Undertable Negotiation

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Abstract

We study a multilateral bargaining game with imperfect information. Players take turns to make offers. Each responder observes only the offer to him, and his response to the offer is also made privately. Agreement is reached if all offers are accepted. Otherwise, the game proceeds to next period, in which another player is called upon to make offers. Previous offers and responses are never revealed, thus they never become common knowledge among players. With a plausible restriction on players' beliefs, we obtain a unique public perfect equilibrium. The uniqueness result remains valid even if we take private strategies into consideration. The bargaining procedure is robust to collusions among any subgroup of players.

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1 Introduction

Whenever a negotiation involves three or more parties, collusion becomes a concern of each party. When a firm is negotiating with several labor unions, each union may be worried about the possibility that the firm will make a secret deal with other unions. While trading a professional athlete, each party, the athlete or the club, may have the concern about being taken advantage by the collusion of other parties. In order to prevent collusion, one could insist on the unanimity rule. If any agreement has to be unanimously approved, every party could effectively protect itself from collusion. By the same token, from policy making perspective, one could argue that every move concerning the negotiation should be made publicly known, and any private deal should be prohibited.

The model that immediately comes to mind is the famous alternating-offer bargaining game proposed by Rubinstein (1982). Unfortunately, it is well-known that the natural extension of the Rubinstein game to multilateral case induces multiplicity. Herrero (1985) shows that in an $N-$player ($N \geq 3$) Rubinstein game, any feasible outcome can be supported by a subgame perfect equilibrium (SPE) when players are sufficiently patient.[1] She also shows that there is a unique SPE in which players' strategies are stationary, and it is also the unique strong equilibrium in the sense of Aumann (1959). In other words, all but one equilibrium are vulnerable to collusion among a subgroup of players. However, it is not clear how one can justify a restriction to either stationary strategies or strong equilibrium.

Thereafter, alternative bargaining procedures have been explored such that, for any number of players, the game possesses a unique SPE (see Jun (1987), Chae and Yang (1988, 1994), Krishna and Serrano (1996), Huang (2002)). Not as in the extended Rubinstein game, unanimity rule is not required in these bargaining procedures. Uniqueness is obtained by introducing partial agreement, or, exit option. That is, at any stage of the bargaining, a

[1] The result is also credited to Shaked. Haller (1986) obtains the same result in a variant of the Rubinstein game. See also Sutton (1986), Osborne and Rubinstein (1990) for elegant elaboration of the result.

player may choose to leave the bargaining table with the share proposed to him. Without unanimity rule, the proposing player can allocate any share of the pie to one of his rivals without the consent of others. Therefore these bargaining procedures are vulnerable to the collusion between any two players. Specifically, two players may "gang up" and take away the whole pie when one of them is in the role of proposer.

An ideal multilateral bargaining procedure shall not provide players incentive and opportunity to collude. On the other hand, it shall possess certain predictive power, if possible, a unique equilibrium outcome. The equilibrium itself shall also be collusion-proof, that is, no subgroup of players can improve their total payoff by coordinating their strategies.[2]

With this as the main motive, we study a multilateral bargaining model with imperfect information, namely, undertable negotiation. In every period of the bargaining, a proposer makes offers to the responders in sealed envelopes, that is, each responder observes only the share proposed for him. The responses to offers are also made privately. The game ends if all responders accept the offers. Otherwise, the game proceeds to next period, in which another player is called upon to make offers. Previous offers and responses are never revealed.

Our bargaining game shares features with repeated games under imperfect monitoring. Following the standard approach in that literature, we focus on the perfect public equilibrium (PPE) of the game. Firstly, we show that any feasible outcome can be supported by a PPE. Yet the multiplicity is caused solely by the "bootstrap" behavior. With a plausible restriction on players' beliefs, bootstrap problem can be avoided, and only one equilibrium remains. The remaining equilibrium is equivalent to the unique stationary SPE of the Rubinstein game, hence it is also collusion-proof. Note that stationarity emerges as an equilibrium behavior in our model. The uniqueness result remains valid even if we take private strategies into consideration.

The intuition is rather straightforward. As shown by Herrero (1985), a $N-$player Rubin-

[2] Note that the collusion-proofness of equilibrium that we articulate here is a stronger requirement than that of Aumann's strong equilibrium.

stein game has multiple equilibria, and only one of them is stationary. In every nonstationary equilibrium, after one player deviates, players switch to a specific continuation equilibrium in which the deviator is punished by being assigned an extremely small share. This requires that players can coordinate their strategies in every subgame, and the coordination is based on that all details of the bargaining history, i.e., offers and responses, are publicly known. In our model, such coordination is simply infeasible since details of the bargaining history never become common knowledge among all players.

Closely related work has been done by Baliga and Serrano (1995, 2001). In their model, the offers are made privately to each responder, but the responses are public and sequential. Moreover, the offers are revealed after every response stage. They consider perfect Bayesian equilibria (PBE), and to avoid bootstrap problem, they make an assumption of optimistic beliefs. It is shown that when players are sufficiently patient, almost all feasible outcomes can be supported in an equilibrium.[3] In the same direction, we go one step further by not revealing details of the bargaining history at any time, and we obtain a uniqueness result.

The rest of the paper is organized as follows. Section 2 presents the multilateral bargaining game with imperfect information. Section 3 characterizes equilibrium of the bargaining game. Section 4 contains a discussion about collusion in multilateral bargaining. Section 5 relates the imperfect information structure to restrictions on strategies in $N-$player Rubinstein game. Section 6 concludes.

2 The Model

A set of players $\mathbb{N} = \{1, 2, ..., N\}$ are bargaining over a pie of size 1. Time is discrete, and players take turns to make proposals. Specifically, in period t $(= 0, 1, 2, ...)$, the proposal is made by player $j(t)$, where $j(t)$ is determined by $j(t) = t + 1 \mod N$. A proposal is a vector $x \in \mathbb{R}^N_+$, and $\sum_{i \in \mathbb{N}} x_i = 1$. The offer to each player i, x_i is enclosed in a sealed envelope.

[3] Interestingly, one can obtain the same set of equilibrium outcomes in a N-player Rubinstaein game with a monotonicity restriction on the strategies. See Section 5 for details.

In other words, player i observes only x_i, the share proposed for him.[4] Responses to offers are made simultaneously and privately. The game ends with an agreement if all offers are accepted. Otherwise, the game proceeds to next period, in which another player is called upon to make a proposal. The bargaining goes on until an agreement is reached.

Players are impatient. For simplicity, we assume that all players have linear utility function and a common discount factor $\delta \in (0,1)$. Denote as (x,t) the outcome that agreement x is reached in period t. Each player i's payoff from (x,t) is $\delta^t x_i$. The outcome can also be a perpetual disagreement, denoted as (D,∞), in which case all receive a payoff of 0.

A key feature of the model is that the offers made in each period and the corresponding responses are never revealed to all players, therefore, the details of the bargaining history never become common knowledge among players. It is easy to see that the sealed offers cannot become commonly known if not being revealed. The responses cannot be commonly known either. To see this, consider a three-player example, when player 2 is called upon to make offers, if he did reject the offer in previous period, he would not know player 3's response; if player 2 did not reject, he would know that player 3 must have rejected, but in that case, player 3 would not know player 2's response.

The bargaining game shares features with repeated games with imperfect monitoring. In particular, it does not have any proper subgame. Following the standard approach in the repeated game literature (see, for example, Abreu, Pearce, and Stacchetti (1990) and Fudenberg, Levine, and Maskin (1994)), we focus on the perfect public equilibrium of the game. A player is using a public strategy if in each period t, his behavior is contingent only on the public history and not on his private history. A PPE is a profile of public strategies such that in any period t and given any public history, the profile of sub-strategies form a Nash equilibrium. Note that the definition of PPE does not preclude a player from deviating to private strategies, but requires that no player can benefit from such deviations.

[4]Note that we assume players cannot make infeasible proposals. Precisely, if player j is making a proposal x, then it must be that $\sum_{i\neq j} x_i \leq 1$, and if the offers are accepted, the proposer receives the remaining share.

3 Equilibrium Characterization

In this section, we characterize all PPEs of the bargaining game. We first present a negative result, i.e., any feasible outcome can be supported in a PPE. The multiplicity is caused by bootstrap behavior. To deal with the problem, we introduce a plausible restriction on players' beliefs, by which we obtain uniqueness, the main result of the paper.

3.1 Multiplicity

In our bargaining game, the only public history is that an agreement has not yet been reached. Thus a public strategy can only depend on t. This feature significantly simplifies the characterization of the PPE. As for player's beliefs, we start with the standard restriction of weak consistency, that is, on the equilibrium path, beliefs should be derived using Bayes' rule. The following lemma says that perpetual disagreement is an equilibrium outcome.

Lemma 1 *The outcome* (D, ∞) *can be supported in a PPE.*

P roof. Consider the following profile of public strategies.[5] Each player, when in the role of a proposer, always offers zero to all responders, and when in the role of a responder, accepts only one.

When a responder is offered zero, it is weakly dominant to reject irrespective of his belief. When a responder receives a positive offer, he believes that other responders will reject their offers with probability one, hence he is indifferent between accepting and rejecting. Since no proposal can be accepted unanimously, it is optimal for the proposer to offer zero to all responders. ■

With Lemma 1 being established, one can see immediately that any efficient outcome can be supported in a PPE.

Lemma 2 *Any efficient outcome* $(x, 0)$ *can be supported in a PPE.*

[5] The equilibrium strategy in Lemma 1 is indeed history-independent.

P roof. Any efficient outcome $(x, 0)$ can be supported by the following equilibrium strategy profile and the corresponding system of beliefs:

In period 0, player 1 makes the proposal x. Each responder i accepts only x_i. After receiving the equilibrium share x_i, player i believes that every other responder has also received the equilibrium share and will accept the offer. If player i receives an offer other than x_i, he believes that at least one other responder did not receive the equilibrium share, and will reject the offer. Finally, if an agreement is not reached in period 0, then all players switch to the strategy profile specified in the proof of Lemma 1.[6] ■

Finally we conclude that any feasible outcome, efficient or inefficient, can be supported in a PPE. Players follow the strategies specified in the proof of Lemma 1 for the first $t-1$ periods, and starting from period t, they switch to the strategies specified in the proof of Lemma 2.

Proposition 1 *Any feasible outcome (x, t) can be supported in a PPE.*

3.2 Uniqueness

It is the so-called bootstrap behavior that causes the multiplicity. Intuitively, a responder may reject an offer because he believes that someone else would reject anyway. However, in such cases, rejection is often a weakly dominated action. For example, in the PPE with perpetual disagreement, a player would reject an offer greater than δ, which seems completely unreasonable.

Let us consider a weak restriction on players' beliefs. We assume that when a player decides whether to accept or reject an offer, he always holds *non-degenerate belief*, that is, he believes that every other responder will accept (or reject) his offer with positive probability. With non-degenerate belief, a player rejects an offer is only because he is looking forward to a strictly higher equilibrium payoff in the continuation game. Hence the bootstrap behavior

[6] Note that the equilibrium strategy is history-dependent and public.

is avoided. A similar restriction on beliefs is made in Baliga and Serrano (1995), and they give a psychological interpretation, that is, each responder optimistically believes that he is a key player in the negotiation.

The non-degenerate belief restriction has a flavor of trembling-hand perfect refinement of Nash equilibria. When a player receives an offer higher than the continuation payoff, it is weakly dominated for him to reject the offer, and to reject is optimal only when there is another responder who will reject his offer with probability 1. Non-degenerate belief can be interpreted as a belief that someone else may "tremble" while responding to his offer. Then dominated action such as rejecting a generous offer can be ruled out.

Under this restriction, perpetual disagreement cannot be supported as an equilibrium outcome by the strategies specified above. If a responder anticipates perpetual disagreement and holds nondegenerate belief, he would accept any positive offer.

Let $\eta = 1 + \delta + \delta^2 + ... + \delta^{N-1}$, and $x^* = \eta^{-1}\left(1, \delta, \delta^2, ..., \delta^{N-1}\right)$. Clearly, x^* is a feasible agreement. It is also the unique stationary SPE outcome of the $N-$player Rubinstein game. Our main result in this section is that $(x^*, 0)$ is the unique PPE outcome that satisfies the nondegenerate belief restriction. Herrero (1985) obtains uniqueness by requiring stationarity. We do not restrict players to use stationary strategies, yet stationarity emerges as equilibrium behavior.

Proposition 2 *There is a unique PPE that satisfies the non-degenerate belief restriction. In the equilibrium, agreement x^* is reached in period* 0.

P roof. Firstly, we claim that (D, ∞) can not be supported in a PPE with nondegenerate beliefs. To see this, consider that when a responder receives a positive offer, if he anticipates perpetual disagreement and holds nondegenerate belief, he shall accept the offer which gives him a positive expected payoff.

Secondly, also due to the non-degenerate belief, in a PPE, a responder's behavior should always exhibit a monotonicity property. Precisely, in any period t, there is a threshold value

z_i^t for each responder i, an offer x_i^t to i will be accepted if and only if $x_i^t \geq z_i^t$. Suppose that player i's continuation payoff is V_i^{t+1}, then the threshold value $z_i^t = \delta V_i^{t+1}$.

Claim 1 *If* (x^1, t) *is a PPE outcome, then* $(x^1, 0)$ *must be a PPE outcome.*

This is trivially true. Let E be the PPE that supports (x^1, t). To construct a new equilibrium $\widetilde{E}$ supporting $(x^1, 0)$, one can simply "truncate" in period t the strategies used in E, and also rotate players such that in $\widetilde{E}$, player 1 uses the truncated strategy used by player $j(t)$, whose proposal is accepted in E.

Claim 2 *If* $(x^1, 0)$ *as a PPE outcome, then* $(x^2, 0)$ *must be a PPE outcome, where* $x_i^2 = x_{i+1}^1/\delta$ *for* $i = 1, ..., N-1$, *and* $x_N^2 = 1 - \sum_{i=1}^{N-1} x_i^2$.

This is true because: (i) acceptance rule is always monotone, and (ii) players use public strategies, therefore they anticipate the same continuation equilibrium.

Following this logic, we can determine a sequence of PPE outcomes, $\left(x^k, 0\right)$ with $x_i^k = x_{i+1}^{k-1}/\delta$ for $i = 1, ..., N-1$, and $x_N^k = 1 - \sum_{i=1}^{N-1} x_i^k$. It is then easy to establish the following difference equations:

$$x_i^{k+N} = \left(\frac{1}{\delta^{N-i}} - \frac{1}{\delta^{N-i+1}}\right) + \frac{1}{\delta^N} x_i^k$$

for $i = 1, ..., N$. Its general solution is

$$x_i^{Nk+1} = \frac{1}{\delta^{Nk}} \left(x_i^1 - \delta^{i-1}\eta^{-1}\right) + \delta^{i-1}\eta^{-1},$$

where $\eta = 1 + \delta + \delta^2 + ... + \delta^{N-1}$.

Then it must be that

$$x_i^k = \delta^{i-1}\eta^{-1} = x_i^*$$

for all i and k. Otherwise, for sufficiently large k, $x_i^k > 1$ for some i, which is infeasible.

Finally, since x^* is the only possible equilibrium agreement, it must be reached at $t = 0$. Thus $(x^*, 0)$ is the unique PPE that satisfies the non-degenerate belief restriction. ■

3.3 Private Strategies

Some recent work on repeated games with imperfect monitoring has taken private strategies into consideration. Kandori and Obara (2006) show that in a repeated prisoners' dilemma game with imperfect monitoring, there may exist private equilibrium outcomes that cannot be supported by public strategies.

Would the uniqueness result remain valid if we take private strategies into consideration? Our conjectured answer is positive. More precisely, we conjecture that there does not exist any private equilibrium. The intuition goes as follows. In a bargaining game, players' interests are in complete conflict. If one player has to choose, based on his private history, between two different continuation plays, he would always choose the one in favor of himself. Also, in repeated games, private strategies might make a difference only when mixed strategies are being considered. We cannot come up with any reason that a player would ever randomize in a bargaining game.

Proposition 3 *(Conjecture) The bargaining game does not have any private equilibrium that satisfies the non-degenerate belief restriction.*

A formal proof has yet to be completed.

4 Collusion

Although the main motive of this paper is to find a collusion-proof bargaining procedure, we did not model collusion explicitly. As an example, let us consider a three-player bargaining game. To incorporate collusion into our model, we can assume that players are randomly drawn from a large population, in which a small fraction of individuals are colluded. Now, if a player, who is not colluded with any of his two rivals, will believe that with a certain probability p, the two rivals are colluded, that is, they will coordinate their strategies to maximize their total payoff, and with probability $1 - p$, they are not colluded, in which case they share the same belief as his.

Now we have a bargaining game with incomplete information, which, in general, is difficult to solve. However, we argue that it is not necessary to do so given the uniqueness result in Section 3. When there is no collusion at all, we obtain a unique PPE outcome, and it is equivalent to the unique SPE outcome when collusion is common knowledge among players. Hence regardless the possibility that two rivals are colluded, a player would behave as if there is no collusion at all.

Without loss of generality, we assume that it is common knowledge player 1 and 2 are colluded. Clearly, when the collusion between player 1 and 2 is commonly known, the three-player bargaining game degenerates into a two-player bargaining game, which has a unique SPE.

Proposition 4 *When it is common knowledge that player 1 and 2 are colluded, the three-player bargaining game has a unique SPE with immediate agreement, by which player 3 receives* $\delta^2/\left(1+\delta+\delta^2\right)$.

The proof is straightforward. When collusion is common knowledge, the three-player bargaining game becomes a bargaining between two parties, and the non-colluding player will be the proposer once every three periods. Hence his equilibrium share is close to 1/3 of the pie.

In general, when the bargaining involves N players, and $M < N$ players are colluded. If the collusion is commonly known, the original bargaining game degenerates into a game among $N - M + 1$ players. It has the equilibrium outcome equivalent to that under the situation where collusion does not exist. Non-colluding players would simply ignore the possibility of collusion.

5 Imperfect Information versus Restriction on Strategies

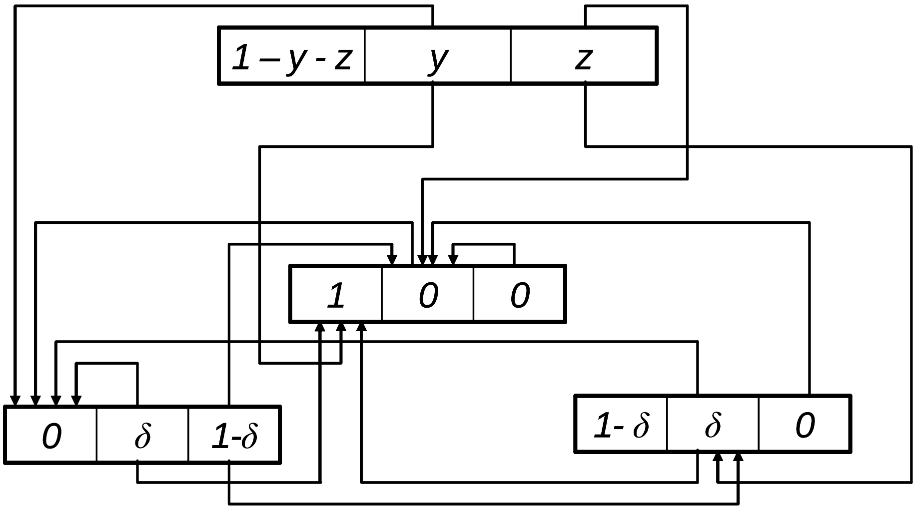
Herrero (1985) obtains uniqueness by a restriction to stationary strategies. Following this approach, one may explore weaker and more plausible restrictions on strategies. Baliga and

Serrano (1995, 2001) and this paper take a different approach, that is, we put restrictions on the information structure of the bargaining game. The two approaches are closely related, yet the latter seems to be more appealing. In this section, we establish the equivalence between the two imperfect information models and the $N-$player Rubinstein game under different restrictions on strategies.

Monotonicity is a plausible restriction on strategies in bargaining. A player's strategy is monotone if, whenever in the role of responder, he specifies a threshold value, and accepts a proposal if and only if it gives him a share not lower than the threshold value. Note that the non-stationary equilibria that Herrero (1985) constructed do not satisfy monotonicity. Under monotonicity restriction, in an equilibrium with immediate agreement, player 2 receives a share at most δ, and player 3 receives at most δ^2, and so on. This immediately rules out the possibility that a responder is offered the whole pie in an equilibrium. It seems that monotonicity has some bite on the multiplicity issue. However, as stated in the following proposition, monotonicity restriction on strategies is equivalent to the information restriction of sealed offers in Baliga and Serrano (1995). The multiplicity does not go away, and when players are extremely patient, almost all feasible outcomes can be supported by a SPE with monotone strategies.

Proposition 5 *Every PBE outcome of the Baliga-Serrano bargaining game can be supported as a SPE outcome of the $N-$player Rubinstein game with a profile of monotone strategies under the same condition on δ.*

The proof of Theorem 2.2 in Baliga and Serrano (1995) can be easily adapted to prove the above proposition. We provide a graphical illustration of the monotone SPEs in a three-player Rubinstein game. Every agreement $(1-y-z, y, z)$ with $y \leq \delta$ and $z \leq \delta^2$ can be supported in a SPE with monotone strategies. The smallest offers that the two responders will accept are y and z respectively. In Figure 1, the connectors point to different continuation equilibrium agreements conditional on whoever deviates. Specifically, if the proposer offers



$y \leq \delta$ and $z \leq \delta^2$ $\qquad \delta \geq \frac{\sqrt{5}-1}{2}$

Figure 1: Subgame Perfect Equilibria with Monotone Strategies

a responder less than the threshold value, the connector below the box then points to the continuation equilibrium share of that responder, who will be rewarded the largest possible share; if a responder rejects an offer higher than the threshold value, the connector above the box points to the continuation equilibrium share of that responder, and in this case he will be punished.

Our undertable bargaining game puts more restrictions on information structure than Baliga and Serrano (1995), thus the equivalent restriction on strategies should be stronger than monotonicity.

Proposition 6 *The unique PPE outcome of the undertable bargaining game is also the unique SPE outcome of the $N-$player Rubinstein game with equilibrium strategies that depend only on t.*

This proposition is self-evident. When the strategies depend only on t, both responders anticipate the same continuation equilibrium, and their acceptance rules must be monotone.

Hence a recursive structure as that in the proof of Proposition 2 can be established. Clearly the t-dependent restriction is weaker than stationarity. One can obtain uniqueness under even weaker restriction on strategies. Indeed it would be sufficient to require that all players' actions in period t does not depend on what happens in period $t-1$. However, we cannot provide an intuitive interpretation of this restriction and its equivalent information structure.

6 Conclusion

In real life situation, negotiating parties may first have to agree on how the negotiation should be conducted. When there are concerns about collusion, it is unlikely that they can agree on a mechanism allowing partial agreements, which happens to be a common feature of the models in previous literature. In order to achieve both uniqueness and collusion-proofness, we propose that multilateral negotiation be conducted undertable. This may appear to be counter-intuitive because people usually relates undertable dealing to collusion. Our argument is that as long as a party is endowed with a veto power, he is effectively protected against collusion among others. Of course, this argument is not necessarily valid if there is incomplete information about the fundamentals of the bargaining situation such as patience, reservation values, and outside options etc.

We obtain a unique equilibrium by keeping details of the bargaining history as private information. When the indeterminacy goes away, so does the possibility of inefficient delay. Therefore, our model can also be viewed as an example illustrating the familiar game theoretic point that players may gain from limiting their information.

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