Optimal Capital Income Taxation and Distributions Service

Akos Valentinyi* Sheikh Selim‡

October 2007

Abstract

We show that in a multisector economy with perfectly competitive markets, in a steady state the optimal capital income tax is in general different from zero. If distributions service is a market good, the difference between buyer price of consumption and buyer price of investment is determined by the unit cost difference of distributing these and the market price of distributions service. We argue that if the buyer price difference is due to the market price of distribution services, it can induce inefficient levels of production, which in turns violates production efficiency. We show that in a steady state of the Ramsey equilibrium, the optimal policy that involves a capital income tax/subsidy and different rates of labour income taxes can undo the relative price difference and can restore production efficiency.

Keywords: Optimal Taxation, Ramsey Equilibrium, Distributions Service, Primal Approach.

JEL Codes E62, H21, H30.

*University of Southampton, CEPR and Hungarian Academy of Science; email: av2@soton.ac.uk

†Cardiff University; email: selimst@cardiff.ac.uk

‡The authors would like to thank Berthold Herrendorf of Arizona State University for important comments on an earlier version of the paper.

1 Introduction

One of the key findings of the Optimal Taxation literature is that taxing capital income is a bad idea. Chamley (1986) and Judd (1985) show that if the equilibrium of a canonical one sector neoclassical growth model has an asymptotic steady state, the optimal policy is eventually to set the tax rate on capital income to zero. Their key argument is that since capital income taxation serves neither efficiency not redistributive purpose in the long run, it is not optimal to tax capital income. Correia (1996) and Judd (1999) argue against taxing capital income because the intertemporal pattern of capital income tax distortions are inconsistent with the well established commodity tax principle. Atkeson, Chari and Kehoe (1999) show that the optimality of zero capital income tax is analytically robust even if one relaxes some of Chamley's (1986) assumptions. They show that this result holds in an economy with heterogeneous agents, or in an economy with endogenous growth, or in an open economy, or in an economy where the agents live in overlapping generations[1].

In this paper, we show that in a neoclassical economy where distributions service is a market good, in a steady state the optimal capital income tax rate depends on the optimal labour income tax rates, and the optimal capital income tax rate is in general different from zero. We thus contribute by presenting an important extension to both the Chamley-Judd result and the Atkeson et al. (1999) analysis of their result. We show that if the unit cost of distributing consumption is different from the unit cost of distributing investment, the buyer price of consumption and the buyer price of investment both depend on the relative price of distributions service. For any difference in the buyer prices of consumption and investment that is due to a change in the price of distribution service, government can tax capital income at the Ramsey optimum, and such a tax restores production efficiency. This policy is supported by differential labour income taxation which undoes the capital income tax distortions. In short, we argue that in a class of general equilibrium models with competitive markets, in a steady state the optimal capital income tax is in general different from zero.

We develop a simple neoclassical model that has a large number of identical utility-maximizing households, two production sectors that produce a composite *consumption-investment* good and *distribution services*, a retail sector that uses distribution services to distribute consumption goods and investment goods to households, perfectly competitive markets, and a government that finances an exogenous stream of purchases by levying a set of distorting flat rate taxes on income from two factors,namely, labour and physical capital. We derive the decentralized equilibrium and define the well-known Ramsey (1927) problem: given a preset revenue target, how should the government choose tax rates such

[1] Chari and Kehoe (1999) and Erosa and Gervais (2001) present a comprehensive survey of a class of general equilibrium models where the optimality of zero capital income tax is robust.

that these tax rates maximize social welfare and generate allocations and prices that are implementable in a decentralized equilibrium.

In our model since distribution services are costly and sold in a competitive market, the unit cost difference of distributing consumption and investment drives a difference between buyer price of consumption and buyer price of investment. While this difference is fixed for a given relative price of distributions service, the difference in relative price of consumption and investment that is due to a change in relative price of distributions service can induce inefficiently large or small production of a particular good, which is not consistent with the production efficiency argument. We argue that in a steady state of the Ramsey equilibrium, a capital income tax supported by different rates of labour income tax (across sectors) can undo the relative price difference and restore the production efficiency condition. We thus argue that a capital income tax can be a long run optimal policy if it is supported by differential rates of labour income taxes.

Prior to our paper, several studies have discussed conditions on preferences and technology for which the steady state optimal policy in a competitive economy *may* involve a *nonzero* tax rate on capital income. Jones, Manuelli and Rossi (1997) show that in a model with endogenous growth through human capital accumulation, the optimality of zero capital income tax is in general valid. But they argue that the optimal capital income tax rate may be nonzero if the level of capital stock appears as an argument in the utility function. Atkeson et al. (1999) argue that in a one sector economy where capital and labour income taxes are same for all *types* of agents, in a steady state zero capital income taxation is optimal if the production function is separable between capital and labour. Chari and Kehoe (1999) show that in an economy where agents live in overlapping generations, the optimality of zero capital income tax is only valid if agents' preferences satisfy certain homotheticity (over consumption) and separability (between consumption and leisure) properties[2].

In the current paper, we argue that a more useful approach to validate the optimality of capital income taxation is to explain its efficiency properties without any explicit restrictions on preferences or technology. We develop a model with the most standard utility function and production functions, and we argue that the underlying intuition of the optimality of capital income taxation is much broader and clearer than that discussed solely on the basis of preference and technology specification. In our analysis, the optimality of setting a tax on capital income stems from the difference in relative price of distributions service and the consumption-investment good. This idea is primarily due to the interdependence of capital and labour margins.

[2]Jones et al. (1997) argue that if one imposes the same homotheticity and separability restrictions on preferences, in a steady state of an endogenously growing economy the optimal policy involves zero tax rates on all transactions. The government in such an economy would frontload revenue and use bonds to finance future consumption stream.

In a competitive equilibrium of a multisector neoclassical model, capital and labour margins are in general interdependent. Their interdependence makes the steady state optimal capital income tax rate and optimal labour income tax rate interdependent. We first show such an interdependence in a setting where we explicitly model retail services as a market good. With a competitive retail market, the buyer price of consumption and the buyer price of investment both are increasing functions of their corresponding costs of distribution, and the market price for distributions service. Quite clearly, in a competitive equilibrium the relative price difference of consumption and investment is due to the unit cost difference of distributing them. With fixed marginal cost of distribution, this difference is in general fixed for a given level of the market price for distributions services. In a competitive equilibrium with efficient production, the relative price difference of consumption and investment should only be characterized by this fixed difference; this is because with efficient production, due to the interdependence of capital and labour margins the equilibrium prices of the consumption-investment good and distributions service would reflect efficient allocation of capital and labour across the two sectors. Thus in a competitive equilibrium, if allocations of capital and labour satisfy the production efficiency condition, price of the consumption-investment good and price of distributions service are same (and the difference in buyer price of consumption and investment is solely due to unit cost difference).

We argue that if the relative price difference of consumption and investment varies for any change in the market price of distributions service, it generates inefficient allocations of capital across sectors — a problem that can be solved by optimal taxation. For instance, if in a competitive equilibrium price of distributions service is lower than that consistent with production efficiency (or more simply, lower than the price of consumption-investment good), the inefficiency is due to higher allocation of capital and labour in distributions service production and lower allocation of these factors in consumption-investment good production. The optimal policy should tax capital income and set higher labour income tax in the distributions service sector, which would encourage agents to shift capital and working time in the sector that produces the consumption-investment good.

2 The Environment

Time is discrete and runs forever. There are two goods traded in sequential markets in each period: a consumption-investment good, z_t, and distribution services, s_t. The consumption-investment good can be used for private consumption, c_t, for government consumption, g_t, and for investment, x_t. We assume that producers sell the goods on the wholesale market. Retailers deliver the goods to the consumer or the government with a technology that combines one unit of consumption with ψ_c units of distribution services, or one unit of investment with ψ_x units of distribution services. Consequently, the economy's resource constraints can be written as:

$$c_t + g_t + x_t \leq f^z(k_{zt}, n_{zt}) \tag{1}$$
$$\psi_c(c_t + g_t) + \psi_x x_t \leq f^s(k_{st}, n_{st}) \tag{2}$$

where $f^i(k_{it}, n_{it})$, $i = \{z, s\}$ is constant returns to scale technology defined over capital and labour inputs, and they satisfy standard regularity conditions. Assuming that capital depreciates at rate $\delta \in (0, 1)$, the law of motion for capital accumulation can be written as:

$$k_{zt+1} + k_{st+1} = x_t + (1 - \delta)(k_{zt} + k_{st}) \tag{3}$$

The economy is populated by a continua of measure one of identical households. Households are endowed with $\Gamma > 0$ units of time at each period, $k_0 > 0$ units of capital at period 0, and property rights of firms. They supply working time and capital to firms in the productions sectors. They derive utility from consumption and leisure. The preferences of the representative household are given by the utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, \Gamma - n_{zt} - n_{st}) \tag{4}$$

where $\beta \in (0, 1)$ is the discount factor, $\Gamma > 0$ is the time endowment of the households. The utility function is strictly increasing in consumption and leisure, and satisfies standard regularity conditions.

We assume that government's revenue target is exogenously given, and in order to raise this preset revenue target the government levies flat rate taxes on the income from the two capital stocks at rates τ^k_{zt} and τ^k_{st}, and on the labour income from the two sectors at rates τ^n_{zt} and τ^n_{st}. We ommit consumption taxes for simplicity. As it is well known consumption and labour taxes are equivalent in this framework. We assume that the government runs a balanced budget each period, and there is an effective *commitment technology* which makes the government choose and announce a tax policy once and for all time. The government's budget constraints for all t are:

$$p_{ct} g_t \leq \tau^n_{zt} w_{zt} n_{zt} + \tau^n_{st} w_{st} n_{st} + \tau^k_{zt} r_{zt} k_{zt} + \tau^k_{st} r_{st} k_{st} \tag{5}$$

where w_{it} is the wage rate and r_{it} is the rental rate of capital for $i = \{z, s\}$, and p_{ct} is the relative price of consumption (the buyer price) in terms of the consumption-investment good. The representative household chooses the allocation $\{c_t, n_{zt}, n_{st}, k_{zt+1}, k_{st+1}\}_{t=0}^{\infty}$ taking taxes, prices and $k_0 = k_{z0} + k_{s0}$ as given so as to maximize (4) subject to the budget constraints:

$$p_{ct}c_t + p_{xt}\left[k_{zt+1} + k_{st+1}\right] \leq R_{zt}p_{xt}k_{zt} + R_{st}p_{xt}k_{st} + (1-\tau_{zt}^n)w_{zt}n_{zt} + (1-\tau_{st}^n)w_{st}n_{st} \tag{6}$$

with $R_{it} \equiv 1 + (1-\tau_{it}^k)\left(\frac{r_{it}}{p_{xt}} - \delta\right)$, $i = \{z, s\}$, and p_{xt} is the relative price of investment (the buyer price) in terms of the consumption-investment good. The capital income tax rate is a tax rate net of depreciation, such that after tax income to capital is equal to $\left[(1-\tau_{it}^k)\frac{r_{it}}{p_{xt}} + 1 - \delta\right] p_{xt}k_{it} + \delta p_{xt}k_{it}\tau_{it}^k$.

With $\beta^t\lambda_t$ as the Lagrange multiplier on period t version of (6), optimality of consumer's decision implies that:

$$\frac{u_c(t)}{p_{ct}} = \lambda_t \tag{7}$$

$$\frac{u_l(t)}{u_c(t)} = \frac{(1-\tau_{zt}^n)w_{zt}}{p_{ct}} = \frac{(1-\tau_{st}^n)w_{st}}{p_{ct}} \tag{8}$$

$$\frac{u_c(t)}{\beta u_c(t+1)} = \left(\frac{p_{ct}}{p_{ct+1}}\right)\left(\frac{p_{xt+1}}{p_{xt}}\right)R_{t+1} \tag{9}$$

with $R_t \equiv R_{zt} = R_{st}$, and where $u_c(t)$ and $u_l(t)$ are the partial derivatives of the utility function with respect to consumption and leisure, respectively.

In the retail sector, retailers purchase distribution services and the consumption-investment good, and sells consumption (to households and to the government) and investment (to households) with a technology that combines one unit of consumption with ψ_c units of distribution services, or one unit of investment with ψ_x units of distribution services. Optimal resource allocation in the retail sector equates marginal revenues with marginal cost, implying that:

$$p_{ct} = 1 + \psi_c p_{st} \tag{10}$$

$$p_{xt} = 1 + \psi_x p_{st} \tag{11}$$

where p_{st} is the relative price of distribution services in terms of the consumption-investment good. Thus, the purchase price of each good depends on the price of distribution services, and the unit cost of distributing these goods. With constant marginal cost of distribution, the difference between buyer price of consumption goods and buyer price of investment goods is simply a function of the price of distributions service.

Firms in production sectors own nothing except the technology. They hire capital and labour from households, and competitively maximize profits. Optimality in the production sectors requires that marginal products are equated with the rental prices of the production factors, implying that equilibrium factor prices satisfy:

$$\begin{aligned} r_{zt} &= f_k^z(k_{zt}, n_{zt}) && (12) \\ w_{zt} &= f_n^z(k_{zt}, n_{zt}) && (13) \\ r_{st} &= p_{st} f_k^s(k_{st}, n_{st}) && (14) \\ w_{st} &= p_{st} f_n^s(k_{st}, n_{st}) && (15) \end{aligned}$$

Definition 1 *A* feasible allocation *is a sequence* $\{k_{zt}, k_{st}, c_t, g_t, n_{zt}, n_{st}, x_t\}_{t=0}^{\infty}$ *that satisfies equations (1), (2) and (3).*

Definition 2 *A* price system *is a 7-tuple of nonnegative bounded sequences* $\{p_{ct}, p_{xt}, p_{st}, r_{zt}, r_{st}, w_{zt}, w_{st}\}_{t=0}^{\infty}$.

Definition 3 *A* government policy *is a 5-tuple of sequences* $\left\{g_t, \tau_{zt}^n, \tau_{st}^n, \tau_{zt}^k, \tau_{st}^k\right\}_{t=0}^{\infty}$.

Definition 4 (Competitive Equilibrium) *A* competitive equilibrium *is a feasible allocation, a price system, and a government policy such that (a) given the price system and the government policy, the allocations solves the firms' problems in production sector, the retailer's problem, and the household's problem; and (b) given the allocation and the price system, the government policy satisfies the sequence of government budget constraints (5).*

Given the government's preset revenue target, $k_0 = k_{z0} + k_{s0} > 0$, and the set of parameters of the model, for a particular sequence of tax rates $\left\{\widetilde{\tau}_{zt}^n, \widetilde{\tau}_{st}^n, \widetilde{\tau}_{zt}^k, \widetilde{\tau}_{st}^k\right\}_{t=0}^{\infty}$, a competitive equilibrium for this economy can be characterized by the solution to the system of equations that includes transversality conditions, (1), (2), (3), (5), (8), (9), (10), (11), (12), (13), (14), and (15), in the set of unknowns $\{k_{zt+1}, k_{st+1}, c_t, g_t, n_{zt}, n_{st}, x_t, p_{ct}, p_{xt}, p_{st}, r_{zt}, r_{st}, w_{zt}, w_{st}\}_{t=0}^{\infty}$. The equations (1), (2), and (3) as part of the competitive equilibrium conditions characterize that a competitive equilibrium allocation satisfy market clearing conditions for the consumption-investment good and distributions service. Equation (5) says that the government's budget is balanced each period, and this equation is part of the competitive equilibrium conditions because the allocations, prices and the government policy must satisfy either the household's or the government's budget constraint. The remaining equations simply say that allocations and prices in a competitive equilibrium must satisfy household's, retailer's and firms' optimal decisions.

3 The Ramsey Problem

Notice that indexed by different government policies, there can be many competitive equilibria. This multiplicity motivates the optimal taxation problem, i.e. choosing taxes that *(a)* maximize welfare, and *(b)* can be implemented in a competitive equilibrium. More formally:

Definition 5 (Ramsey Problem) *Given the government's preset revenue target and the household's initial stock of capital, the* Ramsey Problem *is to choose a competitive equilibrium that maximizes (4).*

We use the primal approach to derive the conditions that characterize the Ramsey allocation. Then we look for the taxes that can implement the second-best wedges. This approach was primarily proposed by Atkinson and Stiglitz (1980); we closely follow the approach presented in Ljungqvist and Sargent (2000, ch. 12). The Ramsey allocation can be characterized by choosing an allocation. The planner chooses the allocation $\{c_t, n_{zt}, n_{st}, k_{zt+1}, k_{st+1}\}_{t=0}^{\infty}$ so as to maximize (4) subject to the resource constraints (1), (2), and (3), and an *implementability constraint* that ensures that resulting taxes, allocations and prices are consistent with the competitive equilibrium. In order to derive the implementability constraint, we first iterate the household's budget constraint (6) backwards in order to derive its present value version. We then rerun the representative household's optimization problem using the present value budget constraint. Using the set of competitive equilibrium conditions, we substitute out prices and taxes in the present value budget constraint in order to derive an intertemporal constraint that involves initial conditions and allocations.

More precisely, we evaluate (6) at period T, and divide by the term $p_{xt} \prod_{j=1}^{T} R_j$. We then evaluate the resulting expression at period $T-1$, and add the two. We iterate this process backwards from $T-2$, and finally add period 0 version of (6). We impose terminal conditions to derive:

$$\sum_{t=0}^{\infty} \frac{[p_{ct}c_t - (1-\tau_{zt}^n)w_{zt}n_{zt} - (1-\tau_{st}^n)w_{st}n_{st}]}{p_{xt} \prod_{j=1}^{t} R_j} = R_0(k_{z0} + k_{s0}) \qquad (16)$$

Define the Arrow-Debreu price $q_t^o = p_{ct} \left[p_{xt} \prod_{j=1}^{t} R_j \right]^{-1}$, and rewrite (16) as:

$$\sum_{t=0}^{\infty} q_t^o c_t - \sum_{t=0}^{\infty} \frac{q_t^o}{p_{ct}} \left[(1-\tau_{zt}^n) w_{zt} n_{zt} + (1-\tau_{st}^n) w_{st} n_{st} \right] = R_0 (k_{z0} + k_{s0}) \qquad (17)$$

The representative household chooses allocations $\{c_t, n_{zt}, n_{st}\}_{t=0}^{\infty}$ so as to maximize (4) subject to the constraint (17). The optimality conditions include (8), (17) and:

$$q_t^o = \beta^t \frac{p_{c0} u_c(t)}{p_{x0} u_c(0)} \tag{18}$$

Use (18) and (8) in (17) in order to derive the implementability constraint:

$$\sum_{t=0}^{\infty} \beta^t \{u_c(t)c_t - u_l(t)(n_{zt} + n_{st})\} - \Omega\left(c_0, n_{z0}, n_{s0}, \tau_{i0}^k, \tau_{i0}^n\right) = 0 \tag{19}$$

with $\Omega\left(c_0, n_{z0}, n_{s0}, \tau_{i0}^k, \tau_{i0}^n\right) = R_0 \frac{p_{x0}}{p_{c0}} u_c(0)(k_{z0} + k_{s0})$, taking the government's preset revenue target, capital income tax rates in period 0, i.e. τ_{i0}^k, and the household's stock of capital in period 0, i.e. $k_0 = k_{z0} + k_{s0}$, as given.

Define the Pseudo utility function:

$$G(c_t, n_{zt}, n_{st}, \eta) \equiv u(c_t, \Gamma - n_{zt} - n_{st}) + \eta\left[u_c(t)c_t - u_l(t)(n_{zt} + n_{st})\right] \tag{20}$$

where $\eta \geq 0$ is the Lagrange multiplier associated with the implementability constraint, (19). Since $G_{nz} = G_{ns}$ for all t, we use $G_n(t)$ to denote the partial derivative of G with respect to labour in period t. Combine (1) and (3) in order to derive the resource constraint:

$$c_t + g_t + k_{zt+1} + k_{st+1} \leq f^z(k_{zt}, n_{zt}) + (1-\delta)(k_{zt} + k_{st}) \tag{21}$$

Let μ_{zt} and μ_{st} be the Lagrange multipliers associated with the resource constraints (21) and (2), respectively. The Ramsey equilibrium conditions involve (21), (2), (19), and:

$$
\begin{aligned}
G_c(t) &= \mu_{zt}\left(1+\psi_c\frac{\mu_{st}}{\mu_{zt}}\right);\ t\geq 1 && (22)\\
-G_n(t) &= \mu_{zt}f_n^z(t); t\geq 1 && (23)\\
-G_n(t) &= \mu_{st}f_n^s(t);\ t\geq 1 && (24)\\
\frac{\mu_{zt}}{\mu_{zt+1}} &= \beta\left[\frac{(1-\delta)\left(1+\psi_x\frac{\mu_{st+1}}{\mu_{zt+1}}\right)+f_k^z(t+1)}{\left(1+\psi_x\frac{\mu_{st}}{\mu_{zt}}\right)}\right];\ t\geq 1 && (25)\\
\frac{\mu_{zt}}{\mu_{zt+1}} &= \beta\left[\frac{(1-\delta)\left(1+\psi_x\frac{\mu_{st+1}}{\mu_{zt+1}}\right)+f_k^s(t+1)\frac{\mu_{st+1}}{\mu_{zt+1}}}{\left(1+\psi_x\frac{\mu_{st}}{\mu_{zt}}\right)}\right];\ t\geq 1 && (26)\\
G_c(0) &= \mu_{z0}+\mu_{s0}\psi_c+\Omega_{c0} && (27)\\
G_n(0) &= -\mu_{z0}f_n^z(0)+\Omega_{nz0} && (28)\\
G_n(0) &= -\mu_{s0}f_n^s(0)+\Omega_{ns0} && (29)
\end{aligned}
$$

The Ramsey allocation is an allocation $\{c_t, n_{zt}, n_{st}, k_{zt+1}, k_{st+1}\}_{t=0}^{\infty}$ and a multiplier $\eta \geq 0$ that satisfies the system of difference equations formed by (22)-(29), (21), (2), and (19). After solving for the Ramsey allocation, we can solve for the prices generated by Ramsey tax plans, and the Ramsey tax plans that implement the Ramsey allocations and the prices in a competitive equilibrium.

4 Analytical Results

We will discuss optimal policy in an asymptotic steady state. We consider a case in which there is a $T \geq 0$ for which government's revenue target is asymptotically constant for all $t \geq T$. We will also assume that the solution to the Ramsey problem converges to a time-invariant allocation, so that consumption, capital allocation and working time are constant after some time.

Proposition 1: *In a steady state, the Ramsey policy generates allocations that satisfy production efficiency, i.e. the Ramsey policy generates allocations for which the ratio of marginal product of capital across sectors equals the ratio of marginal product of labour across sectors.*

Proof. The steady state versions of (25) and (26) are:

$$
\begin{aligned}
\frac{1}{\beta} &= 1-\delta+\frac{f_k^z}{1+\psi_x\frac{\mu_s}{\mu_z}} && (30)\\
\frac{1}{\beta} &= 1-\delta+\frac{f_k^s\frac{\mu_s}{\mu_z}}{1+\psi_x\frac{\mu_s}{\mu_z}} && (31)
\end{aligned}
$$

Together they imply $\frac{\mu_s}{\mu_z} = \frac{f_k^z}{f_k^s}$. From steady state versions of (23) and (24), $\frac{\mu_s}{\mu_z} = \frac{f_n^z}{f_n^s}$. ■

Proposition 1 says that if the Ramsey equilibrium has a steady state, the corresponding Ramsey policy will generate allocations that will satisfy the productive efficiency condition, implying that any implementable optimal policy would not violate the productive efficiency argument.

Proposition 2: *In a steady state the optimal capital income tax rate is in general different from zero. The optimal capital income tax rate is zero if and only if optimal labour income tax rates are equal across sectors.*

Proof. Using the fact that after-tax wages are equalized across sectors (consider steady state version of (8)), and using the steady state versions of (10), (11) and (12)-(15), the steady state version of the competitive equilibrium condition (9) can be written as:

$$\frac{1-\beta}{\beta} = \left(1-\tau_z^k\right)\left[\frac{f_k^z}{1+\psi_x\left(\frac{1-\tau_z^n}{1-\tau_s^n}\right)\frac{f_n^z}{f_n^s}} - \delta\right] \tag{32}$$

$$\frac{1-\beta}{\beta} = \left(1-\tau_s^k\right)\left[\frac{f_k^s\left(\frac{1-\tau_z^n}{1-\tau_s^n}\right)\frac{f_n^z}{f_n^s}}{1+\psi_x\left(\frac{1-\tau_z^n}{1-\tau_s^n}\right)\frac{f_n^z}{f_n^s}} - \delta\right] \tag{33}$$

(30) and (32) together imply that

$$\left(1-\tau_z^k\right) = \left[\frac{f_k^z - \delta\left(1+\psi_x\frac{f_n^z}{f_n^s}\right)}{\left(1+\psi_x\frac{f_n^z}{f_n^s}\right)}\right]\left[\frac{1+\psi_x\left(\frac{1-\tau_z^n}{1-\tau_s^n}\right)\frac{f_n^z}{f_n^s}}{f_k^z - \delta\left(1+\psi_x\left(\frac{1-\tau_z^n}{1-\tau_s^n}\right)\frac{f_n^z}{f_n^s}\right)}\right] \tag{34}$$

(34) implies that $\tau_z^k = 0 \Longleftrightarrow \tau_z^n = \tau_s^n$. It can be shown from (31) and (33) that $\tau_s^k = 0 \Longleftrightarrow \tau_z^n = \tau_s^n$. ■

In proposition 2, we show that in a steady state of the Ramsey equilibrium, the optimal capital income tax rate depends on the optimal labour income tax rates, and the optimal capital income tax rate is in general different from zero. As agreed before, we present this result as a result independent of any explicit restrictions on preferences and technology[3]. In what follows in this section, we will discuss different analytical properties of this Ramsey policy. We begin with

[3]We have simplified our algebra by imposing $G_{nz} = G_{ns}$ for all t, and if one removes this simplification, the correspondence between optimal capital income tax rates and optimal labour income tax rates is still there, although in a less clear form. It is obvious that given the current analysis, if one imposes further restrictions on preferences and technology, proposition 1 and 2 are unchanged.

the simplest question: what is the correspondence between the optimal labour income tax rates and the optimal capital income tax rates?

We attempt to answer this question in proposition 3. As we will discuss later, proposition 3 also explains how the optimal policy and the correspondence between the two sets of income tax rates undo the relative price difference. In addition, we will argue that this correspondence between the two sets of income taxes assists in explaining how the optimal policy undoes the capital income tax distortions.

Proposition 3: *In the multisector economy, in a steady state if it is optimal to set higher labour income tax in sector $i \in \{z, s\}$, it is optimal to tax capital income in the same sector $i \in \{z, s\}$, and set zero tax on capital income from the other sector.*

Proof. Divide (32) by $\left(1-\tau_z^k\right)$ and add δ. Divide (33) by $\left(1-\tau_s^k\right)$ and add δ. Then divide the former expression with the latter, and impose the Ramsey condition $\frac{f_n^z}{f_n^s} = \frac{f_k^z}{f_k^s}$ to derive:

$$\frac{\frac{1-\beta}{\beta}\left(\frac{1-\tau_s^k}{1-\tau_z^k}\right) + \delta\left(1-\tau_s^k\right)}{\frac{1-\beta}{\beta} + \delta\left(1-\tau_s^k\right)} = \frac{1-\tau_s^n}{1-\tau_z^n} \tag{35}$$

Since (35) combines both sets of Ramsey conditions (30)-(31) with corresponding competitive equilibrium conditions, (32) and (33), one can analyze optimal policy in a steady state from (35). Following proposition 2, we will hold that for a particular set of equilibrium allocation and prices, there is an optimal policy that sets $\tau_s^n = \tau_z^n, \tau_s^k = \tau_z^k = 0$ in (35), since the policy satisfies (35).

Say for another set of equilibrium prices and allocations, the government decides to implement another optimal policy that uses capital income tax/subsidy and differential labour income tax rates. We will consider the case where the government decides to hold τ_s^n fixed at its previous level, but decides to increase τ_z^n. Given (35), setting τ_z^n at a higher level increases the right hand side. At the Ramsey optimum, any set of implementable tax rates must sastify (35), and thus increasing the labour income tax rate in sector z must accompany an increase in the capital income tax rate in the same sector from zero level, holding the capital income tax rate in sector s at the zero level. ■

With proposition 3, we can now analyze the efficiency property of the optimal policy and distortion neutralizing property of the correspondence between the two sets of income tax rates. Say there is low production of distributions service, and thus the relative price of distributions service is higher than the relative price of the consumption-investment good. This is due to underaccumulation of capital and working time in distributions service production sector, which violates the production efficiency condition. The optimal policy in this case should set higher labour income tax and a capital income tax in the sector that produces consumption-investment good, and should set zero capital income tax in the sector that produces distributions service. This policy is optimal since the households will shift capital and working time to the distributions service sector, which will increase production of distributions service and undo the relative price difference. Due to the correspondence between labour income taxes and capital income taxes, and since the optimal policy would always set zero capital income tax in one sector, the differential labour income taxation will undo the capital income tax distortions.

In what follows, we will discuss some technical properties of the optimal policy in a steady state. We will first discuss the steady state optimal policy of taxing labour income when it is optimal to tax capital income at zero rate. We argue that if utility is linear in leisure, with this policy the optimal labour income tax rule follows Ramsey's inverse elasticity rule. After that, we will discuss the steady state optimal policy of taxing labour income when it is optimal to tax capital income.

Proposition 4: *If utility is linear in leisure, the steady state optimal plan that involves zero capital income tax involves labour income taxes that satisfy Ramsey's inverse elasticity rule, i.e. the optimal labour income tax is high if the elasticity of marginal utility with respect to consumption is high.*

Proof. Following proposition 2, the steady state optimal plan that involves zero capital income tax also involves same labour income tax rates for two sectors. For this particular policy, we simplify $\tau_s^n = \tau_z^n = \tau^n$. Combine steady state versions of (8) and (10) to derive:

$$u_l = \frac{u_c \left(1 - \tau^n\right) f_n^z}{1 + \psi_c p_s} \tag{36}$$

If utility is linear in leisure, (23) and (24) imply that in a steady state:

$$\left(1 + \eta\right) u_l = \mu_i f_n^i; \; i \in \{z, s\} \tag{37}$$

Also with $\tau_s^n = \tau_z^n = \tau^n$, (8), (13), (15) and proposition 1 imply that in a steady state, $\frac{\mu_s}{\mu_z} = p_s$. Since $G_c = u_c \left(1 + \eta\right) + \eta u_{cc} c$, (22) therefore implies that in a steady state:

$$u_c \left(1 + \eta\right) = \mu_z \left(1 + \psi_c p_s\right) - \eta u_{cc} c \tag{38}$$

Substituting (38) and (37) in (36), it is straightforward to show that

$$1 - \tau^n = 1 - \left(\frac{\eta}{1 + \eta}\right) \left(-\frac{u_{cc} c}{u_c}\right) \tag{39}$$

The term $\left(-\frac{u_{cc} c}{u_c}\right)$ in (39) is the elasticity of marginal utility with respect to consumption, and the right hand side and left hand side of (39) are decreasing in this elasticity and the tax rate, respectively. Consequently, optimal labour income taxes are higher for higher elasticity of the marginal utility with respect to consumption.

The higher the elasticity of the marginal utility with respect to consumption, the lower is the elasticity of demand with respect to prices. This is because the marginal utility of consumption is closely related to the relative price of consumption. So, high elasticity of marginal utility with respect to consumption means a high elasticity of the price with respect to quantity, implying that one percentage point change in the quantity leads to a large change in the price. But this also means that a one percentage point change in the price induces only a small change in consumption implying the demand is inelastic. Since the economy does not have a consumption tax, the labour income tax does the equivalent; i.e. if demand for consumption is less elastic, the optimal labour income tax is higher. This is perfectly consistent with Ramsey's inverse elasticity rule. ■

We now propose a lemma which illustrates a technical property of the steady state optimal labour income tax rates if the optimal plan involves capital income taxation.

Lemma 1: *If utility is linear in leisure, the steady state optimal plan that involves a tax on capital income must involve optimal labour income taxes that satisfy*

$$\frac{1-\tau_z^n}{1-\tau_s^n} = \frac{f_n^s}{f_n^z} p_s \tag{40}$$

It therefore implies that in a steady state it is optimal to tax capital income and set differential rates of labour income taxes if the relative price of distributions service is not equal to the ratio of marginal product of labour. It also implies that for this optimal policy the correspondence between the optimal labour income tax ratio and the optimal capital income tax rate is given by

$$\left(\frac{1-\tau_z^n}{1-\tau_s^n}\right)\frac{f_n^z}{f_n^s} = \frac{\beta\left(1-\tau_z^k\right)\left(f_k^z-\delta\right)-(1-\beta)}{\psi_x\left[1-\beta-\delta\beta\left(1-\tau_z^k\right)\right]} = \frac{\left[1-\beta-\delta\beta\left(1-\tau_s^k\right)\right]}{\beta\left(1-\tau_s^k\right)\left(f_k^s-\psi_x\delta\right)-\psi_x\left(1-\beta\right)} \tag{41}$$

Proof. If utility is linear in leisure, in a steady state (37) holds. With (8) and (10), it therefore implies that in a steady state of the Ramsey equilibrium,

$$\left(1-\tau_z^n\right) = \frac{\mu_z\left(1+\psi_c p_s\right)}{u_c\left(1+\eta\right)} \tag{42}$$

$$\left(1-\tau_s^n\right) = \frac{\mu_s\left(1+\psi_c p_s\right)}{u_c p_s\left(1+\eta\right)} \tag{43}$$

(42), (43) and proposition 1 imply (40). Notice that (40) holds for the steady state of the Ramsey equilibrium irrespective of the optimal policy; i.e. this general condition shows the correspondence between the optimal labour income tax policy and the relative price of distributions service. For instance, if the steady state optimal policy is to set zero tax on capital income and same labour income tax rates (which is one optimal policy from a set of many), the optimal policy implements allocations and prices such that $\frac{f_n^z}{f_n^s} = p_s$ (see proposition 4).

By contrast, if the optimal policy is to tax capital income and set differential labour income tax rates, the left hand side of (40) is not equal to one, and such a policy implements allocations and price for which $\frac{f_n^z}{f_n^s} \neq p_s$. Furthermore, using the steady state versions of (9), (12) and (14), one can derive

$$p_s = \frac{\beta\left(1-\tau_z^k\right)\left(f_k^z-\delta\right)-(1-\beta)}{\psi_x\left[1-\beta-\delta\beta\left(1-\tau_z^k\right)\right]} = \frac{\left[1-\beta-\delta\beta\left(1-\tau_s^k\right)\right]}{\beta\left(1-\tau_s^k\right)\left(f_k^s-\psi_x\delta\right)-\psi_x\left(1-\beta\right)} \tag{44}$$

which together with (40) implies (41). ■

Lemma 1 explains two important findings: it first shows that the optimal policy depends on the relative price of distributions service, and then it explains the correspondence between optimal labour income tax ratio and the optimal capital income tax rates.

5 Conclusion

In this paper we propose an important extension to the Chamley-Judd result: we show that in an otherwise standard neoclassical economy where distributions service is a market good, any difference in buyer price of consumption and investment can be undone by the optimal policy which taxes/subsidizes income from capital. This policy is supported by differential labour income taxation which undoes the intertemporal distortion potential of capital income taxation. We propose some qualitative characteristics of this result. We argue that in a steady state of this economy a capital income tax/subsidy is optimal because it can restore the production efficiency. The model we propose can recover the standard zero capital income tax result. For instance, if one assumes that unit cost of distributing consumption and unit cost of distributing investment is same, the model is a simple one sector neoclassical growth model where relative price of consumption and relative price of investment are same. In such a model the Chamley-Judd result holds unconditionally. We show that in an economy where there is a unit cost difference of distributing consumption and investment, the Chamley-Judd result is a special case.

(MORE TO BE WRITTEN).

6 Bibliography

1. Atkeson, A., Chari, V. V. & Kehoe, P. (1999). 'Taxing Capital Income: A Bad Idea', Federal Reserve Bank of Minneapolis Quarterly Review, Vol. 23, pp. 3-17.

2. Atkinson, A. B. & Stiglitz, J. E. (1980). Lectures on Public Economics, McGraw-Hill, New York, 1980.

3. Chamley, C. (1986). 'Optimal Taxation of Capital Income in general Equilibrium with Infinite Lives', Econometrica, Vol. 54, pp. 607-622.

4. Chari, V. V. & Kehoe, P. J. (1999). 'Optimal Fiscal and Monetary Policy', in J. B. Taylor and M. Woodford, eds., Handbook of Macroeconomics, Vol. 1. Amsterdam: North-Holland, 1999, pp.1671-1745.

5. Correia, I. H. (1996). 'Should Capital Income be Taxed in the Steady State?', Journal of Public Economics, Vol. 60, pp. 147-151.

6. Erosa, A. & Gervais, M. (2001). 'Optimal Taxation in Infinitely-Lived Agent and Overlapping Generations Models: A Review', Federal Reserve Bank of Richmond Economic Quarterly, Vol. 87/2, pp. 23-44.

7. Jones, L. E., Manuelli, R. E. & Rossi, P. E. (1997). 'On the Optimal Taxation of Capital Income', Journal of Economic Theory, Vol. 73, pp. 93-117.

8. Judd, K. L. (1985). 'Redistributive Taxation in a Simple Perfect Foresight Model', Journal of Public Economics, Vol. 28, pp. 59-83.

9. Judd, K. L. (1999). 'Optimal Taxation and Spending in General Competitive Growth Models', Journal of Public Economics, Vol. 71, pp. 1-26.

10. Ljungqvist, L. & Sargent, T. J. (2000). Recursive Macroeconomic Theory. London: The MIT Press, 2000.

11. Ramsey, F. P. (1927). 'A Contribution to the Theory of Taxation', The Economic Journal, Vol. 37, pp. 47-61.