OPENNESS AND ECONOMIC GROWTH

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Abstract

Four different versions of the macroeconomic models popular in policy analysis are specified and numerically implemented. Each provides a framework to experiment fiscal, monetary and trade policy issues for a small open economy or for economies that are interdependent in the global economy. Simulations and non-linear optimisation techniques are employed to find optimal tax, trade and monetary policies. Results in the dynamic model are founded on prices that guarantee the general equilibrium in each period that satisfy inter-temporal budget constraint for households, government and the economy as a whole.

Key words: growth, open economy, economic policy
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I. Introduction

Global economy is interdependent. Growth rates of output in one economy affect that in other economies. Policy makers like to introduce economic policies to achieve a higher rate of growth at home country given their understanding of the structure of the global economy and the interaction of home economy with other economies. Paper builds on four different modelling frameworks widely used in the international macroeconomics literature to explain such interdependence. First one is a small open economy model solved numerically for assessing the impacts of expansionary fiscal, monetary and fiscal-monetary policy mix scenarios. The next one is the model for an interdependent global economy in which policies of one economy can influence another economy. Third model is based on actual time series data set of five major industrial economies on growth rates estimated using single equation and simultaneous equation econometric techniques. Finally one is a micro founded dynamic general equilibrium model formulated and solved to explain how prices, exchange rates, real exchange rates affect major macroeconomic variables in a global economy. In each case assessment is made on how policy actions can alter the course of an economy by discussing model generated results in response to specific policy chosen by policy makers.

II. Small Open Economy Model

Economic activities of a small open economy have very limited impacts on other economies. According to the standard Keynesian and Mundell-Flemming (Keynes (1936), Mundell (1962), Fleming (1962)) models external influences occur through the inflow and outflow of capital as dictated by the movements in the foreign exchange rate in response to the underlying trends of inflation and the interest rates.

Such a small open economy macroeconomic model includes consumption, investment, tax or spending, net exports, real exchange rate, money demand and money supply and aggregate supply. As a work-horse model for analysing open economy issues, I illustrate this model using following eleven equations.

Consumption is determined by the disposable income as:

$$C = a + b(Y - T) \quad (1.1)$$

Tax revenues are proportional to income:

$$T = tY \quad (1.2)$$

Investment is inversely related to the interest rate:

$$I = I_0 - I_1 r \quad (1.3)$$

Imports respond positively to the domestic income and negatively to the exchange rate:

$$IM = m_0 + \mu_1 Y + \mu_2 ER \quad (1.4)$$

Exports relate positively to the exchange rate and foreign income:

$$X = \eta_1 ER + \eta_2 Y^* \quad (1.5)$$

Money demand depends positively on income and negatively to the interest rate:

$$\frac{M}{P} = fY - kr \quad (1.6)$$

Using the market clearing condition $Y = C + I + G + (X - M)$ this model is solved as:

$$Y = \frac{a + I_0 + \frac{I_1}{k}\frac{M}{P} + G + \eta_1 ER + \eta_2 Y^* - m_0 - \mu_2 ER}{\left(1 - b + bt + \mu_1 - \frac{I_1}{k} f\right)} \text{ and } r = \frac{1}{k}\left[fY - \frac{M}{P}\right]$$

Trade balance defined as the difference between exports and imports:

$$TB = X - M \quad (1.7)$$

Budget deficit is the excess of government spending over the tax revenue:

$$BD = T - G \quad (1.8)$$

Savings represents residual income after meeting the consumption and tax spending:

$$S = Y - C - T \quad (1.9)$$

Private plus public saving equals net exports equals net capital flow:

$$(S - I) + (T - G) = (X - M) = KA \quad (1.10)$$

Prices move according to the classical money market:

$$MV = PY \quad (1.11)$$

Above eleven equations are regular text book versions of an open economy macroeconomic model (Blanchard (2003), Mankiw (2003), Miles and Scott (2002), Bradford and De Long (2003)). Model is demand determined and closed by the saving investment identity as given in equation (10). In order to solve all eleven equations simultaneously, model parameter, *a*, *b*, *t*, i_0, i_1, m_0, μ_0 μ_1, μ_1, η_1, η_2, f, k , v, G, M/P and *Y** need to be specified as I have presented in Table 1 below.

Table 1
Parametric Specification of the Small Open Economy ISLM model

Consumption, investment and government spending	Imports, exports and capital flows	Money market

C0 = 200;
b=0.75;
t1=0.3;
i0= 50;
i1=-5;
G= 100;
	μ_0 =20;
	μ_1 =0.2; μ_2 =-3;
	η_1 =5; η_2 =0.2;
	KA =-150;
	Y*=200;
		M/P=100;
		P = 1;
		f=0.2;
		k=-60;
		v=5;

Table 2
Macroeconomic Impacts of Fiscal and Monetary Policy in the small open economy ISLM model

Key Model variables	Base case Solution (G=100, M=100)	Fiscal expansion (G=200)	Monetary Expansion (M =300)	Fiscal and Monetary Expansion (G=200, M=300)
Consumption (C)	742.8	1222.8	956.0	1100.0
Income (y)	1033.9	1604.4	1260.0	1500.0
Interest rate (r)	1.8	3.7	0.8	0.0
Imports (m)	100.5	171.8	128.8	158.8
Tax revenue (T)	310.2	240.7	252.0	300.0
Investment (I)	310.2	240.7	252.0	300.0
Trade-gap (X-M)	150.0	150.0	150.0	150.0
Budget-gap (T-G)	210.2	40.7	152.0	100.0
Saving-investment gap (S-I)	-60.2	109.3	-2.0	50.0
Exchange rate (ER)	42.1	56.4	47.8	53.8

A small open economy model is solved for the base case using above values of parameters. Then model is solved for three different policy scenarios (a) fiscal expansion (b) monetary expansion and (c) both fiscal and monetary mix. Model assumes a flexible exchange rate regime in all above scenarios. The classical scenario in which prices are perfectly flexible is also computed at the end. The results of the model are given in the Table 2.

All of these results make intuitive sense. Increase in public spending raises income and consumption but crowds out the private investment. Levels of tax collection and saving rise in response to increase in income but the budget surplus significantly deteriorates as expansion in spending is much higher than extra revenue raised after increase in income. When capital outflow is fixed at 150, increase in public spending puts pressure in the exchange rate.

The monetary expansion lowers both the interest rate and the exchange rates but has less impact on output and consumption than of the expansionary fiscal policy. Though the lower interest rate has impact on investment, tax collection is more sensitive to income than the investment is to the interest rate. Consequently tax revenue rises and dampens the expansionary impact of the monetary policy.

The accommodative fiscal and monetary policies generate better results both in terms of increase in consumption and income as well as in terms of saving, investment gaps and the trade gaps.

III. Global Economy model of Two Economies

Small open economy model presented above assumes foreign income as given, which may be realistic for a very small economy whose trade links are trivial in comparison to its trading partners. In most cases one economy has discernible influence in another economy. Such inter dependency requires a model for inter-dependent economies. Similarly time factor need to be taken into account as it takes time for dynamic adjustment. The static model presented above is not enough in analysing these dynamic issues. More realistic modelling requires capturing the inter-dependency among economies over time. These dimensions can be extended to the small open economy model presented above by adding time and country specific subscripts to the variables and parameters of the above model. This section illustrates how these features can be added to make the model more appropriate for analysis of the interdependent global economy. For simplicity the global economy is assumed to consist of two inter-dependent economies and the economic horizon is of twenty seven years from 2003 to 2027. Both economies have standard macroeconomic set up with product, labour and money markets. Phillip's curve explains the trade-off between unemployment and inflation. Fiscal and monetary policies can be adopted to influence the growth path of the economy. This model is in spirit of Mundel-Fleming specification of an open economy with flexible exchange rate regime. Two economies are linked by international trade in goods and capital. Subscript *i* in a variable denote country *i*, and subscript *t* denotes time period *t*. Symbol "*" is used to represent a foreign variable to economy i. Model is kept simple to make results more appealing.

The first equation gives the national income identity in terms of consumption, investment, government spending and net exports:

$$Y_{i,t} = C_{i,t}\left(Y_{i,t} - T_{i,t}\right) + I_{i,t} + G_{i,t} + X_{i,t} - M_{i,t} \qquad (2.1)$$

where *Y* is output, *C* consumption, *I* investment, *G* government spending, *X* is exports and M is imports and *T* is tax revenue.

Current consumption depends on current disposable income:

$$C_{i,t} = a + b\left(Y_{i,t} - T_{i,t}\right) \qquad (2.2)$$

Tax revenue at period *t* depend on income at period *t*:

$$T_{i,t} = tY_{i,t} \qquad (2.3)$$

Investment of country I is inversely related to interest in country i:

$$I_{i,t} = I_0 - I_1 r_{i,t} \quad (2.4)$$

I assume that the public spending grows by rate *gr* in each period:

$$G_{i,t} = G_1 (1 + gr_i)^t \quad (2.5)$$

Exports depend on current exchange rate and current foreign income:

$$X_{i,t} = \eta_1 ER_{i,t} + \eta_2 Y^*_{i,t} \quad (2.6)$$

Similarly imports depend on current domestic output and current exchange rate:

$$M_{i,t} = m_0 + \mu_1 Y_{i,t} + \mu_2 ER_{i,t} \quad (2.7)$$

Net exports is the difference between current exports and imports:

$$TB_{i,t} = X_{i,t} - M_{i,t} \quad (2.8)$$

This model includes a version of the Lucas aggregate supply function with actual and expected price levels

$$Y_{i,t} = \overline{Y}_{i,t} + \alpha\left(P^e_{i,t} - P_{i,t}\right) \quad (2.9)$$

π inflation, Y^* foreign income, and P domestic price P^e expected inflation.

Output is linked to the employment in a Keynesian fashion:

$$Y_{i,t} = \beta L_{i,t} \quad (2.10)$$

The model assumes a standard money demand function:

$$\left(\frac{M}{P}\right)_{i,t} = fY_{i,t} + kr_{i,t} \quad (2.11)$$

Budget deficit is the excess of government spending over tax revenue:

$$BD_{i,t} = T_{i,t} - G_{i,t} \quad (2.12)$$

Level of savings in period t is residual of income after consumption spending and tax collection:

$$S_{i,t} = Y_{i,t} - C_{i,t} - T_{i,t} \quad (2.13)$$

Saving investment identity $\left(S_{i,t} - I_{i,t}\right) + \left(T_{i,t} - G_{i,t}\right) = \left(X_{i,t} - M_{i,t}\right) = KA_{i,t}$ (2.14)

The trade off between unemployment in inflation is given by the Phillips curve:

$$P_{i,t} = \gamma\left(U_{i,t} - U_{i,t-1}\right) \quad (2.15)$$

The external debt accumulates if the trade balance is positive:

$$D_{i,t} = D_{i,0} + TB_{i,t} + 0.1 * \left(1 + r_{i,t}\right) * D_{i,t-1} \quad (2.16)$$

Real exchange rate: $\lambda_{i,t} = \dfrac{ER_{i,t} P^*_{i,t}}{P_{i,t}}$ (2.17)

where ER nominal exchange rate, λ real exchange rate, P^* foreign price level

Financial integration condition: $i_{i,t} = i^*_{i,t}$ (2.18)

Where *i* interest rate, i* is the foreign interest rate .

The influence of the external sector at the home economy occurs through foreign income, real exchange rate, the interest rate that is determined in the global market. The level of consumption, investment and net exports and the national income are determined once values of the E, P^* and Y^*, and the domestic and foreign interest rates i^* are known. The model is closed by making net national saving equal to net exports. This model can be applied to assess impacts of increase in the domestic price level, a rise in government spending, increase or decrease in real money balances and the interest rate, the real exchange rate ,increase in the capital and labour inputs a rise in the level of foreign income, change in both taxes and spending.

Table 3
Parameters of the Two Country ISLM Open Economy Model

Consumption / investment			Imports and Exports			Money Market		
Parameter	eco1	Eco2	Parameter	Eco1	Eco 2	Parameter	Eco1	Eco2
C0	200	20	mu0	10	20	mp	100	20
b	0.75	0.8	mu1	0.3	0.3	v	3	3
t1	0.05	0.05	mu2	-1.3	-1.3	f	0.2	0.3
g	100	20	eta	1.55	1.5	k	-30	-40
i0	50	25	eta1	0.2	0.2	lf	1000	500
i1	-5	-5.0	KA	-150	-70	gr	0.05	0.03

Table 4
Key Variables from the Solutions of the Dynamic Open Economy ISLM Model

Model Generated Solutions for Economy 1									
	Consumption	Income	Imports	Exports	Investment	Interest Rate	Gov exp	Tax Revenue	budget-gap
2003	1027	1161	301	301	29	4.24	105	58	-47
2005	1055	1200	311	311	29	4.142	116	60	-56
2010	1138	1316	340	340	31	3.852	148	66	-82
2015	1244	1465	378	378	33	3.481	189	73	-115
2020	1379	1654	426	426	35	3.007	241	83	-158
2025	1551	1896	488	488	38	2.403	307	95	-212
2030	1771	2205	365	365	42	1.631	392	110	-282
	Model Generated Solutions for Economy 2								
2003	207	246	73	73	18	1.328	21	12	-8
2005	211	251	74	74	18	1.334	22	13	-9
2010	221	265	78	78	18	1.353	25	13	-12
2015	234	281	82	82	18	1.375	29	14	-15
2020	248	300	87	87	18	1.4	34	15	-19
2025	265	322	92	92	18	1.43	39	16	-23
2030	284	348	67	67	18	1.464	46	17	-28

Two country interdependent global economy model is twice as complex as the small open economy model to solve. More often it is important to consider more than two countries either to analyse the international policy coordination issue or to assess impacts of development in the international economies at home. Models become analytically intractable as more and more countries are added into it. Simultaneous solution of all of the model equations requires numerical techniques as illustrated below. Mixed complementarity solver in GAMS is used to solve this model. Parameterisation and the some results of this model are given in following tables.

This model tracks economic events well in economy 1 and economy 2 over time. Economy 1 seems to be more flexible than the economy 2 in terms of saving and investment activities and this in turn affects the level of income. Economy 1 is bigger and growing faster than economy 2 which is less flexible as shown by the labour market variables in the following table.

Table 5

Model Generated Solutions for Economy 1								
	Exchange Rate	Money	Price	Unemployed	Employed	Wage Income	Debt	Real Wage Rate
2005	45.6	115.8	14.2	-442.6	1600.2	1220.2	89.1	0.0540
2010	49.7	147.7	18.1	-277.8	1755.3	1328.7	97.8	0.0420
2015	54.9	188.6	23.1	-67.6	1953.2	1467.3	91.7	0.0320
2020	61.6	240.7	29.5	200.8	2205.8	1644.1	84.4	0.0250
2025	70.0	307.2	37.7	543.3	2528.2	1869.7	76.6	0.0200
2030	235.6	392.0	48.1	980.5	2939.6	2157.7	68.4	0.0150
Model Generated Solutions for Economy 2								
2005	16.1	21.9	6.2	212.0	334.3	334.0	64.4	0.1600
2010	16.6	25.3	7.2	280.2	353.2	347.2	65.4	0.1360
2015	17.2	29.4	8.4	359.3	375.0	362.5	65.6	0.1150
2020	17.9	34.0	9.7	451.0	400.3	380.2	65.8	0.0980
2025	18.6	39.5	11.3	557.2	429.6	400.7	66.0	0.0830
2030	44.4	45.8	13.1	680.4	463.6	424.5	66.3	0.0700

These results are just indicative of how a global economy model can be constructed in the spirit of the Keynesian model. Even a slight difference in the specification of the model is enough to cause a large variation in the economic variables. It is very important, therefore, to establish cause and effect among these economic variables over time.

IV. Econometric Time Series Model of Global Economy

In addition to a small open economy and inter-dependent global economy models presented above economists often analyse inter dependency among various economies using panel data on economic growth across countries. Above illustrations were on how macro economic time series can be generated by model based solutions. These model generated series need to be tested against actual series of an economy. One method for testing such inter-dependency is to use the pooled cross section and time series data set for a number of economies. For this time series based multiple economy model is estimated in which actions in one country have impact on actions of other countries. For instance growth rates assumed to be inter-dependent in five major economies in the world:

UK: $$g_{uk,t} = \alpha_{1,0} + \alpha_{1,1} g_{US,t} + \alpha_{1,2} g_{Gr,T} + \alpha_{1,4} g_{Fr,T} + \alpha_{1,5} g_{JP,T} + e_{1,i} \quad (3.1)$$

US: $$g_{US,t} = \alpha_{2,0} + \alpha_{2,1} g_{UK,t} + \alpha_{2,2} g_{Gr,T} + \alpha_{2,4} g_{Fr,T} + \alpha_{2,5} g_{JP,T} + e_{2,i} \quad (3.2)$$

Germany: $$g_{GR,t} = \alpha_{3,0} + \alpha_{3,1} g_{US,t} + \alpha_{3,2} g_{UK,T} + \alpha_{3,4} g_{Fr,T} + \alpha_{3,5} g_{JP,T} + e_{3,i} \quad (3.3)$$

Japan: $$g_{JP,t} = \alpha_{4,0} + \alpha_{4,1} g_{US,t} + \alpha_{4,2} g_{UK,T} + \alpha_{4,4} g_{Gr,T} + \alpha_{4,5} g_{FR,T} + e_{4,i} \quad (3.4)$$

France: $$g_{FR,t} = \alpha_{1,0} + \alpha_{5,1} g_{US,t} + \alpha_{5,2} g_{UK,T} + \alpha_{5,4} g_{Gr,T} + \alpha_{5,5} g_{JP,T} + e_{5,i} \quad (3.5)$$

Though it was possible to include other exogenous variables such as the interest rate or the exchange rate or the budget and trade deficits among the explanatory variables I used single equation and VAR based estimation for analysing the impact of openness among various economies.

III. Data

Actual data on growth rate of output, ratios of savings and investment to the GDP, real interest rate, growth rates of population and per capita output and ratio of trade to GDP is obtained from the World Band CD 2002 and have used the Shazam programme to estimate the model (..to be updated and reestimation)

Table 6:Pooled estimation 1

Interdependence in economic Growth: Evidence from the Pooled Regression

Pooled Sample size 154 (31 observations for: UK,US, Japan Germany and France)

Dependent variable	Coefficient	T-ratio	P-value
Real interest rate	0.17439E-01	0.3806	0.704
Population growth rate	1.3682	0.3806	0.008
Investment ratio	-0.14586	2.677	0.230
Saving ratio	0.21144	-1.204	0.032
Trade ratio (openness)	-0.15425E-02	-0.1025	0.918
Constant	0.37826	0.2766	0.782

R-SQUARE = 0.1570 R-SQUARE ADJUSTED = 0.1285 F-Value =5.511

Other exogenous variables such as the interest rate or the exchange rate or the budget and trade deficits also can be used as the explanatory variables as necessary.

OLS (corrected for the autocorrelation, heteroroscedasticity, or multicollinearity can be applied). The data need to be checked for stationarity using unit roots or other Methods (Nelson and Plosser (1982)).

Table 7: Pooled estimation 2

Interdependence in economic Growth: Evidence from the Pooled Regression

Pooled Sample size 154 (31 observations for: UK,US, Japan Germany and France)

Dependent variable	Coefficient	T-ratio	P-value
Lagged growth rate	0.16047	2.029	0.044
Lagged investment ratio	-0.18579	-3.708	.000
Saving ratio	0.19154	4.220	0.000
Trade ratio (openness)	-0.26965E-01	-2.499	0.014
Constant	2.9901	3.080	0.002

R-SQUARE = 0.2340 R-SQUARE ADJUSTED = 0.2134

F-Value =11.376 DURBIN-WATSON = 1.8810

Table 8: Pooled estimation 3

Determinants of Economic Growth: Country and Time specific effects

	RR	UK	US	GRM	FR	T25	T27	T28	T29	T30	T31	CONSTANT
Coefficient	0.054	-3.272	-2.591	-3.759	-3.206	1.234	-1.194	-2.430	-0.809	-4.298	-4.173	5.484
T-ratio	1.032	-3.653	-2.762	-3.821	-3.554	1.312	-0.998	-1.983	-0.661	-3.519	-3.458	6.766
P-values	0.304	0.000	0.007	0.000	0.001	0.192	0.320	0.049	0.510	0.001	0.001	0.000

R-SQUARE = 0.2091 R-SQUARE ADJUSTED = 0.1483 F = 3.438

Above estimates were obtained by using the single equation method. However, simultaneity among the growth variables implies that there is systematic bias in estimates from the single equation method. Therefore an interdependent global multi-country model is also estimated in which growth in one country not only depends on internal factors but also external factors. Estimates of this multi-country growth model explains growth rates of each of above five major industrial economies, UK, Germany, France, Japan and USA simultaneously. Dummy variables can be used to pick of the country specific effects.

Under the seemingly unrelated regression equation technique growth equations for each country are independent except that the there is contemporaneous correlation among the error terms. These correlations reflect a common shock to these economies such as the increase in the oil prices, terrorist attacks, spread of SARS various or floods or draughts or any other natural disaster that were unpredictable and could not be controlled as exogenous variables.

SURE or 3SLS estimates of growth rates among the interdependent global economy.

Table 9
Determinants of Growth in Five Major Industrial Economies

	UK				Germany		
Variable	Coefficient	T-ratio	P-value	Variable	Coefficient	T-ratio	P-value
GRGR	-0.571	-2.2	0.037	GRUK	-0.295	-2.2	0.037
GRFR	0.453	1.796	0.084	GRFR	0.646	4.273	0
GRJP	0.249	1.912	0.067	GRJP	0.094	1.005	0.324
GRUS	0.88	6.777	0	GRUS	0.513	4.528	0
CONSTANT	-1.351	-2.165	0.04	CONSTANT	-0.874	-1.93	0.065
R-SQUARE = 0.34				R-SQUARE = 0.4920			
	France				Japan		
Variable	Coefficient	T-ratio	P-value	Variable	Coefficient	T-ratio	P-value
GRUK	0.24415	1.796	0.084	GRUK	0.50281	1.912	0.067
GRGR	0.67492	4.273	0	GRGR	0.36707	1.005	0.324
GRJP	0.2453	2.88	0.008	GRFR	0.91758	2.88	0.008
GRUS	-0.30823	-2.246	0.033	GRUS	-0.52336	-1.956	0.061
CONSTANT	0.74417	1.712	0.099	CONSTANT	0.88842	1.011	0.322
	R-SQUARE = 0.4826			R-SQUARE = 0.3656			

	USA		
Variable	Coefficient	T-ratio	P-value
GRUK	0.863	6.777	0
GRGR	0.973	4.528	0
GRFR	-0.56	-2.246	0.033
GRJP	-0.254	-1.956	0.061
CONSTANT	1.502	2.586	0.016
R-SQUARE = 0.4803			

The three stage least square method involves estimating the reduced form using the GLS procedure. Three stages are following (i) apply OLS to individual reduced form equations and get the predicted values of endogenous variables (ii) use those predicted values to estimate the variance covariance matrix of the errors in the system (iii) transform the original model using the variance covariance matrix obtained in the stage (ii). The 3SLS estimator is asymptotically consistent and removes all the biases that may arise due to application of the single equation or the 2SLS estimator in the system of equations (Look Judge-Hill-Griffith-Lutkepoll-Lee (646-652), Wallis (1969), Pagan and Wickens (1989), Garret et. al. (2003), Holly and Sean (2000)) for details.

Both single and system methods of estimations in above equations show that growth rates are inter-dependent among UK, Germany, France, Japan and the USA. These economies are very closely integrated in goods, financial and to some extent also in the labour market. When one economy expands its imports from other countries rises, this in turn raises the aggregate demand of that economy. When the US growth rate is high European economies also tend to grow and similarly when European economies grow that may pull the US economy out of recession. Monetary and fiscal policies as well as trade policy arrangements among these countries, or policy coordination has significant impact in each other. The empirical estimates presented above testify this fact at least for the sample period and thus suggest positive gains of policy coordination among these big economies.

V. Micro founded Model for Inter-dependent Economies

All of the above models, though popular in the literature, do not have explicit optimisation by households and firms in the economy (Lucas (2000)). A model based on optimisation decisions of households who consume both domestic and foreign goods is specified. The exchange rates and prices at home and abroad are determined endogenously. Model solutions differ in autarky, when two countries do not cooperate each other or from trade equilibrium when they cooperate (Mirrlees(1971), Miller and Spencer (1977), Auerbach and Kotlikoff (1985), Rutherford (1995) and Bhattarai (1999)).

Specification of Micro-founded Macro Model of Two Country Global Economy

There are two countries in the world *i*, and *j*. Household utility function contains goods produced at home, imported and leisure. Government uses taxes on consumption, imported goods and labour income. With the Cobb-Douglas production function, the household problem can be stated as:

$$\text{Max } U_0^i = \sum_{t=0}^{\infty} \theta^t \left(C_t^{\alpha} M_t^{\beta} l_t^{\gamma} \right) \quad \text{where } \alpha + \beta + \gamma = 1;\ 0 < \alpha, \beta, \gamma < 1;\ 0 < \theta < 1 \quad (3.1)$$

Subject to life time budget constraint

$$\sum_{t=0}^{\infty} \left[P_{i,t}(1+tc_i)C_{i,t} + P_{j,t}(1+tm_i)M_{i,t} + w_{i,t}(1-tw_i)l_{i,t} \right] \le \sum_{t=0}^{\infty} \left[w_{i,t}(1-tw_i)\overline{L}_{i,t} + r_{i,t}(1-tk_i)K_{i,t} \right] \quad (3.2)$$

Firms maximise profit in each period

$$\text{Max} \quad \Pi_{i,t} = P_{i,t}Y_{i,t} - r_{i,t}K_{i,t} - w_{i,t}LS_{i,t} \quad (3.3)$$

Subject to

$$Y_{i,t} = K_{i,t}^{\eta_i} L_{i,t}^{(1-\eta_i)} \quad (3.4)$$

$$I_{i,t} = K_{i,t} - (1-\delta)K_{i,t-1} \quad (3.5)$$

Government Sector

$$R_{i.t} = P_{i,t}tc_iC_{i,t} + tm_iP_{j,t}M_{i,t} + tw_iLS_{i,t} + tr_iK_{i,t} \le G_{i,t} \quad (3.6)$$

Market clearing

$$Y_{i,t} = C_{i,t} + X_{i,t} + G_{i,t} \quad (3.7)$$

There can be two different ways of trade balance

Period by period trade balance: $M_{i,t} = X_{i,t}$ (3.8)

$$\sum_{t=0}^{\infty}\theta^t\left(M_{i,t}-X_{i,t}\right)=\sum_{t=0}^{\infty}\theta^t\left(TB_{i,t}\right) \quad (3.9)$$

$$\left(S_{i,t}-I_{i,t}\right)+\left(X_{i,t}-M_{i,t}\right)=0 \quad (3.10)$$

$$\left(\overline{L}_{i,t}-I_{i,t}\right)=LS_{i,t} \quad (3.11)$$

Prices from the inter temporal arbitrage condition

$$P_{i,t}=\frac{P_{i,t+1}}{1+r_{i,t}} \quad (3.12)$$

Exchange rates: $E_{i,t}=\dfrac{P_{i,t}}{P_{j,t}}$ (3.13)

A competitive economy is the sequence of prices $P_{i,t}\ P_{j,t}\ r_{i,t}\ r_{j,t}\ w_{i,t}\ w_{j,t}\ E_{i,t}\ E_{j,t}$ and public policy $tc_{i,t}\ tc_{j,t}\ tm_{i,t}\ tm_{j,t}\ tw_{i,t}\ tw_{j,t}\ tr_{i,t}\ tr_{j,t}$ in which allocation of $C_{i,t}\ M_{j,t}\ l_{i,t}, C_{j,t}\ M_{i,t}\ l_{j,t}$ maximise the lifetime utility of households U_0^i and U_0^j and $LS_{i,t}\ K_{i,t}, LS_{j,t}\ K_{j,t}$ that maximise firms profit and government expenditures are $G_{i,t}\ G_{j,t}$ are compatible with the government revenue $R_{i,t}\ R_{j,t}$ and exports $X_{i,t}\ X_{j,t}$ are compatible with imports $M_{i,t}, M_{i,t}$.

For empirical implementation the infinite horizon problem is reduced to finite horizon by fixing the terminal period to be some T in the far distance in the future. Similarly the labour endowment in each period $\overline{L}_{i,t}$ and $\overline{L}_{j,t}$ are taken as given as are the model parameters α, β, γ and θ and the policy parameters $tc_{i,t}$, $tc_{j,t}$, $tm_{i,t}$, $tm_{j,t}$, $tw_{i,t}$, $tw_{j,t}$, $tr_{i,t}$ and $tr_{j,t}$.

Analytical Results from Optimisation

The infinite horizon problem is analytically intractable. Such problems are solved using the first order inter temporal optimisation for any two time intervals with generalisation that solutions that satisfy any two periods can be extended to any other periods. First order conditions for households for two periods are:

$$C_t: \qquad \alpha_i\theta^t\left(C_t^{\alpha_i-1}M_t^{\beta_i}l_t^{\gamma_i}\right)=\lambda_t P_{i,t}\left(1+tc_i\right) \quad (3.14)$$

$$C_{t+1}: \qquad \alpha_i \theta^{t+1}\left(C_{t+1}^{\alpha_i - 1} M_{t+1}^{\beta_i} l_{t+1}^{\gamma_i}\right) = \lambda_t P_{i,t+1}\left(1 + tc_i\right) \tag{3.15}$$

$$M_t: \qquad \beta_i \theta^{t}\left(C_{t}^{\alpha_i} M_{t}^{\beta_i - 1} l_{t}^{\gamma_i}\right) = \lambda_t P_{j,t}\left(1 + tm_i\right) \tag{3.16}$$

$$M_{t+1}: \qquad \beta_i \theta^{t+1}\left(C_{t+1}^{\alpha_i} M_{t+1}^{\beta_i - 1} l_{t+1}^{\gamma_i}\right) = \lambda_t P_{j,t+1}\left(1 + tm_i\right) \tag{3.17}$$

$$l_t: \qquad \gamma_i \theta^{t}\left(C_{t}^{\alpha_i} M_{t}^{\beta_i} l_{t}^{\gamma_i - 1}\right) = \lambda_t w_{i,t}\left(1 - tw_i\right) \tag{3.18}$$

$$l_{t+1}: \qquad \gamma_i \theta^{t+1}\left(C_{t+1}^{\alpha_i} M_{t+1}^{\beta_i} l_{t+1}^{\gamma_i - 1}\right) = \lambda_t w_{i,t+1}\left(1 + tw_i\right) \tag{3.19}$$

$$\lambda_t: \; P_{i,t}\left(1 + tc_i\right)C_{i,t} + P_{j,t}\left(1 + tm_i\right)M_{i,t} + w_{i,t}\left(1 - tw_i\right)l_{i,t} = w_{i,t}\left(1 - tw_i\right)\overline{L}_{i,t} + r_{i,t}\left(1 - tk_i\right)K_{i,t} \tag{3.20}$$

$$\lambda_{t+1}: \; P_{i,t+1}\left(1 + tc_i\right)C_{i,t+1} + P_{j,t+1}\left(1 + tm_i\right)M_{i,t+1} + w_{i,t+1}\left(1 - tw_i\right)l_{i,t+1} = w_{i,t+1}\left(1 - tw_i\right)\overline{L}_{i,t+1} + r_{i,t+1}\left(1 - tk_i\right)K_{i,t+1} \tag{3.21}$$

Above first order conditions can be simplified in terms of Euler equations as:

$$\frac{C_{i,t}}{C_{i,t+1}}: \qquad \frac{1}{\theta}\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{(\alpha_i - 1)}\left(\frac{M_{i,t}}{M_{i,t+1}}\right)^{\beta_i}\left(\frac{l_{i,t}}{l_{i,t+1}}\right)^{\gamma_i} = \frac{P_{i,t}}{P_{j,t}} \tag{3.22}$$

$$\frac{M_{i,t}}{M_{i,t+1}}: \qquad \frac{1}{\theta}\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\alpha_i}\left(\frac{M_{i,t}}{M_{i,t+1}}\right)^{(\beta_i - 1)}\left(\frac{l_{i,t}}{l_{i,t+1}}\right)^{\gamma_i} = \frac{P_{j,t}}{P_{j,t+1}} \tag{3.23}$$

$$\frac{M_{i,t}}{M_{i,t+1}}: \qquad \frac{1}{\theta}\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\alpha_i}\left(\frac{M_{i,t}}{M_{i,t+1}}\right)^{(\beta_i - 1)}\left(\frac{l_{i,t}}{l_{i,t+1}}\right)^{(\gamma_i - 1)} = \frac{w_{i,t}}{w_{i,t+1}} \tag{3.24}$$

$$\frac{C_{i,t+1}}{M_{i,t+1}}: \qquad \frac{\alpha_i}{\beta_i}\left(\frac{M_{i,t+1}}{C_{i,t+1}}\right) = \frac{P_{i,t+1}\left(1 + tc_i\right)}{P_{j,t+1}\left(1 + tm_i\right)} \tag{3.25}$$

$$\frac{l_{i,t+1}}{M_{i,t+1}}: \qquad \frac{\alpha_i}{\gamma_i}\left(\frac{l_{i,t+1}}{C_{i,t+1}}\right) = \frac{P_{i,t+1}\left(1 + tc_i\right)}{w_{i,t+1}\left(1 + tw_i\right)} \tag{3.26}$$

$$\frac{M_{i,t+1}}{l_{i,t+1}}: \qquad \frac{\beta_i}{\gamma_i}\left(\frac{l_{i,t+1}}{M_{i,t+1}}\right) = \frac{P_{j,t+1}\left(1 + tm_i\right)}{w_{i,t+1}\left(1 + tw_i\right)} \tag{3.27}$$

Similarly the first order conditions for firms are:

$$\Pi_{i,t} = P_{i,t} K_{i,t}^{\eta_i} L_{i,t}^{(1 - \eta_i)} - r_{i,t} K_{i,t} - w_{i,t} LS_{i,t} \tag{3.28}$$

$$K_{i,t}: \ \eta_{i,t} P_{i,t} K_{i,t}^{\eta_i - 1} L_{i,t}^{(1-\eta_i)} = r_{i,t} \ \text{ or } \ \frac{\eta_{i,t} P_{i,t} Y_{i,t}}{K_{i,t}} = r_{i,t} \tag{3.29}$$

$$K_{j,t}: \ \eta_{j,t} P_{j,t} K_{j,t}^{\eta_i - 1} L_{j,t}^{(1-\eta_i)} = r_{j,t} \ \text{ or } \ \frac{\eta_{j,t} P_{j,t} Y_{j,t}}{K_{j,t}} = r_{j,t} \tag{3.30}$$

$$L_{i,t}: \ (1-\eta_{i,t}) P_{i,t} K_{i,t}^{\eta_i - 1} L_{i,t}^{-\eta_i} = w_{i,t} \quad \text{ or } \quad \frac{(1-\eta_{i,t}) P_{i,t} Y_{i,t}}{L_{i,t}} = w_{i,t} \tag{3.31}$$

$$L_{j,t}: \ (1-\eta_{j,t}) P_{j,t} K_{j,t}^{\eta_i - 1} L_{j,t}^{-\eta_i} = w_{j,t} \ \text{ or } \ \frac{(1-\eta_{j,t}) P_{j,t} Y_{j,t}}{L_{j,t}} = w_{j,t} \tag{3.32}$$

Initial condition $K_{i,0}$ $K_{j,0}$ and (3.33)

Terminal conditions $I_{i,T} = (g+\delta) K_{i,T-1}; I_{j,T} = (g+\delta) K_{j,T-1}$. (3.34)

Whether the wages rates and the interest rates are same or differ from one country to another partly depends upon the mobility of factors and partly to the tariff rates across countries. If labour and capital are perfectly mobile then the ratios of use of labour and capital across two countries depend on ratios of production.

$$\frac{\eta_{j,t} P_{j,t} Y_{j,t}}{\eta_{i,t} P_{i,t} Y_{i,t}} \frac{K_{i,t}}{K_{j,t}} = \frac{r_{j,t}}{r_{i,t}} \tag{3.35}$$

$$\frac{(1-\eta_{j,t}) P_{j,t} Y_{j,t}}{(1-\eta_{i,t}) P_{i,t} Y_{i,t}} \frac{L_{i,t}}{L_{j,t}} = \frac{w_{j,t}}{w_{i,t}} \tag{3.36}$$

The exchange rate between two countries should be compatible with goods, labour and capital markets.

$$E_{j,t} = \frac{P_{j,t}}{P_{i,t}} = \frac{r_{j,t}}{r_{i,t}} \frac{K_{j,t}}{K_{i,t}} \frac{\eta_{i,t} Y_{i,t}}{\eta_{j,t} Y_{j,t}} = \frac{(1-\eta_{i,t}) Y_{i,t}}{(1-\eta_{j,t}) Y_{j,t}} \frac{L_{j,t}}{L_{i,t}} \frac{w_{j,t}}{w_{i,t}} = \frac{\alpha_i}{\beta_i} \frac{M_{i,t}}{C_{i,,t}} \frac{(1+tm_i)}{(1+tc_i)} \tag{3.37}$$

These analytical results are significantly different than found in the literature (Dornbusch (1976), Taylor (1995)).

Numerical Results

A simple version of the above model is implemented with the following parameters.

Table 10
Parametric specification of micro-founded macro model of the global economy

Parameters	Country 1	Country 2
Alpha	0.6	0.8
Beta	0.4	0.2
gm	0.2	0.2
K0	2000	1000
Th	0.95	0.95
Tc	0.15	0.2
Tm	0.05	0.1
Tw	0.25	0.15
Tr	0.01	0.15
Gr	0.03	0.04
D	0.01	0.02
Lbar	4000	2000
R	0.05	0.03
Nu	0.4	0.3
Q0	2000	1000
P0	1	1

Table 11
Time series of macro economic variables in micro founded dynamic general equilibrium model of the real exchange rate.

Model Generated Solutions for Economy 1												
Years	consum	govex	exports	imports	utility	incom	Exp	taxrev	intrest	price	mprice	rexrate
5	2185	1171	2185	2185	2185	2444	3916	1171	0.05	0.815	0.815	1.000
10	3800	1182	2534	2534	3231	4412	4897	1182	0.05	0.697	0.697	1.000
15	2937	1368	2937	2937	2937	4352	4831	1368	0.05	0.889	0.593	1.500
20	3405	919	3405	3405	3405	3212	3565	919	0.05	0.566	0.377	1.500
25	3947	755	3947	3947	3947	2881	3198	755	0.05	0.438	0.292	1.500
Model Generated Solutions for Economy 2												
5	1360	904	1125	1125	1309	2391	3391	904	0.03	1.406	0.885	1.589
10	1369	1039	1369	1369	1369	2772	3822	1039	0.03	1.63	0.76	2.145
15	1665	1305	1665	1665	1665	3213	4972	1305	0.03	1.89	0.653	2.895
20	2032	1662	2026	2026	2031	5678	6594	1662	0.03	2.191	0.561	3.908
25	2465	2171	2465	2465	2465	7825	9234	2171	0.03	2.54	0.635	4.000

Higher level of utility of the households in country 1 than that in country 2 reflects not only the larger amount of capital and labour endowments in country 1 but also the good policy measures in that country. Price level in country two almost doubles and real exchange rate increases by three times in country 2 while the price level keeps falling in country 1. Inflationary pressure in country 2 are due to expansionary nature of public and private expenditure pushed up by lower interest rate than in country1. These results, however, can change significantly when the above set of parameters change.

VI. Conclusion

Four different versions of the macroeconomic model that are often used for policy analysis are specified and numerically implemented. Each provide a framework to experiment of fiscal, monetary and trade policy issues from a point of view of the small open economy or from the perspective of economies in an interdependent global economy. The dynamic general equilibrium model uses non-linear optimisation technique and results are founded on prices that guarantee the general equilibrium in each period and satisfy inter-temporal budget constraint of households, government and the economy as a whole.

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