Labor-Market Frictions, Human Capital Accumulation, and Long-Run Growth: Positive Analysis and Policy Evaluation

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Abstract: We construct a dynamic search model with endogenous human capital accumulation to study the long-run growth effects of short-run labor-market frictions and to evaluate the effectiveness of human capital policy. We assume that both vacancy creation and job search are costly and that vacancies and job seekers are brought together by a matching technology exhibiting constant returns. Our model departs from the prototypical labor search literature by allowing each firm to create multiple vacancies and each household to choose labor-market participation and search intensity endogenously. We find that employment, learning effort and output growth rise with an increase in the effectiveness of human capital accumulation or the degree of labor-market matching efficacy, or a decrease in the separation rate or the vacancy creation cost. Our calibration exercises suggest that output growth, employment, vacancy creation, and learning and search effort are most responsive to changes in the discretionary human capital accumulation parameter. We also find that, a discretionary human capital enhancement policy is more effective in promoting economic growth but need not be more beneficial in the welfare sense. Moreover, the effects of these public policy programs become larger as the severity of labor-market frictions rises, indicating that a quantitative evaluation of the effectiveness of human capital policy in a frictionless Walrasian world is expected to be severely downward biased.

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"[T]here is an ... obvious need for someone to synthesize the theory of growth, which takes full employment for granted, with the shorter-run macroeconomics whose main subject is variation of the volume of employment." (Robert M. Solow, Radcliffe Lectures, University of Warwick, 1969)

1 Introduction

Since the pivotal work by Romer (1986) and Lucas (1988), the endogenous growth framework has become a useful tool to evaluate the long-run growth consequences of public policy. A partial list of policy instruments evaluated by previous studies includes various forms of taxes, subsidies and economic reforms – some of which focus upon human, physical and research capital, while others on economic and political institutions. Following this convention, we reevaluate the effectiveness of some forms of human capital-related policies by developing an endogenous growth model in which the labor market is no longer frictionless. There are substantial informational and institutional barriers to labor search, recruiting, and job creation. While it is well-documented that these types of frictions can have important effects on individual decisions and economic performance in business cycles, the macroeconomic consequences and policy implications of labor-market frictions in a perpetually growing economy have not been fully explored. Our paper attempts to fill this gap.

Specifically, in terms of primary methodological issues, our paper is almost exactly the opposite to the conventional real business cycle (RBC) theory. The premise of the RBC theory is to argue that *long-run* technological changes can generate *short-run* fluctuations at the business cycle frequency. In the present paper, we instead hypothesize that *short-run* labor market frictions and the resulting temporary frictional unemployment can have *long-run* growth and welfare implications. To accomplish this task, we establish an endogenous growth model with physical and human capital accumulation in which the labor market is subject to search, matching and entry frictions. Following Diamond (1982), Mortensen (1982) and Pissaridis (1984), we postulate that both vacancy creation and job search are costly and that vacancies and job seekers are brought together by a matching technology exhibiting constant returns.[1] Our main departure from this labor search literature is the consideration of "large" firms and "large" households in the sense that each firm can create multiple vacancies and each household can choose search intensity endogenously. These features allow us to move one step closer to the canonical endogenous growth framework for conducting policy analysis

[1] The reader is referred to a recent comprehensive survey by Rogerson, Shimer and Wright (2005).

quantitatively.[2]

In terms of the methodology of modelling labor-market frictions in a dynamic setting, our paper is closely related to the RBC search model developed by Merz (1995) and Andolfatto (1996).[3] One major difference is that the rate of growth is driven by exogenous technological advancement in their models, while we allow the rate of growth to be *endogenously* determined by human capital investment decision. Moreover, their papers study how labor market frictions influence the propagation mechanism of technology shocks over the business cycle, whereas our paper examines the interactions between short-run market frictions and *long-run* economic performance. Also in contrast to their setups, *both* vacancy creation and job search are modeled in terms of labor and time allocation.[4] This latter feature enables us to illustrate how labor-leisure-learning-search trade-offs and endogenous labor-market participation in the presence of labor-market frictions may influence the effectiveness of public policy in the long run.

In terms of policy analysis in optimal growth models with labor search, our paper is related to a recent paper by Mortensen (2005). Our framework is very different from Mortensen's, however. While both papers allow for endogenous growth, Mortensen's is based on the quality ladder *without* physical or human capital accumulation. In contrast, we construct a two-sector endogenous growth framework in which both physical and human capital are endogenously accumulated and in which labor-leisure-learning-search trade-offs play central roles in our analysis. Our policy experiments also differ from Mortensen's. Specifically, Mortensen evaluates wage taxes and employment protection, whereas we assess two forms of *human capital policy* programs. Such a task is feasible and interesting because we model explicitly endogenous human capital accumulation and intratemporal/intertemporal time allocation trade-offs.

Upon developing an endogenous growth model with labor-market frictions, we calibrate the model to match the U.S. economy and then perform comparative-static analysis and policy evalu-

[2] Although Aghion and Howitt (1994) and Laing, Palivos and Wang (1995) generalize the conventional labor search literature to permit sustained growth, it is difficult to calibrate their models to assess quantitatively the role of market frictions played in the long-run performance of the economy and the effectiveness of public policy.

[3] There is a larger but only remotely related literature on growth and cycles. For example, Boldrin and Rustichini (1994) show that positive production externalities in Romer (1986)-Lucas (1988) convention can be sources of persistent economic growth as well as endogenous fluctuations. Matsuyama (1999) formalizes the notion of Schumpeterian growth via creative destruction where innovation serves to promote future growth in the low-growth phase under monopolistic competition. In contrast to their technological considerations, our paper focuses on search, matching and entry frictions originated in the labor market.

[4] In Merz, both activities require only real resources of goods. In Andolfatto, vacancy creation requires only real resources of goods, where job search requires time.

ation. We find that employment, labor-market participation, vacancy creation, learning effort and output growth rise with (i) an increase in the effectiveness of human capital accumulation (either uniformly or discretionarily) or the degree of labor-market matching efficacy, or (ii) a reduction in the job separation rate, and the vacancy creation cost. Moreover, any shift in these parameters fostering long-run growth is always accompanied by a higher unemployment rate, which may be regarded as "creative destruction" as a result of human capital accumulation with frictional labor markets, even without disembodied technological advancements. In response to such shifts in growth-enhancing parameters, the labor market also becomes tighter from the firm's view point. Furthermore, our numerical experiments suggest that output growth, employment, vacancy creation, and learning and search effort are most responsive to changes in the discretionary human capital accumulation parameter that influences the intensive margin of learning and labor-market participation benefit, followed by the degree of labor-market matching efficacy and the job separation rate. While an enhancement in discretionary human capital investment is more effective in fostering growth, it is also associated with a larger decline in effective consumption and leisure, as well as a larger increase in the unemployment rate.

In terms of policy evaluation, we provide a quantitative assessment of the *relative effectiveness* of two human capital policy programs: a uniform human capital enhancement policy and a discretionary human capital enhancement policy. While the uniform human capital enhancement policy may be thought of as general training, the discretionary human capital enhancement policy captures both job-specific training and post-schooling executive learning that are more sensitive to job-related learning effort.[5] Under the "tax incidence" exercises by maintaining a constant government budget, a discretionary human capital enhancement policy is found more effective in promoting labor-market participation, learning, employment and economic growth than a uniform human capital enhancement policy. However, a discretionary human capital enhancement policy also leads to a larger drop in effective consumption and aggregate leisure for the employed, thereby reducing economic welfare despite its strong positive growth effect. As the severity of labor-market frictions diminishes, the effects of these human capital policy programs become smaller. This suggests that a quantitative evaluation of the effectiveness of labor-related policy in a frictionless Walrasian world is expected to be downward biased.

[5] The reader is referred to Becker (1962) and Pencavel (1972) for a discussion on general versus job-specific training and to Werther, Wachtel and Veale (1995) for issues concerning executive learning. Because our focus is on human capital accumulation on-the-job, our human capital policy should not be viewed as programs related to pre-employment formal education.

2 The Model

Time is discrete. The basic economy features three theaters of economic activities: a continuum of identical infinitely lived competitive firms (of measure one), a continuum of identical infinitely lived households (of measure one) and a fiscal authority. All individual agents have perfect foresight. There are two productive factors: capital and labor, both owned by households. Firms and households exchange in both goods and factor markets. The goods market is Walrasian and the capital market is perfect, but the labor market exhibits search/entry frictions. While each firm can create multiple vacancies and each household can choose search intensity endogenously, both vacancy creation and search intensity are costly.

To avoid unnecessary complexity involved in managing the distribution of the employed, the unemployed and their respective cash holdings, we adopt the "large households" framework proposed by Lucas (1990). Specifically, each household can be thought of containing a continuum of "members" who are either employed (engaged in production, on-the-job learning, or leisure activity) or nonemployed (engaged in job seeking or leisure activity), with the sum of their mass normalized to unity (see Figure 1). All members pool their income as well as their enjoyment of the fruit of employment (consumption) and unemployment (leisure). Vacancies and job seekers are brought together through a Diamond (1982) type matching technology, where the flow matches depend on the masses of both matching parties. Each vacancy can be filled by exactly one searching workers. At an exogenous rate, filled vacancies and workers are separated every period and separated workers immediately become job seekers.

Finally, the benevolent fiscal authority determines tax rates and human capital enhancement policies by maintaining periodic budget balance.

2.1 Firms

A representative firm, at a particular period t, rents capital k_t (beginning-of-period measure) from households at a gross rental rate r_t and employs labor of mass n_t with effort ℓ_t at a real market wage rate w_t to produce a single final good y_t under a constant-returns-to-scale Cobb-Douglas technology. However, not all employed workers at the representative firm are devoted to production. A mass of workers of measure Φ are employed solely to maintain the vacancies v_t, which can be thought of covering the costs of posting vacancies, managing personnel-related documentations, as well as providing and maintaining the office space. This labor input will be shortly referred to as the vacancy creation cost. We postulate:

$$\Phi(v_t) = \phi v_t^{\varepsilon}$$

where $\varepsilon > 1$ reflects the convexity of the vacancy creation cost and $\phi > 0$ captures any exogenous shift in such a cost. Accordingly, the measure of workers used for manufacturing is $n_t - \Phi(v_t)$, which is augmented by their corresponding effort ℓ_t and human capital h_t. The output of the representative firm can now be specified as:

$$y_t = Ak_t^{\alpha} \left[(n_t - \Phi(v_t))\, \ell_t h_t \right]^{1-\alpha} \tag{1}$$

where $\alpha \in (0,1)$ is the output elasticity of capital and $A > 0$ denotes the scaling factor of the production technology.

The shadow rate of return on capital is defined as:

$$r_{k_t} = A\alpha \left[\frac{k_t}{(n_t - \Phi(v_t))\ell_t h_t} \right]^{\alpha-1} \tag{2}$$

which is a decreasing function of the effective capital-labor ratio alone. Because one can establish a one-to-one relationship between the shadow capital rental rate and the endogenously determined balanced growth rate of the economy, it will be convenient to invert (2) to write the effective capital-labor ratio q_t as a function of the shadow rental rate and then express other production-related terms in q_t (and hence as functions of the economic growth rate in balanced growth equilibrium):

$$q_t = \frac{k_t}{(n_t - \Phi(v_t))\ell_t h_t} = \left(\frac{A\alpha}{r_{k_t}} \right)^{\frac{1}{1-\alpha}} \tag{3}$$

2.2 Households

Facing a pooled resource, a representative "large" household has a unified preference capturing enjoyment of all its members: the employed, whose fraction is n_t, and the nonemployed, whose fraction is $1-n_t$. To simplify the analysis, we restrict our attention primarily to on-the-job learning. That is, only the employed will devote time to accumulating human capital. This simplifying assumption is innocuous particularly because of strong empirical evidence documented in labor economics that human capital is depreciated severely for dispensed workers (e.g., see Jacobson, LaLonde, and Sullivan 1993).[6]

Thus, employed members divide their time into production ℓ_t (work effort), human capital investment e_t (learning effort) and leisure $1 - \ell_t - e_t$. Nonemployed members divide their time

[6] By allowing the unemployed to learn at a different effort will increase the complexity without generating additional insights toward understanding the long-run growth and welfare effects of labor-market frictions.

only into job search s_t (search effort or search intensity) and leisure $1 - s_t$. The search intensity augmented unemployment measure is defined as $u_t = s_t(1 - n_t)$. Figure 1 shows the time allocation for households.

In addition to leisure, members of the representative household also value their pooled consumption c_t. The representative household's periodic felicity function is given by,

$$U(c_t, \ell_t, e_t, s_t, n_t) = u(c_t) + n_t\Lambda^1(1 - \ell_t - e_t) + (1 - n_t)\Lambda^2(1 - s_t)$$

where employed and nonemployed members need not value their leisure time equally (Λ^1 and Λ^2 may differ), particularly because the nonemployed need not voluntarily take leisure (e.g., a nonemployed member may be involuntarily unemployed as a result of job separation). Functions u and Λ^1 and Λ^2 are strictly increasing and concave. Accordingly, the representative household's preference can be written in a standard time-additive form as:

$$\Omega = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t U(c_t, \ell_t, e_t, s_t, n_t)$$

where Ω is the lifetime utility and $\rho > 0$ is the subjective rate of time preference.

Finally, we extend Lucas (1988) and Lucas (1993) to specify the human capital evolution equation as:

$$h_{t+1} = (1 + \zeta + Dn_te_t)\,h_t \tag{4}$$

where ζ denotes the exogenous component of the rate at which human capital is accumulated, $D > 0$ measures the maximum rate of endogenous human capital accumulation (i.e., the endogenous human capital accumulation rate with maximal learning and full employment, $n_te_t = 1$), and $h_0 > 0$ represents initial human capital prior to entry to the labor market after completing mandatory formal schooling (K-12). While the endogenous choice of e resembles the labor-human capital investment tradeoff in Lucas (1988), the positive dependence of incremental human capital on n captures experience-driven learning on the job postulated by Lucas (1993).

In our policy analysis, we shall refer to any policy to increase ζ as a uniform human capital enhancement policy and that to raise D as a discretionary human capital enhancement policy. More specifically, government-sponsored general training may be viewed as to enhance human capital uniformly. In contrast, government-sponsored job-specific training and post-schooling executive learning are more responsive to employment status and job-related learning effort; thus, these policy programs are expected to affect human capital accumulation in a discretionary fashion. Furthermore, as argued by Heckman (1976), better formal education not only leads to a higher *level* of initial human capital prior to entering the labor market (h_0) but also raise the *rate* at which human capital is accumulated. One may thus regard mandatory K-12 education as to increase ζ and college education as to increase D.

2.3 The Aggregate Economy

Because there is only a single good in the economy, the resource constraint requires that aggregate goods supply must be equal to aggregate goods demand, which is the sum of households' consumption and gross investment:

$$c_t + [k_{t+1} - (1-\delta)k_t] = Ak_t^{\alpha}\left[(n_t - \Phi(v_t))\,\ell_t h_t\right]^{1-\alpha} \tag{5}$$

where $\delta \in (0,1)$ denotes the constant rate of capital depreciation.

While the capital market is perfect as in the conventional Walrasian models, the labor market exhibits search frictions. Similar to Diamond (1982), the aggregate flow matches depend on the masses of both matching parties, namely, search intensity augmented job seekers, $s_t(1-n_t)$, and vacancies, v_t. Assume the matching technology exhibits constant-returns-to-scale property, as suggested by the empirical evidence in Blanchard and Diamond (1990) using the U.S. data. We can specify:

$$m_t = B\left[s_t(1-n_t)\right]^{\beta} v_t^{1-\beta} \tag{6}$$

where $B > 0$ measures the degree of matching efficacy and $\beta \in (0,1)$.

Let ψ be the (exogenous) job separation rate, $\eta_t = \frac{m_t}{v_t}$ be the firm recruitment rate and $\mu_t = \frac{m_t}{s_t(1-n_t)}$ be the job finding rate. Since each vacancy can be filled by only one worker, the inflow of workers to employment is m_t and the outflow is ψn_t. Employment within the economy thus evolves according to the following birth-death process: $n_{t+1} - n_t = m_t - \psi n_t$, or, by rearranging terms and using (6),

$$n_{t+1} = (1-\psi)n_t + B\left[s_t(1-n_t)\right]^{\beta} v_t^{1-\beta} \tag{7}$$

3 Optimization and Equilibrium

Following the conventional wisdom (cf. Hosios1990, Pissarides1990 and Mortensen and Pissarides 2003), we consider a dynamic search equilibrium associated with a wage bargaining outcome that supports the solution of the pseudo social planner's problem on which we shall focus.

Notably, the policy evaluation herein is on contrasting a discretionary with a uniform human capital policy, where the former favors those devoted more time in learning and the latter treats everyone identically. Because both policy instruments only affect the technology of human capital accumulation (i.e., equation (4)) rather than households' budget constraints or firms' flow profits, it is valid to conduct equilibrium and welfare analysis based exclusively on the pseudo social planner's problem.[7] Notice also that the optimization problem to be solved is a "pseudo" social planner's problem in the sense that the social planner cannot fully coordinate search/matching and that the social planner takes prices as given when considering policy programs.

We will proceed as follows in the next three subsections. To begin, we will derive the pseudo social planner's optimizing conditions. Then, we will define the dynamic search equilibrium as well as the balanced growth equilibrium. Finally, we will illustrate how to determine the balanced growth values of the key macroeconomic variables such as employment, output, capital as well as the variables related to search such as probabilities of job matching, search intensity and vacancy rate.

3.1 Optimization

This dynamic programming problem can be specified in the Bellman equation form as:

$$\Omega(k_t, h_t, n_t) = \max_{c_t,\, \ell_t, e_t,\, s_t, v_t} U\left(c_t, \ell_t, e_t, s_t, n_t\right) + \frac{1}{1+\rho}\Omega(k_{t+1}, h_{t+1}, n_{t+1}) \tag{8}$$

subject to constraints (4), (5), and (7).

In the Appendix, we present first-order conditions with respect to consumption (c), work effort (ℓ), learning (e), and search intensity (s) and vacancy creation (v), as well as the Benveniste-Scheinkman conditions governing the two capital stocks and the level of employment (k, h, n). Let us suppress the time subscripts and use "prime" to indicate the next period values. Further denote the marginal valuation of additional human capital accumulated for the next period and the marginal valuation of additional employment to be used in next period production as $MVH' = \frac{1}{1+\rho}\Omega_h(k', h', n')$ and $MVN' = \frac{1}{1+\rho}\Omega_n(k', h', n')$, respectively, where all subscripts attached to functionals are derivatives. As shown in the Appendix, one can manipulate the first-order conditions to obtain the following intratemporal and intertemporal trade-off relationships:

$$-\frac{U_\ell}{U_c} = (1-\alpha)Aq^{\alpha}(n-\Phi)h \tag{9}$$

$$MVH' \cdot (Dnh) = -U_e \tag{10}$$

$$MVN' \cdot [\beta\mu\,(1-n)] = -U_s \tag{11}$$

$$MVN' \cdot [(1-\beta)\,\eta] = U_c \cdot [(1-\alpha)Aq^{\alpha}\ell h \Phi_v(v)] \tag{12}$$

[7] Should one intend to study distortionary factor income taxes/subsidies, a decentralized optimization problem must be used.

While equation (9) displays a standard consumption-leisure trade-off by equating the marginal rate of substitution with the marginal product of labor, others require further elaboration. Concerning the other relationships, we begin by noting that Dnh measures incremental human capital accumulated as a result of learning. Moreover, $\beta\mu(1-n)$ and $(1-\beta)\eta$ represent the incremental employment as a consequence of, respectively, more effort devoted to finding a job and more vacancy created to recruit workers. Furthermore, $(1-\alpha)Aq^{\alpha}\ell h\Phi_v(v)$ is the marginal cost of vacancy in units of goods due to a loss of labor productivity. The intuition underlying the remaining three equations are now clear-cut. Equation (10) requires that the future net gain from learning, by enhancing human capital and hence productivity, be equal to the current loss from a reduction in leisure. Equation (11) states that the employment gain next period from a marginal increase in search intensity this period equals the disutility from the corresponding reduction in leisure. Equation (12) indicates that the marginal benefit of vacancy as a result of a successful recruitment equals the sacrifice in the labor used for production in order to maintain the additional vacancy created.

Also as shown in the Appendix, we can manipulate the first-order and the Benveniste-Scheinkman conditions to obtain the following intertemporal trade-off relationships:[8]

$$(1+\rho)\frac{U_c}{U_{c'}} = (1-\delta)+\alpha A(q')^{\alpha-1} \tag{13}$$

$$MVH\cdot h = -U_\ell \ell + \left(1+\frac{1+\zeta}{Dne}\right)(-U_e e) \tag{14}$$

$$MVN\cdot n = U_n n - U_e e + \frac{n}{n-\Phi}(-U_\ell \ell) + \frac{n}{1-n}\frac{1-\psi-\beta\mu s}{\beta\mu s}(-U_s s) \tag{15}$$

Equation (13) is a standard intertemporal consumption-saving tradeoff condition, equating the marginal rate of intertemporal substitution with the rate of returns on capital. While (14) governs the evolution of human capital, (15) governs the evolution of employment. These relationships equate next period's marginal valuation of incremental human capital and incremental employment, respectively, with the corresponding net marginal opportunity cost from the productivity loss today. It should be noted that, if the employed value leisure more than the nonemployed, the marginal opportunity cost of incremental employment is dampened by an increase in the marginal utility of leisure resulting from having more employed members in the large household (measured by $U_n n$).

[8] As shown in the Appendix, the second-order conditions are met. Thus, the first-order conditions and the Benveniste-Scheinkman conditions, together with the transverality conditions associated with the three state variables, are necessary and sufficient for the interior solution(s) to be the maximum.

3.2 Equilibrium

A *dynamic search equilibrium* is a tuple of individual choice variables, $\{c_t, \ell_t, e_t, s_t, v_t, y_t\}_{t=0}^{\infty}$, state variables, $\{k_{t+1}, h_{t+1}, n_{t+1}\}_{t=0}^{\infty}$, and aggregate variables, $\{m_t, r_{kt}, q_t\}_{t=0}^{\infty}$, such that:

(i) all individuals optimize, i.e., (A1)-(A5) and (A6)-(A8) are met;

(ii) human capital and employment evolve according to (4) and (7), respectively;

(iii) goods production is given by (1) and the effective capital-labor ratio satisfies (3);

(iv) labor-market matching satisfies (6);

(v) the goods market clears, i.e., (5) holds.

Notably, by construction, the labor market automatically clears. Now, there are a total of 13 equations every period, determining 12 endogenous variables. One can easily verify that goods market clearance condition is automatically met once (A1), (A2), (A5), (A6), (A7) and (A8) are met. Thus, Walras' law holds in our economy.

The model economy exhibits perpetual growth and hence we cannot simply analyze the economic aggregates without transforming perpetually growing quantities into stationary ratios. Throughout the remainder of the paper, we focus on a *balanced growth path* (BGP) along which consumption, physical and human capital, and output all grow at positive constant rates. Since the production function is homogeneous of degree one in reproducible factors (k and h) and the human capital accumulation equation is linear (in h), these quantities (c, k, h and y) must all grow at a common rate, g, on a BGP, whereas other quantities are all constant.

Along a BGP, the labor market must satisfy the steady-state matching (Beveridge curve) relationships given by,

$$\psi n = \mu s(1-n) = \eta v = B\left[s(1-n)\right]^{\beta} v^{1-\beta} \tag{16}$$

That is, the equilibrium outflows from the matched pool (ψn) must equal the inflows from either the unmatched worker pool ($\mu s(1-n)$) or the unmatched job vacancy pool (ηv).

For analytical convenience, we assume the felicity function to take the following form: $u(c_t) = \ln c_t$, $\Lambda^1(1-\ell_t-e_t) = \gamma_1 \frac{(1-\ell_t-e_t)^{1-\sigma}}{1-\sigma}$ and $\Lambda^2(1-s_t) = \gamma_2 \frac{(1-s_t)^{1-\sigma}}{1-\sigma}$, where $\gamma_i > 0$ and $\sigma > 0$. Thus, employed and nonemployed members value leisure differently only by a scaling factor of γ_1 versus γ_2. For reason to be seen in the calibration analysis below, it is convenient to write the ratio of the marginal utility of leisure of employed to unemployed members as $R =$

$\frac{\gamma_1(1-\ell-e)^{-\sigma}}{\gamma_2(1-s)^{-\sigma}}$. Hence, the marginal utility of additional employment can be expressed as: $U_n = \Lambda^1 - \Lambda^2 = \gamma_2 \frac{(1-s)^{-\sigma}}{1-\sigma}\left[(1-\ell-e)R-(1-s)\right]$, which is expected to be positive in our benchmark economy.

Along a BGP, we can rewrite the two evolution equations, (4) and (5), as:

$$e = \frac{g-\zeta}{Dn} \tag{17}$$

$$\frac{c}{h} = \left[Aq^{\alpha} - (\delta+g)q\right](n-\Phi)\ell \tag{18}$$

Next, we show in the Appendix that

$$g = \frac{r_k - (\delta+\rho)}{1+\rho} \tag{19}$$

$$\rho(1+g) = Dn\ell \tag{20}$$

$$\frac{\rho+\psi}{\beta\mu} + \frac{1-\sigma s}{1-\sigma} = R\left(\frac{n\ell}{n-\Phi} + \frac{1-\ell-\sigma e}{1-\sigma}\right) \tag{21}$$

Equation (19) gives the prototypical Keynes-Ramsey relationship that governs consumption growth. While (20) is a relationship based upon intertemporal human capital accumulation, (21) is one based on intertemporal employment evolution.

Using (19) and (3), we have:

$$r_k = (\delta+\rho) + (1+\rho)g \tag{22}$$

$$q = \left[\frac{A\alpha}{(\delta+\rho)+(1+\rho)g}\right]^{\frac{1}{1-\alpha}} \tag{23}$$

Both relationships are standard in discrete-time optimal growth models with a Cobb-Douglas production technology. As shown in the Appendix, we can further substitute out $\frac{c}{h}$ and q in (18) to yield:

$$\gamma_1(1-\ell-e)^{-\sigma}n = \frac{1}{\ell}\frac{(1-\alpha)\left[(\delta+g)+\rho(1+g)\right]}{(1-\alpha)(\delta+g)+\rho(1+g)} \tag{24}$$

where the righthand side is increasing in g and the lefthand side may also be locally increasing in g. One may then see that the fixed point mapping may lead to multiple solutions for the balanced growth rate of the economy. In practice, reducing the system to one dimension will not only be overly complicated but also lose economics insights for explaining the underlying results. We will therefore try to reduce the system to two dimensions to which we now turn.

3.3 Reducing the System to Two-by-Two

The equations determining the BGP can be re-arranged in a recursive fashion that is conducive to perform comparative statics. Essentially, we can reduce the system to 2×2 in (μ, n) space. Once the BGP values of (μ, n) are pinned down, the rest of endogenous variables can then be derived recursively.

To see this, we use (16) to derive:

$$\eta = B^{\frac{1}{1-\beta}} \mu^{\frac{-\beta}{1-\beta}} = \eta(\mu; B) \tag{25}$$

$$v = B^{\frac{-1}{1-\beta}} \mu^{\frac{\beta}{1-\beta}} \psi n = v(\mu, n; B, \psi) \tag{26}$$

$$s = \frac{\psi n}{(1-n)\mu} = s(\mu, n; \psi) \tag{27}$$

where it is clear that $\eta_\mu < 0$, $\eta_B > 0$, $v_\mu > 0$, $v_n > 0$, $v_B < 0$, $v_\psi > 0$, $s_\mu < 0$, $s_n > 0$, and $s_\psi > 0$. The properties regarding (25) are standard: while an increase in B represents an outward shift in the Beverage Curve that tends to raise both job finding rate and firm recruitment rate, any other parameter changes cause a movement along the Beverage Curve in (μ, η) space and hence affect the job finding rate and firm recruitment rate differently. Accordingly, an increase in B fosters more matches and hence reduces unfilled vacancies; however, an increase in the job finding rate is associated with a reduction in the firm recruitment rate, leading to more unfilled vacancies. Additionally, a higher job separation rate raises unfilled vacancies whereas an increase in employment requires creation of more vacancies to match. The last relationship is a direct consequence of the first equality in (16): a higher job finding rate enables workers to devote less effort to job search and a higher job separation rate requires workers to spend more search effort.

Then, from (20) and (17), we can write learning effort e as:

$$e = \frac{\ell}{\rho} - \frac{1+\zeta}{Dn} \tag{28}$$

which is positively related to both employment and work effort. We then show in the Appendix to pin down work effort as:

$$\ell \left(1 + \frac{1+\zeta}{Dn} - \frac{1+\rho}{\rho}\ell\right)^{-\sigma} = \frac{(1-\beta)\eta}{\beta\mu} \frac{n-\Phi}{n\Phi_v} \frac{\gamma_2(1-s)^{-\sigma}}{\gamma_1}$$

which can be rewritten as an implicit function:

$$\ell = \ell(\mu, n; B, \psi, \phi, D, \zeta) \tag{29}$$

where $\ell_\mu < 0$, $\ell_n \lesseqgtr 0$, $\ell_B > 0$, $\ell_\psi \lesseqgtr 0$, $\ell_\phi < 0$, $\ell_D > 0$, and $\ell_\zeta < 0$.[9] That is, work effort can be expressed as a function of (μ, n) alone. A higher job finding rate fosters more matches and, as a result

[9] Notice that, in addition to endogenous variables, we have only written down a function in terms of parameters of interest.

of diminishing returns, lowers the marginal benefit of additional employment (measured by $\Omega_n(\mathcal{H}'))$. In our production function specification, employment and work effort are Pareto complements, so the marginal benefit of work effort decreases. This explains why work effort is negatively related to the job finding rate. An increase in employment creates two opposing effects. It, on the one hand, lowers the marginal benefit of employment (by diminishing returns) and hence the marginal benefit of work effort. On the other, it increases the marginal benefit of work effort as a result of Pareto complementarity. On balance, we have an ambiguous relationship between work effort and employment. Since the effects of exogenous parameters are all partial effects for given values of (μ, n), we will not devote our time to discussing the details here but will return to these issues in the numerical analysis after solving each of the endogenous variables purely in terms of exogenous parameters.

We next substitute (29) into (20) and then (22) and (23) to derive:

$$g = g(\mu, n; B, \psi, \phi, D, \zeta) \tag{30}$$

$$r_k = r_k(\mu, n; B, \psi, \phi, D, \zeta) \tag{31}$$

$$q = q(\mu, n; B, \psi, \phi, D, \zeta) \tag{32}$$

where $g_\mu < 0$, $g_n \lesseqgtr 0$, $g_B > 0$, $g_\psi \lesseqgtr 0$, $g_\phi < 0$, $g_D > 0$, $g_\zeta < 0$, as do the functions r_k and q. We would like to restrict our attention to the balanced growth rate that is of greater interest. Since the growth rate is positively related to work effort and work effort is negatively related to the job finding rate, we immediately establish the relationship between the growth rate and the job finding rate for a given level employment. The ambiguity between work effort and employment is also carried over, leading to an ambiguous relationship between growth and employment.

To the end, we substitute (29), (30), (31) and (32) into (21) and (24), which constitute two fundamental relationships to jointly pin down (μ, n). The relationship derived from (21) can be referred to as the *pseudo labor supply locus* (LS) and the relationship obtained from (24) can be called the *pseudo labor demand locus* (LD). Intuitively, the LS locus represents how labor supply responds to a better labor market condition as a result of a higher job finding rate (higher μ), whereas the LD locus indicates how labor demand changes in response to a tighter labor market from the viewpoint of employers (higher μ or lower η). These schedules are named as "pseudo" demand and supply because both schedules are in terms of a job matching probability μ in lieu of labor wages and because both relationships have incorporated goods market clearance and labor matching equilibrium conditions. While the direct effects are to yield an upward-sloping LS locus and a downward-sloping LD locus, there are several indirect effects present in our dynamic general equilibrium models making the net effects ambiguous. The ambiguity of the underlying indirect effects include the potential conflicts between (i) the substitution and the wealth effects, (ii) the employed and the nonemployed within each households, and (iii) households and firms. Of course, the elastic work effort and learning effort as well as the variable vacancies created by each firm lead to further complexity and ambiguity. Nonetheless, one assumes log-linear utility to remove the first potentially conflicting forces and restricts the nonemployed to have less marginal enjoyment in leisure to remove the second ambiguity. If some forms of normality in matching and in labor allocation are further imposed, one may then expect an upward-sloping LS locus in conjunction with a downward-sloping LD locus.

Due to the aforementioned complication in general, we will not perform any further analytic characterization, but instead defer the comparative static analysis to the next section using a numerical method by calibrating the model based on the U.S. data. As will be illustrated, our calibrations will reconfirm the benchmark case with well-behaved upward-sloping LS locus and downward-sloping LD locus.

Remark: In our economy, one can verify that Hosios rule holds (see the Appendix for a detailed proof of this claim). The decentralized supporting prices, capital rental and wage rates, can be shown to take the following forms:

$$1 + r = 1 + r_k = (1 + \rho)\frac{U_c}{U_{c'}} \tag{33}$$

$$w = \left[\beta + (1-\beta)\left(1 - \frac{\Gamma}{1-\beta}\right)\right]\overline{w} > \overline{w} \tag{34}$$

where competitive wage is $\overline{w} = (\frac{n-\Phi}{n})MPL$ and the wage discount is $\Gamma = \frac{1-\beta}{\beta}\frac{r_k+\psi-g(1-\psi)}{1+r_k}\frac{1+\rho}{R\ell\mu} > 0$. Thus, the supporting wage in the presence of frictional labor markets is lower than the competitive wage.

4 Numerical Analysis

We now turn to quantifying our results in the previous section by calibration analysis. Moreover, we provide a policy analysis by assessing the growth effects and the welfare consequences of an array of labor-market related subsidies.

4.1 Calibration

We calibrate parameter values to match the U.S. quarterly data over the period of 1951 to 2003. In particular, the quarterly per capita real GDP growth rate is set to $g = 0.45\%$ and the quarterly depreciation rate of capital is set to 0.02 to match the annual per capita real GDP growth rate of 1.8% and the annual depreciation rate of capital in the range of $5 - 10\%$, respectively. The rate of time preference is assigned to 0.01 (which is equivalent to an annual time preference rate of 4%, as used by Kydland and Prescott (1991))[10]. Then we can calculate from (22) the shadow capital return as $r_k = 0.0345$, along the balanced growth path. Set the capital share to the commonly used value $\alpha = 0.36$, which gives the calibrated capital-real GDP ratio (k/y) at 10.4 and the calibrated consumption-real GDP ratio (c/y) at 0.745, both are very close to the observed value in quarterly data. Based on Kendrick (1976), human capital is as large as physical capital, so we set the physical to human capital ratio at $k/h = 1$.

We calibrate the search intensity augmented unemployment measure ($u = s(1-n)$) to 0.065 to match the labor force participation rate of 61.5% (by setting $1 - (1-s)(1-n) = n + u = 0.615$). Accordingly, the employment rate can be calibrated to 0.57. In terms of time allocation, an average worker spends approximately 10% of time for advanced learning (including all post-mandatory schooling learning and training). Also, the average work time is about 1/3 (see a discussion in Andolfatto 1996). Thus, we set $e = 0.1$. Letting $\frac{\ell}{1-e} = \frac{1}{3}$, we can obtain work effort as $\ell = 0.3$. Substituting these into (20) and (17), we get $D = 0.0609$ and $\zeta = 0.0012$. So the exogenous rate of human capital accumulation is at a low rate just about 0.1%.

Based on the study by Shimer (2005), the monthly separation rate is 0.034, the monthly job finding rate is 0.45, and the elasticity parameter of matching is $\beta = 0.72$. Therefore, the quarterly separation rate $\psi = 1 - (1 - 0.034)^3$ and the quarterly job finding rate $\mu = 1 - (1 - 0.45)^3$ are computed as 0.986 and 0.834, respectively. Shimer (2005) also normalizes the vacancy-searching worker ratio ($\frac{v}{u}$) as one, which we follow. Thus, we apply the first equality of (16) to set $v = (1-n)s = 0.065$. Using the (25) and (26), we calibrate $\eta = B = 0.834$. From (27), we have the value of search intensity: $s = 0.145$. Accordingly, employed members allocate about 60% of their time ($1 - \ell - e = 0.6$) to leisure whereas the comparable figure for nonemployed members is about 85% ($1 - s = 0.855$).

We then assign a reasonable labor cost of vacancy creation and management as a percentage of employment (Φ/n) at 2.5%. This gives $\Phi = 0.025 \cdot 0.55 = 0.0138$, which can be plugged into (2)

[10] In Kydland and Prescott (1991), the quarterly time preference rate is 0.01.

to obtain $A = 0.309$. Since learning effort is non-separable from work effort, we cannot compute directly the labor supply elasticity, but the learning-augmented labor supply elasticity is given by $\frac{1}{\sigma}\left(\frac{1}{\ell} - 1\right)$. While the labor literature estimates the labor supply elasticity around 0.5, the home production literature gets a higher value at 1.7. We select $\sigma = 1.1$, which yields a reasonable learning-augmented labor supply elasticity about 2.121.[11] Now, we can use (A10) to calibrate $\varepsilon = 1.741$ and from the definition of Φ, we obtain $\phi = 1.601$. Next, we use (21) to compute the BGP value of R at 3.434. We then apply (24) to calculate $\gamma_1 = 1.763$, which together with the definition of R implies $\gamma_2 = 1.089$. That is, the employed value their leisure time more than the nonemployed, an intuitive result due to the fact that the nonemployed may be forced to take leisure involuntarily. Finally, these calibrated parameters can be substituted into (34) to obtain $w = 0.351$ and $\Gamma = 0.057$. Thus, the wage discount from its competitive counterpart ($\overline{w} = 0.372$), as a consequence of labor-market frictions, is about 5.7%, which seems quite reasonable.

We summarize the observables, benchmark parameter values and calibrated values of key endogenous variables in Table 1.

4.2 Comparative Statics

We are now ready to simulate the model to examine quantitatively the effects of uniform and discretionary human capital accumulation parameters (ζ and D) and labor-market (B, ψ, and ϕ) parameters on an array of endogenous variables of interest, including the balanced growth rate (g), effective consumption (c/h), physical-human capital ratio (k/h), effective output (y/h), employment (n), unemployment (measured by search intensity augmented job seekers, $s(1-n)$), work effort (ℓ), learning effort (e), search effort (s), workers' job finding rate (μ), firms' employee recruitment rate (η), and firms' vacancies (v). The results are reported in Table 2.

Under the benchmark parametrization, the value function ($\Omega(\mathcal{H})$) is strictly increasing and strictly concave in each argument (see the Appendix). It turns out that the LD locus is downward-sloping and the LS locus is upward-sloping (see Figure 2). While there are many underlying forces driving this outcome, one may identify the dominant forces to gain some intuition. When the job finding rate is higher, the marginal benefit of employment is lower, thereby leading to a downward-sloping pseudo labor demand locus. Turning next to the pseudo labor supply locus, one can see from (29) work effort decreases, which causes leisure to rise. A dominant force to offset this effect to restore the equilibrium is to increase investment in human capital, which can be accomplished

[11] It is difficult to conclude whether the learning-augmented elasticity should be larger or smaller – it all depends on whether education effort is more or less senstive to market wages.

by raising employment according to (28). Thus, the LS locus slopes upward. In response to human capital accumulation and labor market-improving parameter shifts, our numerical results suggest that there is a large outward shift in the LD locus which outweighs the shift in the LS locus (the BGP equilibrium shifts from E_0 to E_1 or E_2 in Figure 2), thus raising both the job finding rate and employment.

Moreover, it is noted that in the calibrated equilibrium, Pareto complementarity between employment and work effort in production is a dominant force; as a consequence, the relationship between growth and employment given in the human capital envelope condition, (30), is always positive. We may therefore characterize the growth effects of parameter changes based on their direct effects through (30) as well as their indirect effects via the job finding rate and employment in (30). The numerical results suggest that any shift in human capital-enhancing and labor market-improving parameters always create a negative *free-rider effect from thick matching* (through μ) and a positive *employment creation effect* (through n): the former reduces growth whereas the latter raises it. All but a shift in the uniform human capital accumulation parameter also generates a positive *direct human capital effect* through (30). On balance, each of such shifts affects the growth rate positively. That is, in response to an increase in the uniform human capital accumulation parameter, the positive employment creation effect dominates the negative direct human capital effect and the negative free-rider effect from thick matching. In response to other human capital-enhancing and labor market-improving parameter shifts, the positive employment creation effect and the positive direct human capital effect together dominate the negative free-rider effect from thick matching.

An increase in either the uniform human capital accumulation parameter (ζ) or the discretionary human capital accumulation parameter (D) raises learning effort, thus raising employment and economic growth. Not surprisingly, the discretionary human capital accumulation parameter creates stronger employment and growth effects compared to the uniform human capital accumulation parameter. Since both parameters raise labor productivity, they also induce labor-market participation and encourage workers to devote greater effort to job search and firms to create more vacancies. While higher search effort raises the unemployment rate, higher employment lowers it.[12] Around the calibrated equilibrium, the search effort effect dominates and hence the unemployment rate is higher in response to an increase in either human-capital enhancing parameter; this may be regarded as "creative destruction" as a result of human capital accumulation with frictional

[12] The positive effect of training on job search intensity is consistent with empirical findings in Barron, Black and Loewenstein (1989).

labor markets in the absence of technological advancements. Due to these offsetting forces, the net increase in unemployment is not as much as the increase in vacancies, thus leading to a higher job finding rate and a lower firm recruitment rate. Additionally, as a result of higher learning and search effort, work effort decreases. Since accumulating human capital is more profitable, there is a factor substitution from physical to human capital. This latter outcome, together with lower work effort and higher vacancy costs, causes the level of effective output to fall, despite a positive growth effect. The fall in effective output subsequently leads to a decrease in effective consumption.

An increase in the degree of labor-market matching efficacy (B), or a reduction in the separation rate (ψ) or the vacancy creation cost (ϕ) raises employment and job finding rates. While the induced wage incentive effect encourages labor-market participation, learning and search effort, individual workers may free-ride on the thickness of the labor market that in turns reduces learning and search effort. In the calibrated BGP equilibrium, the wage incentive effect dominates the free-rider effect and, as a result, both output growth and unemployment rates are higher. Moreover, an increase in B shifts the Beverage Curve outward but a decrease in ψ and ϕ induce a downward movement along the Beverage Curve in (μ, η) space. Thus, the former results in higher job finding rate and firm recruitment rate whereas the latter raises job finding rate but reduces firm recruitment rate. While it is obvious that more effective matching or less costly vacancy creation induces more vacancies, a lower separation rate implies that firms retain current employees without the need for creating more vacancies. Similar to the increase in human capital accumulation parameters, these labor-market improvements also cause work effort to fall as a result of higher learning and search effort. For similar arguments, the levels of effective output and effective consumption decrease as well.

Generally speaking, economic growth, employment, labor-market participation, vacancy creation, and learning and search effort are most responsive to changes in the discretionary human capital accumulation parameter (D), followed by job matching and separation rates (B and ψ). The positive growth effect of an increase in the uniform human capital accumulation parameter (ζ) is by far the smallest, which is not surprising because of the presence of a negative direct human capital effect. While an enhancement in discretionary human capital accumulation parameter is most effective in fostering growth, it is also associated with the largest decline in work effort, effective output, effective consumption and leisure, as well as the largest increase in the unemployment rate. While job finding rate responds most sensitively to the job matching rate followed by the discretionary human capital accumulation parameter, firm recruitment rate responds most sensitively to the discretionary human capital accumulation parameter followed by the job separation rate and the vacancy creation cost parameter (ϕ).

It is worth noting that, in response to shifts in any learning and labor-matching parameters, growth and employment always move in the same direction, as do growth and the search intensity augmented unemployment measure (u). Thus, if one measures unemployment by purely head counts ($1-n$), there is a negative relationship between growth and unemployment in the long run. If one instead measures unemployment by taking search intensity into account, the long-run relationship between growth and unemployment becomes positive.

4.3 Human Capital Policy

In this simple model economy, we may consider two labor-related public policy programs of particular interest:

- a uniform human capital policy enhancing the exogenous component of human capital growth (ζ), which is uniform to all agents (e.g., general training);
- a discretionary human capital policy raising the marginal benefit of human capital accumulation (D), which favors agents devoting more effort to learning (e.g., job-specific training and executive learning).

Such training programs have been commonly employed in practice and some programs may be intense.[13]

Notably, in order to highlight the role of labor-market frictions, we have abstracted from considering any other imperfections or distortions such as human capital externalities or factor tax distortions. Thus, when the Hosios rule of efficiency bargaining holds, it is expected that any public policy will not improve upon the decentralized market equilibrium. Nonetheless, one may still compare the growth and welfare effects of the two above-mentioned policies in the revenue-neutral tax-incidence context.

Denote the rate of the respective human capital "subsidy" as a. In each of the policy experiments, the subsidy is financed by a lump-sum tax whose value in effective unit is fixed at 1% of the benchmark value of effective output (this effective lump-sum tax turns out to be $T/h = 0.00096$).

Two preliminary tasks are now in order prior to policy evaluation. First, we must compute the welfare along the BGP. Setting $h_0 = 1$, we can derive the welfare measured by the lifetime utility,

$$\Omega = \frac{1+\rho}{\rho}\left[\ln(\frac{c}{h})^* + \frac{1}{\rho}\ln(1+g) + n\gamma_1\frac{(1-\ell-e)^{1-\sigma}}{1-\sigma} + (1-n)\gamma_2\frac{(1-s)^{1-\sigma}}{1-\sigma}\right] \tag{35}$$

[13] For example, in terms of firm training alone, Barron, Black and Loewenstein (1989) document that it takes about 29% of the total employment hours for new hires over the first three months since the start of the jobs.

where we need to modify (5), using (3), (22), (23) and the definition of BGP, to derive the after-tax effective consumption:

$$\left(\frac{c}{h}\right)^{*} = Aq^{\alpha}\left(n-\Phi\right)\ell - \left(\delta+g\right)\frac{k}{h} - \frac{T}{h} \tag{36}$$

Second, we must compute the relative price of human capital investment in order to compute the rate of subsidy for the two human capital policy experiments. Notice that individual optimization implies that the relative price of human capital investment in unit of outputs (P_h) multiplied by the marginal utility of consumption must be equal to the marginal valuation of human capital, which can be used to derive: $P_h = \frac{MVH'}{U_c}$. By utilizing (14) and (A9), this reduces to,

$$P_h = \frac{\overline{w}}{D} \tag{37}$$

Table 3A summarizes the results of our key endogenous variables in response to each of the two human capital policies subject to the government budget constraint at a given effective value of lump-sum tax. More specifically, the government budget constraint in each case is given by,

- a uniform human capital enhancement policy that increases ζ to $(1+a)\zeta$: $a\zeta P_h h = T$;
- a discretionary human capital enhancement policy that increases D to $(1+a)D$: $aP_h hDne = T$.

The required rates of subsidy for the two experiments are about 13.7% and 2.3%, respectively. Notice that these policies can be evaluated based only on the relative price of human capital investment.

Overall, under the benchmark parameterization, a discretionary human capital enhancement policy is more effective in promoting human capital accumulation and economic growth. In particular, such a subsidy amounted to 1% of effective output evaluated at the benchmark value can raise output growth by 59.1% (which is about 0.265 percentage point increase). This is far more than the effect of a uniform human capital enhancement policy (4.9%). Of course, this stronger welfare-enhancing growth effect is accompanied by a larger drop in effective consumption which is welfare-reducing.

However, due to its encouragement for household to participate in the labor market, to seek jobs and to spend time on learning, a discretionary human capital enhancement policy also generates larger drops in leisure for each of the employed and the unemployed members of the large household ($1-\ell-e$ and $1-s$). Since the calibrated value of σ exceeds one, the "aggregate value" of leisure of the employed ($n\gamma_1\frac{(1-\ell-e)^{1-\sigma}}{1-\sigma}$) is decreasing in n while the "aggregate value" of leisure of the unemployed ($(1-n)\gamma_2\frac{(1-s)^{1-\sigma}}{1-\sigma}$) is increasing in n. Thus, the aggregate leisure effect for the employed in response to a discretionary human capital enhancement policy is negative, but that for the unemployed is ambiguous. Around the calibrated equilibrium with public policies (see the last column of Table 3B), it turns out that the aggregate leisure effect for the unemployed is positive.

We summarize in Table 3B the four components of changes in welfare according to (35): that due to changes in effective consumption, that due to changes in the rate of human capital accumulation, that due to changes in the aggregate leisure effect for the employed and that due to changes in the aggregate leisure effect for the unemployed. In response to a discretionary human capital enhancement policy, the negative welfare effect via the aggregate leisure effect for the employed is large, which in conjunction with the negative welfare effect via effective consumption dominates the positive welfare effects via the accumulation of human capital and the aggregate leisure effect for the unemployed. As a result, a discretionary human capital enhancement policy reduces economic welfare despite its stronger positive effect on the balanced growth rate. For similar arguments, a uniform human capital enhancement policy also generate qualitatively similar component effects on welfare, leading to a net reduction in our benchmark economy.[14] Quantitatively, the growth-promoting policy instrument by subsidizing human capital discretionarily is associated with higher welfare cost than subsidizing human capital uniformly.

To highlight the role played by labor-market frictions, we repeat the policy experiments presented in Table 3A in an alternative economy in which such frictions are less severe. We do so by raising the degree of labor-market matching efficacy (B) by 5 percent while maintaining a constant government budget at the value computed from the benchmark economy. The numerical results are summarized in Table 4. By comparing the results with their counterparts in Table 3A, a strong conclusion arrives. That is, as the severity of labor-market frictions diminishes, the effects of these human capital policy programs on key variables all become smaller.[15] Quantitatively, such policy consequences are noticeably smaller even with only a moderate improvement in the job-matching conditions. For example, in this alternative economy with 5% less severe labor-market frictions, the policy effects on learning, output growth and employment reduce, on average, by about 50% and 35% and 40%, respectively. This suggests that a quantitative evaluation of the effectiveness of public policy in a

[14] Recall that the dynamic search equilibrium features efficient wage bargaining. In the absence of preference/production externalities, distortionary taxes, or other imperfections, education and investment subsidies are not expected to improve welfare. Should one include uncompensated human capital spillovers (cf. Lucas 1988) or factor income taxation (cf. Bond et al. 1996), these subsidy programs may become welfare-enhancing. Thus, our discussion here only focuses on relative welfare comparisons between different policies, rather than the absolute welfare gains/losses associated with each policy.

[15] This conclusion applies to all individual macroeconomic variables. Here, we exclude the welfare measure because it is an aggregator of several macroeconomic variables.

frictionless Walrasian world is expected to be biased downward severely. This finding is noteworthy because there is a call for reevaluating such human capital policies when the labor market is not frictionless.

4.4 Sensitivity Analysis

In the above calibration exercises, all the pre-set parameters are well-justified. However, one may argue that some of the calibration criteria are possibly questionable. Thus, we conduct a sensitivity analysis, examining the qualitative as well as quantitative implications of taking alternative calibration criteria in fairly wide ranges. Specifically, we consider the following perturbations.

- We allow the amount of physical capital to be twice as large as or half of the amount of human capital.
- We allow the employment ratio to fall in $[0.5, 0.6]$, the work effort in $[0.15, 0.45]$, the learning effort in $[0.08, 0.12]$, the vacancy-unemployment ratio in $[0.85, 1.15]$ and the cost of vacancy creation and management as a percentage of employment in $[0.02, 0.03]$.
- We also consider a wide range ($[0.5, 1.7]$) of labor supply elasticities to encompass both micro labor and macro literature.

The results of our sensitivity analysis suggest that, by changing the calibration criteria and re-calibrate the model, the changes in equilibrium outcomes are either non-existent or inessential (see the Appendix Table A). As a consequence, our findings concerning the long-run growth and welfare effects of uniform and discretionary human capital policy remain robust.

5 Concluding Remarks

In this paper, we develop an endogenous growth model where sustained human capital accumulation and labor search, matching and entry frictions are integral parts of the economy. Our analysis demonstrates the significant role of labor market frictions in assessing macroeconomic performance and policy effectiveness. We find that an increase in the effectiveness of human capital accumulation or a reduction in the job separation rate or the vacancy creation cost will raise employment, vacancy creation, learning effort and output growth. By conducting two policy experiments that enhance human capital accumulation, we find that a discretionary human capital policy is more growth-promoting than a uniform human capital policy, though such a discretionary policy is also associated with a higher welfare cost. Our numerical results also suggest that the effects of these public policy programs become larger as the severity of labor-market frictions increase, which reconfirms the important role of labor market frictions.

Our model is subject to several qualifications which calls for future research. For brevity, we would only mention four possible extensions. First, to simplify the analysis, we assume that the accumulation of human capital only depends on learning effort. It would be interesting to consider the case in which physical capital also contributes to human capital accumulation as modeled by Bond et al. (1996). One may then conduct a full tax-incidence analysis on labor and capital income taxes in the presence of labor-market frictions and compare the results with findings obtained in canonical growth models without frictions. Second, in the present framework, job separation is assumed to be exogenous. It may be extended to allow the separation rate to depend on on-the-job learning effort, as postulated by Mortensen (1988). Such generalization yields an additional margin that may differentiate uniform and discretionary human capital policy via endogenous layoff. Third, our framework is ready to be extended to one with credit-market imperfections. The resulting credit constraints may affect human capital investment financing and/or vacancy creation, so the effectiveness of subsidies to learning and vacancy creation need be reevaluated. Finally, in this study, we focus on the long-run implications of an endogenously growing economy with labor market frictions. Our model may be modified to include technological shocks, as in Merz (1995) and Andolfatto (1996), for quantifying the short-run effects of education and labor-market policies over the business cycle.

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Appendix

(A major portion of the Appendix is not intended for publications.)

In this Appendix, we provide mathematical details of the theoretical results, including the optimization conditions, the second-order conditions, some key equilibrium relationships, the equilibrium price support and and the properties of the value function. At the end of the paper, we also present an Appendix Table A, summarizing the equilibrium outcomes obtained from the sensitive analysis.

1. Optimization Conditions

Let $\mathcal{H}$ denote the vector of state variables this period, namely, $\mathcal{H} = (k, h, n)$, and $\mathcal{H}'$ denote the triplets in the next period.

The first-order conditions with respect to c, ℓ, e, s and v can be derived as:

$$U_c = \frac{1}{1+\rho}\Omega_k(\mathcal{H}') \tag{A1}$$

$$\frac{1}{1+\rho}\Omega_k(\mathcal{H}')(1-\alpha)Aq^{\alpha}(n-\Phi)h = -U_\ell \tag{A2}$$

$$\frac{1}{1+\rho}\Omega_h(\mathcal{H}')Dnh = -U_e \tag{A3}$$

$$\frac{1}{1+\rho}\Omega_n(\mathcal{H}')\beta\mu(1-n) = -U_s \tag{A4}$$

$$\Omega_n(\mathcal{H}')(1-\beta)\eta = \Omega_k(\mathcal{H}')(1-\alpha)Aq^{\alpha}\ell h\Phi_v(v) \tag{A5}$$

Combining (A1) and (A2) gives (9), while rewriting (A3)-(A5) yields (9)-(12).

The Benveniste-Scheinkman conditions governing (k, h, n) are given as follows (after making use of the first order conditions above):

$$\Omega_k(\mathcal{H}) = \frac{1}{1+\rho}\Omega_k(\mathcal{H}')\left[(1-\delta) + \alpha Aq^{\alpha-1}\right] \tag{A6}$$

$$\Omega_h(\mathcal{H}) = \frac{1}{1+\rho}\left[\Omega_k(\mathcal{H}')(1-\alpha)Aq^{\alpha}(n-\Phi)\ell + \Omega_h(\mathcal{H}')\left(1+\zeta+Dne\right)\right] \tag{A7}$$

$$\Omega_n(\mathcal{H}) = U_n + \frac{1}{1+\rho}\left[\Omega_k(\mathcal{H}')(1-\alpha)Aq^{\alpha}\ell h + \Omega_h(\mathcal{H}')Deh + \Omega_n(\mathcal{H}')\left(1-\psi-\beta\mu s\right)\right] \tag{A8}$$

Manipulating (A6) and using (A1) give (13). Substituting (4), (A2) and (A3) into (A7), one obtains (14). Similarly, we can substitute (7) and (A2)-(A4) into (A8) to yield ((15).

2. Second-Order Conditions

We next turn to the second-order conditions with respect to choice varialbes $\{c, \ell, e, s, v\}$. Denote MP as marginal product. By differentiating (A1)-(A5) and using the following relationships,

$$\begin{aligned}
\frac{1}{1+\rho}\Omega_k(\mathcal{H}')\cdot MP_\ell &= -U_\ell \\
\Omega_n(\mathcal{H}')(1-\beta)\eta &= \Omega_k(\mathcal{H}')\cdot(-MP_v) \\
MP_v &= Ak^{\alpha}(\ell h)^{1-\alpha}(1-\alpha)(n-\Phi)^{-\alpha}(-\phi\varepsilon v^{\varepsilon-1}) \\
MP_{vv} &= Ak^{\alpha}(\ell h)^{1-\alpha}(1-\alpha)\phi\varepsilon v^{\varepsilon-1}(n-\Phi)^{-\alpha-1}[\frac{\varepsilon-1}{v} + \alpha\phi\varepsilon v^{\varepsilon-1}]
\end{aligned}$$

$$\mu = \frac{m}{s(1-n)} = \frac{B[s(1-n)]^{\beta} v^{1-\beta}}{s(1-n)}$$
$$\eta = \frac{m}{v} = \frac{B[s(1-n)]^{\beta} v^{1-\beta}}{v}$$

we can derive the following second-order conditions with respect to $\{c, \ell, e, s, v\}$:

$$\begin{aligned}
U_{cc} &< 0 \\
\frac{1}{1+\rho}\Omega_k(\mathcal{H}') \cdot MP_{\ell\ell} + U_{\ell\ell} &< 0 \\
U_{ee} &< 0 \\
\frac{1}{1+\rho}\Omega_n(\mathcal{H}')\beta(1-n)\frac{\partial \mu}{\partial s} + U_{ss} &< 0 \\
\Omega_n(\mathcal{H}')(1-\beta)\frac{\partial \eta}{\partial v} + \Omega_k(\mathcal{H}') \cdot MP_{vv} &< 0
\end{aligned}$$

That is, all second-order conditions are met.

3. Fundamental Equilibrium Relationships

By applying the specific functional forms, (9) becomes:

$$\frac{c}{h} = (1-\alpha)Aq^{\alpha}\left(\frac{n-\Phi}{n}\right)[\gamma_1(1-\ell-e)^{-\sigma}]^{-1} \tag{A9}$$

Since the felicity function is separable in consumption and leisure, we can see from (A1), (A3) and (A4) that, along a BGP, $\Omega_n(\mathcal{H}')$ is constant whereas $\Omega_k(\mathcal{H}')$ and $\Omega_h(\mathcal{H}')$ are decreasing at the common growth rate. In turn, we can utilize these equations to derive:

$$\begin{aligned}
\Omega_k(\mathcal{H}') &= \frac{1+\rho}{c} = \frac{\Omega_k(\mathcal{H})}{1+g} \\
\Omega_h(\mathcal{H}') &= \frac{(1+\rho)\,\gamma_1(1-\ell-e)^{-\sigma}}{Dh} = \frac{\Omega_h(\mathcal{H})}{1+g} \\
\Omega_n(\mathcal{H}') &= \frac{(1+\rho)\,\gamma_2(1-s)^{-\sigma}}{\beta\mu} = \Omega_n(\mathcal{H})
\end{aligned}$$

These, together with (2), (10), (11), (17) and (A9), can be substituted into (12) and (13)-(15) to obtain:

$$\frac{\Phi_v n \ell R}{n-\Phi} = \frac{(1-\beta)\eta}{\beta\mu} \tag{A10}$$

and (19)-(21). We can then substitute (17), (20) and (25)-(27) into (A10) to yield (29). Finally, manipulation of (18), (A9) and (23) leads to (24).

4. Equilibrium Price Support

We now derive equilibrium price support. Since the capital market is perfect, the capital rental r is equal to its shadow rate of return and the intertemporal relative price of consumption. In contrast, the labor market is frictional; thus, the wage rate w derived from efficient bargaining that supports the pseudo social planner's solution is generally different from the competitive wage $\overline{w}$.

To derive this wage support, it is convenient to write out the marginal product of labor, $MPL = dy/d[(n-\Phi(v))\,\ell h] = (1-\alpha)Aq^{\alpha}$, and the competitive wage rate, $\overline{w} = (\frac{n-\Phi}{n})MPL$. We turn now to deriving firms' unmatched value (Π^U) and matched value (Π^M) accrued from a successful bargain with their employees. Consider a representative firm which is currently unmatched. Its flow profit is negative due to costly vacancy creation and maintenance (VC). However, at probability η, it will change the state from unmatched to matched next period with a value $\Pi^{M'}$; at probability $1-\eta$, it will remain unmatched next period with a value $\Pi^{U'}$. Thus, the value of an unmatched firm is given by,

$$\Pi^U = -VC + \frac{1}{1+r}\left[\eta\Pi^{M'} + (1-\eta)\Pi^{U'}\right]$$

where the marginal vacancy cost is $VC = -dy/dv = \Phi_v \ell h \cdot MPL$. In the absence of entry costs, we must have $\Pi^{U'} = \Pi^U = 0$ in balanced growth equilibrium. We can therefore use (33) to rewrite the Bellman equation above as:

$$\Pi^{M'} = \left(\frac{1+r_k}{\eta}\right)\Phi_v \ell h \cdot MPL \tag{A11}$$

We can also specify the Bellman equation concerning the value of a matched firm below:

$$\Pi^M = \pi + \frac{1}{1+r}\left[(1-\psi)\Pi^{M'} + \psi\Pi^{U'}\right] \tag{A12}$$

where the flow profit per vacancy is governed by, $\pi = \max_{k/n}\left\{\frac{y}{n} - r_k\frac{k}{n} - w\ell h\right\}$. By the constant-returns property of the production function, we have:

$$\pi = \left(MPL \cdot \frac{n-\Phi}{n} - w\right)\ell h \tag{A13}$$

The matching surplus accrued from a successful hire of an additional employment is $\frac{\Omega_n}{U_c}$ (which is in unit of outputs). Denoting the share of this surplus to workers as b (to be determined below) and hence the remaining $(1-b)$ to firms, we can write: $\Pi^M - \Pi^U = (1-b)\frac{\Omega_n}{U_c}$. With $\Pi^U = 0$, this implies:

$$\Pi^M = (1-b)\frac{\Omega_n}{U_c} \tag{A14}$$

Also, updating (A14) by one period and using (33) and (A11) to eliminate $\Pi^{M'}$ and $U_{c'}$, we can get:

$$\Omega_n(\mathcal{H}')(1-b)\eta = \Omega_k(\mathcal{H}')(1-\alpha)Aq^\alpha \ell h \Phi_v(v)$$

Comparing this expression with (A5), one can easily solve $b = \beta$, which implies that Hosios rule holds in our model economy.

From (A14), we know that $\Pi^M = (1-b)\frac{\Omega_n(\mathcal{H})}{U_c} = (1-\beta)c\Omega_n$ and $\Pi^{M'} = (1-b)\frac{\Omega_n(\mathcal{H}')}{U_{c'}} = (1-\beta)c\Omega_n(1+g)$. Substituting these into (A12), we have:

$$\pi = (1-\beta)\frac{r_k + \psi - g(1-\psi)}{1+r_k}c\Omega_n \tag{A15}$$

Combining (A13) and (A15) to eliminate π, we derive the wage support:

$$w = \overline{w} - (1-\beta)\frac{r_k + \psi - g(1-\psi)}{1+r_k}\frac{c}{\ell h}\Omega_n$$

Thus, the presence of labor-market frictions results in a wage discount, $\Gamma = \frac{\overline{w}-w}{\overline{w}} > 0$. When matching becomes less responsive to vacancies ($1-\beta$ is lower), firm's share of matching surplus shrinks, as does the wage discount. We can further substitute Ω_n and (A9) into the wage expression above to obtain:

$$w = \overline{w}\left[1-(1-\beta)\frac{r_k+\psi-g(1-\psi)}{1+r_k}\frac{1+\rho}{R\ell\beta\mu}\right]$$

which is a weighted average of the competitive wage and the outside option facing each worker, as given in the main text. This finding is similar to one obtained by Merz (1995) and Andolfatto (1996).

5. Properties of the Value Function

Using (A1)-(A8) and the evolution equations of $\{k, h, n\}$, we can express choice variables c, ℓ, e, s, v as functions of $\mathcal{H}$. Concerning the co-state variables $\left\{\lambda^k, \lambda^h, \lambda^n\right\}$ associated with the evolution equations of $\{k, h, n\}$, we have:

$$\begin{aligned}
\lambda^k(\mathcal{H}') &= \Omega_k(\mathcal{H}') = \frac{1+\rho}{c} \\
\lambda^h(\mathcal{H}') &= \Omega_h(\mathcal{H}') = \frac{(1+\rho)\,\gamma_1(1-\ell-e)^{-\sigma}}{Dh} \\
\lambda^n(\mathcal{H}') &= \Omega_n(\mathcal{H}') = \frac{(1+\rho)\,\gamma_2(1-s)^{-\sigma}}{\beta\mu}
\end{aligned}$$

Hence, we can calculate their derivatives with respect to $\{k, h, n\}$ under our calibrated benchmark economy as follows:

$$\begin{aligned}
\lambda^k_k(\mathcal{H}') &= -\frac{(1+\rho)c_k}{c^2} < 0 \\
\lambda^k_h(\mathcal{H}') &= -\frac{(1+\rho)c_h}{c^2} < 0 \\
\lambda^k_n(\mathcal{H}') &= -\frac{(1+\rho)c_n}{c^2} < 0 \\
\lambda^h_k(\mathcal{H}') &= \frac{(1+\rho)\gamma_1(1-\ell-e)^{-\sigma-1}}{Dh}\sigma h(\ell_k+e_k) < 0 \\
\lambda^h_h(\mathcal{H}') &= \frac{(1+\rho)\gamma_1(1-\ell-e)^{-\sigma-1}}{Dh^2}[\sigma h(\ell_h+e_h)-(1-\ell-e)] < 0 \\
\lambda^h_n(\mathcal{H}') &= \frac{(1+\rho)\gamma_1\sigma(1-\ell-e)^{-\sigma-1}}{Dh}[\ell'(n)+e'(n)] < 0 \\
\lambda^n_k(\mathcal{H}') &= \frac{(1+\rho)\gamma_2(1-s)^{-\sigma-1}}{\beta\mu^2}[\sigma\mu s_k(n)-(1-s)\mu_k] < 0 \\
\lambda^n_h(\mathcal{H}') &= \frac{(1+\rho)\gamma_2(1-s)^{-\sigma-1}}{\beta\mu^2}[\sigma\mu s_h(n)-(1-s)\mu_h] > 0 \\
\lambda^n_n(\mathcal{H}') &= \frac{(1+\rho)\gamma_2(1-s)^{-\sigma-1}}{\beta\mu^2}[\sigma\mu s_n(n)-(1-s)\mu_n] < 0
\end{aligned}$$

It is not difficult to derive

$$q_k = \frac{(n-\Phi(v))\ell + k[\Phi_v v_k \ell - (n-\Phi(v))\ell_k]}{[(n-\Phi(v))\ell]^2 h} > 0$$

$$q_h = \frac{-k\{(n-\Phi(v))\ell + [(n-\Phi(v))\ell_h - \Phi_v v_h \ell]h\}}{[(n-\Phi(v))\ell h]^2} < 0$$

$$q_n = \frac{-k\{[1-\Phi_v v_n]\ell + (n-\Phi(v))\ell_n\}}{[(n-\Phi(v))\ell]^2 h} < 0$$

From (A6) and the properties above, we can derive:

$$\Omega_{kk}(\mathcal{H}) = \frac{1}{1+\rho}\{\lambda_k^k(\mathcal{H}')\left[(1-\delta)+\alpha A q^{\alpha-1}\right]^2 + \Omega_k(\mathcal{H}')\alpha A(\alpha-1)q^{\alpha-2}q_k\} < 0$$

From (A7) and the properties above we get:

$$\Omega_{hh}(\mathcal{H}) = \frac{1}{1+\rho}\{term1 + term2 + term3 + term4\} < 0$$

where

$$\begin{aligned}
term1 &= \lambda_k^k(\mathcal{H}')\,[(1-\alpha)Aq^\alpha(n-\Phi)\ell]^2 < 0\\
term2 &= \lambda^k(\mathcal{H}')(1-\alpha)A\alpha q^{\alpha-1}(n-\Phi)\ell q_h < 0\\
term3 &= \lambda_h^h(\mathcal{H}')\,(1+\zeta+Dne)^2 < 0\\
term4 &= \left[\lambda_h^k(\mathcal{H}')+\lambda_k^h(\mathcal{H}')\right](1+\zeta+Dne)\,(1-\alpha)Aq^\alpha(n-\Phi)\ell < 0
\end{aligned}$$

From (A8) and the properties above, we have:

$$\begin{aligned}
\Omega_{nn}(\mathcal{H}) = {} & U_{nn} + U_{nc}c_n + U_{n\ell}\ell_n + U_{ne}e_n + U_{ns}s_n\\
& + \frac{1}{1+\rho}\{\lambda_k^k(\mathcal{H}')[(1-\alpha)Aq^\alpha \ell h]^2 + \lambda^k(\mathcal{H}')(1-\alpha)Ah[\alpha q^{\alpha-1}q_n\ell + q^\alpha\ell_n]\\
& + \lambda_h^h(\mathcal{H}')(Deh)^2 + \Omega_h(\mathcal{H}')Dhe_n\\
& + \lambda_n^n(\mathcal{H}')\,(1-\psi-\beta\mu s)^2 - \lambda^n(\mathcal{H}')\beta\,(\mu_n s + \mu s_n)\\
& + \left[\lambda_h^k(\mathcal{H}')+\lambda_k^h(\mathcal{H}')\right][(1-\alpha)Aq^\alpha\ell h](Deh) + \left[\lambda_n^h(\mathcal{H}')+\lambda_h^n(\mathcal{H}')\right](Deh)\,(1-\psi-\beta\mu s)\\
& + \left[\lambda_n^k(\mathcal{H}')+\lambda_k^n(\mathcal{H}')\right][(1-\alpha)Aq^\alpha\ell h]\,(1-\psi-\beta\mu s)\}
\end{aligned}$$

Note $U_{nn} = U_{nc} = 0$, $U_{n\ell} = U_{ne} < 0$ and $U_{ns} > 0$.Define $TE1 = \frac{(1+\rho)(1-\alpha)Aq^{\alpha-1}(\alpha q_n\ell + q\ell_n)}{c/h} < 0$, $TE2 = (1+\rho)\,\gamma_1(1-\ell-e)^{-\sigma}e_n > 0$ and $TE3 = (1+\rho)\,\gamma_2(1-s)^{-\sigma}\left(\mu_n\frac{s}{\mu} + s_n\right) > 0$. Then we can rewrite:

$$\Omega_{nn}(\mathcal{H}) = \frac{1}{1-(1-\psi-\beta\mu s)^2}\,(term5 + term6 + term7 + term8)$$

where

$$\begin{aligned}
term5 &= U_{n\ell}[\ell'(n) + e'(n)] + U_{ns}s'(n) < 0\\
term6 &= \frac{1}{1+\rho}\left\{\Omega_{kk}(\mathcal{H}')[(1-\alpha)Aq^\alpha\ell h]^2 + \Omega_{hh}(\mathcal{H}')(Deh)^2\right\} < 0\\
term7 &= \frac{1}{1+\rho}\{TE1 + TE2 - TE3\} > 0\\
term8 &= \frac{1}{1+\rho}\left[\lambda_h^k(\mathcal{H}')+\lambda_k^h(\mathcal{H}')\right][(1-\alpha)Aq^\alpha\ell h](Deh) < 0\\
term9 &= \frac{1}{1+\rho}\left[\lambda_n^h(\mathcal{H}')+\lambda_h^n(\mathcal{H}')\right](Deh)\,(1-\psi-\beta\mu s) < 0\\
term10 &= \frac{1}{1+\rho}\left[\lambda_n^k(\mathcal{H}')+\lambda_k^n(\mathcal{H}')\right][(1-\alpha)Aq^\alpha\ell h]\,(1-\psi-\beta\mu s) < 0
\end{aligned}$$

Since under our calibration, $1-(1-\psi-\beta\mu s)^2 > 0$ and $term5 + term7 < 0$. Hence, $\Omega_{nn}(\mathcal{H}) < 0$.

Figure 1: Labor Allocation for Households

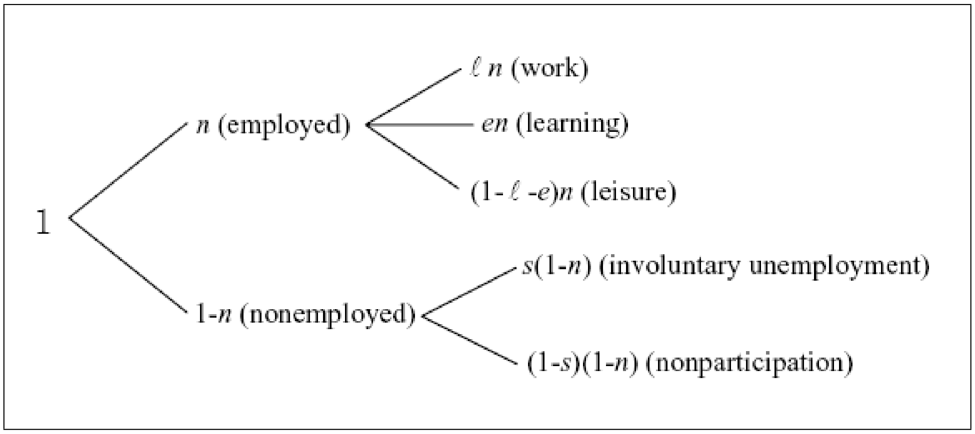

Figure 2: Balanced Growth Equilibrium and Effects of Educational/Labor-Market Improvements

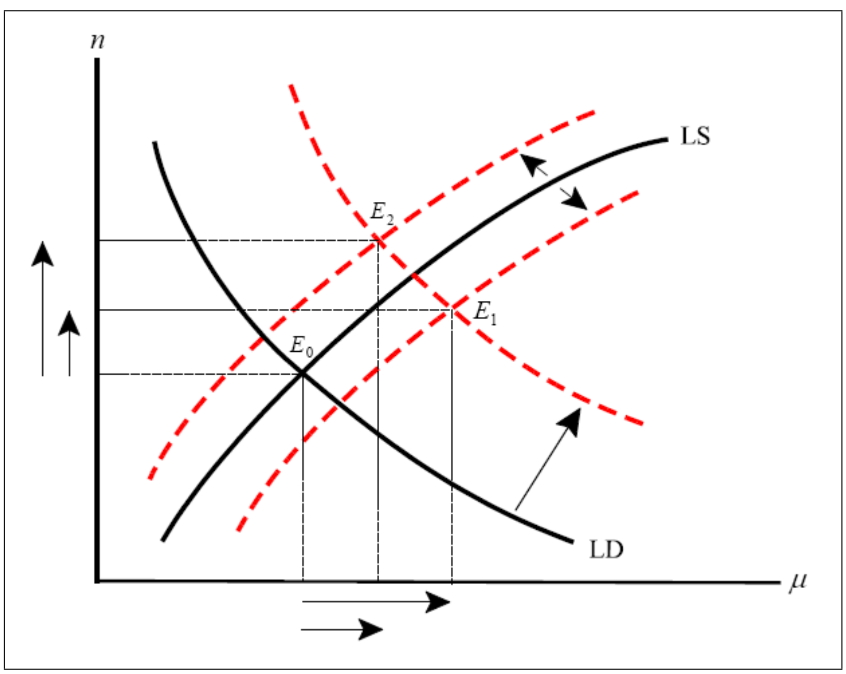

Table 1 Benchmark Parameter Values and Calibration

Benchmark Parameters and Observables		
per capita real economic growth rate	g	0.0045
capital's depreciation rate	δ	0.0200
time preference rate	ρ	0.0100
physical capital-human capital ratio	k/h	1.0000
fraction of time devoted to work	l	0.3000
fraction of time devoted to education	e	0.1000
capital's share	α	0.3600
labor searcher's share in matching production	β	0.7200
job separating rate	ψ	0.0986
job finding rate	μ	0.8336
labor force participation rate	$n+u$	0.6150
vacancy-searching worker ratio	v/u	1.0000
labor supply elasticity	$(1/\ell-1)/\sigma$	1.1000
vacancy creation cost per employment	Φ/n	0.0250
Calibration		
coefficient of goods technology	A	0.3090
coefficient of matching technology	B	0.8336
capital-output ratio	k/y	10.4212
aggregate consumption-aggregate output ratio	c/y	0.7447
consumption-human capital ratio	c/h	0.0715
coefficient of the cost of vacancy creation and management	ϕ	1.6014
exogenous human capital accumulation rate	ζ	0.0012
maximum rate of endogenous human capital accumulation	D	0.0609
rate of return of capital	r_k	0.0345
elasticity of substitution of leisure	σ	2.1212
unemployment measure	u	0.0650
fraction of time devoted to employment	n	0.5500
search intensity	s	0.1445
vacancy creation	ν	0.0650
cost elasticity of vacancy creation and management	ε	1.7409
employee recruitment rate	η	0.8336
coefficient in the utility function	γ_1	1.7626
coefficient in the utility function	γ_2	1.0894

Table 2 Quantitative Comparative Static Results

	g	c/h	k/h	y/h	n	l	e	s	μ	η	v	u	$n+u$
Benchmark	**0.004500**	**0.071458**	**1.000000**	**0.095958**	**0.549969**	**0.300000**	**0.100000**	**0.144503**	**0.833625**	**0.833625**	**0.065031**	**0.065031**	**0.615000**
ζ up by 1%	0.003604	-0.000326	-0.000736	-0.000262	0.000329	-0.000313	0.001075	0.000685	0.000047	-0.000120	0.000449	0.000283	0.000324
D up by 1%	0.302278	-0.038316	-0.070270	-0.033295	0.081141	-0.082969	0.287827	0.183106	0.014404	-0.036108	0.121641	0.065789	0.079518
B up by 1%	0.058667	-0.005463	-0.012053	-0.004427	0.019348	-0.018723	0.058369	0.029634	0.013985	-0.000176	0.019528	0.005289	0.017861
ψ up by 1%	-0.059250	0.005596	0.012416	0.004524	-0.019551	0.019670	-0.061276	-0.029062	-0.003906	0.010114	-0.019661	-0.005864	-0.018104
ϕ up by 1%	-0.009837	0.000926	0.002046	0.000749	-0.003245	0.003211	-0.010007	-0.004946	-0.002248	0.005803	-0.008996	-0.001000	-0.003008

Note: Numbers reported in rows 3-7 are percentage changes of key variables from their benchmark values (presented in row 2) due to each exogenous shift.

Table 3 Policy Experiments

Table 3A Percentage Changes in Key Variables

	a	*g*	*(c/h)**	*(y/h)**	*n*	*l*	*e*	*1-l-e*	*s*	*μ*	*η*	*v*	*n+u*	Ω
Benchmark	**NA**	**0.004500**	**0.071458**	**0.095958**	**0.549969**	**0.300000**	**0.100000**	**0.600000**	**0.144503**	**0.833625**	**0.833625**	**0.065031**	**0.615000**	**-428.600561**
Subsidizing human capital uniformly: ζ	0.136798	0.048998	-0.017839	-0.013544	0.004402	-0.004164	0.014333	-0.000307	0.009206	0.000624	-0.001602	0.006013	0.004336	-0.000338
Subsidizing human capital discretionarily: *D*	0.023179	0.590883	-0.089265	-0.076695	0.149093	-0.147212	0.525965	-0.014055	0.365916	0.028691	-0.070156	0.235790	0.145704	-0.005319

Note: Variables *(c/h)** and *(y/h)** represent after-tax effective consumption and output, respectively; see also Table 2.

Table 3B Decomposition of Changes in Welfare

Welfare Decomposition	Ω	(1) Effective Consumption	(2) Human Capital Growth	(3) Leisure of the Employed	(4) Leisure of the Unemployed
Subsidizing human capital uniformly: ζ	-0.000338	-0.004242	0.005172	-0.001715	0.000447
Subsidizing human capital discretionarily: *D*	-0.005319	-0.022034	0.062296	-0.060502	0.014921

Note: See Table 2.

Table 4 Policy Experiments for an Alternative Economy: Percentage Changes in Key Variables

	a	*g*	*(c/h)**	*(y/h)**	*n*	*l*	*e*	*1-l-e*	*s*	*μ*	*η*	*v*	Ω
Equilibrium (B=0.8336*1.05)	**NA**	**0.005653**	**0.069805**	**0.094155**	**0.596401**	**0.276961**	**0.123969**	**0.599070**	**0.163437**	**0.891227**	**0.835662**	**0.070349**	**-428.514429**
Subsidizing human capital uniformly: ζ	0.139517	0.035909	-0.017618	-0.013312	0.002657	-0.002448	0.006726	-0.000260	0.006170	0.000436	-0.001120	0.003780	-0.000299
Subsidizing human capital discretionarily: *D*	0.019032	0.335155	-0.069828	-0.059896	0.092257	-0.099870	0.276593	-0.011065	0.238852	0.020837	-0.051650	0.151744	-0.004268

Note: Numbers reported in rows 3-4 are percentage changes of key variables from their equilibrium values with B=0.8336*1.05 (presented in row 2) due to each educational subsidy.

Table 5 Sensitivity Analysis – Policy Analysis

	Percentage Change in g in Response to		Percentage Change in Ω in Response to	
Human Capital Enhancement Policy	ζ	D	ζ	D
Benchmark	**0.048998**	**0.590883**	**-0.000338**	**-0.005319**
k/h=0.50	0.097723	0.262668	-0.000616	-0.006691
k/h=2.00	0.024535	0.365888	-0.000196	-0.003743
n=0.50	0.074035	0.211570	-0.000480	-0.004310
n=0.60	0.041147	0.469093	-0.000294	-0.004463
ℓ=0.15	0.036656	0.183539	-0.000291	-0.002204
ℓ=0.45	0.056972	0.227334	-0.000103	-0.001208
e=0.08	0.042600	0.563998	-0.000284	-0.004793
e=0.12	0.080176	0.185913	-0.000562	-0.004473
v/u=0.85	0.048998	0.587221	-0.000338	-0.005314
v/u=1.15	0.048998	0.593631	-0.000338	-0.005323
Φ/n=0.02	0.046634	0.515189	-0.000323	-0.005025
Φ/n=0.03	0.052178	0.546091	-0.000358	-0.005424
LSE=0.5	0.035346	0.184991	-0.000262	-0.000274
LSE=1.7	0.041890	0.525352	-0.000122	-0.000185

Appendix Sensitivity Analysis – Equilibrium Outcome

	g	*c/h*	*k/h*	*y/h*	*n*	*l*	*e*	*s*	*μ*	*η*	*v*	*u*
Benchmark	**0.004500**	**0.071458**	**1.000000**	**0.095958**	**0.549969**	**0.300000**	**0.100000**	**0.144503**	**0.833625**	**0.833625**	**0.065031**	**0.065031**
k/h=0.50	0.000000	-0.500000	-0.5000000	-0.500000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
k/h=2.00	0.000000	1.000000	1.00000	1.00000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
n=0.50	0.000000	0.000000	0.000000	0.000000	-0.091057	0.000000	0.000000	-0.182074	0.000000	0.000000	-0.091057	-0.091057
n=0.60	0.000000	0.000000	0.000000	0.000000	0.091057	0.000000	0.000000	0.227670	0.000000	0.000000	0.091057	0.091057
ℓ=0.15	0.000000	0.000000	0.000000	0.000000	0.000000	-0.500000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
ℓ=0.45	0.000000	0.000000	0.000000	0.000000	0.000000	0.500000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
e=0.08	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.200000	0.000000	0.000000	0.000000	0.000000	0.000000
e=0.12	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.200000	0.000000	0.000000	0.000000	0.000000	0.000000
v/u=0.85	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.176471	-0.150000	0.000000
v/u=1.15	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.130435	0.150000	0.000000
Φ/n=0.02	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Φ/n=0.03	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
LSE=1.05	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
LSE=1.15	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Note: Numbers reported in rows 3-7 are percentage changes of key variables from their benchmark values (presented in row 2) due to each exogenous change.