Ramsey Pricing Equilibria
In Commercial TV Broadcasting

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Abstract

We consider a model of commercial television market, where private broadcasters coexist with a public television broadcaster. Assuming that the public TV station follows a policy of Ramsey pricing whereas the private stations are profit maximizers, we consider the equilibria in this market and compare with a situation where the public station is privatized and acts as another private TV broadcaster. A closer scrutiny of the market for commercial television leads to a distinction between target rating points, which are the prime unit of account in TV advertising, and net coverage, which is the final goal of advertisers. Working with net coverage as the fundamental concept, we exploit the models of competition between public and private price and quantity in order to show that privatization of the public TV station entails a welfare loss and results in TV advertising becoming more expensive.

Keywords: TV broadcasting, imperfect competition, Ramsey pricing, welfare comparison.

JEL classification: L11, L82, L33

1. Introduction

During the last decades, the European markets for commercial television broadcasting has undergone profound changes. In many countries, public television stations with little if any income from advertising have lost their monopoly status through the opening up of competition from commercial TV stations, and former budget financing of TV broadcasting has now been supplemented or replaced by commercial financing. As a consequence, the market for TV broadcasting services has obtained several characteristics of the American

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market; the concern for number of viewers has come to the forefront, with its consequences for choices of program profile and program quality, and the market for TV advertisement has developed into a complex one with its own special features. On the other hand, most European countries still retain some presence of public activity in this field, and the resulting mixture of public and private activity adds new features to the market.

Commercial TV broadcasting is an example of a fixed cost technology; the program structure has to be established, with its ability to catch and retain audiences, before advertising time can be sold. As is well known to the academic profession, fixed cost technologies have the drawback that textbook models of competition work poorly, and we should expect some kind of imperfect competition in such markets. Moreover, attempts to establish perfect competition in this market are futile due to the purely technological conditions.

It might be added that TV markets display other forms of unusual performance; since commercial TV broadcasting involves the general public which are important as potential audiences but are not paying for what they are viewing, we may run into the situations originally investigated by Steiner (1952), where a monopoly broadcaster is able to provide a better program profile to the totality of viewers than competitive broadcasters having to their disposal the same number of channels. We shall not involve this type of welfare problem in the present investigation, since we do not introduce any measure of program diversity, but such problems clearly add to the caveats in connection with privatizing public TV stations.

In the paper, we begin our discussion with a treatment of markets with both public and private television broadcasters by concentrating on the particular form of imperfect competition prevailing, so that the peculiarities of the specific field of business, TV broadcasting, is kept in the background. This leads to the concept of Ramsey-Cournot or Ramsey-Bertrand competition, where public firms are guided by Ramsey pricing whereas private firms are profit maximizers. When moving to the Ramsey-Bertrand situation (in Section 3), we add the particular feature of the TV markets that production decisions (choice of programs) have to be made before the sales can take place.

As might be expected, the presence of a public firm whoose choices are guided by consumer welfare rather than by profits has considerable influence on the outcome; it might however be argued that Ramsey pricing remains a theoretical construction and that real life public TV stations can hardly be assumed to follow principles that they have never heard about. We show in Section 4 that replacing Ramsey pricing by other objectives ("viewer satisfaction") does not change the results in a fundamental way.

In the sections to follow we turn to a more detailed consideration of the specific features of commercial TV broadcasting, the production of audiences with the purpose of putting them to the disposal of advertisers. While most of the literature on imperfect competition in TV broadcasting has used standard functional forms for cost and demand functions, we have included the specific features of the sector to derive production and cost functions which are relevant for the questions at hand. In Section 5 we discuss the fundamental unit of account in TV advertising, the *target rating point* (TRP), as well as the concept of net coverage which is not directly observable (as the TRP) but is what matters to advertisers. Then, in Section 6 we return to the consideration of the market, showing that with the specific properties of the market added to the general structure as described in the earlier sections, the conclusions get enforced so that the changes from public/private to fully private TV broadcasting emerges as having negative consequences for all involved parties except the existing private broadcasters. We conclude in Section 7 by indicating some directions of further investigation.

2. Ramsey-Cournot equilibria

In this section, we consider a situation where three firms, one public firm (indexed by $i = 1$) and two profit maximizing private firms (indexed by $i = 2, 3$), compete in the market for a consumption good. We assume that the public firm is suggested to supply the good to the public in such a way that consumer welfare is maximized under the constraint that the cost incurred should be covered by incomes from selling to the public. This would be a standard case for Ramsey pricing (which of course in this initial case with a single good reduces to a simple pricing rule), except for the existence of private firms competing for market shares.

For reasons to become clearer in the following sections, we assume that there is Cournot competition in the market, i.e. that the strategic variables in the market are the quantities q_1, q_2, q_3 supplied by the three firms. Actually this is in line with most of the recent contributions to the literature on competition among TV broadcasters, cf. e.g. Masson e.a. (1990), Papandrea (1997), Nilssen and Sørgaard (2000), Bourreau (2003), Mangàni (2003). Let the demand of the consumers be given by

$$q = D(p),$$

where p is the market price of the good, q is total quantity demanded, and $D(\cdot)$ is assumed to be strictly decreasing, so that its inverse D^{-1} is well-defined. As is usual in models of market behaviour, we measure consumer welfare at the price p with associated consumption $q = D(p)$ by consumer surplus

$$S(p) = \int_p^\infty D(p)\, dp.$$

Then we may formulate a Ramsey-Cournot equilibrium in this market as a triple

(q_1^0, q_2^0, q_3^0), with total supply $q^0 = q_1^0 + q_2^0 + q_3^0$ and associated price $p^0 = D^{-1}(q^0)$, such that

(i) q_1^0 maximizes $S(p^0)$ under the budget constraint

$$D^{-1}(q^0)q_1^0 - C_1(q_1^0) \geq 0,$$

(ii) for $i = 2, 3$, q_i^0 maximizes profits $D^{-1}(q_i + \sum_{j\in\{1,2,3\}\setminus\{i\}} q_j^0)q_i - C_i(q_i)$.

As mentioned above, the public pricing rule becomes very simple indeed in the present case: The choice q_1^0 by the public firm should satisfy the first order condition

$$S'(D^{-1}(q^0))\frac{1}{D'(p^0)} - \lambda\left(D^{-1}(q^0) + q_1^0\frac{1}{D'(p^0)} - C_1'(q_1^0)\right) = 0,$$

where λ is the Lagrange multiplier associated with the budget constraint, or, after inserting $S'(p^0) = -q^0$ and rearranging

$$\frac{p^0 - C_1'(q_1^0)}{p^0} = \frac{1}{\epsilon_1} + \frac{1}{\lambda}\frac{1}{\epsilon}, \tag{1}$$

where $\epsilon = -pD'(p)/D(p)$ is the elasticity of demand, and

$$\epsilon_1 = -\frac{pD'(p)}{D(p) - q_2^0 - q_3^0} = -\frac{pD'(p)}{q_1}$$

is the elasticity of (perceived) demand for firm 1. For the profit maximizing firms, first order conditions are

$$D^{-1}(q^0) + q_i^0\frac{1}{D'(p^0)} - C_i'(q_i^0) = 0$$

which transform to the well-known condition

$$\frac{p^0 - C_i'(q_i^0)}{p^0} = \frac{1}{\epsilon_i}, \tag{2}$$

where again $\epsilon_i = -pD'(p)/q_i$ is the elasticity of perceived demand of firm i (given the quantities supplied by the other firms).

Using that $\epsilon_i = (q/q_i)\epsilon$ we see from (1) and (2) that the mark-ups are proportional to $\left(d_1 - \frac{1}{\lambda}, d_2, d_3\right)$, where $d_i = q_i^0/q^0$ is the market share of firm i. This simple result is formulated as a proposition.

PROPOSITION 1. *The mark-up $M_i = p^0 - C_i'(q_i^0)$, for $i = 1, \ldots, m$, satisfies*

$$M_1 = -\left[d_1 + \frac{1}{\lambda}\right]\frac{q^0}{D'(p^0)},\ M_i = -\frac{q^0}{D'(p^0)}d_i,\ i = 2, 3.$$

If marginal cost functions are identical and constant, then

$$d_1 = d_2 - \frac{1}{\lambda}.$$

We note that with identical and constant marginal cost, the market share of the public firm must be smaller than that of each of the private firms, which consequently is smaller than 1/3. Also it is seen that if all three firms are active in the equilibrium, then $\lambda \geq 1$; since, the Lagrange multiplier expresses the marginal benefit, in terms of consumer surplus, of an infinitesimal budget increase, this means that increasing the budget of the public station and allowing it to supply more units according to this budget increase will lead to a price reduction so that the gain to the consumers is greater than the cost of producing the additional supply, once more reflecting the small market share of the public station.

If variable cost differs between firms, the market shares may be different; indeed, for a smaller M_i in the private firms we may have that

$$\frac{d_1}{d_2} = \frac{M_1}{M_2} - \frac{1}{\lambda d_2}$$

becomes smaller than 1. This may happen for example in the case where there are different choices of technique, and where the public firm has chosen a technique with high fixed cost and small marginal costs, whereas the private firms have chosen low fixed cost and high marginal cost. That such a choice may indeed be a rational one from the point of view of the profit-maximizing firms, is seen when we extend the model in the next section to take capacity cost into consideration.

Before doing so, we notice that if the public firm is turned into another private firm, thus transforming the market to a standard case of Cournot competition, one might expect that consumer welfare (as measured by consumer surplus) will not increase (since the Ramsey-Cournot equilibrium maximizes consumer welfare). However, this reasoning does not take into consideration the special structure of the equilibrium, according to which each firm must choose optimally *given the quantities of the others.* In this context, turning to another allocation such as that of a Cournot-Nash equilibrium, the market shares of the firms may have become more equal which may in its turn have an effect on the price level. In the following example, this is indeed what happens.

EXAMPLE 1. Let $p = 1 - q$ be the demand function, and assume that the common cost function is given by $C(q) = c_0$ (so that marginal cost is 0). In a symmetric (with respect to private firms) Ramsey-Cournot equilibrium the price p and the quantity q_1 supplied by the public firm must satisfy

$$pq_1 = c_0, \tag{3}$$

and the optimal choice q_2 of the private firm 2 (equal to q_3 by symmetry) satisfies

$$p = \frac{1 - (q_1 + q_2)}{2} = \frac{1 - \frac{c_0}{p} - q_2}{2}, \tag{4}$$

where we have inserted (3). From this equation and the demand relationship

$$\frac{c_0}{p} + 2q_2 = 1 - p$$

we get that

$$p = q_2 = q_3 = \frac{1}{6} \pm \sqrt{\frac{1}{36} - \frac{c_0}{3}} \tag{5}$$

for $c_0 \leq \frac{1}{12}$; for a Ramsey-Cournot equilibrium to exist we must further demand that $p \cdot q_2 = p^2 \geq c_0$. For $c_0 = 0$, the public firm does not supply anything, turning the equilibrium into a Cournot-Nash equilibrium with two firms. For $c_0 = \frac{1}{16}$, we get a Ramsey-Cournot equilibrium with

$$p = q_1 = q_2 = q_3 = \frac{1}{4},$$

and it is seen that for values of c_0 greater than $1/16$ we would get a smaller price, meaning that profits of private firms cannot cover the fixed cost.

Incidentally, this equilibrium corresponds to the symmetric Cournot-Nash equilibrium with 3 firms. If fixed costs are smaller than $\frac{1}{16}$, then the Ramsey-Cournot equilibrium has a higher price and the public firm a smaller market share, meaning that private profits are higher, and consumer welfare lower, than in the Cournot-Nash equilibrium. ○

3. Ramsey-Bertrand equilibria and capacity choice

The somewhat unintuitive feature of Ramsey-Cournot equilibria, according to which equilibria with welfare optimizing behaviour from some of the actors in the market may result in lower welfare than profit maximization, may to some extent be a consequence of the rigid structure of the model, where the fixed cost is not open to choice. We now open up for this possibility, using the well-known two-period framework where capacity is chosen at $t = 0$, market price at $t = 1$. This slight extension of the model will also shed some light on the question of whether smaller private market shares could be expected in some cases.

Suppose that at time $t = 0$, each firm chooses a technique, formalized as a triple (c_0, c, y), where c_0 is a fixed cost, c is the associated unit cost of production at $t = 1$, and y is capacity, meaning that production of firm i must satisfy $q_i \leq y$; there is a given set $\mathcal{T}$ of techniques available to all the firms. We assume that firms choose prices at $t = 1$, and that consumers choose the firm with lowest price and turn to sellers with higher price only if demand is rationed at the lowest price. For simplicity, we assume that there is no discounting between periods.

In this context, Ramsey pricing by firm 1 implies a choice (c_{01}, c_1, y_1) at $t = 0$, and a price p_1 at $t = 1$, such that the resulting consumption pattern at $t = 1$ maximizes consumer welfare given the prices chosen by the two other firms at $t = 1$. For the private firms i, $i = 2, 3$, the choice of technique (c_{0i}, c_i, y_i) and the price p_i should be such that profits are maximized.

To characterize the Ramsey-Nash equilibrium in this two-period game, we begin by analyzing the situation at $t = 1$. First of all we notice that all firms produce at capacity, i.e. $q_i = y_i$ for all i; if $i = 2, 3$, this follows immediately from profit maximization and the fact that reduced y_i allows for a smaller c_{0i}. For the public firm, reducing unused capacity makes it possible to lower prices so that capacity is used up, and since some consumers get lower prices and no consumers get higher (this follows from Bertrand competition between the two private firms given their capacities and cost) this is an improvement for the public firm.

It remains to find the equilibrium choices at $t = 0$, given that the capacity will be used up in the next and final period. The cost of capacity y is the smallest number r such that $r = c_0 + cy$ and $(c_0, c, y) \in \mathcal{T}$; defining the cost function

$$C(y) = \min\{c_0 + cy \mid (c_0, c, y) \in \mathcal{T}\}, \tag{6}$$

we see that the equilibrium choice of technique at $t = 0$ and price at $t = 1$ corresponds to a choice of capacity, thus reducing the model to one of quantity choices in a one-period setting. This is not surprising given that the extension to two periods follows the classical interpretation of Cournot equilibria as subsequent choice of capacity and price (cf. e.g., Tirole (1988)). What is new is that the Ramsey-Bertrand equilibrium gives another solution than the one found previously, since in the present setup all costs are variable.

In order to analyze Bertrand competition in the market considered, we need to specify how consumers react on non-identical prices charged by the firms. For this we make the standard assumption that consumers share the supply of the cheapest firms and only then move to the more expensive, so that if prices charged are $p = (p_1, p_2, p_3)$ and capacities are $y = (y_1, y_2, y_3)$, then demand is

$$D_i(p, y) = \min\left\{ y_i\,, \frac{1}{|I(p_i)|}\ \max\left\{0, D(p_i) - \sum_{h \in I_-(p_i)} y_h\right\}\right\},$$

where $I(p_i) = \{j \mid p_j = p_i\}$ and $I_-(p_i) = \{j \mid p_j < p_i\}$. Thus, firms sell to capacity unless the consumers have already been served by firms charging lower prices. To define consumer surplus in a situation with non-identical prices, we similarly assume that consumers are served by lowprice firms first, so that if e.g. prices are $p_{i_1} < p_{i_2} < p_{i_3}$

for some permutation (i_1, i_2, i_3) of $(1, 2, 3)$, and all three firms are active, then

$$S(p, y) = \int_0^{y_{i_1}} [D^{-1}(q) - p_{i_1}]\, dq + \int_{y_{i_1}}^{y_{i_1}+y_{i_2}} [D^{-1}(q) - p_{i_2}]\, dq + \int_{y_{i_1}+y_{i_2}}^{\sum_i D_i(p_1,p_2,p_3)} D^{-1}(q)\, dq.$$

The formulation of $S(p_{i_1}, p_{i_2}, p_{i_3})$ in cases where fewer than three firms are active is left to the reader; they are not of central importance since in equilibrium, prices charged by the firms will be the same.

We have the following result.

PROPOSITION 2. *An array* $((c_{01}^0, c_1^0, y_1^0), (c_{02}^0, c_2^0, y_2^0); p, q_1, q_2)$ *is a symmetric Ramsey-Bertrand equilibrium if and only if* (y_1^0, y_2^0, y_2^0) *is a Ramsey-Cournot equilibrium in the one-period model with cost function* $C(y)$ *given in (6), and* $c_{0i}^0 + c_i^0 y$ *supports* $C(y)$ *at* y_i^0.

PROOF: If $((c_{01}^0, c_1^0, y_1^0), (c_{02}^0, c_2^0, y_2^0); p, q_1, q_2)$ is a symmetric Ramsey-Bertrand equilibrium, then by the reasoning above, all firms sell at capacity, i.e., $q_1 = y_1^0$, $q_2 = y_2^0$. If firm 1 could increase consumer surplus or if any firm $i = 2, 3$ could increase profits by choosing another quantity with resulting market price (given that the others sell at capacity) and cost according to $C(y)$, then this would imply that another technique (c'_{0i}, c'_i, y'_i) would be better, given the techniques of the others, contradicting equilibrium. The converse is shown by a similar argument. □

EXAMPLE 2. The impact of our change of model framework can be assessed if we reconsider Example 1 in the present context. We assume that marginal cost c is 0 for any choice of technique, but that there is a simple linear connection

$$c_0 = \frac{1}{6} y$$

between c_0 and y in any $(c_0, 0, y) \in \mathcal{T}$. Thus, capacity at $t = 1$ can be acquired at $t = 0$, but at a linear cost.

To find the Ramsey-Bertrand equilibrium in the two-period model, the simplest approach is to use the computations in Example 1, which involve only the fixed cost of firm 1. The highest consumer surplus is achieved at the smallest price satisfying (5) for some c_0. This value is seen to be $p = \frac{1}{6}$, obtained by setting $c_{01} = \frac{1}{12}$. The corresponding capacity is $y_1 = q_1 = \frac{1}{2}$.

For the two other firms, optimal capacity choice is $y_2 = y_3 = \frac{1}{6}$, which requires a period 0 investment of $c_{02} = c_{03} = \frac{1}{36}$. Incidentally, this investment is exactly recovered by the sales revenue at $t = 1$.

It might be instructive to compare this equilibrium with the equilibrium which we expect to be the outcome if the three firms all aimed at maximizing profits; the equilibrium is the same as in Example 1, with $p = q_1 = q_2 = q_3 = \frac{1}{4}$, and the total period 0 investment is

$$c_0 = 3 \cdot \frac{1}{6} \cdot \frac{1}{4} = \frac{1}{8},$$

which is smaller than the investment in the Ramsey-Bertrand equilibrium, which is

$$c_0 = \frac{1}{12} + 2 \cdot \frac{1}{36} = \frac{5}{36},$$

an observation which will be useful at a later stage. Finally, consumer surplus changes from

$$\frac{1}{2} \cdot \left(1 - \frac{1}{6}\right)^2 = \frac{25}{72} \quad \text{to} \quad \frac{1}{2} \cdot \left(1 - \frac{1}{4}\right)^2 = \frac{9}{32}.$$

The total profits have changed from 0 to $\frac{3}{16} - 3 \cdot \frac{1}{6} \cdot \frac{1}{4} = \frac{1}{16}$. ○

For the following, we need a generalization of the model which points to the application to commercial television broadcasting. For this, we must introduce some interdependency of firms in their choice of capacity and its associated cost, reflecting the specific way of producing audiences through television broadcasting. The capacity (interpreted as number of viewers available for advertisers) depends on the program profiles chosen by the broadcaster in question *and* the program profiles of the other broadcasters. Therefore, capacity is not chosen directly by the firm but emerges as a result of the joint strategy choices of all broadcasters.

Thus, for each i we let Σ_i be an abstract set of strategies for firm i, and we let $\Sigma = \Sigma_1 \times \Sigma_2 \times \Sigma_3$. Let $K_i : \Sigma \to \mathbb{R}_+$ and $c_i : \Sigma \to \mathbb{R}_+$ be the capacity and cost mappings, so that if $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ is the array of strategies chosen by the public and the private broadcasters, then the capacities are $(K_1(\sigma), K_2(\sigma), K_3(\sigma))$ and the associated costs $(c_1(K_1(\sigma)), c_2(K_2(\sigma)), c_3(K_3(\sigma)))$.

As previously, we shall assume that at $t = 1$, firms compete by choosing prices when selling the available capacity, and equilibrium (to be referred to as a Ramsey-Bertrand equilibrium) obtains when each firm i chooses strategy σ_i^0 at $t = 0$ and price p_i^0 at $t = 1$ so that

(1) the public firm maximizes consumer surplus under a budget constraint, that is $S(p, K(\sigma))$ is maximal at σ_1^0, p_1^0 (over all σ_1, p_1 given σ_i^0, p_i^0, $i = 2, 3$) under the constraints

$$p_1 D_1(p, K(\sigma)) - c_1(K_1(\sigma)) \geq 0, \; D_1(p, K(\sigma)) \leq K_1(\sigma),$$

(2) the private firms maximize $p_i D_i(p, K(\sigma)) - c_i(K_i(\sigma))$ over σ_i, p_i given σ_j, p_j, under the constraint $D_i(p, K(\sigma)) \leq K_i(\sigma)$, $j \neq i$, $i = 2, 3$.

An Edgeworth-Bertrand equilibrium in this model is an array (σ^0, p^0) such that all the choices of the firms satisfy (2) above.

The game runs over two periods as previously, so that the strategy choices determine capacity, whereas the second period this capacity is sold under price competition up to capacity; for simplicity, we assume that period 2 cost is zero. Since the basic decision variables are not capacities but strategies which in their turn determine capacities of all firms, the standard approach using marginal revenue and cost with respect to capacity changes will not work, and we need another way of assuring that equilibria exist in this model.

To formulate the basic existence result, we need some more notation: Let $R_i((p_j)_{j\neq i}, \sigma_i', (\sigma_j)_{j\neq i})$ be the revenue to firm i given prices and strategies of the others, when the price for firm i is such that capacity $K_i(\sigma_i, (\sigma_j)_{j\neq i})$ is sold; for given prices and strategies of the others, this is a function of the strategy σ_i alone (since the price is given by the demand condition); it corresponds to the total revenue function in classical partial monopoly models.

PROPOSITION 3. *Assume that for each i, Σ_i is convex and compact, and that for each i,*

(i) K_i is convex in σ_i for fixed values of σ_j, $j \neq i$,

(ii) c is a convex function of capacity,

(iii) R_i is a convex function of σ_i for fixed values of p_j and σ_j;

Then there exist Ramsey-Bertrand and Edgeworth-Bertrand equilibria in the model, and these equilibria are such that $p_1^0 = p_2^0 = p_3^0$ if all firms produce nonzero quantities of output.

The proof of Proposition 3 relies on standard fixed-point techniques.

PROOF OF PROPOSITION 3: For $i = 1, 2, 3$, define the correspondences $\varphi_i : \Sigma \times [0, \overline{P}]^3 \to \Sigma_i$ by

$$\varphi_i(\sigma, p) = \{\sigma_i' \mid R_i((p_j)_{j\neq i}, \sigma_i', (\sigma_j)_{j\neq i}) - c(K_i(\sigma_i', (\sigma_j)_{j\neq i})) > \pi_i(\sigma, p)\},$$

where

$$\pi_i(\sigma, p) = p_i \min\{K_i(\sigma), D_i(p)\} - c(K_i(\sigma)),$$

and where $R_i((p_j)_{j\neq i}, \sigma_i', (\sigma_j)_{j\neq i})$ is the revenue to firm i given prices and strategies of the others, when the price for firm i is such that capacity is sold. Next, define $\lambda_i : \Sigma \times [0, \overline{P}]^3 \to [0, \overline{P}]$ by

$$\lambda_i(\sigma, p) = \{p_i' \mid D_i(p_i', (p_j)_{j\neq i}) > K_i\},$$

and let $\psi_i : \Sigma \times [0, \overline{P}]^3 \to [0, \overline{P}]$ be given by

$$\psi(\sigma, p) = \begin{cases} \{p_i' \in \lambda_i(\sigma, p) \mid p_i' > p_i\} & \text{if } p_i \in \text{cl}\lambda_i(\sigma, p) \\ \lambda_i(\sigma, p) & \text{otherwise.} \end{cases}$$

For $i = 1$, we define $\gamma_1 : \Sigma \times [0, \overline{P}]^3 \to \Sigma_1$ by

$$\gamma_1(\sigma, p) = \{\sigma_1' \mid p_1 \min\{K_1', D_1(p)\} < c(K_1')\}$$

and $\kappa_1 : \Sigma \times [0, \overline{P}]^3 \to \Sigma_1$ by

$$\kappa_1(\sigma, p) = \begin{cases} \{\sigma_1' \in \gamma_1(\sigma, p) \mid S(p, K_1', K_2', K_3') > \\ \qquad\qquad\qquad\qquad S(p, K_1, K_2, K_3)\} & \text{if } \sigma_1 \in \text{cl}\gamma_1(\sigma, p), \\ \gamma_1(\sigma, p) & \text{otherwise.} \end{cases}$$

where we have used shorthand notation $K_i' = K_i(\sigma_1', \sigma_2, \sigma_3)$, $K_i = K_i(\sigma)$, $i = 1, 2, 3$.

It is easily checked that each of the correspondences φ_i, ψ_i, $i = 1, 2, 3$, as well as the correspondences κ_1, have convex (possibly empty) values and open graph. Moreover, they are irreflexive in the sense that for all (σ, p), $\sigma_i \notin \varphi_i(\sigma, p)$, $p_i \notin \psi_i(\sigma, p)$, $\sigma_1 \notin \kappa_1(\sigma, p)$. We now proceed to exhibit arrays (σ, p) where all relevant correspondences have empty values, and study their properties:

Edgeworth-Bertrand equilibrium: Consider the correspondence

$$\varphi_1 \times \varphi_2 \times \varphi_3 \times \psi_1 \times \psi_2 \times \psi_3 : \Sigma \times [0, \overline{P}]^3 \to \Sigma \times [0, \overline{P}]^3.$$

By the fixed point theorem of Ky Fan (see e.g. Borglin and Keiding, 1976) there is (σ^0, p^0) such that $\varphi_i(\sigma^0, p^0) = \emptyset$, and $\psi_i(\sigma^0, p^0) = \emptyset$, $i = 1, 2, 3$. From the latter property, we have that $D_i(p^0) = K_i(\sigma^0)$, and since $\varphi_i(\sigma^0, p^0) = \emptyset$, we have that there is no other strategy σ_i' for firm i which would increase profits. Thus, the array (σ^0, p^0) satisfies the conditions for an Edgeworth-Bertrand equilibrium.

Assume that there are $i \neq j$ with $p_i^0 > p_j^0$ and both firms active. Then we would have a contradiction to $\phi_j(\sigma^0, p^0) = \emptyset$ or $D_j(p^0) = K_j(\sigma^0)$ since consumers will want to buy more at the lower price before using the seller with a higher price. We conclude that $p_1^0 = p_2^0 = p_3^0$ if all firms are active.

Ramsey-Bertrand equilibrium: Here we work with the correspondence

$$\kappa_1 \times \varphi_2 \times \varphi_3 \times \psi_1 \times \psi_2 \times \psi_3 : \Sigma \times [0, \overline{P}]^3 \to \Sigma \times [0, \overline{P}]^3,$$

for which there is (σ^*, p^*) such that $\kappa_1(\sigma^*, p^*) = \emptyset$, $\varphi_i(\sigma^*, p^*) = \emptyset$, $i = 2, 3$, and $\psi_i(\sigma^*, p^*) = \emptyset$, $i = 1, 2, 3$. Reasoning as above, we have that the profit maximization conditions are satisfied for each private firm. For $i = 1$, we have from $\psi_1(\sigma^*, p^*) = \emptyset$ that $D_i(p^*, K(\sigma^*)) = K_i(\sigma^*)$ and that σ_1^* maximizes consumer surplus given p_i^* and σ_i^*, $i \neq 1$, which are the conditions for a Ramsey-Bertrand equilibrium. The fact that $p_1^* = p_2^* = p_3^*$ if all firms are active follows by the same reasoning as above. □

The interdependence of the production decisions of the firms – the strategy chosen by one firm affects not only its own capacity, but also those of the other firms – makes the situation slightly different from standard partial models of incomplete competition, and therefore we have chosen to give the full proof of the proposition. We shall make use of the proposition in Section 6 where the programming outlays of the broadcasters take the role of the strategies, and where the program quality of one broadcaster affects the sizes of not only its own audience but also of the audiences of the competing broadcasters.

4. The case of a non-welfare-maximizing public firm

In the previous sections, we have been considering Ramsey-type equilibria where the public firm is guided in its choice by the welfare of its costumers. While important as a benchmark for pricing decisions in public enterprises, this equilibrium may not be convincing as a description of actual market behaviour, given that the agents may have slight if any knowledge of the welfare of its consumers. Nevertheless, some of the insights gained in previous sections may still hold provided that the public firm acts sufficiently different from ordinary profit maximizing behaviour (in which case we would be back in the standard Cournot oligopoly). Here we consider the assumption of maximizing output, also an objective which could hardly be defended as *the* objective of a public enterprise, but on the other hand it has been used freely in the literature (which makes it non-exotic as an assumption of behaviour), and in the application which we have in mind, that of commercial television, it may even turn out to be a reasonably good approximation to what public TV stations actually do.

Thus, in the present section, we retain the model of the previous section, where decisions about capacity are taken at $t = 0$ and where prices are chosen and sales take place at $t = 1$. There is one public firm and $n \geq 1$ private firms, all with access to the same technology $\mathcal{T}$; in the present case, we allow for the case of only one private competitor to the public firm. As in the previous section, private firms are assumed to maximize (intertemporal) profits, while the public firm maximizes output, or, what amounts to the same in the present model, capacity. A symmetric equilibrium with public output maximization is an array $((K_1^0, c_1^0, y_1^0), (K_2^0, c_2^0, y_2^0); p, q_1, q_2)$ (where, as before, subscript 1 indicates variables related to the public firm and subscript 2 variables related to the private firms), such that

(i) there is no alternative strategy choice $((K_1, c_1, y_1), p')$ with $q_1' > q_1$, $(K_1, c_1, y_1) \in \mathcal{T}$ and $(p - c_1)q_1' - K_1 \geq 0$, where q_1 (q_1') is the sale of firm 1 given the strategy (K_2^0, c_2^0, y_2^0) of the private firms and the price p (p'),

(ii) each private firm maximizes profits by choosing $((K_2^0, c_2^0, y_2^0), p)$, given the choices of the other firms, public or private.

The following is an obvious and well-known consequence of the sales-maximizing behaviour of the public firm, and we omit the proof.

PROPOSITION 4. *In any symmetric equilibrium with public output maximization total sales exceed that of an Edgeworth-Cournot equilibrium.*

EXAMPLE 3. Let us find a symmetric equilibrium with public output maximization in the special case considered in Example 2 of the previous section, for which we have already found the Edgeworth-Cournot equilibrium, which was symmetric with $K = 1/24$, $q = 1/4$, and $p = 1/4$.

To find the equilibrium where the public firm maximizes its output, we may use (5) once again, since maximizing q under the relevant constraints corresponds to finding the maximal feasible value of K, which is the same value $1/12$ that we found in the previous section, something which is due to the extreme simplicity of the example and will not be the case in general. But at least here we obtain that the otherwise rather irrationally behaving public firm will achieve a welfare maximum that would not be realized if the firms were all private and maximized profits. ○

It might be noticed that apart from the effects considered above, the output maximizing public firm will be induced to keep cost down, both that of capacity building and the current cost at $t = 1$. Indeed, if these costs were to increase due to organizational slack, then this would have reverse effects on output, and the competing private firms would increase their market shares.

5. The cost structure of a television broadcaster

In the present section we begin a closer investigation of the cost structure of a commercial television broadcaster. The aim of this is not only to justify the model of market competition and pricing discussed in the previous sections, but also to obtain some more insight with regard to the cost structure and its impact on the market behaviour. Shortly speaking, the costs of a television broadcaster is connected with the programs that are broadcasted (we are here neglecting administration costs, which in real life are by no means negligible, but which do not enter into our arguments in a way different from what is completely standard), but the output of the television broadcaster, to which these costs should be ultimately assigned to get a model of the type considered above, are audiences created by these programs and delivered to advertisers. We begin our discussion with these latter aspects of the production process.

As is well known, what is sold by television broadcasters to the advertisers (or rather, to the agents commissioned by the advertisers to take care of their advertising program) is *target rating points* (TRP), usually measured as 1 percent of the relevant population. In real markets, TRPs may be distinguished as to age, sex and other characteristics, and the broadcasters sell several products which are derived from the basic TRPs; we shall assume here that there is only one type of TRP sold in the market.

We assume that a television broadcaster has a total capacity of M spots, advertisements which for simplicity are assumed all to have the same length in time. In order to sell a certain number τ of TRPs, the advertisement may have to be shown several times. If π is the proportion of all television viewers watching the channel, then the number of times the advertisement must be shown to reach the proportion τ is

$$k = \tau/\pi \tag{7}$$

(which for π small compared to τ may be considered as integer valued). When buying TRPs, the advertiser obtains *gross coverage* in the sense that some viewers may have seen the spot several times. Clearly, the proportion of viewers becomes a crucial parameter when selling TRPs. The advertisers may buy more than 100 TRP (as a matter of fact, they most often do) for several reasons, the simplest being that they are interested in sending repeated messages to the viewers.

Matters are, however, slightly more complicated than that; the advertiser on her side is not merely interested in TRPs, which are the units purchases, but is oriented towards the *net coverage* obtained, which is different from the gross coverage expressed by τ. Indeed, if π expresses the probability that a television viewer watches the given channel, and the event of watching a channel is independent over days, then the probability of having seen the advertisement after k broadcasts is

$$\nu(k) = 1 - (1 - \pi)^k, \tag{8}$$

which gives us the net coverage in this particular situation.

Let p be the price of a TRP. Using (7) we get that the cost of reaching ν per cent of the population, whereby all the reached individuals are different, is found by solving the equation

$$1 - (1 - \pi)^{\frac{\tau}{\pi}} = \nu \tag{9}$$

for τ and multiplying by p, the price of TRPs with the channel considered. We obtain the expression

$$C(\nu) = p\pi \frac{\log(1 - \nu)}{\log(1 - \pi)}$$

for the cost function as seen from the view of the advertiser. If, as assumed in the present model, π is a constant, then the cost to the advertiser of buying television audiences is not linear in the size of the audience but has the shape of $\log(1-\nu)$ which is a convex function of ν.

In this calculation, we have taken the price of a TRP as given and computed a non-linear payment scheme for net coverage. Alternatively, we might also take the viewpoint that what the advertiser buys is net coverage, and comparing the cost of advertisement in different competing channels, what should matter is the cost of net coverage rather than the somewhat irrelevant cost of TRP. If the television channels are responsive to this advertiser behaviour, then they should operate with a price of net coverage and then work back to the TRP price. Indeed, if ρ is the price per percentage of net coverage, then the implicit price per TRP connected with a purchase of net coverage ν is

$$p(\nu) = \frac{\rho \log(1-\pi)}{\pi} \frac{\nu}{\log(1-\nu)}.$$

Clearly, the function $p(\nu)$ is non-linear, with a price per unit decreasing in ν, and inserting (8), we get a price schedule by which the price paid per TRP depends on the number of TRPs purchased, allowing for substantial rebates when this number is large.

This theoretical price schedule corresponds rather well – the extreme simplicity of the underlying assumptions taken into account – to the price structure seen in real television advertising markets. The non-linear price structure arises, at least in the present model, from a linear price structure on the commodity that is really traded. The fact that purchases are made in terms of TRP is connected with the observability problem: TRPs may be verified (statistically), while net coverage cannot be verified, at least not at the current state of the technology. Since net coverage is what the advertisers want, the competition among TV broadcasters must be phrased in terms of prices on net coverage rather than on TRPs which anyway are not easily comparable between broadcasters.

6. Programming outlays and market equilibria

In order to get the full picture of the cost structure of the television broadcaster, we need to take the program cost into account. Programs are used to attract audiences which then are put into the disposal of advertisers. The seminal model by Steiner (1952) of viewer choices emphasizes that viewers have different preferences with respect to different types of programs, and that the choice of the broadcaster reflects these differences; this point of view has been further developed in the literature, see e.g. Spence and Owen (1977), Owen and Wildman (1992). For our present study, we shall be satisfied with a much more simplistic way of representing the strategic aspects of program choice: We assume that there is a simple way of attracting viewers, namely by choosing more expensive programs, so that the probability π_j that the representative viewer is watching the programs of broadcaster j is given by

$$\pi_j = \frac{\hat{\delta}_j}{\sum_{h \in J} \hat{\delta}_h} \text{ for } \delta_j \geq \delta_0, \tag{10}$$

and $\pi_j = 0$ for $\delta_j < \delta_0$. Here J is the (finite) set of broadcasters, δ_h is the program cost of broadcaster h, $h \in J$, and $\hat{\delta}_h = \max\{\delta_0, \delta_h\}$; we assume thus that there is a minimal outlay

δ_0 which is necessary in order to perform broadcasting, and that viewers are attracted to the different broadcasters actually in business in proportion to their programming outlays. Throughout the following, we shall assume that $\delta_j \geq \delta_0$ for all j.

Needless to say, this formalization of the relationship between program cost and size of audience does away with many interesting aspects of real life, such as the complicated choice of program profile and program qualities (as discussed e.g. by Papandrea (1997), Bourreau (2003) and Mangàni (2003). On the other hand, it may be considered a first approximation, and similar approaches have been found useful in the treatment of related problems (such as patent races, see e.g. Reinganum (1989), where the relative amounts of money spent on research are assumed to determine the probability of getting the patent).

In order to relate the cost of attracting audiences to the entities which are sold to advertisers, we must use the formalism developed in the previous section. We have assumed that there is a given number M of broadcasts available (typically fixed by rules determining the maximal amount of advertisement admissible) which is common to all broadcasters, which by (7) corresponds to the TRP capacity $M\pi$. This total number of broadcasts can then be divided into a number k of *campaigns;* these are sold to the advertisers, and the viewer probability π_j determines the number $T(\pi_j)$ of campaigns and hence the capacity of the broadcaster in the following way: Assume that there is fixed value ν_0 of net coverage to be attained by a campaign. Given the probability π_j, we find the number $k(\pi_j)$ of broadcasts by solving for k in

$$1 - (1 - \pi_j)^k = \nu_0,$$

which gives us

$$k(\pi_j) = \frac{\log(1 - \nu_0)}{\log(1 - \pi_j)}.$$

The capacity of broadcaster j may now be found as $T(\pi_j) = M/k(\pi_j)$, and conversely, we may find the viewer probability π_j associated with a capacity T_j as

$$\pi_j = 1 - (1 - \nu_0)^{\frac{T_j}{M}}. \tag{11}$$

To find the cost associated with building capacity T_j we use (10) to determine the programming outlays δ_j of broadcaster j needed to achieve viewer probability π_j given the outlays δ_h for $h \neq j$,

$$\delta_j = \left(\frac{1}{\pi_j} - 1\right)^{-1} \sum_{h \neq j} \delta_h;$$

inserting from (11) we have a cost function $C(T_j)$ for achieving the capacity T_j of the size

$$C_j(T_j) = \left((1 - \nu_0)^{-\frac{T_j}{M}} - 1\right) \sum_{h \neq j} \delta_h. \tag{12}$$

It may be noticed that cost of capacity depends linearly on the outlays of the other broadcasters; in our model broadcaster competition works on several planes, not only through the prices (here the price of campaigns yielding the standardized net coverage) but also through the cost externality – a broadcaster increasing the outlays on programming to obtain a larger capacity automatically increases capacity costs of all the competing broadcasters. Since $\kappa = (1-\nu_0)^{-1/M}$ is a constant, the expression in the bracket takes the form $\kappa^{T_j} - 1$ which shows that cost is exponential in capacity T_j for any given level of programming outlays of the other broadcasters.

With the structure of production and cost as outlined above, the model fits into the framework developed in Section 3. Strategy sets of the broadcasters are programming outlays determining capacity together with price decisions when capacity is sold. We need the revenue assumption from Proposition 3: Define $R_i((\rho_j)_{j\neq i}, \delta_i', (\delta_j)_{j\neq i})$ as the revenue for broadcaster i when the competitors have chosen prices ρ_j and outlays δ_j, $j \neq i$. The assumption that this function is convex in δ_i' is basically an assumption on the well-behavedness of the demand function and will be satisfied for simple specifications of demand as those of the examples in previous sections. The remaining assumptions of the proposition follow from the specific properties of the model. Application of Proposition 3 then gives us the following:

PROPOSITION 5. *Assume that* $R_i((\rho_j)_{j\neq i}, \delta_i', (\delta_j)_{j\neq i})$ *is convex in* δ_i'. *Then there are Edgeworth-Bertrand and Ramsey-Bertrand equilibria, and all such equilibria are characterized by equal prices for all active firms.* □

While existence of equilibria is reassuring, it does not tell us much about the properties of each type of equilibrium and how they compare. For this, we need some additional assumptions. The one which we use is a counterpart – for the present model with interdependent supply – of classical conditions that marginal revenue is smaller than demand and hence smaller than marginal consumer surplus. Note that revenue and consumer surplus are both homogeneous of degree zero in $\delta = (\delta_1, \delta_2, \delta_3)$, depending only on the distribution of programming outlays but not on its overall level.

PROPOSITION 6. *Suppose that at all* $\delta = (\delta_1, \delta_2, \delta_3)$ *and all prices* ρ *(common to all broadcasters)*

$$\frac{\partial}{\partial \delta_1} S(\rho, T_1(\delta), T_2(\delta), T_3(\delta)) - \frac{\partial}{\partial \delta_1} R_1(\rho, \delta_1, \delta_2, \delta_3)$$

is positive, and that both quantities are decreasing in $\delta_1/(\delta_2+\delta_3)$. *Let* (δ^*, ρ^*) *be a Ramsey-Bertrand equilibrium. Then there is an Edgeworth-Bertrand equilibrium* (δ^0, ρ^0) *with* $\max_i \delta_i^0 < \max_i \delta_i^*$ *and* $\rho^0 > \rho^*$.

PROOF: Let $\underline{\delta}^* = \delta_2^* = \delta_3^*$; by our assumption on the quantity in (12), we have that $\underline{\delta}^* < \delta_1^*$. Define $R(t)$ for $t \geq 1$ as

$$R(t) = D^{-1}(3T(t\underline{\delta}^*, t\underline{\delta}^*, t\underline{\delta}^*))T(t\underline{\delta}^*, t\underline{\delta}^*, t\underline{\delta}^*),$$

that is the revenue to any of the broadcasters at the programming outlay $(t\underline{\delta}^*, t\underline{\delta}^*, t\underline{\delta}^*)$, and let MR$(t)$ be the partial derivative of $R(t)$ with respect to programming outlay of any broadcaster, evaluated at $(t\underline{\delta}^*, t\underline{\delta}^*, t\underline{\delta}^*)$. Clearly, at $t = 1$, marginal revenue from changing programming outlays, $MR(1)$ exceeds the cost of increasing programming outlays by a small unit, which is 1. Next, let $\overline{\delta}^* = \delta_1^*$; then at the programming outlays $(\overline{\delta}^*, \overline{\delta}^*, \overline{\delta}^*)$, we would have that marginal revenue with respect to programming outlays is smaller than marginal cost, so that MR$(\overline{\delta}^*/\underline{\delta}^*) < 1$, and it follows by continuity that there is a symmetric Edgeworth-Bertrand equilibrium with outlays $(t^0\delta^*, t^0\delta^*, t^0\delta^*)$ and some price ρ^0. Since MR(t^0) equals marginal revenue w.r.t. capacity multiplied by the derivative of capacity with respect to programming outlay at $(\delta^0, \delta^0, \delta^0)$, and the latter is higher at δ^0 than at $\underline{\delta}^*$, we conclude that marginal revenue w.r.t. capacity is higher at δ^0 than at $\underline{\delta}^*$, so that $\rho^* < \rho^0$. □

7. Concluding comments

In the preceding sections, we have treated a model of imperfect competition between commercial television broadcasters. The specific feature of the model has been the presence of one broadcaster under public control, which consequently acts according to other objectives than the purely private broadcasters. Given this situation, we have had to investigate not only the workings of the market for commercial television broadcasting, but also the consequences in terms of equilibrium concepts of the presence of both public and private firms with each their specific objectives.

The latter problem has been confronted by considering the notion of a Ramsey equilibrium, where the public firm chooses Ramsey prices facing residual demand from the private firms, which on their side maximize profits on their residual demand schedules. In the present paper we have added a feature which seems central to markets for commercial television broadcasting, namely that caåacity (total amount of possible audiences to be exposed to commercials) must be established before the sale can take place and cannot be changed without unreasonable cost once it is put into place. This means that the markets are more properly studied in the context of a two-period model with an initial decision on capacity-building followed by a second period where the given capacity is sold, now under price competition.

Clearly, many simplifications have to be accepted in order to keep the model tractable; among these simplifications is our assumption that there are only standardized campaigns for sale, where the standard is a given percentage of the population actually reached (in terms of net coverage). On the other hand, with this assumption we can specify a model of production of commercial television broadcasting which reflects the basic facts of the sector in a more direct way than most contributions which are satisfied with abstractly specified demand and cost schedules not derived from the underlying technological conditions.

The outcome of our modelling is that production is inherently interrelated; the amount of advertising campaigns that can be sold depends on the attractiveness of programs to viewers and this again depends on programming outlays; increasing program quality not only increases your audience but at the same time diminishes those of your competitors. This interrelationship is not entirely standard in the industrial organization literature, and therefore we have felt that it was necessary to check the existence of the equilibria which we study, even though we deal with simple partial models.

The ultimate goal of this modelling was of course to compare the situation sketched initially, where one broadcaster was public and the remaining private, with the situation after privatization. Even if such a comparison has not really been on the agenda in connection with political decision making, given that the received wisdom prescribes privatization as a means towards increased competition and thereby to enhanced welfare and effectiveness, it nevertheless seems worthwhile to see what would emerge from using economic theory. And indeed the results are not quite in line with the conventional wisdom: Privatization will lead to higher prices for advertising and to lower quality of television programs. Shortly speaking, this privatization issue is for once a very clear-cut case, since everybody looses except the existing private television broadcasters (and possibly the new one if this will not be one conglomerate of the existing ones).

There are of course several reasons why it works this way, but none of them are particularly subtle. For one thing, the commercial television broadcasting market is not one of perfect competition. In smaller countries, the market is so small that only a few broadcasters can operate with profits, and it is sufficiently complicated that competition authorities have little chance of tracing what is actually going on; even so, tacit agreements would tend to send the prices towards a monopoly level. This situation is on the other hand upset by the presence of a non-profit organization, which under suitable circumstances will create competition as a by-product of its activities. Eliminating this organization or turning it into a standard profit maximizer, the effects disappear and the market turns into a traditional oligopolistic one, with all the well-known effects for prices and consumer welfare.

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