Altruism, Liquidity Constraint, and Education Investment

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Abstract

In Japan and other East Asian societies, the household's education expenditure (especially the private tutoring expenditure) has sharply increased. The purpose of this paper is to provide a rationale behind the fact that a number of families very actively invest in education. Introducing altruism and liquidity constraints into a simple parent-child model, we show that the investment in education can be higher or lower than the parent's first best, depending on the income level of the family. Our model also has an implication for the rotten-kid theorem (Becker, 1974). There exist families such that the parental welfare in the equilibrium is higher than that in the parent's second best if the liquidity constraint is binding.

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1. Introduction

Many societies are experiencing an aging population, which is caused by falling fertility rates as well as increasing longevity. In particular, Japan is an extreme case of fertility rates falling dramatically, and it is often pointed out that one of the crucial factors in such a trend is the increasing costs of educating a child. The education expenditure per household in Japan continued to increase until the end of the 1990s, and has stabilized since then. However, noting the fact that the number of children per household has fallen consistently, we can see that the education expenditure per child still continues to increase. In addition, the expenditure on private supplementary tutoring has a quite large share of the education expenditure. Such a phenomenon prevails also in other East Asian societies such as South Korea, Taiwan and Hong Kong (Bray and Kwok, 2003). Since private tutoring is freely chosen by households, it is suggested that they very actively invest in education.

To evaluate whether households expend too much on private education or not, the rate of return to investment in education provide useful information. Based on Japanese cross-sectional data from 1986 to 1995, Arai (2001) finds that the average internal rate of return to university education is 5.93-6.42% for women and 4.81-5.36% for men. Also for Japan, Cabinet Office, Government of Japan (2005) estimates that the rate of return from university education for men born in 1975 is 5.7%. For other countries, a large number of studies have been done since the late 1950s. Psacharopoulos and Patrinos (2004), who review the resent empirical results for a wide variety of countries, summarize that the rate of return to another year of schooling is 10% on world average, and it is lower than the average for the high-income countries of the OECD. According to the cross-country analysis by Trostel et al. (2002), the rate of return to schooling is less than 4% for several countries such as Germany (West), Netherlands, Norway, Sweden and Canada. From these results, it is difficult to judge whether the rate of return to investment in education is disproportionately high or low relative to investments in physical capital, several factors causing upward bias in estimation of the rate of return to education investment have been pointed out, such as a correlation between years of schooling and innate ability to earn income, the effects of liquidity constraints on education decisions, and direct costs of education (including private tutoring).[1] Furthermore, the downward trend in the rate of return to education (Psacharopoulos and Patrinos, 2004; Cabinet Office, Government of Japan, 2005) implies that children in this time may face lower rates of return to education than estimated in the previous studies.

Despite these facts, most of the economic literature argues that the private investment in education tends to be insufficient due to the external effect on economic growth, the liquidity constraint (Barham et al., 1995; De Fraja, 2002), the family constitution (Balestrino, 1997; Anderberg and Balestrino, 2003), and the strategic bequest motives (Cremer and Pestieau, 1992). An exception is Cremer and Pestieau (2006), in which the joy of giving is considered as the motivation behind parental involvement in their children's education. They show that, if the joy of giving term is not included in the social welfare function, parents may invest in their children's education more than the social optimum.

In order to investigate education decisions of a family, we consider a simple model of families, each of which consists of a parent and a child. Key features of the model are as follows. First, the parents are purely altruistic to their children, and this motivates the parents to involve in their child's education. Second, the parents differ in income, which is exogenously determined. Third, the children can borrow to finance education investment (and consumption) within the bounds, which differ between children, depending on their parent's income: a child whose parent earns higher income can borrow larger amount.

Forth, while the children choose the level of their education investment, its cost is shared between them and their parents, and the share is determined

[1] In many studies (especially those using the Mincer specification), the cost of education is measured by forgone earnings alone.

by the parents. Namely, the parents choose how much they pay for their children's education. Therefore, the children must borrow to pay their share of the cost of education investment. Who chooses the level of education investment in a family is a modeling issue. In the literature, while Balestrino (1997), De Fraja (2002), Anderberg and Balestrino (2003), and Cremer and Pestieau (2006) suppose that the parents do, Barham et al. (1995) and Boldrin and Montes (2005) suppose that the children do. In our model, while the children's education investment is their own choice, the parents can influence it through their decisions on the share of the cost. This means that the education investment is determined as a result of interactions between parents and children. Such a set-up seems to be in line with the practice in many societies, and has a significant effect on the results obtained below.

Fifth, while the children's wage income is determined by their education investment, the parents make transfers to their children after the children's wage income is realized. Such ex-post transfers, which are motivated by the parental altruism, provide an incentive for the child to consume too much in her youth so as to receive more transfers, namely engender the Samaritan's dilemma (Lindbeck and Weibull,1988).

Our main results are as follows. First, the investment in education can be too much or too little, depending on the income level of the family. We can distinguish three categories of families, according to their income level. In the families belonging to the first category, who are the wealthy and are not liquidity constrained in the equilibrium, the level of education investment is either equal to or higher than the parental first-best level. In the families belonging to the second category, who are the middle class and are liquidity constrained in the equilibrium, the level of education investment is higher than the parental first-best level. In the families belonging to the third category, who are the poor and are liquidity constrained in the equilibrium, the level of education investment is lower than the parental first-best level.

While the families in the first and second categories (namely, high and middle income classes) may invest too much in the child's education, the reason differs between categories. As to a family in the first category, if the ex-post transfers are operative, the child chooses the efficient level of education investment because her liquidity constraint is not binding, but the Samaritan’s dilemma arises. On the other hand, if the parent makes sufficiently large transfers in the form of education expenditures and the ex-post transfers are made inoperative, the efficient intertemporal allocation of consumption is achieved, but the child chooses too much education investment.[2] As to a family in the second category, since the liquidity constraint is binding, inoperative ex-post transfers do not lead to the efficient consumption allocation, and also the child adjust her consumption allocation through the education investment marginally. Therefore, the level of education investment that attaines the efficient consumption allocation does not coincide with the first-best level (namely, the level where the marginal return to education equals to the market interest rate) in general. Since the Samaritan’s dilemma arises under the first-best level of education investment in a family in the second category, the parent behaves so as to induce her child to receive higher education. This is because the education investment reallocates resources forwards, and counteracts the Samaritan’s dilemma.

The results obtained in this paper also have implications for the rotten kid theorem (Becker, 1974). In some of families with binding liquidity constraint, the parental welfare in the equilibrium is higher than that in the parent’s second best.

2. Model

Consider an economy, which consists of two generations: parents’ generation and children’s generation. A parent lives for three periods of equal length: youth (period 0), middle-age (period 1) and old-age (period 2), and a child also lives for three periods: youth (period 1), middle-age (period 2) and old-age (period 3), with overlapping of periods 1 and 2. Each member of the parents’ generation is heterogeneous with respect to their income level. The

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[2] This result is similar to that of Bruce and Waldman (1990).

population of the parents' generation is N, and each parent produces one child exogenously.

We focus on the periods in which two generations overlap, i.e., periods 1 and 2. In period 1, the parent in family i allocates her income $Y_{p,i}$, which is determined by the education investment made in period 0 and thus exogenous in period 1, among consumption $C^1_{p,i}$, savings $S_{p,i}$ and financial contribution to the cost of her child's education investment. We assume that the investment in the child's education k_i is partially financed by her parent, and the rest is financed by her own. In period 2, the parent observes her child's income, and allocates her savings carried over from period 1 between her own consumption $C^2_{p,i}$ and ex-post transfers toward her child $A_i(\geq 0)$. Thus, the parent's budget constraints in periods 1 and 2 are

(1) $$Y_{p,i} = C^1_{p,i} + S_i + p_i k_i, \qquad (i = 1,...N)$$

(2) $$(1+r)S_i = C^2_{p,i} + A_i, \qquad (i = 1,...N)$$

where p_i $(0 \leq p_i \leq 1)$ is the parental share of education expenditure, and r is the interest rate, which is determined exogenously.

The child has no income in period 1, and thus must borrow to finance consumption $C^1_{k,i}$ and education expenditure $(1-p_i)k_i$. In period 2, the child receives her income $Y_{k,i}$, which is a function of k_i satisfying $Y'_{k,i}(k_i) > 0,\ Y''_{k,i}(k_i) < 0$. She repays the borrowings out of the sum of her income and the ex-post transfers from her parent, and allocates the rest among consumption $C^1_{k,i}$ and savings $S_{k,i}$. Thus the child's budget constraints in periods 1 and 2 are

(3) $$D_i = C^1_{k,i} + (1-p_i)k_i, \qquad (i = 1,...N)$$

(4) $$Y_{k,i}(k_i) - (1+r)D_i + A_i = C^2_{k,i} + S_{k,i}, \qquad (i = 1,...N)$$

where D_i is the child's borrowings.[3] We assume that the amount the child can borrow has the upper bound $\bar{D}_i$, which depends positively on her parent's income:

(5) $$D_i \leq \bar{D}(Y_{p,i}), \qquad \bar{D}'(Y_{p,i}) > 0, \qquad (i = 1,...N) .$$

[3] Without a loss of the generality of the model, we neglect the children's old-age (period 3). Namely, $S_{k,i} = 0$ is assumed hereafter.

The parent is altruistic and her utility function is given by

(6) $$U_{p,i} = u_p(C^1_{p,i}) + v_p(C^2_{p,i}) + \delta U_{k,i},$$

where δ is the weight attached to her child's utility $U_{k,i}$. We assume that $u'_p > 0$, $u''_p < 0$, $v'_p > 0$ and $v''_p < 0$.

The child is selfish and cares only about her own consumption, and her utility function is given by

(7) $$U_{k,i} = u_k(C^1_{k,i}) + v_k(C^2_{k,i}).$$

We assume that $u'_k > 0$, $u''_k < 0$, $v'_k > 0$ and $v''_k < 0$. We hereafter omit the subscript i as long as that does not cause a misunderstanding.

The timing of the game is as follows: (i) the parent chooses C^1_p, S and p; (ii) the child chooses C^1_k, D and k; (iii) the child's income $Y_{k,i}$ is realized, and the parent chooses C^2_p and A. (As a result, C^2_k is determined.)

3. First best for the parent

As a benchmark, we start by deriving the first-best allocation for the parent in a family. The parent, who implements the optimal allocation with respect to $\left\{C^1_p,\ C^2_p,\ C^1_k,\ C^2_k,\ k\right\}$, maximizes her utility subject to the overall feasibility constraint:

(8) $$\max_{C^1_p, C^2_p, C^1_k, C^2_k, k} \ u_p(C^1_p) + v_p(C^2_p) + \delta[u_k(C^1_k) + v_k(C^2_k)]$$

(9) $$s.t.\ \ C^1_p + \frac{C^2_p}{1+r} + C^1_k + \frac{C^2_k}{1+r} + k = Y_p + \frac{Y_k(k)}{1+r}.$$

The first-order conditions (FOCs) for this problem are given by

(10) $$u'_p(C^1_p) = \delta u'_k(C^1_k),$$

(11) $$v'_p(C^2_p) = \delta v'_k(C^2_k),$$

(12) $$\frac{u'_p(C^1_p)}{v'_p(C^2_p)} = \frac{u'_k(C^1_k)}{v'_k(C^2_k)} = 1 + r,$$

(13) $$Y'_k(k) = 1 + r.$$

The above optimality conditions (10)-(13) and the feasibility condition (9) determine the Pareto efficient allocation as the benchmark case. In the rest of the paper, the first best is denoted by the superscript F.

4. Families with non-binding liquidity constraint

From now on, we examine the behavior of families in the competitive equilibrium. Since the parental income Y_p differs with the family, we can consider two types of families: families with non-binding liquidity constraint and families with binding liquidity constraint. (We will show that families with lower Y_p face the binding liquidity constraint while families with higher Y_p do not in the section 6.) In this section, we deal with families with non-binding liquidity constraint.

4-1. Second and Third Stages: Ex-post transfers, borrowings and education investment

We first examine the optimizing behavior of the parent in a family with non-binding liquidity constraint at the third stage of the game. In period 2, the parent transfers $A(\geq 0)$ toward her child so as to maximize $v_p((1+r)S-A)+\delta v_k(Y_k(k)-(1+r)D+A)$, given k, D and S. Noting the non-negativity constraint on ex-post transfers, the FOC with respect to A is

(14) $-v_p'((1+r)S-A)+\delta v_k'(Y_k(k)-(1+r)D+A)\leq 0$ (with equality if $A>0$).

From (14), we obtain the parent's reaction function:

(15) $$A=A(k,\ D,\ S)=\begin{cases} A^+(k,\ D,\ S), & \text{if (15) holds with equality,} \\ 0, & \text{if (15) holds with strict inequality.} \end{cases}$$

The properties of $A(k,\ D,\ S)$ are summarized as follows:

(16) $$\frac{\partial A^+}{\partial k}=-\eta Y_k'(k)<0,$$

(17) $$\frac{\partial A^+}{\partial D}=(1+r)\eta>0,$$

(18) $$\frac{\partial A^+}{\partial S}=(1-\eta)(1+r)>0,$$

where

$$\eta \equiv \frac{\delta v_k''}{v_p'' + \delta v_k''} > 0, \ (0 < \eta < 1).$$

We next examine the second stage of the game. In period 1, anticipating the parent's reaction function (15), the child solves the following problem, given p and S:

$$\max_{D,\, k} \ u_k(D-(1-p)k) + v_k(Y_k(k)-(1+r)D + A(k,\, D,\, S))$$
$$s.t. \ D \le \bar{D}(Y_p).$$

Since we suppose the liquidity constraint is not binding in this case, the FOCs for this problem are given by

(19) $$u_k'(D-(1-p)k) - v_k'(Y_k(k)-(1+r)D + A(k,\, D,\, S)) \cdot \left[(1+r) - \frac{\partial A}{\partial D}\right] = 0,$$

(20) $$-u_k'(D-(1-p)k)\cdot(1-p) + v_k'(Y_k(k)-(1+r)D + A(k,\, D,\, S)) \cdot \left[Y_k'(k) + \frac{\partial A}{\partial k}\right] = 0.$$

Equations (19) and (20) imply

(21) $$Y_k'(k) - (1+r)(1-p) = 0.$$

From (19) and (21), we obtain the child's reaction functions:

(22) $$D = D(p,\, S) = \begin{cases} D^{+}(p,\, S), & \text{if } \partial A/\partial D = \partial A^{+}/\partial D, \\ D^{0}(p,\, S), & \text{if } \partial A/\partial D = 0, \end{cases}$$

(23) $$k = k(p,\, S) = \begin{cases} k^{+}(p,\, S), & \text{if } \partial A/\partial k = \partial A^{+}/\partial k, \\ k^{0}(p,\, S), & \text{if } \partial A/\partial k = 0. \end{cases}$$

The properties of $D(p,\, S)$ and $k(p,\, S)$ are summarized as follows:

(24) $$\frac{\partial D^{+}}{\partial p} = -k\rho - \frac{Y_k'(k)}{Y_k''(k)},$$

(25) $$\frac{\partial D^{+}}{\partial S} = 1-\rho > 0,$$

(26) $$\frac{\partial D^{0}}{\partial p} = -\frac{Y_k'(k)}{Y_k''(k)} > 0,$$

(27) $$\frac{\partial D^{0}}{\partial S} = 0,$$

(28) $$\frac{\partial k^{+}}{\partial p}=\frac{\partial k^{0}}{\partial p}=-\frac{1+r}{Y_k''(k)}>0,$$

(29) $$\frac{\partial k^{+}}{\partial S}=\frac{\partial k^{0}}{\partial S}=0,$$

where

$$\rho \equiv \frac{u_k''}{u_k''+(1-\eta)^2(1+r)^2 v_k''}>0 \quad (0<\rho<1).$$

When $A>0$, from (19) and $\partial A/\partial D=\partial A^{+}/\partial D>0$, we obtain

(30) $$u_k'(C_k^1)-(1+r)v_k'(C_k^2)<0.$$

Comparing (12) with (30) derives the following proposition:

Proposition 1 (Lindbeck and Weibull, 1988)

If $A>0$, then the child in families with non-binding liquidity constraint consumes too much in period 1, and consumes too little in period 2. Hence, Samaritan's Dilemma arises in the competitive equilibrium.

Furthermore, comparing (13) with (21) derives the following proposition:

Proposition 2

If $p=0$, then the child chooses her parent's first-best level of education investment. If $p>0$, the child chooses the level of education investment higher than the first-best level for her parent.

4.2 First Stage: Parental savings and parental share of education expenditures

We now examine the optimizing behavior of the parent at the first stage. The parent maximizes

$$
\begin{aligned}
U_p &= u_p[Y_p - S - pk(p,\ S)] \\
&\quad + v_p[(1+r)S - A(k(p,\ S),\ D(p,\ S),\ S)] \\
&\quad + \delta\{u_k[D(p,\ S) - (1-p)k(p,\ S)] \\
&\quad + v_k[Y_k(k(p,\ S)) - (1+r)D(p,\ S) + A(k(p,\ S),\ D(p,\ S),\ S)]\}
\end{aligned} \tag{31}
$$

with respect to S and p.

4.2.1 The case in which the non-negativity constraint on A is not binding

In this case, the parent's problem is

$$
\begin{aligned}
&\underset{p\geq 0,\ S}{Max}\ u_p[Y_p - S - pk^+(p,\ S)] \\
&\quad + v_p[(1+r)S - A^+(k^+(p,\ S),\ D^+(p,\ S),\ S)] \\
&\quad + \delta\{u_k[D^+(p,\ S) - (1-p)k^+(p,\ S)] \\
&\quad + v_k[Y_k(k^+(p,\ S)) - (1+r)D^+(p,\ S) + A^+(k^+(p,\ S),\ D^+(p,\ S),\ S)]\}.
\end{aligned} \tag{32}
$$

Noting (29), the FOC with respect to S is

$$
\begin{aligned}
&-u'_p + v'_p \cdot [(1+r) - A^+_D D^+_S - A^+_S] \\
&+\delta\{u'_k \cdot D^+_S - v'_k \cdot [(1+r)D^+_S - A^+_D D^+_S - A^+_S]\} = 0.
\end{aligned} \tag{33}
$$

Using (14) with equality and (19) with $\partial A / \partial D = \partial A^+ / \partial D$, (33) can be rewritten as

$$
-u'_p + v'_p \cdot [(1+r) - A^+_D D^+_S] = 0\,. \tag{34}
$$

From $A^+_D D^+_S > 0$, we derive the following proposition:

Proposition 3

If $A > 0$, the parent consumes too much relative to her first-best level in period 1, and consumes too little relative to her first-best level in period 2.

In order to derive the parental share of education expenditure in the equilibrium, we examine the form of (32) in the pU_p-plain. Differentiating (32) with respect to p yields

$$\frac{\partial U_p}{\partial p} = -u'_p \cdot [k^+(p,\ S) + pk^+_p] - v'_p \cdot [A^+_k k^+_p + A^+_D D^+_p]$$

(35)
$$+\delta\{u'_k \cdot [D^+_p + k^+(p,\ S) - (1-p)k^+_p] + v'_k \cdot [Y'_k(k)k^+_p - (1+r)D^+_p + A^+_k k^+_p + A^+_D D^+_p]\}.$$

Using (19) with $\partial A/\partial D = \partial A^+/\partial D$ and (20) with $\partial A/\partial k = \partial A^+/\partial k$, (35) can be rewritten as

(36)
$$\frac{\partial U_p}{\partial p} = (-u'_p + \delta u'_k)k - v'_p \cdot [A^+_k k^+_p + A^+_D D^+_p] - pk^+_p u'_p.$$

The first term in (36) represents the effect of a marginal increase in p and the second term represents the effect of the marginal decrease in A, which is a reaction to the increase in p. Both effects cancel out each other and the sum of the first and second terms always becomes zero.[4] The third term in (36) shows that, when p rises, the parent's utility decreases because the parent's expenditure on education increases and her consumption decreases. Hence, from (36), we obtain

(37)
$$\left(\frac{\partial U_p}{\partial p}\right)_{p=0} = 0, \quad \left(\frac{\partial U_p}{\partial p}\right)_{p>0} = -pk^+_p u'_p < 0\,.$$

4.2.2 The case in which the non-negativity constraint on A is binding

In this case, the parent's problem is

(38)
$$\max_{p\geq 0,\ S}\ u_p[Y_p - S - pk^0(p,\ S)] + v_p[(1+r)S] + \delta\{u_k[D^0(p,\ S) - (1-p)k^0(p,\ S)] + v_k[Y_k(k^0(p,\ S)) - (1+r)D^0(p,\ S)]\}.$$

[4] Substituting (14) and (19) with $\partial A/\partial D = \partial A^+/\partial D$ into (34) yields

$$-u'_p + \delta u'_k + v'_p \rho A^+_D = 0\,.$$

Substituting the above equation into the first and second terms in (36) yields

$$(-u'_p + \delta u'_k)k^+ - v'_p \cdot (A^+_k k^+_p + A^+_D D^+_p) = -v'_p \cdot [A^+_k k^+_p + A^+_D(D^+_p + \rho k^+)] = 0.$$

From (27) and (29), the FOC with respect to S is

(39) $$-u_p' + (1+r)v_p' = 0.$$

Differentiating (32) with respect to p yields

(40) $$\frac{\partial U_p}{\partial p} = -u_p' \cdot (k^0 + pk_p^0) + \delta\{k^0 u_k' + [u_k' - v_k' \cdot (1+r)]D_p^0 + [-u_k' \cdot (1-p) + v_k' \cdot Y_k'(k)]k_p^0\}.$$

Using (19) with $\partial A / \partial D = 0$ and (20) with $\partial A / \partial k = 0$, (40) can be rewritten as

(41) $$\left(\frac{\partial U_p}{\partial p}\right)_{p \geq 0} = (-u_p' + \delta u_k')k^0 - pk_p^0 u_p'.$$

From (14) with strict inequality, (19) with $\partial A / \partial D = 0$ and (39), we have

(42) $$-u_p' + \delta u_k' < 0.$$

From (28), (41) and (42), we have

(43) $$\left(\frac{\partial U_p}{\partial p}\right)_{p \geq 0} < 0.$$

4.2.3 Competitive equilibrium

We define p_0 as p that satisfies (14) with equality when $A = 0$:

(44) $$-v_p'[(1+r)S] + \delta v_k'[Y_k(k(p,S)) - (1+r)D(p,S)] = 0.$$

Thus, $A = 0$ is the interior solution under $p = p_0$ [5].

Figure 1 shows that U_p jumps at $p = p_0$. The Samaritan's Dilemma arises for $0 \leq p \leq p_0$ because (14) holds with equality and thus $\partial A / \partial D = \partial A^+ / \partial D > 0$ holds in (19), while the Samaritan's Dilemma is dissolved for $p_0 < p \leq 1$ because (14) holds with strict inequality and thus $\partial A / \partial D = 0$ holds in (19). On the other hand, k is excessive relative to the parental first best at $p = p_0$, and the increase in p from p_0 to $\lim_{\varepsilon \to 0}(p_0 + \varepsilon)$ raises k further. Although this has an negative effect on the parental welfare, this effect is very small and can be neglected because ε is infinitely close to zero. Thus, the parent's utility at $p = \lim_{\varepsilon \to 0}(p_0 + \varepsilon)$ higher than that at $p = p_0$.

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[5] The existence of p_0 such that $p_0 < 1$ is proved in the Appendix.

This is formally stated in the following lemma.

Lemma 1. $\lim_{\varepsilon \to 0}[U_p \big|_{p=p_0} - U_p \big|_{p=p_0+\varepsilon}] < 0$

Proof: See the Appendix.

From(37), (43) and Lemma 1, we obtain the form of (31) as shown in Figure 1. The following proposition summarizes the results obtained in this section:

Proposition 4

The parental share of education expenditure in the competitive equilibrium is either $p* = 0$ or $p* = p_0(+\lim_{\varepsilon \to 0} \varepsilon)$. If $p* = 0$, the child consumes too much in period 1 (the Samaritan's dilemma arises) while the child chooses her parent's first-best level of education investment. If $p* = p_0(+\lim_{\varepsilon \to 0} \varepsilon)$, the child chooses the level of the education investment higher than her parent's first-best level while the child's consumption allocation is efficient.

5. Families with binding liquidity constraint

In this section, we examine the behavior of families whose borrowings take corner solutions.

5.1. Second and third stages: Ex-post transfers, borrowings and education investment

The third stage in this case is same as that in the case of families with non-binding liquidity constraint described in the previous section, except that the first-order condition for A is assumed to be satisfied with equality here[6].

[6] While the Samaritan's dilemma disappears if the parents choose $A = 0$ in the case of

In the second stage, the child in each family faces the following problem:

$$\max_{D,\, k}\; u_k(D-(1-p)k)+v_k(Y_k(k)-(1+r)D+A(k,\ D,\ S))$$
$$s.t.\ D\le \bar{D}(Y_p).$$

Since we suppose the liquidity constraint is binding here, the FOCs for this problem are as follows:

$$(45)\qquad u_k'(D-(1-p)k)-v_k'(Y_k(k)-(1+r)D+A(k,\ D,\ S))\cdot\left[(1+r)-\frac{\partial A}{\partial D}\right]>0,$$

$$(46)\ -u_k'(D-(1-p)k)\cdot(1-p)+v_k'(Y_k(k)-(1+r)D+A(k,\ D,\ S))\cdot\left[Y_k'(k)+\frac{\partial A}{\partial k}\right]=0.$$

From (46), we obtain the child's reaction function:

$$(47)\qquad k=k^{+}(p,\ S,\ \bar{D}(Y_p)),$$

where

$$(48)\qquad \frac{\partial k^{+}}{\partial p}=\frac{u_k''k(1-p)-u_k'}{SOC(k)}>0,$$

$$(49)\qquad \frac{\partial k^{+}}{\partial S}=\frac{-v_k''\cdot(1-\eta)^2(1+r)Y_k'}{SOC(k)}<0,$$

$$(50)\qquad \frac{\partial k^{+}}{\partial \bar{D}}=\frac{u_k''\cdot(1-p)+v_k''\cdot(1+r)(1-\eta)^2Y_k'}{SOC(k)}>0,$$

and $SOC(k)=u_k''\cdot(1-p)^2+v_k''\cdot(1-\eta)^2(Y_k')^2+v_k'\cdot(1-\eta)Y_k''<0$.

While (45) cannot determine the sign of $u_k'(c_k^1)-v_k'(c_k^2)(1+r)$ in this case, we can rewrite (46) as

$$(51)\qquad \frac{u_k'(c_k^1)}{v_k'(c_k^2)}=\frac{(1-\eta)Y_k'(k)}{1-p},$$

and (51) derives the following proposition:

Proposition 5

1. The child consumes too little in period 1 if $\dfrac{(1-\eta)Y_k'(k)}{1-p}>1+r$.

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non-binding liquidity constraint, such a choice cannot lead to an efficient intertemporal consumption allocation in the case of binding liquidity constraint. We hence assume that A takes an interior solution in the equilibrium in this section.

2. The child's consumption allocation is optimal if $\frac{(1-\eta)Y_k'(k)}{1-p} = 1+r$.

3. The child consumes too much in period 1 if $\frac{(1-\eta)Y_k'(k)}{1-p} < 1+r$.

In the last case in proposition 5, the Samaritan's dilemma arises even though the liquidity constraint is binding. This is because the strategic incentive of the child to obtain more transfers from the parent is strong enough to surpass the effect of the binding liquidity constraint in this case.

Equations (45) and (46) imply

$$Y_k'(k) > (1-p)(1+r), \tag{52}$$

which derives the following proposition:

Proposition 6

If $p = 0$, the child chooses the level of education investment less than the first-best level for her parent.

It is noted that whether the level of education investment is too high or too low is indeterminate if $p > 0$.

5.2 First Stage

In the first stage, the problem for the parent in each family is to maximize

$$\begin{aligned} U_p = {} & u_p[Y_p - S - pk^+(p,\ S,\ \bar{D}(Y_p)] \\ & + v_p[(1+r)S - A^+(k^+(p,\ S,\ \bar{D}(Y_p)),\ \bar{D}(Y_p),\ S)] \\ & + \delta\{u_k[\bar{D}(Y_p) - (1-p)k^+(p,\ S,\ \bar{D}(Y_p))] \\ & + v_k[Y_k(k^+(p,\ S,\ \bar{D}(Y_p))) - (1+r)\bar{D}(Y_p) + A^+(k^+(p,\ S,\ \bar{D}(Y_p)),\ \bar{D}(Y_p),\ S)]\} \end{aligned} \tag{53}$$

with respect to S and p.

5.2.1 Parental savings

The FOC with respect to S is as follows:

(54)
$$\begin{aligned}&-u_p' + v_p' \cdot (1+r) + (-v_p' + \delta v_k') A_S^+ \\ &+\{(-u_p' p - v_p' A_k^+) + \delta[-u_k' \cdot (1-p) + v_k' \cdot (Y_k' + A_k^+)]\} k_S^+ = 0.\end{aligned}$$

Substituting (14) with equality and (46) into (54) yields

(55)
$$-u_p' + v_p' \cdot (1+r) - (u_p' p + v_p' A_k^+) k_S^+ = 0.$$

5.2.2 Parental share of education expenditure

In order to derive the parental share of education expenditure in the equilibrium, we examine the form of (53) in the pU_p-plain. Differentiating (53) with respect to p yields

(56)
$$\begin{aligned}\frac{\partial U_p}{\partial p} &= -u_p' \cdot [k^+(p,\ S,\ \bar{D}(Y_p)) + p k_p^+] - v_p' A_k^+ k_p^+ \\ &\quad + \delta\{u_k' \cdot [k^+(p,\ S,\ \bar{D}(Y_p)) - (1-p) k_p^+] \\ &\quad + v_k' \cdot [Y_k'(k) k_p^+ + A_k^+ k_p^+]\}.\end{aligned}$$

Substituting (46) into (56) yields

(57)
$$\frac{\partial U_p}{\partial p} = (-u_p' + \delta u_k') k - (u_p' p + v_p' A_k^+) k_p^+.$$

Using (55), (57) can be rewritten as

(58)
$$\begin{aligned}\frac{\partial U_p}{\partial p} &= (-u_p' + \delta u_k') k - (-u_p' + v_p' \cdot (1+r)) \left(\frac{k_p^+}{k_S^+} \right) \\ &= \frac{v_p'}{(1+\hat{p} k_S^+)(1-p)} \{[-(1-p)(1+r) + (1-\eta) Y_k' + (p-\eta) Y_k' k_S^+] k \\ &\quad + (1-p)[\eta Y_k' - p(1+r)] k_p^+\}.\end{aligned}$$

We now evaluate $\partial U_p / \partial p$ for $p=0$. From (58), we have[7]

(59)
$$\left(\frac{\partial U_p}{\partial p} \right)_{p=0} = v_p' \cdot \{-(k k_S^+ - k_p^+ + k) \eta Y_k' + [Y_k' - (1+r)] k\} > 0.$$

We define k when $p=0$ as k_0. Noting that $Y_k'(k_0) > 1+r$ (proposition 6), we have two cases concerning the relative magnitude of $Y_k'(k_0)$,

[7] The derivation of (59) is shown in the Appendix.

$(1-\eta)Y_k'(k_0)$ and $1+r$, namely

(60) $$(1-\eta)Y_k'(k_0) < 1+r < Y_k'(k_0),$$

(61) $$1+r < (1-\eta)Y_k'(k_0) < Y_k'(k_0).$$

Since differentiating (46) with $p=0$ with respect to k and Y_p yields

(62) $$\frac{\partial k_0}{\partial Y_p} = \frac{u_k'' + v_k'' \cdot (1+r)(1-\eta)^2 Y_k'}{u_k'' + v_k'' \cdot (1-\eta)^2 (Y_k')^2 + v' \cdot (1-\eta) Y_k''} \bar{D}'(Y_p) > 0,$$

we have

(63) $$\frac{\partial Y_k'(k_0)}{\partial Y_p} < 0, \quad \frac{\partial (1-\eta) Y_k'(k_0)}{\partial Y_p} < 0.$$

Therefore, (60) holds for families with higher Y_p and (61) holds for families with lower Y_p.

First, we consider families with (60) satisfied. The changes in $Y_k'(k(p))$ and $(1-\eta)Y_k'(k(p))/(1-p)$ when p rises are given by

(64) $$\frac{\partial Y_k'(k(p))}{\partial p} = Y_k''(k)\frac{\partial k^+}{\partial p} < 0,$$

(65) $$\begin{aligned}\frac{\partial}{\partial p}\left(\frac{(1-\eta)Y_k'(k(p))}{(1-p)}\right) &= \frac{(1-\eta)Y_k''(k)k_p^+}{1-p} + \frac{(1-\eta)Y_k'(k)}{(1-p)^2} \\ &= \frac{(1-\eta)}{(1-p)^2 SOC(k)}\left[u_k'' \cdot (1-p)^2 Y_k'(1-\sigma) + v_k'' \cdot (1-\eta)^2 (Y_k')^3\right] > 0.\end{aligned}$$

The sign of (65) depends on the assumption that $\sigma \equiv -kY_k''/Y_k' < 1$.[8] Under (64) and (65), as p increases from zero, both $Y_k'(k(p))$ and $(1-\eta)Y_k'(k(p))/(1-p)$ approach to $1+r$. Therefore, noting that Y_p determines $Y_k'(k_0)$ and $(1-\eta)Y_k'(k_0)$, we can distinguish three categories of families in this case, according to Y_p:

(i) $(1-\eta)Y_k'(k(p))/(1-p) < 1+r$ holds for p that satisfies $Y_k'(k(p)) = 1+r$,

(ii) $(1-\eta)Y_k'(k(p))/(1-p) = 1+r$ holds for p that satisfies $Y_k'(k(p)) = 1+r$,

(iii) $(1-\eta)Y_k'(k(p))/(1-p) > 1+r$ holds for p that satisfies $Y_k'(k(p)) = 1+r$.

Defining Y_p of families categorized into (ii) as $\tilde{Y}_p$, (63) implies that (i)

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[8] When $Y_k(k)$ takes the Cobb-Douglas functional form, we have $\sigma < 1$. Given $Y_k = Bk^{\alpha}$ $(B > 0,\ 0 < \alpha < 1)$, $\sigma = -Y_k''k / Y_k' = 1-\alpha < 1$ is obtained.

corresponds to families with $Y_p > \tilde{Y}_p$ and (iii) corresponds to families with $Y_p < \tilde{Y}_p$.

We examine the determination of p in each category in turn. The following lemma is useful for this.

Lemma 2

1. If $Y_k'(k) = 1+r$ and $\dfrac{(1-\eta)Y_k'(k)}{1-p} \geq (\leq) 1+r$, then $\dfrac{\partial U_p}{\partial p} \leq (\geq) 0$,

2. If $\dfrac{(1-\eta)Y_k'(k)}{1-p} = 1+r$ and $Y_k'(k) \geq (\leq) 1+r$, then $\dfrac{\partial U_p}{\partial p} \geq (\leq) 0$.

Proof: See the Appendix

(i) $Y_p > \tilde{Y}_p$

Figure 2 illustrates the parental utility function (53) in the pU_p-plain. When $p = 0$, we have $\partial U_p / \partial p > 0$ from (59). Raising p from zero, we reach to p_1, which represents p satisfying $Y_k'(k) = 1+r$. When $p = p_1$, $(1-\eta)Y_k'(k)/(1-p) < 1+r$ holds and Lemma 2.1 implies that $\partial U_p / \partial p > 0$. Raising p further from p_1, we reach to p_2, which reptresents p satisfying $(1-\eta)Y_k'(k)/(1-p) = 1+r$. When $p = p_2$, $Y_k'(k) < 1+r$ holds and Lemma 2.2 implies that $\partial U_p / \partial p < 0$. Therefore, under an assumption that $\partial^2 U_p / \partial p^2 < 0$, p^* must be located between p_1 and p_2, and thus $(1-\eta)Y_k'(k)/(1-p) < 1+r$ and $Y_k'(k) < 1+r$ simultaneously hold for p^*. From Proposition 5, this implies that Samaritan's dilemma arises (the child over-consumes in her youth), and too much amounts are invested in education in the equilibrium.

(ii) $Y_p = \tilde{Y}_p$

When $p = 0$, we have $\partial U_p / \partial p > 0$ from (59). Raising p from zero, we reach to p^*, where $(1-\eta)Y_k'(k(p))/(1-p) = 1+r$ and $Y_k'(k(p)) = 1+r$ are simultaneously satisfied. Therefore, both the intertemporal allocation of consumption and the education investment are efficient. In addition, since

$p = \eta$ holds, the parental first-best is achieved in the equilibrium.[9]

(iii) $Y_p < \tilde{Y}_p$

When $p = 0$, we have $\partial U_p / \partial p > 0$ from (59). Raising p from zero, we reach to a level that satisfies $(1-\eta)Y_k'(k)/(1-p) = 1+r$. At this level of p, $Y_k'(k) > 1+r$ holds and Lemma 2.2 implies that $\partial U_p / \partial p > 0$. Raising p further, we reach to a level that satisfies $(1-\eta)Y_k'(k)/(1-p) > 1+r$ and $Y_k'(k) = 1+r$. At this level of p, Lemma 2.1 implies that $\partial U_p / \partial p < 0$. Therefore, under an assumption that $\partial^2 U_p / \partial p^2 < 0$, $(1-\eta)Y_k'(k)/(1-p) > 1+r$ and $Y_k'(k) > 1+r$ simultaneously hold at the equilibrium level of p (p^*). From Proposition 5, this implies that the child consumes too little in her youth, and the education investment is insufficient in the equilibrium.

As to the families with (61) satisfied, their Y_p is lower than that of families in category (iii), and the property of equilibrium in (iii) can be applied to them.

The following proposition summarizes the above analysis:

Proposition 7

1. The Samaritan's dilemma and the over-investment in education arise for families with $Y_p > \tilde{Y}_p$.
2. The parental first best is achieved for families with $Y_p = \tilde{Y}_p$.
3. The insufficiency both in the filial consumption in period 1 and in the education investment arises for families with $Y_p < \tilde{Y}_p$.

In contrast to the case of non-binding liquidity constraint (Proposition 4), Proposition 7 suggests that, in the case of binding liquidity constraint, the

9 From (55), we obtain $-u_p' + v_p' \cdot (1+r) = [v_p' / 1 + pk_S^+][-\eta Y_k' + p(1+r)]k_S^+$. Substituting $p = \eta$, $Y_k' = 1+r$ and $(1-\eta)Y_k'/(1-p) = 1+r$ into the above equation and (51) yields the first-best conditions.

property of equilibrium is different from that in Bruce and Waldman (1990). While either the intertemporal allocation of consumption or the choice of filial action is efficient in the family perspective in Bruce and Waldman (1990), neither of them is inefficient in this case (except for families with $Y_p = \tilde{Y}_p$).

6. The effects of income on education and welfare

In this section, based on the results obtained in sections 4 and 6, we clarify the differences in education and welfare between the families with different income.

First of all, we show that, Y_p of the families with non-binding liquidity constraint is higher than Y_p of the families with binding liquidity constraint, under a quite natural assumption. Define $\hat{Y}_p$ as Y_p of families whose FOC with respect to D, (19), is satisfied with $D = \bar{D}(Y_p)$. In other words, the most-preferred level of borrowings for the child in a family with $\hat{Y}_p$, which is denoted by $D^{**}(\hat{Y}_p)$, is just equal to the upper limit on how much she can borrow $\bar{D}(\hat{Y}_p)$. Assuming that $d\bar{D}(Y_p)/dY_p > dD^{**}(Y_p)/dY_p$,[10] we have that the liquidity constraint is not binding if $Y_p \geq \hat{Y}_p$, and is binding if $Y_p < \hat{Y}_p$.

Using Proposition 4 and Proposition 7, we obtain the following proposition

Proposition 8

Whether the investment in the child's education is too much or too little depends on her parent's income level:

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[10] From (22), (25) and (34) we can derive

$$\frac{dD^{**}(Y_p)}{dY_p} = \frac{\partial D^{**}}{\partial p^*}\frac{\partial p^*}{\partial Y_p} + \frac{\partial D^{**}}{\partial S^*}\frac{\partial S^*}{\partial Y_p}$$

$$= (1-\rho)\frac{u_p''}{u_p'' + (1+r)^2(1-\eta)[1-\eta(1-\rho)]v_p''}.$$

This is smaller than 1, and likely to be very small. Hence, to assume that $d\bar{D}(Y_p)/dY_p > dD^{**}(Y_p)/dY_p$ may be acceptable.

1. k^* is equal to or greater than k^F, if $Y_p \geq \hat{Y}_p$,
2. k^* is greater than k^F, if $\tilde{Y}_p < Y_p < \hat{Y}_p$,
3. k^* is equal to k^F, if $Y_p = \tilde{Y}_p$,
4. k^* is smaller than k^F, if $Y_p < \tilde{Y}_p$.

Our model also has an implication for the rotten-kid theorem (Becker, 1974). This is shown in the following proposition, which is derived as a consequence of Proposition 7:

Proposition 9

For families whose Y_p is $\tilde{Y}_p$ or in the neighborhood of $\tilde{Y}_p$, the utility level of the parent in the equilibrium is higher than that in the second best. [11]

Proposition 9 has an implication for the incentive problems in the family, especially for the rotten-kid theorem (Becker, 1974), which states that a child, who is not altruistic toward her parent, chooses actions that would be chosen by the parent if the parent made all choices. In the families with binding liquidity constraint, if the parent made all choices, the parent's second best would be the solution. However, the parent may obtain the welfare higher than that in her second best in the incorporative game with her child.

7. Conclusion

Considering pure altruism as the relevant transfer motives, we show that the investment in education can be too much or too little, depending on the income level of the family. In obtaining such a result, the child's strategic behavior in consumption allocation plays a large part. In our model, two types of transfers from the parent to the child are introduced: one is the financial contribution of the parent to the child's education cost in the child's youth, and the other is ex-post transfers, which occurs after the child's income is realized. The latter transfers provide an incentive for the child to consume too

[11] The parental second best is defined in the Appendix.

much in her youth, namely engenders the Samaritan’s dilemma (Lindbeck and Weibull,1988). Whether the parent faces the Samaritan’s dilemma or not depends on the parent’s income in our model, and, in families with the Samaritan’s dilemma, the parent behaves so as to induce her child to receive higher education. This is because the education investment reallocates resources forwards, and counteracts the Samaritan’s dilemma.

The results obtained in this paper also have implications for the rotten kid theorem (Becker, 1974). In some of families with binding liquidity constraint, the parental welfare in the equilibrium is higher than that in the parent’s second best.

Appendix

Proof of the existence of p_0 such that $p_0 < 1$

First, we show the negative relationship between A and p, which is a necessary condition for the existence of p_0. If $A > 0$, from (15), (22) and (23), we have $A = A^+(k^+(p, S), D^+(p, S), S)$. Differentiating this equation with respect to p and substituting (16)-(18), (24), (25), (28), (29) yields

$$\frac{dA^+}{dp} = \frac{\partial A^+}{\partial k}\frac{\partial k^+}{\partial p} + \frac{\partial A^+}{\partial D}\frac{\partial D^+}{\partial p} + \left(\frac{\partial A^+}{\partial k}\frac{\partial k^+}{\partial S} + \frac{\partial A^+}{\partial D}\frac{\partial D^+}{\partial S} + \frac{\partial A^+}{\partial S}\right)\frac{\partial S}{\partial p}$$

(A1) $$= \eta Y_k'(k)\frac{(1+r)}{Y_k''(k)} + (1+r)\eta\left(-k\rho - \frac{Y_k'(k)}{Y_k''(k)}\right) + [(1+r)\eta(1-\rho) + (1-\eta)(1+r)]\frac{\partial S}{\partial p}$$

$$= -(1+r)\eta\rho k + (1+r)(1-\eta\rho)\frac{\partial S}{\partial p} < 0,$$

where

$$\frac{\partial S}{\partial p} = \frac{-u_p''(k + pk_p) - v_p''(1+r)^2\eta\rho k(1-\eta(1-\rho))}{u_p'' + v_p''(1+r)^2\eta\rho(1-\eta(1-\rho))}$$

$$= -k - \frac{u_p'' pk_p}{u_p'' + v_p''(1+r)^2\eta\rho(1-\eta(1-\rho))} < 0.$$

From (A1), we have $dA^+/dp < 0$.

Next, we prove $p_0 < 1$. From the definition, p_0 satisfies

(A2) $$-v_p'[(1+r)S] + \delta v_k'[Y_k(k(p,S)) - (1+r)D(p,S)] = 0.$$

Since the child is selfish, k that satisfies (21) must be equal to Y_p for some $p \in (0, 1)$. We define such p as $\overline{p}$. Substituting $k = Y_p$, $S = 0$ and $p = \overline{p}$ into the LHS of (A2) yields

(A3) $$-v_p'(0) + \delta v_k'[Y_k(Y_p) - (1+r)D(\overline{p},0)] < 0.$$

Differentiating the LHS of (A2) with respect to p yields

(A4) $$\frac{\partial(-v_p' + \delta v_k')_{A=0}}{\partial p} = -v_p'' \cdot (1+r)\frac{\partial S}{\partial p} + \delta v_k''[Y_k'\frac{\partial k^0}{\partial p} - (1+r)\frac{\partial D^0}{\partial p}]$$

$$= -v_p'' \cdot (1+r)\frac{\partial S}{\partial p} < 0.$$

Thus, from (A2)-(A4), we obtain $p_0 < \overline{p} < 1$.

Proof of *Lemma 1*

For $p = p_0$, we define the following function with dummy variable θ:

(A5) $$u_k'(D-(1-p_0)k) - v_k'(Y_k(k)-(1+r)D)\left[(1+r)-\theta A_D^+\right] = 0,$$

(A6) $$-(1-p_0)u_k'(D-(1-p_0)k) + v_k'(Y_k(k)-(1+r)D)\left[Y_k'(k)+\theta A_k^+\right] = 0,$$

(A7) $$u_p'(Y_p - S - p_0 k) - v_p'((1+r)S)\left[(1+r)-\theta A_D^+ D_S^+\right] = 0.$$

Equations (A5) and (A6) imply

(A8) $$Y_k'(k)-(1+r)(1-p_0) = 0.$$

From (A5)-(A7) (or (A5), (A7) and (A8)), we obtain $(D(\theta), k(\theta), S(\theta))$. From (A8), k is independent of θ. D^* and S^* satisfy (A5) and (A7) with $\theta = 0$ when $\partial A/\partial D = 0$, whereas satisfy (A5) and (A7) with $\theta = 1$ when $\partial A/\partial D = \partial A^+/\partial D$.

Differentiating (A5), (A7) and (A8) yields

(A9) $$\frac{dk}{d\theta}(\equiv k'(\theta)) = 0,$$

(A10) $$\frac{dD}{d\theta}(\equiv D'(\theta)) = \frac{-(1+r)\eta v_k'}{u_k'' + (1+r)^2(1-\theta\eta)v_k''} > 0,$$

(A11) $$\frac{dS}{d\theta}(\equiv S'(\theta)) = \frac{(1+r)(1-\rho)\eta v_p'}{u_p'' + (1+r)^2[1-\theta\eta(1-\rho)]v_p''} < 0.$$

Noting that $A = 0$ if $p = p_0$, the parent's utility function is given by

(A12) $$\begin{aligned} U_p\big|_{p=p_0} &= u_p[Y_p - S(\theta) - p_0 k(\theta)] + v_p[(1+r)S(\theta)] \\ &\quad + \delta\{u_k[D(\theta)-(1-p_0)k(\theta)] + v_k[Y_k(k(\theta)-(1+r)D(\theta)]\}. \end{aligned}$$

Differentiating (A12) with respect to θ yields

(A13) $$\begin{aligned} \left.\frac{dU_p}{d\theta}\right|_{p=p_0} &= p_0 k'(\theta) + [-u_p' + (1+r)v_p']S'(\theta) \\ &\quad + \delta\{\left[u_k' - v_k'\cdot(1+r)\right]D'(\theta) + \left[-u_k'\cdot(1-p_0) + v_k'\cdot\left(Y_k'(k)\right)\right]k'(\theta)\}. \end{aligned}$$

From (A5)-(A7), (A9)-(A11) and (14), we have

(A14) $$\left.\frac{dU_p}{d\theta}\right|_{p=p_0} = [D_S^+ S'(\theta) v_p' - D'(\theta) v_k']\theta A_D^+ < 0.$$

When θ moves from $\theta = 1$ to $\theta = 0$, the parent's utility increases.

Derivation of sign of (59)

Substituting (46), (48) and (49) into $kk_S^+ - k_p^+ + k\big|_{p=0}$ yields

(A15) $$kk_S^+ - k_p^+ + k\big|_{p=0} = \frac{1}{SOC(k)\big|_{p=0}}\{v_k' \cdot (1-\eta)Y_k'(1-\sigma) + kv_k'' \cdot (1-\eta)^2 Y_k'[Y_k' - (1+r)]\},$$

where $SOC(k)\big|_{p=0} = u_k'' + v_k'' \cdot (1-\eta)^2 (Y_k')^2 + v_k' \cdot (1-\eta)Y_k'' < 0$ and $\sigma = -Y_k''k / Y_k'$.

Substituting (A15) into (59) yields

$$\begin{aligned}
\left(\frac{\partial U_p}{\partial p}\right)_{p=0} &= v_p' \cdot \{-(kk_{S_p}^+ - k_{\hat{p}}^+ + k)\eta Y_k' + [Y_k' - (1+r)]k\}\big|_{p=0} \\
&= \frac{v_p'}{SOC(k)}\{-[v_k' \cdot (1-\eta)Y_k'(1-\sigma) + kv_k'' \cdot (1-\eta)^2 Y_k'(Y_k' - (1+r))]\eta Y_k' \\
&\qquad + SOC(k)\big|_{p=0}[Y_k' - (1+r)]k\}\big|_{p=0} \\
&= \frac{v_p'}{SOC(k)}\{-[v_k' \cdot (1-\eta)Y_k'(1-\sigma)] + [kv_k'' \cdot (1-\eta)^3 (Y_k')^2 (Y_k' - (1+r))]\eta Y_k' \\
&\qquad + [u_k'' + v_k' \cdot (1-\eta)Y_k''][Y_k' - (1+r)]k\}\big|_{p=0}.
\end{aligned}$$

Since $\sigma < 1$ is assumed and $Y_k' > 1+r$ holds for $p = 0$ (Proposition 6), $SOC(k)\big|_{p=0} < 0$ implies

(A16) $$\left(\frac{\partial U_p}{\partial p}\right)_{p=0} > 0.$$

Proof of *Lemma 2*

1. Substituting $Y_k' = 1+r$ into (58) yields

$$\text{(A17)}\qquad \begin{aligned} \frac{\partial U_p}{\partial p}\Big|_{Y_k'=1+r} &= \frac{v_p' \cdot (1+r)(p-\eta)}{(1+pk_S^+)(1-p)}[(1+k_S^+)k-(1-p)k_p^+] \\ &= \frac{v_p' \cdot (1+r)(p-\eta)}{(1+pk_S^+)(1-p)} \cdot \frac{v_k' \cdot (1-\eta)Y_k'(1-\sigma)}{SOC(k)}. \end{aligned}$$

From (A17), noting that $\sigma < 1$ is assumed and $1+pk_S^+ > 0$ holds when $Y_k' = 1+r$, the sign of $\partial U_p / \partial p\big|_{Y_k'=1+r}$ depends on the sign of $p-\eta$. Since $(1-\eta)Y_k'(k)/(1-p) \geq (\leq) 1+r$ is equivalent to $p-\eta \geq (\leq) 0$ when $Y_k'(k) = 1+r$, $SOC(k) < 0$ implies

$$\text{(A18)}\qquad \frac{\partial U_p}{\partial p}\Big|_{Y_k'=1+r} \leq (\geq) 0.$$

2. Substituting $(1-\eta)Y_k'/(1-p) = 1+r$ into (58) yields

$$\text{(A19)}\qquad \frac{\partial U_p}{\partial p}\Bigg|_{\frac{(1-\eta)Y_k'}{1-p}=1+r} = \frac{v_p' \cdot (1+r)(p-\eta)}{(1+pk_S^+)(1-\eta)}(kk_S^+ - k_p^+).$$

From (A19), assuming $1+pk_S^+ > 0$, [12] the sign of $\partial U_p / \partial p\big|_{(1-\eta)Y_k'(k)/(1-p)=1+r}$ depends on the sign of $p-\eta$ because $kk_S^+ - k_p^+ < 0$. Since $Y_k'(k) \geq (\leq) 1+r$ is equivalent to $p-\eta \leq (\geq) 0$ when $(1-\eta)Y_k'(k)/(1-p) = 1+r$, $kk_S^+ - k_p^+ < 0$ ($SOC(k) < 0$) implies

$$\text{(A20)}\qquad \frac{\partial U_p}{\partial p}\Bigg|_{\frac{(1-\eta)Y_k'(k)}{1-p}=1+r} \geq (\leq) 0.$$

Second Best for the Parent

Suppose that, under the liquidity constraint $D = \bar{D}$ binding, the parent

[12] If $Y_k' \geq 1+r$, then we have $1+pk_S^+ > 0$, but, if $Y_k' < 1+r$, then we do not necessarily have $1+pk_S^+ > 0$. In this case, however, if $\eta \leq 2-(1+r)/Y_k'$ (if the difference between Y_k' and $1+r$ is small enough), then we have $1+pk_S^+ > 0$.

chooses $\left\{C_p^1,\, C_p^2,\, C_k^1,\, C_k^2,\, k\right\}$ so as to maximize her utility subject to the budget constraints (1)-(4):

$$\text{(A21)} \qquad \max_{S,\, p,\, k,\, A} u_p[Y_p - S - pk] + v_p[(1+r)S - A] + \delta\{u_k[\bar{D} - (1-p)k] + v_k[Y_k(k) - (1+r)\bar{D} + A].$$

FOCs for this problem are given by

$$\text{(A22)} \qquad u_p'(C_p^1) = \delta u_k'(C_k^1),$$

$$\text{(A23)} \qquad v_p'(C_p^2) = \delta v_k'(C_k^2),$$

$$\text{(A24)} \qquad \frac{u_p'(C_p^1)}{v_p'(C_p^2)} = 1 + r,$$

$$\text{(A25)} \qquad \frac{u_k'(C_k^1)}{v_k'(C_k^2)} = Y_k'(k) > 1 + r.$$

Equations (A22)-(A25) are the second best conditions for the parent.

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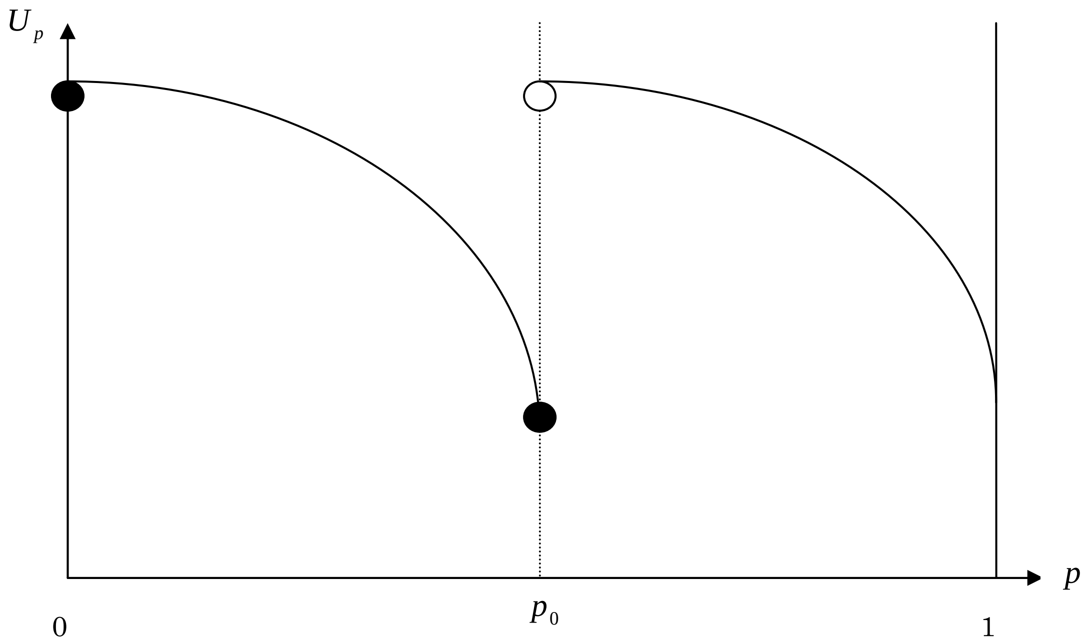
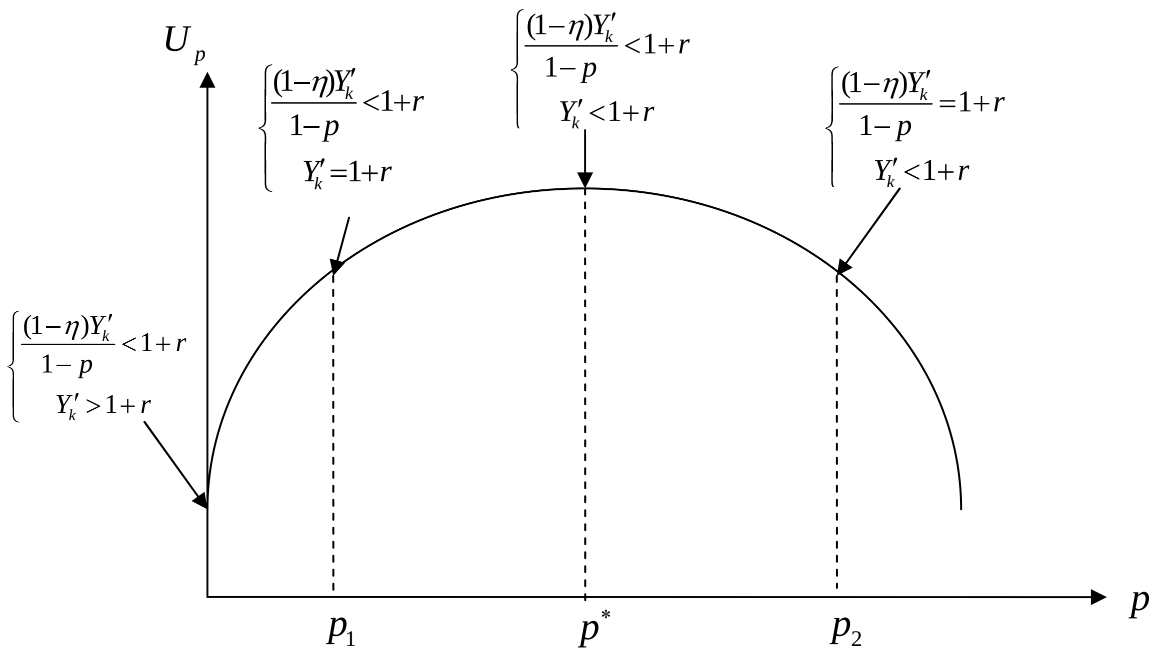


（*Figure* 1） U_p in Families with non-binding liquidity constraint



（*Figure* 2） U_p in Families with binding liquidity constraint when $Y_p > \tilde{Y}_p$