-1}{s_{2A}} > c_{A} + (1-\beta) F_{A}$.

In sum, there must be a vector of threshold defined as $s_{A}^{1} = \left(\overset{-1}{s_{1A}}, \overset{-1}{s_{2A}}\right)$.

As the number of early investors (in A) increase, dynamic increasing differences imply that waiting becomes less desirable. Then we can compute new thresholds vector $\overset{-2}{s_{A}} = \left(\overset{-2}{s_{1A}}, \overset{-2}{s_{2A}}\right)$ such that if $s_{1A}^{i} = \overset{-2}{s_{1A}}$, player i is indifferent between investing (in A) and waiting.

Repeating this procedure, one can get a decreasing sequence of threshold vectors. This sequence must converge to a symmetric switching equilibrium.

For players with sufficiently low signals the dominant strategy is not to invest, even if many other players invest. So one can find an increasing sequence of threshold vectors below which every player would not want to invest. This sequence also converges to a symmetric switching equilibrium.

To complete the proof, suppose the iterative elimination from above and below converge to different symmetric switching equilibria. Then in a lower equilibrium, a player, who receives a signal equal to the first-period threshold level (i.e., k_{1A}), is more optimistic about the fundamental than a player in a higher equilibrium. So a player in the lower equilibrium must expect less investment activity in the higher equilibrium. In the higher equilibrium, a player with a signal equal to the second-period threshold level (i.e., k_{2A}) expects less investment activity only if less players have already invest in the first-period which means that $k_{1A} > k_{2A}$. This requires that $k_{1A} - k_{2A}$ must have a higher value in the higher equilibrium. However, a player with a signal equal to k_{1A} expects a lower level of investment activity only if k_{2A} is relatively higher. This implies that $k_{1A} - k_{2A}$ must have a lower value in the higher equilibrium, which results in contradiction.

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