The Interaction Between Unemployment Insurance and Human Capital Policies.*

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February, 2008

Abstract

In a two period representation of life-cycle choices we show that, in the presence of an optimally designed unemployment insurance (UI) program, it is optimal for the government to encourage human capital acquisition. This raises the opportunity costs for those who intend to free ride on the program. We also show that, at the constrained optimum, human capital investments should be driven up to the point where its expected return equals the risk free rate, even though human capital is 'risky' from a private perspective. *J.E.L. codes: J65, I28.*

1 Introduction

Economists have long recognized the connection between unemployment episodes and an individual's human capital. On the one hand, unemployment episodes are associated with lower returns to human capital investments, since human capital is of limited use when one is jobless.[1] On the other hand, there is substantial evidence that the more educated a person is for the less extent she is unemployed — see, for example, Nickell and Bell (1997).

It is apparent from the previous considerations that optimal unemployment insurance — henceforth, UI — and educational policies may have important interactions. It is not so clear, however, how these forces play out to determine optimal policies. On the one hand, human capital typically pays in the states of nature where the agent is working, i.e., where the marginal utility of consumption is lower. Human capital is, therefore, risky, and we should expect there to be under-investment and shorter unemployment episodes,[2] which drives private investments in the other direction. General statements based on under or over-investment are, therefore, bound to be a poor guide for policy.

This paper analyzes the interaction between educational and UI policies, with particular emphasis on the questions of whether the government should influence private choices of human capital. To the best of our knowledge, Brown and Kaufold (1988) were the first to build a theoretical framework to explore the relationship between human capital formation and UI programs. They showed that the presence of a UI program may lead to increased investment in human capital by its reducing human capital risk. They also explore various channels through which human capital choices affect the optimal design of such program.

We revisit the work of Brown and Kaufold (1988) within the framework of the new dynamic public finance literature, where government optimal policies are derived in a dynamic agency framework. We adopt the simple two period structure of their model, as well as their consideration of non-market activities.[3] By investigating the problem in an agency framework, however, we are able to explicitly consider the labor/leisure trade-offs that are not included in their paper and to emphasize the incentive effects of unemployment insurance in determining the fraction of an agent's productive life in which she is unemployed. At the same time, we are able to derive robust results under weaker assumptions on agents' preferences. The main shortcoming of our approach is that tax systems implicitly derived are more complex and (in some cases) informationally demanding than those considered in Brown and Kaufold (1988).

Interestingly enough, the model generates a complementarity between education and labor market attitude that endogenously produces the negative correlation between education and unemployment which empirical evidence we have mentioned before. The consequence is that the encouragement of human capital formation becomes an important ally for the UI program. Governments that provide insurance networks may often face individuals who claim that they cannot find or keep their jobs when in fact they are not spending enough job-retention and/or job-search effort. It is, therefore, possible for the government to alleviate this problem by raising the opportunity cost of the unemployment spell through higher provision of education.

Notice that the risky or otherwise nature of human capital investment has no baring here. Human capital acquisition is to be encouraged not because agents under or over-invest in it,[4] but exactly because the complementarity of search effort and human capital implies that more education signals a 'good' labor market attitude and helps separating 'unlucky' agents from those who just do not put enough effort in participating in formal markets.

Another interesting finding is that, at the optimum, the expected return of human capital investment is equal to the risk free rate. This is a little surprising since, human capital is risky. That is, because the UI program must take incentives into account, consumption is higher when an agent is employed than when unemployed. Since human capital investment only pays in the first case, the private optimal choice implies a risk adjustment for human capital investment. Yet, government intervention drives human capital investment up to the point where its expected return is equal to the risk free rate of return.

As we have mentioned, we allow here for the existence of informal markets. Agents in our model may participate in the informal sector, both when young, when they are not engaged in human capital formation and, later in life, whenever unemployed, to raise their consumption. In the informal markets, agents face the same labor/leisure choices which they face when participating in the formal markets with the proviso that human capital is of no use and that agents are sheltered from taxes in the former case. Although with different sizes and relative importance, informal sectors are found in all economies in the world, being particularly relevant in the description of underdeveloped and developing countries, which is why we included them in our model. We should remark, however, that all our results are robust to the existence of an informal sector, *not* dependent on it.

Finally, the role of savings is carefully discussed in our paper. Savings represent a very important form of self insurance, and whether they are observed or not will determine to a great extent what the government may accomplish in our setting. We first assume that the government can fully control savings. In this case we recover the inverse Euler equation result of Rogerson, which requires the government to be able to force agents to save less than they would privately wish to do. However, when savings cannot be controlled by the government, simple taxation of capital return is not able to implement the optimal allocations. Non-observable savings lead to changes in the prescriptions regarding the design of labor income taxes used to finance the optimal unemployment insurance program. Nonetheless, the prescriptions for human capital policies are robust to non-observability of savings.

savings are controlled by the government. In section 5 we discuss the role of non-observed savings that arises when the government tries to implement the second-best inter-temporal transfers. This possibility is accompanied by some technical issues which are handled through a series of results that we present in the appendix. Section 6 concludes.

2 The Economy

Ours is a two period economy with an atom of identical agents. The two period assumption — also adopted by Brown and Kaufold (1988) and Bailey (1978), to name a few[5] — is mainly due to our emphasis in the interaction between education and unemployment. Educational choices are usually long term choices, as compared to the length of unemployment spells, and mostly done early in life. They affect later choices regarding work since they change relative payoff of employment *vis à vis* unemployment, as emphasized by Brown and Kaufold (1988).

As for considering identical agents, we assume heterogeneity away not for sake of realism but rather for simplicity. Bailey (1978) has pointed out that non-observed heterogeneity and the possibility of self-selection issues may be the very reason for the non-existence of private unemployment insurance, in the first place. Moreover, the fact that agents are identical means that we disregard the possible interactions between redistributive and insurance motives in government's policy design. Yet, as in Bailey (1978), we remove heterogeneity to focus on the incentive effects associated with the unemployment insurance program.

Preferences Agents are expected utility maximizers with temporary utility given by $u(c)-\zeta\left(\bar{L}-l\right)$, where c is consumption, l is leisure and $\bar{L}$ is the agent's time endowment. We assume that both functions are smooth with u', ζ', ζ'', $-u''>0$ all satisfying the usual Inada conditions.

There is another dimension of effort, not included in the description of temporary utility which is related to the struggle to remain in the formal markets. The fraction of time of an agent's adult life that she spends unemployed is a function of both the (per period) probability of her losing a job when employed and the probability that she gets a new job when unemployed. We capture both transition probabilities with a single variable $p \in [0, 1-\varepsilon]$ which we associate with the agent's attitude toward work: be it her willingness to conform to different rules or environments, encompassed in the general label of job retention effort, be it her search effort whenever unemployed, a utility cost which we represent with the increasing, convex, continuously differentiable function $\varphi(\cdot)$.[6] In section 4.1 we consider the case where, instead of a utility loss, the cost of remaining in the formal markets is earnings loss, which we interpret as a reduced form of a search model.

With all this in mind, we write an agents's life-time utility as

$$u(c) - \zeta\left(\bar{L} - l\right) + \mathbb{E}\left[u(c) - \zeta\left(\bar{L} - l\right)\right] - \varphi(p), \tag{1}$$

where the expectation operator in (1) is with respect to the probability, p

Technology The economy has two sectors: a formal sector and an informal sector. Each sector produces the single consumption good with a linear technology that transforms one efficiency unit of labor into one unit of output. We abuse notation slightly by using Y to represent both, and normalize units in such a way that, in the informal sector, one hour of time produces one efficiency unit of labor. The crucial assumption is, thus, that in the informal sector the rate at which time is transformed in efficiency units, Y, is independent of how educated the agent is, Y, and normalized to one.

As for the formal sector we have $Y = w(h)L$, where $w(h)$ is an agent's productivity in the formal sector. Productivity in the formal sector depends on an agent's human capital, h. We assume $w(0) = 1$ and $w' > 0, w'' < 0$.

It is natural to think that the equilibrium productivity of labor is higher in the formal sector. Absent this, a formal sector would not exist in equilibrium. Our assumptions imply that the productivity is higher for all levels of human capital. Underlying it is the idea that the technology that is available in the informal sector can be adopted by the formal sector, but not necessarily the other way around. The other two assumption are directly related to the role of education on agents' labor market choices; the first guarantees that an agent's productivity is increasing in her human capital, while the second guarantees that human capital increases productivity at decreasing rates.

The informal sector adds an important, often neglected dimension to labor market description. This is particularly true for under-developed economies, but the point is more general.

To acquire human capital an agent must dedicate some of her time at youth to studying, therefore sacrificing her leisure and/or her first period income. This means that, in the first period, and absent government intervention, $c = Y = \bar{L} - l - h$, where l is leisure and $\bar{L}$ is total time endowment.

Second, we have ruled out the participation in the formal sector in the first period: we are concerned with unemployment at an agent's prime age.

Because we have ruled out aggregate risk, p will be associated not only with the fraction of the agent's adult life that she is employed but also with the unemployment rate of the economy. In fact, the steady state ratio unemployment/employment is equal to the ratio of the probability of transition from employment to unemployment to the probability of transition from unemployment to employment which are both captured in our model with the single parameter p.[7]

3 First Best and Autarchy Allocations

In this section we evaluate the first best allocations and the equilibrium allocations in a world where the only form of reducing risk is self-insurance through savings, which we shall refer to as autarchy.

Autarchy Consider the case in which there is no unemployment insurance. Letting s denote savings, we may write the agent's problem as

$$\max_{p,Y,h,s} \left\{ u(Y-s) - \zeta\left(h+Y\right) + pV^{e}(h,s) + (1-p)V^{u}\left(s\right) - \varphi\left(p\right) \right\},$$

where

$$V^{u}\left(s\right) = \max_{Y^{u}} \left\{ u(Y^{u}+s) - \zeta\left(Y^{u}\right) \right\}, \tag{2}$$

and

$$V^{e}(h,s) = \max_{Y^{e}} \left\{ u(Y^{e}+s) - \zeta\left(\frac{Y^{e}}{w\left(h\right)}\right) \right\}, \tag{3}$$

where we use Y to denote output in the first period, Y^{e} to denote output in the second period while employed in the formal markets, and Y^{u} to denote output in the second period when working in the informal markets.

Notice that the agent's problem need not be convex, due to the interplay between p, s and h. Nonetheless, provided that the solution is interior, the following first order conditions are necessary:[8]

$$u'\left(Y+s\right) = p\partial_{s}V^{e}(h,s) + \left(1-p\right)\partial_{s}V^{u}\left(s\right), \tag{4}$$

$$u'(Y+s) = \zeta'(h+Y), \tag{6}$$

and

$$\zeta'(h+Y) = p\partial_h V^e(h,s). \tag{7}$$

Finally, applying the envelope theorem to (3) we may rewrite (7) as

$$\zeta'(h+Y) = p\partial_h V^e(h,s) = p\zeta'\left(\frac{Y^e}{w(h)}\right)\frac{Y^e}{w(h)^2}w'(h). \tag{8}$$

First Best Let c, c^e and c^u denote, respectively, consumption in the first period, in the second period if employed and in the second period if unemployed and consider the first best allocations, in which the possibility of transfers from employed to unemployed agents is given to a social planner.

$$\max\left\{u(c) - \zeta(h+Y) + p\left[u(c^e) - \zeta\left(\frac{Y^e}{w(h)}\right)\right] + (1-p)\left[u(c^u) - \zeta(Y^u)\right] - \varphi(p)\right\},$$

subject to

$$c + pc^e + (1-p)c^u = Y + pY^e + (1-p)Y^u. \qquad [\ \mu\ ]$$

Once again, one should beware with the fact that the problem need not be convex, thus, first order conditions are only necessary (once again, assuming that the solution is interior). They are,

$$u'(c) = u'(c^e) = u'(c^u) = \mu, \tag{9}$$

$$\zeta'\left(\frac{Y^e}{w(h)}\right)\frac{1}{w(h)} = \zeta'(h+Y) = \mu, \tag{10}$$

$$u(c^e) - \zeta\left(\frac{Y^e}{w(h)}\right) - \left[u(c^u) - \zeta(Y^u)\right] + \lambda\left[Y^e - c^e - Y^u + c^u\right] = \varphi'(p), \tag{11}$$

and

$$\zeta'(h+Y) = p\zeta'\left(\frac{Y^e}{w(h)}\right)\frac{Y^e}{w(h)^2}w'(h). \tag{12}$$

The fact that (8) and (12) are identical implies that, *conditional on identical labor supply choices*, educational choice are identical in the first best and in autarchy. However, because labor supply first order conditions does not allow us to tell whether the government should distort human capital choices absent other policies.

In evaluating the unconstrained optimum we implicitly assumed that the planner controls all agent's choices. When we move on to the constrained efficient allocations we have to be explicit about what is under the government's control and what is not. With regards to savings, we start with the benchmark case in which the planner controls agent's savings because it is more tractable and conveys most of the intuition for our results. This implies an identity mapping from after tax income (or labor income in the informal sector) and consumption.

The assumption that the government controls savings may be motivated by the possibility that savings take the form of contracts or purchases of real assets for which observation is possible. This seems to be the underlying assumption adopted in most of the literature. Also, if credit markets are not well developed, and if one takes into account the fact that the bulk of one's income comes later in life, the restriction that $s \geq 0$ should bind.[10] This being the case, we should not expect savings choice to destroy convexity over the relevant range of the main variables. However, if one considers, as we do, that the government instruments include inter-temporal transfers, the same argument may not — will not, indeed — be valid at the optimum.

The question of whether our results depend on this assumption should, therefore, be addressed at some point. We shall come back to it in section 5, when we introduce hidden savings to the problem and compare the results therein with the ones found in the next section.

4 Optimal Policy

We set up the government's program as a mechanism design problem and derive the optimal allocations leaving the policy instruments in the background. Our model is, with regards to human capital choices, close to Hamilton (1987) where the level of human capital chosen by the individuals is compared to the level which the government chooses when it has the power to do so, or when the instruments necessary to induce such choices. In section 4.1, we briefly discuss the implementation of these allocations.

Back to the mechanism design problem, we assume that the government controls are: *i)* a transfer δ to the first-period of each agent's life; *ii)* the unemployment insurance policy, which takes choices Y^e, y^e while employed; *iv)* the 'labor market attitude', p, and; *v)* the agents human capital, h.

The choice of p is made under the restriction that the agent will only choose the level of p prescribed by the government if she finds in her best interest to do so. I.e., p must satisfy the associated incentive compatibility constraint,

$$p \in \arg\max \left\{ p \left[u(y^e) - \zeta \left(\frac{Y^e}{w(h)} \right) \right] + (1-p) V^u(\omega) - \varphi(p) \right\}. \tag{13}$$

The government cannot observe the amount of work the agent supplies in the hidden economy, both when young, Y, and when adult, Y^u. Nonetheless, because the government controls h and s, the problem of the agent is convex in the remaining choice variables, which means that the first order approach may be applied, and (13) replaced with

$$u(y^e) - \zeta \left(\frac{Y^e}{w(h)} \right) - V^u(\omega) = \varphi'(p). \tag{14}$$

The government, thus, solves the following problem,

$$\max_{h,p,y^e,Y^e,\omega,\delta} \left\{ V(h,\delta) + p \left(u(y^e) - \zeta \left(\frac{Y^e}{w(h)} \right) \right) + (1-p) V^u(\omega) - \varphi(p) \right\}, \tag{15}$$

where

$$V(h,\delta) \equiv \max_{Y} \left\{ u(Y+\delta) - \zeta(h+Y) \right\},$$

and

$$V^u(\omega) \equiv \max_{Y^u} \left\{ u(Y^u+\omega) - \zeta(Y^u) \right\},$$

subject to the resource constraint,

$$p(Y^e - y^e) \geq (1-p)\omega + \delta \qquad [\ \mu\ ]$$

and the incentive constraint (14) to which we associate the Lagrange multiplier λ.

Walras' identity allows us to leave the government's budget constraint in the background; if the resource constraint is met, so is the government budget constraint. The underlying assumption is (the multipliers are as shown above) by h to get the first order condition,

$$(p+\lambda)\,\zeta'\left(\frac{Y^e}{w(h)}\right)\frac{Y^e}{w(h)^2}w'(h)=\zeta'\left(h+Y\right). \tag{16}$$

Assume for now that the solution to the agent's optimization problem is interior (the corner solution $h=0$ is obvious). Then, if the government does not intervene in human capital formation, the agent's optimal choice of education would be characterized by the first order condition

$$p\zeta'\left(\frac{Y^e}{w(h)}\right)\frac{Y^e}{w(h)^2}w'(h)=\zeta'\left(h+Y\right). \tag{17}$$

By comparing (16) with (17) it is apparent that the government should distort the agent's choice thus creating a wedge between private marginal costs and private marginal benefits of education.

Proposition 1. *At the (constrained) optimum, the government induces agents to choose a level of human capital, h, higher than what an agent would choose if the government did not intervene in this choice.*

The proof is by comparing (16) and (17), using the convexity and monotonicity of $\zeta(\cdot)$ and the concavity and monotonicity of $w(\cdot)$.

The optimal policy comprises the government inducing agents to increase human capital investment, as formally stated in the following proposition. By inducing agents to over-accumulate human capital, the government increases the opportunity cost of being unemployed, thus alleviating the moral hazard problem that undermines the risk sharing possibilities of the UI program.

Next, notice that the first order conditions with respect to y^e and h can be manipulated to obtain the following alternative representation of the wedge,

$$\frac{1}{1+\lambda/p}=p\frac{u'\left(y^e\right)}{u'(c)}L^e w'(h). \tag{18}$$

The private marginal benefit of education which appears in the right hand side of (18) displays a state-price deflator that adjusts for risk involved in human capital investment. If a moral hazard problem were not present in this setting we would have full insurance and the optimal level of comparing the expected return of human capital with that of the risk free asset. The reason why such comparison is important is because when the expected returns on the two assets are equal, i.e., $1 = pL^e w'(h)$, production is taking place at the technological frontier in the sense that there is no way for aggregate consumption to be increased in both periods, at current choices of leisure — or, for that matter, for leisure to be increased at the current level of consumption.

This is where the possibility of inter-temporal transfers, as represented by δ, becomes important. Differentiating the government's problem with respect to δ and using the envelope theorem we have $u'(c) = \mu$. Thus, $u'(y^e)/\,u'(c) = (1 + \lambda/p)^{-1}$, which yields the following proposition.

Proposition 2. *Investment in human capital is driven up to the point where its expected return is equal to that of a risk free asset.*

Thus, despite the fact that there is no full insurance — which makes human capital investment risky —, government policies induce agents to increase investment in human capital up to the point where its expected return is equal to the risk free rate.

In passing, it is also worth mentioning the fact that the marginal tax on labor income is zero. This is immediate from the first order conditions with respect to y^e and Y^e, respectively,

$$(p + \lambda)\, u'(y^e) = p\mu, \text{ and, } (p + \lambda)\, \zeta' \left(\frac{Y^e}{w(h)} \right) \frac{1}{w(h)} = p\mu.$$

There is a sense in which the result is to be expected since agents are homogeneous and lump-sum taxes, feasible. However, labor income taxation may still help if labor income is used by the agents in connection with savings to allow for free riding in the unemployment insurance program. This is exactly the case in section 5. As we shall see, no-distortions at the margin ceases to be optimal.

4.1 Discussions and a Caveat

The Role of Informal Markets. The inclusion of an informal market as part of the description of the economy may lead one to wonder how important this is for the results we obtain — and, consequently, how relevant this may be for developed economies. The answer is that all the results remain valid if we remove the informal sector.

subject to the appropriately modified constraints, the exact same expressions, (16) and (18), obtain. Hence, the result is not dependent on the existence of an informal market but rather robust to its existence!

Search. We have modeled the cost of 'having the right attitude' as an additive utility cost, $\varphi(p)$. In many studies in which one concentrates on the transition from unemployment to employment (taking the employment tenure as given) search is modeled as a sequence of unobserved (by the planner) wage offers that the agent may or may not accept. In this case, higher effort means accepting lower wages, which means that, in a reduced form, we should write the agent's life-time utility as

$$u(y) - \zeta\,(Y + h) + p\left(u(c^e) - \zeta\left(\frac{Y^e + \varphi(p)}{w(h)}\right)\right) + (1-p)V^u.$$

where $Y^e = \hat{Y}^e - \varphi(p)$ is observed but not $\hat{Y}^e$ and $\varphi(p)$ in isolation.[11] In this case, $c^e = Y^e - T\left(Y^e\right) = \hat{Y}^e - \varphi(p) - T\left(\hat{Y}^e - \varphi(p)\right)$.

The result regarding human capital formation not only survives this change but is actually strengthened by the fact that education also reduces the marginal cost of search. It is also easy to verify that the inverse Euler equation still characterizes the inter-temporal distribution of consumption.

The only major change is with regards to the zero marginal tax on labor income which is now replaced by a negative marginal income tax.

Our reduced form interpretation should be viewed with some caution. If $\varphi(p)$ were appropriated by the firms, we would have to model explicitly their decision and define how expected profits are determined in equilibrium. To avoid this, we are considering the source of $\varphi(p)$ to be a mismatch between workers and firms that leads to less efficiency units being supplied at the same cost for the agent. This, however, is not completely satisfactory. If longer spells are associated with a better matching, agents' search might produce some positive externalities which we have not considered. In fact, by waiting longer to find a firm with a better matching the agent could reduce the expected cost of search (in terms of shorter unemployment spells) for other agents who are trying to find their preferred match. If such externalities exist they should be taken into account by the planner.

Implementation. So far, we have not said a word about how these optimal policies may be implemented. That is, what do we mean by having the government choose h and s? Even though

We shall not discuss in detail the issue of implementation, not because we do not think that they are interesting but because the type of problems that arise here are well understood, and careful discussions are found in the literature — see Chiappori, Macho, Rey, and Slanié (1994) for the problem of double deviation in a moral hazard context, and da Costa and Maestri (2007) for tax systems that allow the government to effectively control savings and human capital investment. We shall, however, point out to the fact that linear, non-stochastic taxes on both forms of investment need not substitute for compulsory choices. That is, a simple subsidy on h and a simple tax on s may not implement the optimal choices because of the double-deviation problem discussed in Chiappori, Macho, Rey, and Slanié (1994). Some form of non-linear or state-dependent tax may be necessary, in this case.

Inter-temporal transfers. An important caveat associated with our main result concerns the role of inter-temporal transfers. When arguing that the assumption that the government controls savings is not a very restrictive one, we used the fact that credit markets may not be generous enough to allow for negative savings when such long horizons are considered. With inter-temporal transfers, however, the optimal policy is characterized by an inverse Euler equation that implies[12]

$$u'(Y+\delta) < pu'(y^e) + (1-p)\, u'(Y^u+\omega)\,.$$

The consequence is that the non-negativity restriction on savings ceases to be relevant and the potential non-observability of savings has important consequences for policy design.

There are two possible reactions to this issue. First we may argue that non-observability is not important so that, in practice, the government controls savings and all our results remain valid.[13] The second possibility is to recognize that non-observability is important and optimal policies should take this into account. At an extreme, the existence of perfect capital markets eliminates the very reason for the existence of a UI program. As Levine and Zame (2002) have shown, with infinite lives and purely idiosyncratic shocks, the equilibrium allocation of an economy with perfect capital markets can be made arbitrarily close to the complete markets allocation by making the discount rate sufficiently close to zero. In an economy with finite lives and empirically sound discount rates (and capital markets), however, unemployment *does* matter even when savings are used to smooth consumption:[14] without full insurance consumption is history dependent and decreasing in the expected amount of time that one is unemployed.[15]

In the next section we characterize optimal policies under hidden savings. In this case, the problem that the agent solves is not convex, and we may no longer rely on a first order approach. We handle the non-convexity problem and show that, even though hidden savings do alter other dimensions of policy — e.g., the marginal tax rate for labor income —, it is still the case that education should be encouraged.

5 Hidden Savings

There is a growing literature dealing with the effects of hidden savings on the design of unemployment insurance programs — e.g., Kocherlakota (2004), Werning (2002), **?**. Our two period framework does not allow us to discuss how savings affect the pattern of transfers along an unemployment spell.[16] Nevertheless, we share with this literature, the concern with how incentives are affected by savings and how this feeds back to the design of government policies.

We first define the indirect utility functions of an unemployed agent,

$$V^u(s,\omega) \equiv \max_{Y^u} \{u(Y^u+\omega+s) - \zeta(Y^u)\},$$

and that of an employed agent,

$$V(s,\delta,h) \equiv \max_{Y} u(Y-s+\delta) - \zeta(Y+h).$$

With these definitions, the government's program is

$$\max\left\{V(s,\delta,h) + p\left[u(y^e+s) - \zeta\left(\frac{Y^e}{w(h)}\right)\right] + (1-p)V^u(Y,s,\omega) - \varphi(p)\right\}, \tag{19}$$

subject to the resource constraint,

$$p(Y^e - y^e) \geq (1-p)\omega + \delta, \tag{20}$$

program and 22.2 without it).

[15]With multiple periods, the point in time in which one is unemployed is also important. It may very well be the

and to the incentive constraint,

$$(p, s) \in \arg\max_{(\hat{p},\hat{s})} \left\{ V(\hat{s}, \delta, h) + \hat{p} \left[u(y^e + \hat{s}) - \zeta \left(\frac{Y^e}{w(h)} \right) \right] + (1 - \hat{p}) V^u(\hat{s}, \omega) - \varphi(\hat{p}) \right\}. \tag{21}$$

As we have argued, non-observation of both savings and non-markets skills renders the agent's problem potentially non-convex, and makes the use of a first order approach unreliable: since the sets generated by (21) and the sets generated by the agents' first order conditions may differ, one cannot substitute the latter for the former when solving the government's problem, in general.

There are some alternatives for dealing with the issue. Werning (2002) restricts preferences to a class where the first order approach is guaranteed to work. **?** solve the model assuming that the approach works and check whether, for the specific parametrization they have chosen, the first order conditions characterize a maximum at the optimal solution. Both models are substantially more complex than ours since these authors work with fully dynamic problems which require the use of recursive methods. The payoff we obtain from working in a simplified environment is that we are able to adopt a procedure that does not rely on a specific functional form and/or parametrization of the problem.

Our procedure consists in discretizing the effort space by redefining the domain of p as the finite set $\{0, p_1, .., 1\}$. Next, we characterize the optimal deviation strategies and verify which ones may bind at the optimum and what this implies for the design of optimal policies. This procedure mimics, in some sense, a numerical approach with an important advantage: all results derived herein do not depend on any specific parametrization or functional forms beyond the ones we have being using all along!

First, define $W(\delta, h, y^e, Y^e, \omega, p) \equiv$

$$\max_{\hat{s} \in \mathbb{R}_+} \left\{ V(\hat{s}, \delta, h) + p \left[u(y^e + \hat{s}) - \zeta \left(\frac{Y^e}{w(h)} \right) \right] + (1 - p) V^u(\hat{s}, \omega) - \varphi(p) \right\}. \tag{22}$$

the maximum utility the worker attains by optimally choosing her savings, conditional on a given p. The restriction that $\hat{s} \in \mathbb{R}_+$ is due to the credit constraint.

Next, we define for the government a *relaxed* program, account the downward ones: those which guarantee that the agent does not choose a lower level of effort than the optimal, p^*.

Because the government faces fewer constraints, the solution to (23) is not inferior to the solution to the government's problem (19) when constraint (21) is considered. What we show in the appendix is that if $(\delta^*, h^*, y_e^*, Y_e^*, \omega^*, p^*)$ solves (23) then, at this solution, there is no strategy with $p > \hat{p}$ (and associated optimal choices) that yields higher expected utility for the agent. Therefore, $(\delta^*, h^*, y_e^*, Y_e^*, \omega^*, p^*)$ solves government's problem (19) subject to (20) and (21).

For our purposes, the fact that only downward constraints bind will be of paramount importance in identifying the relevant deviating strategies. Along these lines, the next two lemmas, proven in the appendix, are stated here to facilitate the intuition regarding some of the results that follow.

Lemma 1. *In all strategies that contemplate a lower level of effort than the optimal, $p < p^*$, the agent saves at least as much as when she makes the optimal effort, p^*.*

Lemma 2. *In all strategies that contemplate a lower level of effort than the optimal, $p < p^*$, the agent supplies at least as much labor in the first period.*

Underlying these results is the fact that, if the relevant deviating strategies are the ones that contemplate lower effort, savings are complementary to deviant behavior. Agents who do not make enough effort to remain in the formal markets have a higher expected marginal utility of consumption, when compared to agents who choose the optimal effort, p^*. By the same token, higher savings increase first period marginal utility of income thus implying a higher propensity to work in the first period — lemma 2. That is, agents who anticipate being unemployed more often or for longer periods work more on informal activities from very early in their lives, and hold more wealth.[17]

Educational Policy. The qualitative results regarding the educational policy are not altered by the possibility of hidden savings. It is still optimal for the government to encourage the acquisition of human capital. To show this, we write the first order necessary condition with respect to h,

$$-\zeta'(h+Y^*) + p^*\zeta'(L^e)\, L^e \frac{w'(h)}{w(h)} + \sum \lambda(p)[\zeta'(h+Y(p)) - \zeta'(h+Y^*)]$$

noting that we have applied the envelope theorem in (22) to find the partial derivative of W with respect to h.

It is apparent from lemma 2 that the third and the forth terms in (24) are positive. Therefore,

$$p^* \zeta' \left(L^e\right) L^e \frac{w'(h)}{w(h)} < \zeta'(h + Y^*). \tag{25}$$

The inequality above shows that the optimal policy requires the creation of a wedge between optimal private costs and benefits of education, which yields the next proposition.

Proposition 3. *At the optimum,* $h^* > h^o$*, where* $h^o \equiv \arg\max_h W\left(\delta, h, y^e, Y^e, \omega, p^*\right)$*.*

The proof uses convexity of $\zeta\left(\cdot\right)$, concavity of $w\left(\cdot\right)$ and inequality (25). The government must induce a choice of h that is higher than the private optimum. The rationale is once again that, by forcing agents to get more education, the government raises the costs of free riding on the unemployment benefit program.

Labor income taxation and UI. Next, we investigate the consequences of hidden savings for optimal labor income taxes and unemployment benefits. We begin by taking the first order conditions with respect to y^e, ω and δ, respectively,

$$\mu = u'(c_e^*) + \sum\nolimits_{p<p^*} \lambda(p) \left[u'(c_e^*) - \frac{p}{p^*} u'\left(c^e(p)\right)\right],$$

$$\mu = u'(c_u^*) + \sum\nolimits_{p<p^*} \lambda(p) \left[u'(c_u^*) - \frac{1-p}{1-p^*} u'\left(c^u(p)\right)\right],$$

and

$$\mu = u'(c_0^*) + \sum\nolimits_{p<p^*} \lambda(p) \left[u'(c_0^*) - u'(c_0(p))\right].$$

Combining the three first order conditions above, we get

$$u'(c_0^*) - p^* u'(c_e^*) - (1-p^*) u'(c_u^*) = \frac{\sum_{p<p^*} \lambda(p) [u'(c_0(p)) - p u'\left(c^e(p)\right) - (1-p) u'\left(c^u(p)\right)]}{1 + \sum_{p<p^*} \lambda(p)}.$$

conditions with respect to Y^{e*} and w^{e*} yield

$$\frac{\zeta'(L^e)}{w(h)}\left\{p^* + \sum\nolimits_{p<p^*} \lambda(p)\,[p^* - p]\right\}$$
$$= p^* u'(c_e^*) + \sum\nolimits_{p<p^*} \lambda(p)[p^* u'(c_e^*) - p u'(c^e(p))],$$

which finally implies

$$\frac{\zeta'(L^e)}{w(h)} - u'(c_e^*) = \frac{\sum_{p<p^*} \lambda(p)p\,[\zeta'(L^e)/w(h) - u'(c^e(p))]}{p^* + p^* \sum_{p<p^*} \lambda(p)} \geq 0. \tag{26}$$

The marginal tax rate on labor income $\Phi(p^*)$ is proportional to the (implicit) marginal tax rate on agents following all binding strategies, which, as we have proven, contemplate a lower level of effort than p^*. Naturally, no agent actually follows a different strategy. These are off-equilibrium choices which must be well understood for us to access the optimal marginal tax rate on labor income.

What (26) shows is that the marginal tax rate is non-positive. However if we add the following assumption, we may guarantee that the inequality in (26) is strict, which means that the marginal tax rate on labor income is negative.

Assumption φ: *There exists an (arbitrarily small) $p > 0$ such that $\varphi(p) = 0$.*

This assumption guarantees that even if one does not make any effort to find a job there is a positive probability that she will find a job at the legal markets.

Proposition 4. *Under Assumption φ, the marginal tax rate on labor income is negative at the optimum.*

Notice that Assumption φ is sufficient, not necessary, for proposition 4. What is interesting about proposition 4 is the fact that this result was not present in the case where savings were observed. Nor is it part of any optimal unemployment insurance scheme derived in the literature.

Conditional on one's participating in the legal markets her labor supply, L^e is independent of her labor market attitude, p. This explains the zero marginal taxes prescription in the framework of section 4. What is new here is the fact that differences in savings affect the propensity to work

An important caveat is that this result may not be robust to relaxing the two period formulation, *if* one considers the possibility of multiple unemployment spells. However, the result should still be valid in a multi-period setting under the assumption, adopted in most of the literature, that once a worker gets a job she remains employed for the rest of her life.[18]

An issue we have not investigated here is the possibility of hidden human capital investment. When we speak of human capital what we have in mind is more than simply years of schooling, which is what governments usually have some control over. This being the case, the use of the sophisticated tax instruments required to induce the optimal choices of h (see discussion in section 4.1) is not feasible. Government intervention is still possible through subsidies to direct costs of schooling, which we have not allowed for here. The intuition from the previous results are still valid and we do believe that subsidizing schooling will prove to be optimal.

6 Conclusion

In a two period model that subsumes life-long choices, we investigate the interaction between UI programs and educational policies. Agents' employment status is assumed to be affected by labor market attitude, which in its own turn is dependent on the relative cost of being unemployed. Education is important in this world not only because it raises expected income but also because it affects the opportunity cost of unemployment. It is this latter effect that plays the most prominent role in our model.

Our main result is that unemployment insurance and educational policies are complementary, i.e., in order to alleviate the moral hazard which is inherent to UI programs it is always optimal for the government to distort agents choices toward over-investment in human capital. Another important finding is that, despite the fact that there remains some consumption risk at the optimum — due to its being a constrained optimum in which moral hazard plays a role — the expected benefit of education is equal to its expected cost: a form of production efficiency result in our setup.

This latter result, however, depends on the government being able to make optimal inter-temporal transfers. The problem is that, as in Rogerson (1985), optimal policies require the expected marginal utility of consumption in the second period to be higher than marginal utility of consumption in the first period. This raises all types of questions about observability of savings and the this modification in our main setup. Marginal income taxes, however, depend on whether savings are observe or whether they are not.

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A Appendix

The next three lemmas guarantee that labor supply responses are not strong enough to overcome the direct effect of transfers and savings on consumption.

Lemma 3. *At a fixed s, c^e is decreasing and c^u is increasing in transfers.*

Proof. For the first part we just note that $c^e = Y^e + s - \omega$. Since Y^e is chosen by the government, we have $dc^e/d\omega = -1 < 0$. For the second, assume for ease of exposition that $\delta = 0$ and note that $c^u = Y^u + s + \alpha\omega$ where $\alpha = p/(1-p)$ which means that $dc^u/d\omega = \alpha + dY^u/d\omega$. Then, $u'(Y^u + s + \alpha\omega) - \zeta'(Y^u) = 0$, which implies

Lemma 4. *c_0 is decreasing and c_u and c_e are increasing in s.*

Proof. The proof follows the steps of lemma 3. *Q.E.D.*

Lemma 5. *$c_e^* \geq c_u^*$ in any relaxed program.*

Proof. We will consider the relaxed program and we will prove the lemma by showing that, if $c_e^* < c_u^*$, a redistribution of income from the unemployment state to the employment increases welfare and is incentive compatible.

Define $(c_0^*, c_e^*, c_u^*, Y_u^*, Y_0^*, s^*) \equiv$

$$\begin{cases} \arg\max u(c_0) - \zeta(Y_0) + p^* \left[u(c^e) - \zeta\left(\frac{Y^{e*}}{w(h)}\right)\right] + (1-p^*)[u(c^u) - \zeta(Y^u)] - \varphi(p^*) \\ \text{s.t. } c^e = y^{e*} + s, \; c^u = Y^u + s + \omega, \text{ and, } c_0 = Y_0^* - s + \delta, \end{cases} \tag{27}$$

and $(c_0(p), c_e(p), c_u(p), Y_0(p), Y_u(p), s(p)) \equiv$

$$\begin{cases} \arg\max u(c_0) - \zeta(Y_0) + p \left[u(c^e) - \zeta\left(\frac{Y^{e*}}{w(h)}\right)\right] + (1-p)[u(c^u) - \zeta(Y^u)] - \varphi(p^*) \\ \text{s.t. } c^e = y^{e*} + s, \; c^u = Y^u + s + \omega, \text{ and, } c_0 = Y_0^* - s + \delta \end{cases} \tag{28}$$

Since p^* maximizes the relaxed program, we should have, for all $p < p^*$,

$$\begin{aligned} u(c_0^*) - \zeta(Y_0^*) + p^* \left[u(c_e^*) - \zeta\left(\frac{Y^{e*}}{w(h)}\right)\right] + (1-p^*)[u(c_u^*) - \zeta(Y_u^*)] - \varphi(p^*) \geq u(c_0(p)) \\ -\zeta(Y_0(p)) + p\left[u(c^e(p)) - \zeta\left(\frac{Y^e}{w(h)}\right)\right] + (1-p)[u(c^u(p)) - \zeta(Y^u(p))] - \varphi(p) \end{aligned} \tag{29}$$

Now, the fact that choices in (28) are optimal when the probability is p guarantees that

$$\begin{aligned} u(c_0^*) - \zeta(Y_0^*) + p \left[u(c_e^*) - \zeta\left(\frac{Y^{e*}}{w(h)}\right)\right] + (1-p)[u(c_u^*) - \zeta(Y_u^*)] - \varphi(p) \leq u(c_0(p)) \\ -\zeta(Y_0(p)) + p\left[u(c^e(p)) - \zeta\left(\frac{Y^e}{w(h)}\right)\right] + (1-p)[u(c^u(p)) - \zeta(Y^u(p))] - \varphi(p) \end{aligned} \tag{30}$$

From (29) and (30) we have

which implies that

$$\Delta p\left[\zeta(Y_u^*)-\zeta\left(\frac{Y^{e*}}{w(h)}\right)\right]\geq\Delta p[u(c_u^*)-u(c_e^*)]+\varphi(p^*)-\varphi(p), \tag{31}$$

where $\Delta p = p^* - p$. Observe that the deviation strategies generally contemplate different choices of s and Y^u, as long as $c_e^* \neq c_u^*$. Notice, however, that, if $c_e^* = c_u^*$, we have $s(p) = s^*$ and $Y^u(p) = Y_u^*$. Assume that $c_e^* < c_u^*$. We, now, distribute income from the unemployment state to the employment until we have $c_e^* = c_u^*$. This is feasible according to lemma 3. Denoting $\hat{Y}^u$ the choice made by the truth-telling strategy after the reform, we have $\hat{Y}^u > Y_u^*$, (see the proof of lemma 3). We shall prove that the reform does not violate incentive compatibility, i.e.,

$$\begin{aligned}&u(\hat{c}_0^*)-\zeta(\hat{Y}_0^*)+p^*\left[u(\hat{c}_e^*)-\zeta\left(\frac{Y^e}{w(h)}\right)\right]+(1-p^*)[u(\hat{c}_u^*)-\zeta(\hat{Y}_u^*)]-\varphi(p^*)\geq u(\hat{c}_0(p))\\&-\zeta\left(\hat{Y}_0(p)\right)+p\left[u(\hat{c}^e(p))-\zeta\left(\frac{Y^e}{w(h)}\right)\right]+(1-p)[u(\hat{c}^u(p))-\zeta(\hat{Y}^u(p))]-\varphi(p)\end{aligned} \tag{32}$$

Because $\hat{c}_0^* = \hat{c}_0(p)$, $\hat{Y}_0^* = \hat{Y}_0(p)$, $\hat{c}_e^* = \hat{c}_0^e(p)$, $\hat{c}^u(p) = \hat{c}_u^*$ and $\hat{Y}_u^* = \hat{Y}^u\left(p\right)$, since $c_e^* = c_u^*$, after the reform, inequality (32) collapses to

$$\Delta p\left[\zeta(\hat{Y}^{u*})-\zeta\left(\frac{Y^e}{w(h)}\right)\right]\geq\varphi(p^*)-\varphi(p) \tag{33}$$

Now, the right hand side of (33) minus the right hand side of (31) is $\Delta p\int_{Y_u^*}^{\hat{Y}^{u*}}\zeta'(Y)dY > 0$ and the left hand side of (31) minus the left hand side of (33) is $\Delta p[u(c_u^*) - u(c_e^*)] > 0$. Therefore, we conclude that the reform is incentive-compatible and increases welfare, since the utility is strictly concave. *Q.E.D.*

We are now in a position two prove the first two lemmas in Section 5.

Proof of lemma 1. Let s^* and $s\left(p\right)$ be as defined in (27) and (28), respectively, for $p < p^*$. Assume that $s\left(p\right) < s^*$. (In which case $s^* > 0$). From lemma 4, this implies $c_0\left(s\right) > c_0^*$ which, in turn gives $u'\left(c_0\left(p\right)\right) < u'\left(c_0^*\right)$. Now, $u'\left(c_0\left(p\right)\right) \geq pu'\left(c_e\left(p\right)\right) + \left(1-p\right)u'\left(c_u\left(p\right)\right) > pu'\left(c_e^*\right) + \left(1-p\right)u'\left(c_0^*\right) >$

Proof of lemma 2. We have from lemma 1 that $s^* \leq s(p)$ whenever $p < p^*$. But, then, by an argument identical to the one used in the proof of lemma 3, one can easily show the result. *Q.E.D.*

The next lemma proves that resource constraints are binding at the optimum. This is not a trivial issue in a dynamic agency problem so a careful demonstration is needed.

Lemma 6. *The resource constraint multiplier, μ, for the relaxed program, (23), is positive.*

Proof. We first show that $Y^e = 0$ cannot be part of the solution to (23). First note that when $Y^e = 0$ the government only intervenes in the equilibrium of this economy by transferring resources across time. It is clear that at $Y^e = 0$ the welfare is lower than in the competitive equilibrium, when the government plays the same role of transferring resources. Hence, Y^e cannot solve the problem. Consider, then, the case where solution with respect to Y^e is interior. Take the first order condition with respect to Y^e,

$$p^*\mu - \frac{1}{w(h)}\zeta'\left(\frac{Y^e}{w(h)}\right)\left[p^* - \sum\nolimits_{p<p^*} \lambda(p)\,(p^* - p)\right] = 0$$

which, then, implies

$$\mu = \frac{1}{w(h)}\zeta'\left(\frac{Y^e}{w(h)}\right)\left[1 - \sum\nolimits_{p<p^*} \lambda(p)\,(1 - p/p^*)\right] > 0.$$

Q.E.D.

The next two propositions contain the main results regarding the usefulness of our approach. They guarantee that the solution to the relaxed program is the solution to the government's program, and that at least one IC constraint is binding at the optimum.

Proposition 5. *No constraint relative to a strategy that contemplates $p > p^*$ is binding at the optimum.*

Proof. First solve the relaxed problem and find the value p^* that solves (23). If there is no deviation strategy in which the agent chooses a higher level of effort and that yields at least the same utility and let $\bar{W} \equiv \max_p W(p)$ and $\bar{p} \equiv \max\left\{p; W(p) = \bar{W}\right\}$. Next, observe that $\bar{p}(Y^{e*} - w^{e*}) - (1 - \bar{p})w^{u*} - \omega^* > p^*(Y^{e*} - w^{e*}) - (1 - p^*)w^{u*} - \omega^* \geq 0$. Hence, resources are idle, which implies, from lemma 6, that $\{w^{e*}, h^*, w^{u*}, Y^{e*}\}$ is not a solution to the $\bar{p}$- relaxed program. This contradicts the assumption that p^* belongs to the solution of (23). *Q.E.D.*

Proposition 6. *At least one incentive compatibility constraint binds at the optimum.*

Proof. Assume the contrary. It is clear that the government must provide full insurance. Hence, it is obvious that no agent would have any incentive to choose a positive effort. Therefore, this policy would not be feasible. *Q.E.D.*

We may, now strengthen the result in lemma 5.

Lemma 7. *At the optimum $c_e^* > c_u^*$.*

Proof. Recall the Lagrangian for the government's problem,

$$\begin{aligned} \mathcal{L} \equiv\ & V\left(\delta, h, y^e, Y^e, \omega, \hat{p}\right) + \mu[\hat{p}\left(Y^e - y^e\right) - (1 - \hat{p})\omega - w] \\ & + \sum_{p<\hat{p}} \lambda(p)[V\left(\delta, h, y^e, Y^e, \omega, \hat{p}\right) - V\left(\delta, h, y^e, Y^e, \omega, \hat{p}\right)]. \end{aligned}$$

In this case,

$$\left.\frac{\partial \mathcal{L}}{\partial \omega}\right|_{c_e^* = c_u^*} = \frac{\partial V_0}{\partial \omega} - \mu + \sum_{p<p^*} \lambda(p) \left[\frac{\partial V\left(p^*\right)}{\partial \omega} - \frac{\partial V\left(p\right)}{\partial \omega}\right] = -\mu < 0.$$

Q.E.D.

We now show that there is a deviating strategy that binds at the optimum and for which savings are greater than at the equilibrium choices.

Lemma 8. *At the optimum, there is $p < p^*$ such that $s(p) > s(p^*) \geq 0$ and $\lambda(p) > 0$.*

Proof. First, the existence of $\lambda\left(p\right) > 0$ is due proposition 6 and the Kuhn-Tucker theorem. Now, if $u'(c_e^*) = \mathbb{E}u'(c_1^*)$, a slight change in the proof of lemma 5 shows that $s(p) > s(p^*)$. So, let us suppose

$s(p) = 0$ for all $p < p^*$ with $\lambda(p) > 0$, leads to $u'(c_0^*) = u'(c_0(p))$ for all $p < p^*$ such that $\lambda(p) > 0$. Moreover, from the fact that $c_u^* < c_e^*$, it is clear that $\mathbb{E}u'(c_0^*) < \mathbb{E}u'\left(c_0\left(p\right)\right)$. We will now show that for $\varepsilon > 0$ sufficiently low, the policy $\{w^{e*} - \varepsilon, w^{u*} - \varepsilon, \omega^* + \varepsilon\}$ is welfare-improving and clearly does not violate the resource constraint. For ε sufficiently low, $\Delta W \approx$

$$\varepsilon \left\{ u'(c_0^*) - \mathbb{E}u'(c_0^*) + \sum\nolimits_{p<p^*} \lambda(p)u'(c_0^*) - \mathbb{E}u'(c_0^*) - u'(c_0(p)) + \mathbb{E}u'\left(c_0\left(p\right)\right) \right\},$$

or

$$\Delta W \approx \varepsilon \left\{ u'(c_0^*) - \mathbb{E}u'(c_0^*) + \sum\nolimits_{p<p^*} \lambda(p)\mathbb{E}u'\left(c_0\left(p\right)\right) - \mathbb{E}u'(c_0^*) \right\} > 0,$$

which contradicts the optimality of the policy. *Q.E.D.*

Proof of Proposition 4. We re-write (26) more compactly as

$$\Phi(p^*) = \kappa\left(p^*\right) \sum_{p<p^*} \pi\left(p\right) \Phi(p) \geq 0, \tag{34}$$

where

$$\kappa\left(p^*\right) \equiv \frac{\sum_{p<p^*} \lambda(p)pu'(c^e(p))}{\left(p^* + p^* \sum_{p<p^*} \lambda(p)\right) u'(c_e^*)},\ \pi\left(p\right) \equiv \frac{\lambda\left(p\right) pu'(c^e(p))}{\sum_{p<p^*} \lambda(p)pu'(c^e(p))},$$

and

$$\Phi(p) \equiv \frac{\zeta'\left(L^e\right)}{w(h)u'(c^e(p))} - 1.$$

From lemma 8, there is $p < p^*$ such that $s(p) > s(p^*) \geq 0$ and $\lambda(p) > 0$. From the Spence-Mirrlees condition, it is clear that $s(p) > s(p^*) \Rightarrow \Phi(p^*) < \Phi(p)$. Now, if the left hand side of (34) is zero, the right hand side is negative. Therefore, the marginal tax rate can not be zero.
Suppose, however, that $\Phi(p^*) < 0$. In this case,

$$0 < \frac{\sum_{p<p^*} \lambda(p)p}{p^* \left(1 + \sum_{p<p^*} \lambda(p)\right)} < 1.$$

The right hand side is less than the left hand side, hence this can not be the case. *Q.E.D.*