Overlapping soft budget constraints

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Abstract

At a time when the virtues of decentralization are widely extoled, this paper emphasizes an additional limit. The transfer of responsibility from the central to the regional governments for carrying out transfers policy at the local level undermines macroeconomic discipline under decentralized leadership. Empowering regional governments creates an overlapping equalization policy which worsens the soft budget constraint issue in the country. Contrary to Qian and Roland (1998), we also show that the fiscal competition among regional rescuers does not act as a commitment device to harden the local budget constraint.

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1 Introduction

To decentralize or not to decentralize? The question is not an innovative one, at least in a two-tier governmental framework. Decentralization is purported to enhance efficiency when expenditure and taxation decisions are carried out at the level that best respects citizens' preferences (Tiebout (1956)). Decentralization, however, has also been cited as a source of inefficiency when lower-level governments fail to internalize the costs or benefits that their fiscal choices impart on citizens outside of their jurisdiction. An important element of the decentralization debate has been its impact on the strategic behaviour of sub-national governments. That is, lower-level governments are said to have soft budget constraints when they expect and receive additional resources from the central government after they have selected their expenditure, revenue-raising, or borrowing levels. The incentive to bail out *ex post* on the part of the central government has been examined both theoretically and empirically[1], and is attributed to the national government's inability to commit to not bailing out lower-level governments when they have violated their budgets constraints. The systematic intervention from the top is either driven by the presence of interregional spillovers proportional to the size of the region (the "too big to fail" argument of Wildasin (1997)) or by an aim of equalization of marginal utilities from the public good provision across regions (Goodspeed (2002), Köthenbürger (2004), Breuillé, Madiès and Taugourdeau (2006)). Empirical evidence suggests that the soft budget constraint problem can be a very serious one for countries where lower-level governments have broad spending and borrowing powers and the central government has the discretionary power to intervene on their behalf (Rodden (2002), Rodden, Eskeland, and Litvack (2003)).

The passionate debate amongst decentralization's advocates and skeptics has been strikingly restricted to a vertical structure with only one layer of sub-national jurisdiction. Whether more than one layer of sub-national jurisdiction with differing responsibilities affects the advantages or disadvantages of decentralization is still an open question. Which layer should collect which taxes? Which layer should provide which public goods? How should intergovernmental transfers be organized? On this last point, we observe in practice a large variety of intergovernmental transfer designs in countries where multiple subnational levels exist with broad spending powers (OECD (2003)). Whereas in some - often centralized - countries, the top level directly allocates transfers to middle and bottom levels, in some others, transfers to the bottom levels pass through the middle levels. The German intergovernmental fiscal transfer system is an example of this latter scheme whereby vertical transfers from the Bund to the Länder parallel those designed by each Land authority for its municipalities. These vertical transfers co-exist with horizontal transfers amongst the Länder governments.

[1] See Vigneault (2007) for a survey.

A multi-level and overlapping intergovernmental transfer scheme, when viewed from the transfer recipient's perspective, may lead to an overlapping soft budget constraint problem; that is, the dissolution of transfer responsibility may worsen the incentive for lower-level governments to behave strategically in attracting additional resources from higher-level governments. Our paper investigates this issue in a multi-tier fiscal architecture composed of a top layer (central or federal), an intermediate layer (provinces, states or regions) and a bottom layer (cities, municipalities or districts). In this setting, the intermediate layer of jurisdiction is both a "rotten kid" vis-à-vis the top layer and a "good samaritan" vis-à-vis the bottom layer. In our symmetric three-layer framework, transfers are granted according to an overlapping upward equalization scheme. Each region allocates transfers to the cities located in its territory to equalize marginal utilities from the local public good provision. Then, the central government redistributes public funds across regions to equalize marginal utilities from regional public good provision. As in Goodspeed (2002), Köthenbürger (2004), and Breuillé, Madiès and Taugourdeau (2006), equalization schemes in our model constitute an incentive for higher-level governments to provide additional resources to lower-level governments *ex post.* In addition to overlapping equalization schemes, we consider the importance of decentralized leadership vis-à-vis the higher-level governments; that is, we examine regional leadership vis-à-vis the central jurisdiction and local leadership vis-à-vis their regional and central jurisdictions. The redistributive policy, which is implemented *ex post*, is anticipated by the beneficiaries when they make their budgetary choices[2]. By combining these ingredients, *i.e.* overlapping equalization with decentralized leadership, we obtain an overlapping soft budget constraint problem.

Our model of regional decentralized leadership is similar to Köthenbürger (2004), with the important difference that the central transfer scheme now also depends on the regional transfer scheme (in addition to regional tax effort). The regional governments at this intermediate level compete both for extracting transfers from the top and for attracting mobile capital. Regions thus simultaneously choose the (distortive) tax and transfer policy while fully anticipating how the top layer will respond. Unlike in Köthenbürger (2004), we also have cities select their (non-distortive) tax policy while fully anticipating both how the regional layer will respond to local decisions and how these regional decisions will affect decisions at the top.

The first issue examined in the paper is how the overlapping of the equalization policies, i.e. the coexistence of a vertical equalization scheme between the central ju-

[2] Local and regional leadership are particularly appropriate for characterizing the intergovernmental relationships in a bottom-up system, and even more so in a system of multiple mandates. The fact that the regional decision-maker is also a Member of Parliament enables him to expect perfectly the central reaction function. Constitutional reasons, e.g. equity in the country in terms of public goods consumption wherever citizens are located like in Germany, or fixed rules of the equalization schemes also allow the infra-layer to anticipate the determinants of the allocation of public funds.

risdiction and regional jurisdictions on one hand and a vertical equalization scheme between each region and the cities located in its territory on the other hand, affects the soft budget constraint issue in a country with decentralized leadership. We show that regional decentralization, *i.e.* the transfer of responsibility from the central to the regional governments for carrying out transfers policy at the city level, undermines macroeconomic discipline under decentralized leadership. Empowering regional governments creates an overlapping equalization policy which worsens the soft budget constraint issue in the country. We observe a kind of snowball effect - the softer the regional budget constraint, the softer the local budget constraint - that is due to the fact that the region is even more generous towards the cities than the central government is toward the region itself. By extension, increasing the number of layers within a decentralized structure can have a negative impact on macroeconomic discipline. This negative aspect of decentralization had been overlooked in the literature.

The second important issue examined in the paper is whether competition among regional rescuers to attract a mobile factor acts as a commitment device and hardens local budget constraints. This latter issue has been explored by Qian and Roland (1998), who find that decentralization in the presence of regional competition does indeed serve to harden local budget constraints. Like us, they also examine a multi-tier framework, but not a three-tier governmental one. In particular, the bottom layer in their model is comprised of state enterprises, whereas in ours it is local governments. However, like Qian and Roland, our regional government layer competes both for mobile capital and transfers from the central government. Qian and Roland determine the conditions under which the budget constraint is hard or soft. By contrast, we endogenously determine the levels of the central and regional bailouts in the presence of regional tax competition and analyze their impact on regional and local fiscal choices. Our results show that the presence of a three-tier governmental structure combined with decentralized leadership alters Qian and Roland's main finding: horizontal tax competition among regional rescuers does not act as a commitment device to harden the local budget constraint. Whether the bailout to cities is financed by a regional lump-sum tax or a distortive tax on mobile capital has no impact on the inability of the region to commit dynamically not to bail out. This result is explained by the fact that transfers to regions are designed by the central government in a way to internalize externalities due to tax competition. As pointed out by Köthenbürger, decentralized commitment insulates regions from harmful tax competition. The positive "competition effect" put forward by Qian and Roland (1998) therefore does not exist in our model.

The paper is organized as follows. Section 2 presents the model. Section 3 determines the outcome of the three-tier architecture without decentralized leadership at the regional and local levels. Section 4 proceeds to the analysis with decentralized leadership.

2 The model

The territorial architecture is a three-tier intergovernmental structure comprised of a central/federal government indexed by c, n identical regional/state governments indexed by i, and, within each region, m identical local/city governments indexed by j. The local government in ij finances a local public good provided in quantity g_{ij} with a lump-sum tax, t_{ij}, levied on its immobile citizens and with a transfer from the regional government, s_{ij}. The local budget constraint is thus given by:

$$g_{ij} = t_{ij} + s_{ij}. \tag{1}$$

The regional government in i provides a regional public good G_i and transfers $\{s_{ij}\}_j$ to the cities located within the region. These expenditures are financed with a tax τ_i on the capital K_i invested in i and with a transfer from the central government, S_i. The regional budget constraint is thus given by:

$$G_i = \tau_i K_i + S_i - \sum_j s_{ij}. \tag{2}$$

Neither cities nor regions are allowed to run a deficit in this one period model, and so public good provision is adjusted so that the budget is balanced.

The central transfers are distributed according to an horizontal net equalization scheme, where transfers granted to one region are financed by contributions made by the other regions:

$$\sum_i S_i = 0.$$

Such schemes have been introduced in several countries, e.g. Denmark and Germany (Seitz (1999)), leading at times to serious moral hazard issues. The generality of our results is not affected by the amount of resources devoted to transfers by the central government, but a key assumption is that these resources are exogenous.

The representative household located in city ij derives utility from the consumption of a private good, c_{ij}, the local public good, g_{ij}, and the regional public good, G_i, according to the function:

$$U(c_{ij}, g_{ij}, G_i) = u(c_{ij}) + v(g_{ij}) + V(G_i), \tag{3}$$

where $u(.)$, $v(.)$, and $V(.)$ are assumed to be strictly increasing, twice differentiable, and concave. Private consumption for the representative household satisfies the budget constraint:

$$c_{ij} = \frac{\Pi_i}{m} + \rho\frac{\widetilde{K}}{m} - t_{ij}, \tag{4}$$

where $\frac{\widetilde{K}}{m}$ is the household's exogenous initial capital endowment which can be invested in immobile regional firms to earn a net return ρ, $\frac{\Pi_i}{m}$ is the household's share of the income generated by the firm located in region i, and t_{ij} is the local/city lump-sum tax[3].

The firm in region i produces an homogeneous consumption good according to the production function $F(K_i)$, where $F(.)$ is strictly increasing, twice differentiable and concave. The amount of capital K_i borrowed by the firm in i on the national market is remunerated at the gross rate $r_i = \rho + \tau_i$, and maximizes profits given by

$$\Pi_i(r_i, K_i) = F(K_i) - r_i K_i.$$

The resulting demand for capital $K_i(r_i)$ and profit $\Pi_i(r_i)$ are both decreasing functions of the interest rate r_i: $K_i'(r_i) = \frac{1}{F''} < 0$ and $\Pi_i'(r_i) = -K_i < 0$.

Capital is perfectly mobile across regions so that it relocates until it earns the same post-tax return ρ in each region:

$$\rho = r_i - \tau_i = r_{-i} - \tau_{-i}.$$

For the exogenous supply of capital $n\widetilde{K}$ in the country, equilibrium in the capital market is such that the following condition,

$$\sum_i K_i(r_i) = n\widetilde{K},$$

is satisfied. This market-clearing condition implicitly defines the net return to capital $\rho(\boldsymbol{\tau})$, with $\boldsymbol{\tau} = (\tau_1, ...\tau_n)$. Differentiating the market-clearing condition yields:

$$\frac{\partial\rho}{\partial\tau_i} = -\frac{K_i'}{\sum\limits_i K_i'} = -\frac{1}{n}, \quad \frac{\partial r_i}{\partial\tau_i} = 1 + \frac{\partial\rho}{\partial\tau_i} = \frac{n-1}{n}, \quad \frac{\partial r_i}{\partial\tau_{-i}} = \frac{\partial\rho}{\partial\tau_{-i}} = -\frac{1}{n}. \quad (5)$$

Before we proceed to the analysis with decentralized leadership, we first present the outcome of the three-tier architecture without decentralized leadership at the regional and local levels. This outcome will serve as a benchmark for comparison purposes to highlight the impact of overlapping soft budget constraints on the budgetary decisions at the equilibrium.

3 The overlapping HBCs benchmark

In the benchmark case, the central government, the n regional governments and the m local governments simultaneously select their budgetary parameters, with

[3]We implicitly assume that the firm is owned by the households of the region. The number of firms in each region and the number of households in each city are both normalized to one without loss of generality.

each one taking as given the policy choices of the other players. Regardless of the commitment device used by the central (resp. regional) rescuer, the regional (resp. local) government does not manipulate its budgetary decisions in order to attract more transfers. The regional (resp. local) budget constraint is therefore a hard one. The transfers to regions $\{S_i\}_i$ chosen by the central government to maximize national welfare $\sum_i \sum_j U(c_{ij}, g_{ij}, G_i)$, equalize the marginal utilities from the regional public good provision; i.e. $V'(G_i) = V'(G_{-i})\ \forall i$. The transfers to cities $\{s_{ij}\}_j$, chosen by each region i to maximize regional welfare $\sum_j U(c_{ij}, g_{ij}, G_i)$, equalize the marginal utilities from the regional and local public good provision in the region; i.e. $v'(g_{i,j}) = v'(g_{i,-j}) = mV'(G_i)\ \forall i, j$. Note that in the symmetric equilibrium, $S_i = s_{ij} = 0\ \forall i, j$. Each city ij in turn chooses t_{ij} to equalize the marginal utilities for private consumption and the local public good provision in order to maximize local welfare $U(c_{ij}, g_{ij}, G_i)$; i.e. $u'(c_{ij}) = v'(g_{ij})$. The outcome with overlapping hard budget constraints thus satisfies:

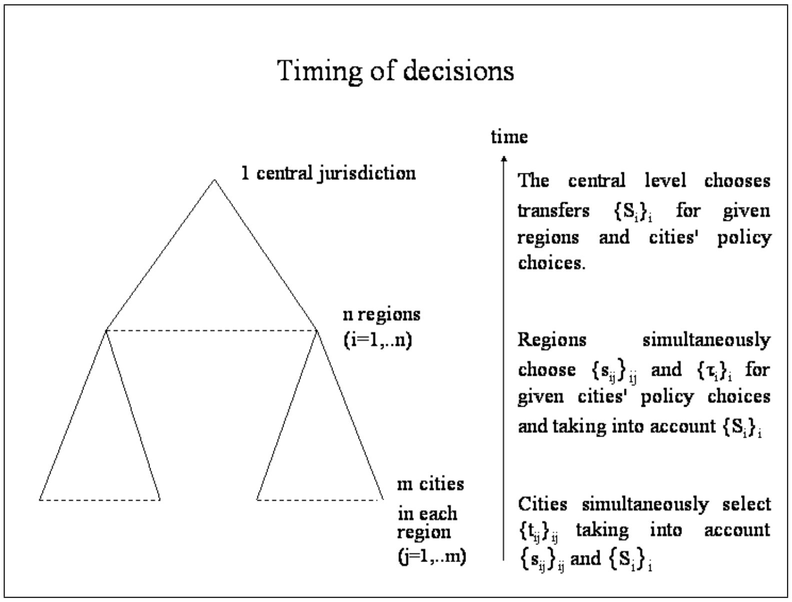
$$
\begin{aligned}
u'(c_{ij}) &= u'(c_{i,-j}) = u'(c_{-i,j}) = u'(c_{-i,-j}) && \text{(6a)}\\
&= v'(g_{ij}) = v'(g_{i,-j}) = v'(g_{-i,j}) = v'(g_{-i,-j}) && \text{(6b)}\\
&= mV'(G_i) = mV'(G_{-i}). && \text{(6c)}
\end{aligned}
$$

4 Overlapping SBCs with regional tax competition

Regions and cities are now assumed to act as Stackelberg leaders vis-à-vis the higher layer. The diagram below provides a visual description of the overlapping of the equalization policy in the three-tier country.

Specifically, regional governments allocate transfers to the local governments within their region to help finance expenditures on the local public good. The central government in turn allocates transfers to the regional governments to help finance expenditures on the regional public good and on transfers to local governments. Neither the central nor the regional governments are able to commit to a transfer scheme in advance of budgetary decisions of lower-level governments.

The timing of decisions is as follows. Firstly, local governments simultaneously select their tax policy to maximize the welfare of the citizens residing within their city, taking into account the reaction of the regional and central governments. In doing so, they play as Nash competitors vis-à-vis each other, but as Stackelberg leaders vis-à-vis the higher layers of government. Secondly, given the local budgetary decisions, the regional governments select their tax rates and transfers to cities to maximize the welfare of citizens residing within their region, taking into account



the reaction of the central government. Regional governments are thus Stackelberg followers vis-à-vis the local governments, Nash competitors with other regional governments and Stackelberg leaders vis-à-vis the central government. Thirdly, the central government allocates transfers to regions to maximize welfare of all citizens countrywide, for given regional and local policy choices. Finally, transfers are paid, taxes are collected, regional and local public goods are provided as residuals of the budgetary decisions and households consume. We determine the subgame-perfect equilibrium by solving the governments' choice problems backwards; i.e. from the top to the bottom.

4.1 The central government's problem

Given the budgetary choices of regions and cities, the central government designs a transfer scheme for regions to maximize national welfare according to:

$$\underset{\mathbf{S}}{Max} \quad \sum_i \sum_j \left[u\left(c_{ij}\right) + v\left(g_{ij}\right)\right] + m \sum_i V\left(G_i\right),$$

subject to

$$
\begin{aligned}
c_{ij} &= \frac{\Pi_i}{m} + \rho\frac{\widetilde{K}}{m} - t_{ij}, \\
g_{ij} &= t_{ij} + s_{ij}, \\
G_i &= \tau_i K_i + S_i - \sum_j s_{ij}, \\
\sum_i S_i &= 0.
\end{aligned}
$$

From the first-order conditions,

$$V'(G_i) = V'(G_{-i}) \qquad \forall\ i, -i, \tag{7}$$

we see that transfers are allocated to regions so as to equalize marginal utility from the regional public good consumption countrywide. This aim of equalization combined with decentralized leadership is the cause of the soft budget constraint problem at the regional level because regions take into account that the central government will respond to their budgetary choices by altering its transfers so that condition (7) holds. They can thus exploit this incentive to their advantage by strategically selecting their policy variables. To derive explicitly the central best-reply to a change in the regional policy, we differentiate the central government's first-order conditions with respect to $\tau_i, \boldsymbol{\tau}_{-i}, s_{ij}, \mathbf{s}_{-i,j}$ and $S_i\ \ \forall i, -i$:

$$
\begin{aligned}
& V''\left[\frac{\partial \tau_i K_i}{\partial \tau_i} d\tau_i + \frac{\partial \tau_i K_i}{\partial \tau_{-i}} d\tau_{-i} + dS_i - \sum_j ds_{ij}\right] \\
= \;\; & V''\left[\frac{\partial \tau_{-i} K_{-i}}{\partial \tau_{-i}} d\tau_{-i} + \frac{\partial \tau_{-i} K_{-i}}{\partial \tau_i} d\tau_i + dS_{-i} - \sum_j ds_{-i,j}\right] \qquad \forall i, -i,
\end{aligned}
$$

and use the budget constraint $\sum\limits_i S_i\left(\tau_i, \boldsymbol{\tau}_{-i}, s_{ij}, \mathbf{s}_{-i,j}\right) = 0$

$$\frac{\partial S_i}{\partial \tau_i} = -\sum_{-i} \frac{\partial S_{-i}}{\partial \tau_i}, \qquad \frac{\partial S_{-i}}{\partial \tau_{-i}} = -\sum_{k \neq i} \frac{\partial S_{-k}}{\partial \tau_{-i}}\ , \quad \text{and} \qquad \frac{\partial S_i}{\partial s_{ij}} = -\sum_{-i} \frac{\partial S_{-i}}{\partial s_{ij}}.$$

Summing, combining equations, and invoking symmetry yields[4]:

[4] We assume that the elasticity of the regional tax base with respect to the regional tax rate, ε_i, belongs to the interval $]-1, 0]$ which is in line with empirical findings (see Chirinko, Fazzari, Steven and Meyer (1999) for instance).

$$\frac{\partial \widehat{S}_i}{\partial \tau_i} = \frac{n-1}{n}\left[-\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial \tau_{-i} K_{-i}}{\partial \tau_i}\right] < 0; \tag{8a}$$

$$\frac{\partial \widehat{S}_i}{\partial \tau_{-i}} = \frac{1}{n}\left[-\frac{\partial \tau_i K_i}{\partial \tau_{-i}} + \frac{\partial \tau_{-i} K_{-i}}{\partial \tau_{-i}}\right] > 0; \tag{8b}$$

$$\frac{\partial \widehat{S}_i}{\partial s_{ij}} = \frac{\partial \widehat{S}_i}{\partial s_{i,-j}} = \frac{n-1}{n} \qquad \text{and} \tag{8c}$$

$$\frac{\partial \widehat{S}_i}{\partial s_{-i,j}} = \frac{\partial \widehat{S}_i}{\partial s_{-i,-j}} = -\frac{1}{n}. \tag{8d}$$

The best-reply of central transfers $\{\widehat{S}_i(\tau_i, \boldsymbol{\tau}_{-i}, s_{ij}, \mathbf{s}_{-i,j})\}_i$ depends both on regional tax rates $\boldsymbol{\tau}$ and regional transfer schemes $\mathbf{s}$. Like Köthenbürger (2004), a change in τ_i exerts two opposite effects on the central transfer to region i: on the one hand, any increase in region i's tax revenues is captured to be redistributed equally among all the regions; on the other hand, any capital outflow from region i, which increases other regions' tax revenues, is partially compensated by contributions made by the other regions, which ensures the equalization of marginal utilities *ex post*. When combining these two effects, the global influence of an increase in region i's tax effort is a reduction in region i's central transfer, which benefits the other regions. In a standard way, the externalities linked to capital mobility across regions are perfectly internalized by the transfer scheme designed by the top layer. But unlike in Köthenbürger (2004), the central transfers also react to the regional transfer policy. When region i's government increases its transfer by one dollar to any of the cities within its jurisdiction there is a reduction in funds available for the regional public good. The central government responds to this reduction by cutting all other regions' transfers by $\frac{1}{n}$ and transfers these funds to region i. Similarly, when another region increases its transfer by one dollar to any of the cities within its jurisdiction, region i's transfer from the central government is reduced by $\frac{1}{n}$.

4.2 The regional government's problem

The government of region i selects its tax rate τ_i and transfer scheme $\{s_{ij}\}_j$ to maximize the welfare of citizens within its jurisdiction. In doing so, it takes into account the central government's reaction to its choices in the next stage of the game and takes as given the policy parameters chosen by the other regions and the cities. The problem for the regional government is thus to:

$$\underset{\tau_i, \mathbf{s}_i}{Max} \quad \sum_j \left[u\left(c_{ij}\right) + v\left(g_{ij}\right)\right] + mV\left(G_i\right),$$

subject to

$$\begin{aligned} c_{ij} &= \frac{\Pi_i}{m} + \rho \frac{\widetilde{K}}{m} - t_{ij}, \\ g_{ij} &= t_{ij} + s_{ij}, \\ G_i &= \tau_i K_i + \widehat{S}_i - \sum_j s_{ij}, \end{aligned}$$

expecting the reaction of the central government according to (8a) to (8d). The first-order conditions are:

$$/\tau_i \quad : \quad \frac{1}{m} \sum_j u' \left[\Pi_i' \frac{\partial r_i}{\partial \tau_i} + \frac{\partial \rho}{\partial \tau_i} \widetilde{K} \right] + mV' \left[\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial S_i}{\partial \tau_i} \right] = 0, \tag{9a}$$

$$/s_{ij} \quad : \quad v' + mV' \left[\frac{\partial S_i}{\partial s_{ij}} - 1 \right] = 0, \tag{9b}$$

which determine the regional government's reaction functions $\widehat{\tau}_i(t_1, ...t_m; \boldsymbol{\tau}_{-i})$ and $(\widehat{s}_{i1}(\cdot), ..\widehat{s}_{im}(\cdot); \mathbf{s}_{-i})$. Solving the first-order conditions for all regions simultaneously determines the Nash equilibrium levels of regional tax rates and transfers.

To gain more insight into the regional governments' incentives at the symmetric equilibrium, we simplify the first-order condition for τ_i to yield[5]:

$$V' = \frac{-u' \left[\Pi_i' \frac{\partial r_i}{\partial \tau_i} + \frac{\partial \rho}{\partial \tau_i} \widetilde{K} \right]}{m((1 + \varepsilon_i) K_i + \frac{\partial S_i}{\partial \tau_i})} = \frac{u'}{m(1 + \varepsilon_i + \frac{1}{\widetilde{K}} \frac{\partial S_i}{\partial \tau_i})} = \frac{n}{m} u', \tag{10}$$

where $\varepsilon_i = \frac{\partial K_i}{\partial \tau_i} \frac{\tau_i}{K_i} < 0$ is the elasticity of capital invested in region i with respect to region i's tax rate. Under a hard budget constraint ($i.e.$ $\frac{\partial S_i}{\partial \tau_i} = 0$), the region has no incentive to internalize the positive tax externality that an increase in its tax rate has on other regions' tax revenues. Under a soft budget constraint, the expectation of a bailout from the central government forces each region to take into account the impact its tax policy has on other regions, and thus absorbs the distortionary effects of tax competition on regional budgetary choices.

Combining the first-order conditions (9a) and (9b) with the central government's aim of equalization $V'(G_i) = V'(G_{-i})$ yields:

$$\begin{aligned} u'(c_{ij}) &= u'(c_{i,-j}) = u'(c_{-i,j}) = u'(c_{-i,-j}) && \text{(11a)} \\ &= v'(g_{ij}) = v'(g_{i,-j}) = v'(g_{-i,j}) = v'(g_{-i,-j}) && \text{(11b)} \\ &= \frac{m}{n} V'(G_i) = \frac{m}{n} V'(G_{-i}). && \text{(11c)} \end{aligned}$$

[5] At the symmetric equilibrium, given $K_i = \widetilde{K}\ \forall i$, $\left[\Pi_i' \frac{\partial r_i}{\partial \tau_i} + \frac{\partial \rho}{\partial \tau_i} \widetilde{K} \right] = -K_i \frac{(n-1)}{n} - \frac{1}{n} \widetilde{K} = -K_i$ and $\left[\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial S_i}{\partial \tau_i} \right] = \left[\frac{1}{n} \frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{n-1}{n} \frac{\partial \tau_{-i} K_{-i}}{\partial \tau_i} \right] = \left[\frac{1}{n} K_i + \frac{1}{n} \tau_i K_i' \frac{\partial r_i}{\partial \tau_i} + \frac{n-1}{n} \tau_{-i} K_{-i}' \frac{\partial \rho}{\partial \tau_i} \right] = \frac{1}{n} K_i$.

A comparison of the equilibrium with overlapping soft budget constraints (11) with the equilibrium with overlapping hard budget constraints (6) shows that for a number of regions $n > 1$, regional decentralized leadership results in too much consumption of the private and local public goods and too little expenditure on the regional public good. Put differently, the region undertaxes capital and overpays transfers to cities. The region wants to extract as many transfers as possible from the central government and knows that the lower the regional public good, the higher the transfer from the top. The softness of the regional budget constraint leads the region to distort its budgetary choices toward more transfers to cities and less taxes, both reducing G_i, as soon as the cost of the bailout is not entirely born by the region itself that is for $n > 1$.

We next examine the best-reply of regional transfers to the local governments' choice of tax rate. Differentiating the first-order conditions and invoking symmetry yields[6]:

$$\frac{\partial \widehat{s}_{ij}}{\partial t_{ij}} = -1 + \frac{B}{\det A} < -1; \tag{12a}$$

$$\frac{\partial \widehat{s}_{i,-j}}{\partial t_{ij}} = \frac{B}{\det A} < 0; \tag{12b}$$

$$\frac{\partial \widehat{\tau}_i}{\partial t_{ij}} = \frac{C}{\det A} < 0; \tag{12c}$$

where

$$\begin{aligned} B &= \frac{\partial c_{ij}}{\partial \tau_i}\left[\frac{\partial S_i}{\partial s_{ij}} - 1\right] u''v''\frac{m}{n}V'' - \left[\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial S_i}{\partial \tau_i}\right] u''v''\frac{m}{n}V'' > 0; \\ C &= -u''\left[v''v'' - \left[\frac{\partial S_i}{\partial s_{i,-j}} - 1\right] v''\frac{m}{n}V'' - \left[\frac{\partial S_i}{\partial s_{ij}} - 1\right] v''\frac{m}{n}V''\right] \\ &\quad + \left[\frac{\partial S_i}{\partial s_{ij}} - 1\right] v''v''\frac{m}{n}V'' > 0; \\ \det A &= \left[\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial S_i}{\partial \tau_i}\right] v''v''\frac{m}{n}V'' - \\ &\quad u''\frac{\partial c_{ij}}{\partial \tau_i}\left[v''v'' - \left[\frac{\partial S_i}{\partial s_{i,-j}} - 1\right] v''\frac{m}{n}V'' - \left[\frac{\partial S_i}{\partial s_{ij}} - 1\right] v''\frac{m}{n}V''\right] < 0. \end{aligned}$$

Ultimately, all of the regional policy parameters respond to a change in the local lump-sum tax t_{ij} with the aim of equalizing marginal utilities according to condition (11). A change in t_{ij} directly affects private consumption (and hence u') and local public good provision (and hence v'). This brings forth changes in τ_i (to offset the

[6] See Appendix A for the derivation of these results.

impact on private consumption) and changes in s_{ij} and $s_{i,-j}$ (to offset the impact on the local public good provision). Then, changes in τ_i, s_{ij}, $s_{i,-j}$ elicit responses from the central government through changes in S_i, S_{-i} to keep $V_i' = V_{-i}'$. All of these responses serve to satisfy the aim of equalization (condition (11)) *ex post*.

4.3 Implications of overlapping equalization with SBCs

We can use the comparative statics of the regional government's choice of transfers to local governments to examine in more detail the role of overlapping soft budget constraints. Recall that a local government faces a soft budget constraint when a reduction in its tax effort elicits an increase in the regional government's transfer. The regional government too faces a soft budget constraint when both a reduction in its tax effort and an increase in its transfers to local governments elicit an increase in the central government's transfer. Consider then the following comparative static exercise:

$$
\begin{aligned}
\frac{\partial}{\partial \frac{\partial \widehat{S}_i}{\partial s_{ij}}} \frac{\partial \widehat{s}_{ij}}{\partial t_{ij}} &= \frac{\partial}{\partial \frac{\partial \widehat{S}_i}{\partial s_{ij}}} \frac{\partial \widehat{s}_{i,-j}}{\partial t_{ij}} = \frac{\frac{\partial c_{ij}}{\partial \tau_i} u'' v'' \frac{m}{n} V'' \det A - B \frac{\partial c_{ij}}{\partial \tau_i} u'' v'' \frac{m}{n} V''}{[\det A]^2} < 0 \\
\frac{\partial}{\partial \frac{\partial \widehat{S}_i}{\partial \tau_i}} \frac{\partial \widehat{s}_{ij}}{\partial t_{ij}} &= \frac{\partial}{\partial \frac{\partial \widehat{S}_i}{\partial \tau_i}} \frac{\partial \widehat{s}_{i,-j}}{\partial t_{ij}} = \frac{-u'' v'' \frac{m}{n} V'' \det A - B v'' v'' \frac{m}{n} V''}{[\det A]^2} < 0 \\
\frac{\partial}{\partial \frac{\partial \widehat{S}_i}{\partial s_{i,-j}}} \frac{\partial \widehat{s}_{ij}}{\partial t_{ij}} &= \frac{\partial}{\partial \frac{\partial \widehat{S}_i}{\partial s_{i,-j}}} \frac{\partial \widehat{s}_{i,-j}}{\partial t_{ij}} = \frac{-B \frac{\partial c_{ij}}{\partial \tau_i} u'' v'' \frac{m}{n} V''}{[\det A]^2} < 0 \\
\frac{\partial}{\partial \frac{\partial \widehat{S}_i}{\partial s_{ij}}} \frac{\partial \widehat{\tau}_i}{\partial t_{ij}} &= \frac{\left[u'' v'' \frac{m}{n} V'' + v'' v'' \frac{m}{n} V''\right] \det A - \frac{\partial c_{ij}}{\partial \tau_i} C u'' v'' \frac{m}{n} V''}{[\det A]^2} > 0 \\
\frac{\partial}{\partial \frac{\partial \widehat{S}_i}{\partial \tau_i}} \frac{\partial \widehat{\tau}_i}{\partial t_{ij}} &= \frac{-C v'' v'' \frac{m}{n} V''}{[\det A]^2} > 0 \\
\frac{\partial}{\partial \frac{\partial \widehat{S}_i}{\partial s_{i,-j}}} \frac{\partial \widehat{\tau}_i}{\partial t_{ij}} &= \frac{u'' v'' \frac{m}{n} V'' \det A - C \frac{\partial c_{ij}}{\partial \tau_i} u'' v'' \frac{m}{n} V''}{[\det A]^2} > 0.
\end{aligned}
$$

This exercise highlights the role of overlapping transfers schemes in worsening the soft budget constraint problem in the country. All of the comparative static results above show that the regional government's incentive to provide additional transfers to cities is worsened when the central government itself is victim of this incentive vis-à-vis regional governments. To summarize,

Proposition 1 *The softer the regional budget constraint, the softer the local budget constraint. The overlapping equalization policy worsens the soft budget constraint* *problem in the country, which provides a strong argument against the decentralization to regional governments of local transfer policy.*

4.4 The local government's problem

As first players of the game, the local governments select their lump-sum taxes to maximize the utility of citizens within their jurisdiction, while expecting the best-reply of the regional and central governments to their local choices in the next stages of the game. Thus, they take into account the subgame-perfect Nash equilibrium of the sequential game played between the central and regional levels of government. The problem for local government in ij is thus to

$$\underset{t_{ij}}{Max} \quad u\left(c_{ij}\right)+v\left(g_{ij}\right)+V\left(G_i\right),$$

subject to

$$\begin{aligned} c_{ij} &= \frac{\Pi_i}{m}+\rho\frac{\widetilde{K}}{m}-t_{ij}, \\ g_{ij} &= t_{ij}+\hat{s}_{ij}, \\ G_i &= \widehat{\tau}_i K_i+\hat{S}_i-\sum_j \hat{s}_{ij}, \end{aligned}$$

expecting the reaction function of the regional government with respect to the local

government's choice of taxation given by (12), as well as the reaction function of the central government with respect to regional budgetary choices given by (8). Differentiating $U(c_{ij}, g_{ij}, G_i)$, invoking symmetry, substituting the values for $\frac{\partial\widehat{S}_i}{\partial s_{ij}} = \frac{\partial\widehat{S}_i}{\partial s_{i,-j}} = \frac{n-1}{n}$, $\frac{\partial c_{ij}}{\partial\tau_i} = \frac{\Pi_i'}{m}\frac{\partial r_i}{\partial\tau_i}+\frac{\partial\rho}{\partial\tau_i}\frac{\widetilde{K}}{m} = -\frac{K_i}{m}$, $\frac{\partial\widehat{S}_i}{\partial\tau_i} = \frac{n-1}{n}\left[-\frac{\partial\tau_i K_i}{\partial\tau_i}+\frac{\partial\tau_{-i}K_{-i}}{\partial\tau_i}\right]$, $\frac{\partial\hat{s}_{ij}}{\partial t_{ij}} = -1+\frac{B}{\det A} < -1$, $\frac{\partial\hat{s}_{i,-j}}{\partial t_{ij}} = \frac{B}{\det A} < 0$, $\frac{\partial\widehat{\tau}_i}{\partial t_{ij}} = \frac{C}{\det A} < 0$ and using $u' = v' = \frac{m}{n}V'$ from the regional government's problem we obtain:

$$\begin{aligned} \frac{\partial U(c_{ij}, g_{ij}, G_i)}{\partial t_{ij}} &= u'\left[\frac{\partial c_{ij}}{\partial\tau_i}\frac{\partial\widehat{\tau}_i}{\partial t_{ij}}-1\right]+v'\left[1+\frac{\partial\widehat{s}_{ij}}{\partial t_{ij}}\right]+ \\ & \quad V'\left[\left[\frac{\partial\tau_i K_i}{\partial\tau_i}+\frac{\partial\widehat{S}_i}{\partial\tau_i}\right]\frac{\partial\widehat{\tau}_i}{\partial t_{ij}}+\left[\frac{\partial\widehat{S}_i}{\partial s_{ij}}-1\right]\frac{\partial\widehat{s}_{ij}}{\partial t_{ij}}+\sum_{k\neq j}\left[\frac{\partial\widehat{S}_i}{\partial s_{i,k}}-1\right]\frac{\partial\widehat{s}_{i,k}}{\partial t_{ij}}\right] \\ &= u'\left[-\frac{K_i}{m}\frac{C}{\det A}-1\right]+v'\left[\frac{B}{\det A}\right]+V'\left[\frac{1}{n}K_i\frac{C}{\det A}+\frac{1}{n}-\frac{m}{n}\frac{B}{\det A}\right] \\ &= -u'+\frac{1}{n}V' = -v'+\frac{v'}{m} < 0 \quad \text{for } m>1. \end{aligned}$$

For $m > 1$, the derivative of utility with respect to t_{ij} being always negative, the only way to raise utility is to lower t_{ij} until the threshold $t_{ij} = 0$[7]. As a consequence, the local public good is entirely financed by transfers from the regional government. The intuition for this result is more easily understood by examining the local government's budget constraint. For $m = 1$, the representative citizen in ij fully bears the cost of the regional transfers so that the local lump-sum tax and the regional transfer are sustitutes in financing the local public good. But when $m > 1$, there is clearly a preference for the local government to finance the public good solely from the regional transfer because the cost of the transfer to the city ij is partially borne by citizens residing in other cities. This negative vertical tax externality results in an incentive to the local government to reduce its tax effort as much as possible - zero in this case.

4.5 Implications of regional tax competition

To challenge Qian and Roland (1998)'s main result according to which "fiscal competition among local governments under factor mobility increases the opportunity cost of the bailout and thus serves as a commitment device", we consider an alternative set-up of the model in which there is no tax competition amongst regional governments. Specifically, region i now levies a lump-sum tax, T_i, on regional citizens to help finance the transfers to cities within the region and expenditure on the regional public good. Budget constraints are thus modified as follows:

$$
\begin{aligned}
c_{ij} &= \frac{\Pi_i}{m} + \rho\frac{\widetilde{K}}{m} - T_i - t_{ij}, \\
g_{ij} &= t_{ij} + s_{ij}, \\
G_i &= mT_i + S_i - \sum_j s_{ij}, \\
\sum_i S_i &= 0.
\end{aligned}
$$

The central government still sets transfers to regions so as to equalize marginal utilities from the regional public good provision, according to condition (7). The best-reply[8] of central transfers $\{S_i^n(T_i, \mathbf{T}_{-i}, s_{ij}, \mathbf{s}_{-i,j})\}_i$ now depends both on regional lump-sum taxes $\mathbf{T}$ and regional transfers schemes $\mathbf{s}$:

[7] We implicitly exclude subsidies to citizens ($t_{ij} < 0$).

[8] Again, the results are obtained by differentiating the first-order condition $V'(G_i) = V'(G_{-i})$ and using the central government's budget constraint $\sum_i S_i (T_i, \mathbf{T}_{-i}, s_{ij}, \mathbf{s}_{-i,j}) = 0$. Let "$n$" be the superscript for the absence of tax competition.

$$\frac{\partial S_i^n}{\partial T_i} = -\frac{(n-1)m}{n}; \tag{13a}$$
$$\frac{\partial S_i^n}{\partial T_{-i}} = \frac{m}{n}; \tag{13b}$$
$$\frac{\partial S_i^n}{\partial s_{ij}} = \frac{\partial S_i^n}{\partial s_{i,-j}} = \frac{n-1}{n}; \tag{13c}$$
$$\frac{\partial S_i^n}{\partial s_{-i,j}} = \frac{\partial S_i^n}{\partial s_{-i,-j}} = -\frac{1}{n}. \tag{13d}$$

The reaction of central transfers to an increase in the lump-sum tax of region i depends both on the size of the regional tax base, m, and the number of regions, n. When the region i's government increases by one dollar the lump-sum tax levied on each of its citizens, *ceteris paribus*, the region i's public good is overprovided in region i. Guided by the aim of equalization among regions, the central government responds by cutting the region i's transfers in proportion to the regional tax base m by $-\frac{(n-1)m}{n}$ to affect these funds to the $(n-1)$ other regions. The reaction of the central transfers to the regional transfers' policy is unchanged.

Solving the maximization program of each region i, we derive that the conditions at the symmetric equilibrium turn out to be the same as the conditions (11). The resulting best-reply of the region i w.r.t. local tax policy[9] are:

$$\frac{\partial s_{ij}^n}{\partial t_{ij}} = -1 + \frac{B'}{\det A'} < -1; \tag{14a}$$
$$\frac{\partial s_{i,-j}^n}{\partial t_{ij}} = \frac{B'}{\det A'} < 0; \tag{14b}$$
$$\frac{\partial T_i^n}{\partial t_{ij}} = \frac{C'}{\det A'} < 0 \tag{14c}$$

where

[9] See Appendix B for a proof of these results.

$$
\begin{aligned}
B' &= -\frac{(m-1)}{n} u'' v'' \frac{m}{n} V'' > 0; \\
C' &= -u'' \left[v''v'' - \left[\frac{\partial S_i^n}{\partial s_{i,-j}} - 1 \right] v'' \frac{m}{n} V'' - \left[\frac{\partial S_i^n}{\partial s_{ij}} - 1 \right] v'' \frac{m}{n} V'' \right] \\
&\qquad\qquad + \left[\frac{\partial S_i}{\partial s_{ij}} - 1 \right] v''v'' \frac{m}{n} V'' > 0; \\
\det A' &= \left[m + \frac{\partial S_i^n}{\partial T_i} \right] v''v'' \frac{m}{n} V'' + \\
&\qquad u'' \left[v''v'' - \left[\frac{\partial S_i^n}{\partial s_{i,-j}} - 1 \right] v'' \frac{m}{n} V'' - \left[\frac{\partial S_i^n}{\partial s_{ij}} - 1 \right] v'' \frac{m}{n} V'' \right] < 0.
\end{aligned}
$$

From the comparison of the best-reply $\{\widehat{s}_{ij}\}_j$ when regional transfers are financed by a tax on mobile capital (12) with the best-reply $\{s_{ij}^n\}_j$ when regional transfers are financed by a lump-sum tax (14), we are able to state that tax competition among regions has no impact on the opportunity cost of the regional bailout provided to cities for the case where $u''(c_{ij}) = u''(c_{ij}^n)$, $v''(g_{ij}) = v''(g_{ij}^n)$, $V''(G_i) = V''(G_i^n)$, which is satisfied under the assumption that the third derivative of the utility functions $u(.), v(.)$ and $V(.)$ is nul. Indeed, we can show[10], using the result that $\det A = \frac{K_i}{m} \det A'$:

$$
\begin{aligned}
\frac{\partial s_{ij}^n}{\partial t_{ij}} &= \frac{\partial \widehat{s}_{ij}}{\partial t_{ij}} \\
\frac{\partial s_{i,-j}^n}{\partial t_{ij}} &= \frac{\partial \widehat{s}_{i,-j}}{\partial t_{ij}} \\
\frac{\partial T_i^n}{\partial t_{ij}} &= \frac{K_i}{m} \frac{\partial \widehat{\tau}_i}{\partial t_{ij}}
\end{aligned}
$$

which can be summarized by the following proposition:

Proposition 2 *The amount of the bailout to cities is the same whether it is financed by a regional lump-sum tax or a regional tax on mobile capital. Contrary to Qian and Roland (1998), horizontal tax competition among the rescuers does not harden the local budget constraint.*

This (apparently surprising) result is explained by the fact that transfers to regions are designed by the central government in a way to internalize externalities due to regional tax competition. The positive "competition effect" put forward by Qian and Roland has thus disappeared because of the existence of equalizing transfers to the regional rescuers. Note that local governments still have an incentive to lower the lump-sum tax until the threshold $t_{ij} = 0$.

[10] See Appendix C for a proof these results.

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5 Appendices

5.1 Appendix A

Differentiating the FOCs w.r.t. $t_{ij}, t_{i,-j}$ and $\tau_i, s_{ij}, s_{i,-j}$:

$$\frac{m}{n}V''\left[\frac{\partial \tau_i K_i}{\partial \tau_i}d\tau_i + \frac{\partial S_i}{\partial \tau_i}d\tau_i + \frac{\partial S_i}{\partial s_{ij}}ds_{ij} + \frac{\partial S_i}{\partial s_{i,-j}}ds_{i,-j} - \sum_j ds_{ij}\right] = u''\left[\frac{\partial c_{ij}}{\partial \tau_i}d\tau_i - dt_{ij}\right]$$

$$v''\left[dt_{ij} + ds_{ij}\right] = \frac{m}{n}V''\left[\frac{\partial \tau_i K_i}{\partial \tau_i}d\tau_i + \frac{\partial S_i}{\partial \tau_i}d\tau_i + \frac{\partial S_i}{\partial s_{ij}}ds_{ij} + \frac{\partial S_i}{\partial s_{i,-j}}ds_{i,-j} - \sum_j ds_{ij}\right]$$

$$v''ds_{i,-j} = \frac{m}{n}V''\left[\frac{\partial \tau_i K_i}{\partial \tau_i}d\tau_i + \frac{\partial S_i}{\partial \tau_i}d\tau_i + \frac{\partial S_i}{\partial s_{ij}}ds_{ij} + \frac{\partial S_i}{\partial s_{i,-j}}ds_{i,-j} - \sum_j ds_{ij}\right]$$

and simplifying it, given the symmetry, yields the following system:

$$\begin{pmatrix} \left[\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial S_i}{\partial \tau_i}\right]\frac{m}{n}V'' - u''\frac{\partial c_{ij}}{\partial \tau_i} & \left[\frac{\partial S_i}{\partial s_{ij}} - 1\right]\frac{m}{n}V'' & \left[\frac{\partial S_i}{\partial s_{i,-j}} - 1\right]\frac{m}{n}V'' \\ -\left[\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial S_i}{\partial \tau_i}\right]\frac{m}{n}V'' & v'' - \left(\frac{\partial S_i}{\partial s_{ij}} - 1\right)\frac{m}{n}V'' & -\left[\frac{\partial S_i}{\partial s_{i,-j}} - 1\right]\frac{m}{n}V'' \\ -\left[\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial S_i}{\partial \tau_i}\right]\frac{m}{n}V'' & -\left[\frac{\partial S_i}{\partial s_{ij}} - 1\right]\frac{m}{n}V'' & v'' - \left[\frac{\partial S_i}{\partial s_{i,-j}} - 1\right]\frac{m}{n}V'' \end{pmatrix}\begin{pmatrix} d\tau_i \\ ds_{ij} \\ ds_{i,-j} \end{pmatrix} = \begin{pmatrix} -u'' \\ -v'' \\ 0 \end{pmatrix} dt_{ij}.$$

We compute the reaction function with the Cramer's rule. *Q.E.D.*

5.2 Appendice B

Differentiating the FOCs w.r.t. $t_{ij}, t_{i,-j}$ and $\tau_i, s_{ij}, s_{i,-j}$

$$\frac{m}{n}V''\left[mdT_i + \frac{\partial S_i}{\partial T_i}dT_i + \frac{\partial S_i}{\partial s_{ij}}ds_{ij} + \frac{\partial S_i}{\partial s_{i,-j}}ds_{i,-j} - \sum_j ds_{ij}\right] = u''\left[-dT_i - dt_{ij}\right]$$

$$v''\left[dt_{ij} + ds_{ij}\right] = \frac{m}{n}V''\left[mdT_i + \frac{\partial S_i}{\partial T_i}dT_i + \frac{\partial S_i}{\partial s_{ij}}ds_{ij} + \frac{\partial S_i}{\partial s_{i,-j}}ds_{i,-j} - \sum_j ds_{ij}\right]$$

$$v'' ds_{i,-j} = \frac{m}{n} V'' \left[m dT_i + \frac{\partial S_i}{\partial T_i} dT_i + \frac{\partial S_i}{\partial s_{ij}} ds_{ij} + \frac{\partial S_i}{\partial s_{i,-j}} ds_{i,-j} - \sum_j ds_{ij} \right]$$

and simplifying it, given the symmetry, yields the following system:

$$\begin{pmatrix} \left[m + \frac{\partial S_i}{\partial T_i}\right] \frac{m}{n} V'' + u'' & \left[\frac{\partial S_i}{\partial s_{ij}} - 1\right] \frac{m}{n} V'' & \left[\frac{\partial S_i}{\partial s_{i,-j}} - 1\right] \frac{m}{n} V'' \\ -\left[m + \frac{\partial S_i}{\partial T_i}\right] \frac{m}{n} V'' & v'' - \left[\frac{\partial S_i}{\partial s_{ij}} - 1\right] \frac{m}{n} V'' & -\left[\frac{\partial S_i}{\partial s_{i,-j}} - 1\right] \frac{m}{n} V'' \\ -\left[m + \frac{\partial S_i}{\partial T_i}\right] \frac{m}{n} V'' & -\left[\frac{\partial S_i}{\partial s_{ij}} - 1\right] \frac{m}{n} V'' & v'' - \left[\frac{\partial S_i}{\partial s_{i,-j}} - 1\right] \frac{m}{n} V'' \end{pmatrix} \begin{pmatrix} d\tau_i \\ ds_{ij} \\ ds_{i,-j} \end{pmatrix} = \begin{pmatrix} -u'' \\ -v'' \\ 0 \end{pmatrix} dt_{ij}.$$

We compute the reaction function with the Cramer's rule. *Q.E.D.*

5.3 Appendice C

Substituting the values of the best-reply of the central government with regional tax competition (8) and without regional tax competition (13) in the respective best-reply of the regional government:

$$\frac{\partial s_{ij}^n}{\partial t_{ij}} = -1 + \frac{-\frac{(m-1)}{n} u'' v'' \frac{m}{n} V''}{\det A'}$$

$$\frac{\partial s_{ij}}{\partial t_{ij}} = -1 + \frac{K_i}{m} \frac{-\frac{(m-1)}{n} u'' v'' \frac{m}{n} V''}{\det A}$$

$$\frac{\partial s_{i,-j}^n}{\partial t_{ij}} = \frac{1}{\det A'} \left[-\frac{(m-1)}{n} u'' v'' \frac{m}{n} V'' \right]$$

$$\frac{\partial s_{i,-j}}{\partial t_{ij}} = \frac{1}{\det A} \frac{K_i}{m} \left[-\frac{(m-1)}{n} u'' v'' \frac{m}{n} V'' \right]$$

$$\frac{\partial T_i^n}{\partial t_{ij}} = \frac{C'}{\det A'}$$

$$\frac{\partial \tau_i}{\partial t_{ij}} = \frac{C}{\det A}$$

and assuming $u''(c_{ij}) = u''(c_{ij}^n)$, $v''(g_{ij}) = v''(g_{ij}^n)$, $V''(G_i) = V''(G_i^n)$, which is the case for the third derivative equal to 0, we obtain $\det A = \frac{K_i}{m} \det A'$, which as a consequence leads to:

$$\frac{\partial s_{ij}^n}{\partial t_{ij}} = \frac{\partial s_{ij}}{\partial t_{ij}}, \qquad \frac{\partial s_{i,-j}^n}{\partial t_{ij}} = \frac{\partial s_{i,-j}}{\partial t_{ij}}, \qquad \frac{\partial T_i^n}{\partial t_{ij}} = \frac{K_i}{m} \frac{\partial \tau_i}{\partial t_{ij}}.$$

Q.E.D.