Spatial Agglomeration with Vertical Differentiation

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Abstract

This paper constructs a model using a two-dimensional framework to take account of both horizontal and vertical differentiation. To examine firms' location configuration, the paper employs a two-stage game, in which firms first simultaneously decide optimal locations and then play Bertrand price competition with three pricing policies taking the degree of vertical differentiation as given. It is shown that the higher the degree of vertical differentiation, the weaker the centrifugal competition effect will be. The Principle of Minimum Differentiation can attain if firms engage in discriminatory pricing. However, the Principle of Maximum Differentiation will never emerge. In addition, firms locate at the market center as long as the degree of vertical differentiation is greater than zero in both cases where firms adopt uniform delivered and mill pricing schemes.

1. Introduction

Hotelling (1929) first proposed that two firms of a homogeneous product agglomerate at the center of the line market under linear transportation costs, which has been termed the *Principle of Minimum Differentiation*. However, D'Aspremont *et al*. (1979) challenge this principle by indicating that there exists no price equilibrium in this case and shows that the two firms will locate at the opposite endpoints of the line market under quadratic transportation cost instead. This has been termed the *Principle of Maximum Differentiation*. From then on, many regional economists have tried to deduce the conditions under which the Principle of Minimum Differentiation can be restored. They include: De Palma *et al*. (1985) and Rhee *et al*. (1992), who introduce heterogeneity in both consumers and firms; Stahl (1982) who considers some harmonious conjectural variations; Anderson and Neven (1991) who assume that firms play Cournot quantity competition instead of Bertrand price competition in the commodity market; Jehiel (1992) and Friedman and Thisse (1993) who adopt price collusion; and Tabuchi (1994) who constructs a model with two dimensions of horizontal differentiation. Tabuchi in particular shows that two firms maximize their distance in one dimension, but minimize their distance in the other dimension. In addition to these researchers, Zhang (1995) imposes a price-matching policy; Mai and Peng (1999) emphasize the importance of the externality-like benefits generated from the exchange of information between firms; Liang and Mai (2006) focus on the crucial influence caused from the vertical subcontracting of the intermediate product; and Matsushima and Matsumura (2006) analyze the mixed-oligopoly economy.

Ferreira and Thisse (1996) employ Launhardt (1885)'s spatial oligopolistic model to examine the decisions of the firms' optimal quality levels (vertical differentiation) taking location (horizontal differentiation) as exogenously given.[1] They use transport rate as a measure of quality, a high (low) transport rate representing a low (high) quality level, and the game employed is a two-stage game, in which firms select the optimal quality levels in the first stage and then engage in Bertrand price competition in the commodity market in the second stage. They find an interesting result that firms select to maximize the vertical differentiation when the horizontal differentiation is minimized, while to minimize the vertical differentiation when the horizontal differentiation is maximized. This result is termed the Max-Min and Min-Max result hereafter.

Ferreira and Thisse (1996)'s model imply that location is a long-term decision, while quality is a short-term decision.[2] However, it can be observed in the real world that there exist many industries whose location choice is regarded as a short-term

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[1] According the definition of Ferreira and Thisse (1996, p. 486), two products are said to be horizontally differentiated when both products have a positive demand whenever they are offered at the same price. Neither product dominates the other in terms of characteristics, and heterogeneity in preferences over characteristics explains why both products are present in the market. We can also find a similar definition in Lancaster (1979).

[2] Most of the literature treat location as a long-term while quality as a short-run decision. They include: Economides (1989), Neven and Thisse (1990), Calem and Rizzo (1995), Mai and Peng (1999), Anderson and De Palma (2001), Valletti (2002), Piga and Poyago-Theotoky (2005), and Brekke *et al*. (2006).

decision, while quality is a long-term decision.[3] Generally, these cases arise in the industries owning low set-up costs such that firms are capable of changing their locations easily with fixed qualities. For example, the horizontal differentiation of a restaurant can be represented by the differentiation in the geographic location, while the quality of the restaurant denoted by Mobil stars represents the vertical differentiation. The quality level of a five Mobil stars restaurant is obviously higher than that of a lower Mobil stars restaurant.[4]

Theoretically, taking into account both horizontal and vertical differentiation allows us to explore the substitutability of quality for location and the strategic interactions between location-quality combinations that firms provide.[5] To the best of our knowledge, the decision of firms' optimal location, in which the level of quality is exogenously determined, has yet to be touched upon.

Based on the above analysis, the purpose of this paper is to determine the conditions under which the Principle of Minimum Differentiation can be restored and in which the firms' quality levels (vertical differentiation) are exogenously given. In order to take into account both horizontal and vertical differentiation, we follow Economides (1993) by introducing a two-dimension model, in which each differentiated product is defined by one feature of location and one feature of quality.

[3] In a theoretical paper, Bonanno and Hawoth (1998) treat quality as a long-term decision, while process R&D as a short-term decision.
[4] See Berry and Waldfogel (2003).
[5] See Economides (1993, p.236).

This facilitates the study of the effect of quality differentiation on firms' location decisions.

The game in question is a two-stage game, in which firms select their optimal locations to maximize their profits, respectively, in the first stage, and then play Bertrand competition in the commodity market in the second stage, given firms' quality levels. Three pricing regimes -- discriminatory, uniform delivered and mill pricing -- are taken into consideration by firms while competing in the commodity market.

We show, in the paper, that there exist a centrifugal competition effect and a centripetal cost-saving effect.[6] The competition effect indicates that as the two firms are more distant from each other, they become more dissimilar and therefore competition lessens.[7] Accordingly, the two firms tend to separate more distantly to reduce the competition for earning higher profits via charging higher prices. We find that the introduction of vertical differentiation mitigates the competition effect due to enlarging the differentiation between firms. On the other hand, the cost-saving effect reflects firm *i*'s desire to move toward the center in order to save on the transportation cost. Therefore, firms would agglomerate if the degree of the vertical differentiation between firms is sufficiently large while separate if firms are less differentiated, as

[6] The idea of the competition effect can also be found in Liang *et al*. (2006).
[7] Contrarily, if the two firms locate at the center of the market, they are symmetric in terms of production cost plus transport cost at any site of the market and the competition is the highest.

firms engage in discriminatory pricing. However, this competition effect vanishes, as firms conduct uniform delivered and mill pricing. This result arises because firms charge the same price for every point over the Hotelling line, which leads to the outcome that firms are unable to increase price and profits via locating further away from each other.

The agglomeration result of firms with vertical differentiation being higher can be supported by the industries of department store and apparel. We can observe that some department stores such as Dillard (higher quality store) and JC Penny (lower quality store) and apparel stores such as Banana Republic (higher quality store) and The Limited (lower quality store) agglomerate at the same mall in many towns of the U.S. On the other hand, the dispersion result of firms with lower vertical differentiation can be supported by the supermarket and electronic appliance businesses. There is significant evidence that Wal-mart and K-mart, as well as Circuit City and local electronic appliance stores, never locate at the same location due to narrow quality differentiation.[8]

The remainder of the paper is organized as follows. Section 2 sets up a spatial model with products exhibiting exogenously vertical differentiation and analyzes the optimal location in the case of discriminatory pricing. Section 3 examines the optimal

[8] The pricing policy of Wal-mart is every day low price, and that of Circuit City is lowest price guaranteed. Both pricing policies demonstrate the feature of Bertrand price competition.

location in the cases of uniform delivered and mill pricing. The final section concludes the paper.

2. The Basic Model with Discriminatory Pricing

Consider a two-dimensional framework, in which the horizontal axis measures the traditional Hotelling line referred to as horizontal characteristic, while the vertical axis measures the tastes of consumers to qualities referred to as vertical characteristic, as shown in Figure 1.[9] Two firms, denoted firm 1 and firm 2, are located at x_1 and x_2, with $x_1 \leq x_2$ along a line segment with length $L = 1$ on the horizontal axis. The firms, whose production cost is for simplicity assumed to be nil, sell products with vertically differentiated qualities, α_1 and α_2 with $\alpha_1 \leq \alpha_2$ respectively, to consumers. In a model with vertically differentiated quality, there must be heterogeneity in consumers' willingness to pay for quality, which is captured by assuming that a continuum of consumers is uniformly distributed over the interval $[\underline{\theta}, \overline{\theta}]$ along the vertical axis with unit density at each point of the Hotelling line.[10] Following Choi and Shin (1987), we assume $\underline{\theta} = \overline{\theta} - 1$, where $\overline{\theta} > 1$. Thus, these two characteristics lead to a rectangular distribution of consumers over $[0, 1] \times [\overline{\theta} - 1, \overline{\theta}]$. A firm faces a

[9] Economides (1993) extends the circular model of variety-differentiated products constructed by Salop's (1979) to a two-dimension model, in which both horizontal and vertical differentiation are taken into consideration.

[10] Given two products of different qualities, all consumers would prefer the product with higher quality to that with lower quality at the same price. In order to keep the two firms survive in the market, the consumers' willingness to pay for quality must be heterogeneous.

continuum of consumers with taste $\theta \in [\bar{\theta}-1, \bar{\theta}]$ at each point of the Hotelling line or a continuum of consumers with different locations for a given taste of the vertical axis. Assume further that the transport cost function of the product is linear and takes the following form: $T(x-x_i)=t|x_i-x|$, where T is the transport cost, and t is the transport rate per unit output per unit distance.

(Insert Figure 1 here)

The game employed in this paper is a two-stage game. Firms simultaneously select their locations to maximize their profits, respectively, in the first stage; and then play Bertrand price competition in the commodity market in the second stage. Prior to the first stage, the quality levels are exogenously determined and can not change in the short run. The sub-game perfect Nash equilibrium can be solved by backward induction, beginning with the final stage.

Suppose that firms adopt discriminatory pricing to charge different prices for consumers residing at different locations. The indirect utility of a consumer residing at the location with combination (x, θ) and purchasing from firm i is taken to be:

$$u(x,\theta)=k+\theta\alpha_i-p_i(x), i=1,2, \tag{1}$$

where $u(x, \theta)$ is the utility function of the consumer with combination (x, θ); and k is the reservation utility of consuming one unit of commodity; and θ denotes the taste of consumers' preference for quality ranging along the interval $[\bar{\theta}-1, \bar{\theta}]$ with $\bar{\theta}$ is the upper bound of the consumers' tastes; and α_i (i = 1, 2) represents the quality level of the product produced by firm i; and p_i (x) is the delivered price charged by firm i at site x.

The taste of the marginal consumer, who is indifferent between buying one unit of the product from either firm, for a continuum of consumers residing at x can be obtained by equaling the utility levels of buying from the two firms as follows:[11]

$$\hat{\theta}(x) = [p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1), \tag{2}$$

where $\hat{\theta}(x)$ denotes the taste of the marginal consumer for a continuum of consumers residing at x.

Each firm's demand function at site x can be derivable as:

$$q_1(x) = \hat{\theta} - \underline{\theta} = \{[p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1) - (\overline{\theta} - 1)\}, \tag{3.1}$$

$$q_2(x) = \overline{\theta} - \hat{\theta} = \{\overline{\theta} - [p_2(x) - p_1(x)]/(\alpha_2 - \alpha_1)\}. \tag{3.2}$$

Assuming that production costs are zero and the quality cost is fixed, firm i's operating profit function at site x can be expressed as:[12]

$$\pi_i(x) = [p_i(x) - t|x - x_i|]q_i(x), i = 1,2, \tag{4}$$

where $\pi_i(x)$ denotes firm i's operating profit at site x.

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[11] Notice that Eq. (2) is derived by assuming that the reservation utility k is sufficiently high such that all consumers buy one unit of product, i.e., the market is covered. However, the main results of the paper remain unchanged if the market is uncovered, i.e., some low taste consumers refuse to purchase any product. For simplicity, we use the covered market assumption for the exclusion of tedious expositions.

[12] Firm i's profit equals its operating profit minus fixed quality cost.

Differentiating (4) with respect to $p_i(x)$ respectively, we can derive the profit-maximizing conditions for prices. Solving these equations, we have:[13]

$$p_1(x) = (1/3)[(\alpha_2 - \alpha_1)(2 - \overline{\theta}) + t(2|x - x_1| + |x - x_2|)], \tag{5.1}$$

$$p_2(x) = (1/3)[(\alpha_2 - \alpha_1)(\overline{\theta} + 1) + t(|x - x_1| + 2|x - x_2|)]. \tag{5.2}$$

Substituting (5) into (3), we obtain:

$$q_1(x) = [1/3(\alpha_2 - \alpha_1)][(\alpha_2 - \alpha_1)(2 - \overline{\theta}) + t(|x - x_2| - |x - x_1|)], \tag{6.1}$$

$$q_2(x) = [1/3(\alpha_2 - \alpha_1)][(\alpha_2 - \alpha_1)(\overline{\theta} + 1) - t(|x - x_2| - |x - x_1|)]. \tag{6.2}$$

It is worth noting that the upper bound of the quality taste ($\overline{\theta}$) must be smaller than 2 to ensure firm 1's demand being positive, as firms locate at the same site, i.e. $x_1 = x_2$. Consequently, the upper bound of the quality taste lies within the interval [1, 2].

Substituting (5) into (2), we can derive the taste of the marginal consumer residing at site x as follows:

$$\hat{\theta}(x) = [1/3(\alpha_2 - \alpha_1)][(\alpha_2 - \alpha_1)(2\overline{\theta} - 1) + t(|x - x_2| - |x - x_1|)], x \in [0,1]. \tag{7}$$

Differentiating (7) with respect to x, yields:[14]

$$\partial\hat{\theta}(x)/\partial x = \begin{cases} 0 & \text{if } \ x \in [0, x_1], \\ -2t/3(\alpha_2 - \alpha_1) < 0 & \text{if } \ x \in [x_1, x_2], \\ 0 & \text{if } \ x \in [x_2, 1]. \end{cases} \tag{8}$$

We see from (8) that given firms' locations x_1 and x_2, the taste of the marginal consumer remains unchanged for $x \in [0, x_1]$ and $x \in [x_2, 1]$, while taste decreases with

[13] Suppose that the second-order conditions are satisfied.

[14] In the second stage, firms' locations x_1 and x_2 have been determined in the first stage. We can thus have $\partial x_1/ \partial x = 0$ and $\partial x_2 /\partial x = 0$.

respect to x within the interval [x_1, x_2]. According to eqs. (7) and (8), the relationship of the taste of the marginal consumer and location x along the Hotelling line is depicted as the broken line on Figure 1. The area above the broken line represents the total output of the high quality firm, while the area below denotes the total output of the low quality firm.

Next, we turn to the first stage. Substituting (5) and (6) into (4), we can derive firm i's reduced aggregate operating profit function as follows:

$$\begin{aligned}\Pi_1 = [1/9(\alpha_2-\alpha_1)]\{&\int_0^{x_1}[(\alpha_2-\alpha_1)(2-\bar{\theta})+t(x_2-x_1)]^2\,dx \\ &+\int_{x_1}^{x_2}[(\alpha_2-\alpha_1)(2-\bar{\theta})+t(x_2+x_1-2x)]^2\,dx \\ &+\int_{x_2}^{1}[(\alpha_2-\alpha_1)(2-\bar{\theta})-t(x_2-x_1)]^2\,dx\},\end{aligned} \tag{9.1}$$

$$\begin{aligned}\Pi_2 = [1/9(\alpha_2-\alpha_1)]\{&\int_0^{x_1}[(\alpha_2-\alpha_1)(\bar{\theta}+1)+t(x_1-x_2)]^2\,dx \\ &+\int_{x_1}^{x_2}[(\alpha_2-\alpha_1)(\bar{\theta}+1)+t(-x_1-x_2+2x)]^2\,dx \\ &+\int_{x_2}^{1}[(\alpha_2-\alpha_1)(\bar{\theta}+1)-t(x_1-x_2)]^2\,dx\}.\end{aligned} \tag{9.2}$$

Differentiating (9) with respect to x_i, respectively, yields the profit-maximizing conditions for locations as follows:

$$\begin{aligned}\partial\Pi_1/\partial x_1 = (t)\{&\text{-}[\,2t/9(\alpha_2-\alpha_1)]\big[(x_2-x_1)(1-x_2+x_1)\big] \\ &+(4/9)(1/2-x_1)(2-\bar{\theta})\}=0,\end{aligned} \tag{10.1}$$

$$\begin{aligned}\partial\Pi_2/\partial x_2 = (t)\{&[2t/9(\alpha_2-\alpha_1)]\big[(x_2-x_1)(1-x_2+x_1)\big] \\ &+(4/9)(1/2-x_2)(\bar{\theta}+1)\}=0,\end{aligned} \tag{10.2}$$

where $1 < \overline{\theta} < 2$ and $0 \leq x_1 \leq x_2 \leq 1$.

Recalling that $0 \leq x_1 \leq x_2 \leq 1$ and $\alpha_1 \leq \alpha_2$, we find that the first term in the brace of the right-hand side of (10.1) is non-positive. This term can be named the competition effect, which shows that as the two firms move apart, the horizontal differentiation between the two products is increased, implying that price competition between firms is mitigated. Consequently, the competition effect attracts firm 1 to move leftward. Moreover, the competition effect is weakened, as the two products become more vertically differentiated (i.e., α_2 - α_1, is larger) or the transport rate is lower. On the other hand, the second term in the brace is denoted as the transportation cost saving effect (for simplicity, the cost-saving effect, hereafter), whose value is nonnegative. This arises because the first term is non-positive. In order to ensure an interior solution, the second term has to be non-negative to make the profit-maximizing condition equal zero. The cost-saving effect reflects firm *i*'s desire to move toward the center in order to save on the transportation cost. Consequently, firm 1's location equilibrium is determined by the balance of the competition and the cost-saving effects. We find from (10.2) that this result applies to firm 2's location equilibrium.

The location equilibria are subject to the second-order and the stability conditions as follows:

$$\partial^2\Pi_1 / \partial x_1{}^2 = [2t/9(\alpha_2-\alpha_1)]\{-2(\alpha_2-\alpha_1)(2-\overline{\theta}) - t[2(x_2-x_1)-1)]\} \le 0, \quad (11.1)$$

$$\partial^2\Pi_2 / \partial x_2{}^2 = [2t/9(\alpha_2-\alpha_1)]\{-2(\alpha_2-\alpha_1)(\overline{\theta}+1) - t[2(x_2-x_1)-1]\} \le 0, \quad (11.2)$$

$$\begin{aligned} J &= (\partial^2\Pi_1/\partial x_1^2)(\partial^2\Pi_2/\partial x_2^2) - (\partial^2\Pi_1/\partial x_1\partial x_2)(\partial^2\Pi_2/\partial x_2\partial x_1) \\ &= [8t^2/81(\alpha_2-\alpha_1)]\{2(\alpha_2-\alpha_1)(2-\overline{\theta})(\overline{\theta}+1) + 3t[2(x_2-x_1)-1]\} \ge 0. \end{aligned} \quad (11.3)$$

In addition, the location equilibria should fulfill the market-serving condition, which requires the output of each firm at its remote endpoint be positive.[15] This can be described as follows:

$$q_1(1;x_1,x_2) = [(\alpha_2-\alpha_1)(2-\overline{\theta}) - t(x_2-x_1)]/3(\alpha_2-\alpha_1) > 0, \quad (12.1)$$

$$q_2(0;x_1,x_2) = [(\alpha_2-\alpha_1)(1+\overline{\theta}) - t(x_2-x_1)]/3(\alpha_2-\alpha_1) > 0. \quad (12.2)$$

Solving (10), we can obtain location equilibria as follows:

$$x_1{}^A = x_2{}^A = 1/2, \quad (13.1)$$

$$\begin{aligned} x_1^D &= [2(\alpha_2-\alpha_1)(2+3\overline{\theta}-\overline{\theta}^3)/9t] + [(1-2\overline{\theta})/6], \\ x_2^D &= [2(\alpha_2-\alpha_1)(-4+3\overline{\theta}^2-\overline{\theta}^3)/9t] + [(7-2\overline{\theta})/6], \end{aligned} \quad (13.2)$$

where the superscript "*A*" ("*D*") denotes the variables associated with the case of the agglomeration (dispersion) equilibrium, respectively.

There are two possible location equilibria, central agglomeration and spatial dispersion. Substituting (13.1) into (11.1) - (11.3), (12.1) and (12.2), we have:

$$\partial^2\Pi_1/\partial x_1{}^2\Big|_{x_1^A,x_2^A} \le 0, \text{if } (\alpha_2-\alpha_1) \ge t/2(2-\overline{\theta}) \equiv \Delta_1^A, \quad (14.1)$$

$$\partial^2\Pi_2/\partial x_2{}^2\Big|_{x_1^A,x_2^A} \le 0, \text{if } (\alpha_2-\alpha_1) \ge t/2(1+\overline{\theta}) \equiv \Delta_2^A, \quad (14.2)$$

[15] See Yang *et al*. (2007).

$$J\big|_{x_1^A, x_2^A} \geq 0, \text{if } (\alpha_2 - \alpha_1) \geq 3t/2(2-\bar{\theta})(1+\bar{\theta}) \equiv \Delta_3^A, \tag{14.3}$$

$$q_1(1; x_2 = x_1 = 1/2) = (2-\bar{\theta})/3 > 0, \tag{15.1}$$

$$q_2(0; x_2 = x_1 = 1/2) = (1+\bar{\theta})/3 > 0. \tag{15.2}$$

Recall that $1 < \bar{\theta} < 2$. We figure out from (14) that $\Delta_3{}^A > \Delta_1{}^A > \Delta_2{}^A$. Thus, central agglomeration arises only if the degree of vertical differentiation is sufficiently large, say $(\alpha_2 - \alpha_1) \geq \Delta_3^A$. The intuition behind this result can be stated as follows. We have argued that the location equilibrium is determined by the competition and the cost-saving effects. We have also shown that the higher the degree of vertical differentiation, the weaker the competition effect will be. Therefore, as the degree of vertical differentiation is no less than the critical value $\Delta_3{}^A$, the competition effect is dominated by the cost-saving effect such that the two firms agglomerate at the center of the Hotelling line.

Next, substituting (13.2) into (11.1) - (11.3), (12.1) and (12.2), we have:

$$\partial^2\Pi_1/\partial x_1{}^2\big|_{x_1^D, x_2^D} \leq 0, \text{if } (\alpha_2 - \alpha_1) \leq 3t/2(2-\bar{\theta})(2\bar{\theta}-1) \equiv \Delta_1^D, \tag{16.1}$$

$$\partial^2\Pi_2/\partial x_2{}^2\big|_{x_1^D, x_2^D} = 2t[-2(\alpha_2-\alpha_1)(1+\bar{\theta})(2\bar{\theta}-1) - 5t]/27(\alpha_2-\alpha_1) \leq 0, \tag{16.2}$$

$$J\big|_{x_1^D, x_2^D} \geq 0, \text{if } (\alpha_2 - \alpha_1) \leq 3t/2(2-\bar{\theta})(1+\bar{\theta}) \equiv \Delta_2^D \equiv \Delta_3^A, \tag{16.3}$$

$$q_1(1; x_1^D, x_2^D) > 0, \text{if } (\alpha_2 - \alpha_1) > 4t/(2-\bar{\theta})(5+2\bar{\theta}) \equiv \Delta_3^D, \tag{17.1}$$

$$q_2(0; x_1^D, x_2^D) > 0, \text{if } (\alpha_2 - \alpha_1) > 4t/(1+\bar{\theta})(7-2\bar{\theta}) \equiv \Delta_4^D. \tag{17.2}$$

We find from (16) that $\Delta_2{}^D < \Delta_1{}^D$, and from (17) that $\Delta_3{}^D > \Delta_4{}^D$. We also find from (14.3) and (16.3) that $\Delta_3^A = \Delta_2^D$. Accordingly, we yield that the spatial dispersion arises as the degree of vertical differentiation lies in between $[\Delta_3^D, \Delta_2^D]$, i.e., $\Delta_3^D \le (\alpha_2 - \alpha_1) \le \Delta_3^A$. The same intuition applies to this result. The competition effect is strong enough to push the two firms apart, as the degree of vertical differentiation is no greater than the critical value, Δ_3^A.

Based on the above analysis, we can establish:

Proposition 1. *Assuming that firms engage in discriminatory pricing, we yield*:

(*i*) *Firms agglomerate at the center of the Hotelling line as the degree of vertical differentiation is high enough, i.e.,* $(\alpha_2 - \alpha_1) \ge \Delta_3^A$.

(*ii*) *Spatial dispersion emerges as the degree of vertical differentiation lies in between* $\Delta_3^D \le (\alpha_2 - \alpha_1) \le \Delta_3^A$.

Next, we explore the invalidity of the Principle of Maximum Differentiation. First of all, we examine this principle as the interior location equilibrium arises, which can be done via (13.2). Recall that $1 < \overline{\theta} < 2$ and $\alpha_1 \le \alpha_2$. We can find from (13.2) that $x_2^D = [2(\alpha_2 - \alpha_1)(-4 + 3\overline{\theta}^2 - \overline{\theta}^3)/9t] + [(7 - 2\overline{\theta})/6] < 5/6$.[16] This shows that firm 2 would never locate at the right end of the Hotelling line. Thus, the Principle of

[16] This arises because $\alpha_2 - \alpha_1 \ge 0, (-4 + 3\overline{\theta}^2 - \overline{\theta}^3) < 0$, and $[(7 - 2\overline{\theta})/6] < 5/6$.

Maximum Differentiation will never emerge in this case. Secondly, we examine this Principle when a corner solution for location equilibrium occurs. This arises as the conditions $\partial\Pi_1/\partial x_1 < 0$ and $\partial\Pi_2/\partial x_2 > 0$ hold. We can calculate from (10.1) that the former condition holds if $(\alpha_2 - \alpha_1) < t\,(x_2 - x_1)(1 - x_2 + x_1) / (1 - 2x_1)(2 - \bar{\theta}) \equiv \Delta_1^c$ and from (10.2) that the latter condition holds if $(\alpha_2 - \alpha_1) < t\,(x_2 - x_1)(1 - x_2 + x_1) / (2x_2 - 1)(1 + \bar{\theta}) \equiv \Delta_2^c$. We find that $\Delta_1^c = \Delta_2^c = 0$ as $x_1 = 0$ and $x_2 = 1$. Thus, the conditions hold only if $\alpha_2 - \alpha_1 < 0$, which contradicts the assumption $\alpha_1 \le \alpha_2$ and excludes satisfaction of the Principle of Maximum Differentiation as the corner solution for location equilibrium emerges.

Based on an analysis of eqs. (13) – (17), we can depict the relationship between firms' location equilibria and the degree of vertical differentiation as shown in Figure 2. The locus D_1AE represents firm 1's location equilibrium, while locus D_2AE denotes firm 2's location equilibrium.[17] Figure 2 shows that as the degree of vertical differentiation, $\alpha_2 - \alpha_1$ is no less than Δ_3^A, two firms agglomerate at the center of Hotelling line, while they take apart as the degree of vertical differentiation lies in between (Δ_3^D, Δ_3^A).

[17] Manipulating eq. (13.2), we yield

$\partial x_1^D / \partial(\alpha_2 - \alpha_1) = 2(2 + 3\bar{\theta} - \bar{\theta}^3)/9t > 0,$

$\partial x_2^D / \partial(\alpha_2 - \alpha_1) = 2(-4 + 3\bar{\theta}^2 - \bar{\theta}^3)/9t < 0,$ and

$\partial^2 x_i^D / \partial(\alpha_2 - \alpha_1)^2 = 0, i = 1,2.$

Accordingly, we find that D_1A and D_2A are linear, the slope of D_1A (D_2A) is positive (negative) and D_2A is steeper than D_1A due to the absolute value of the slope of D_2A larger than that of D_1A.

(Insert Figure 2 here)

Accordingly, we have:

Proposition 2. *Assuming that firms engage in discriminatory pricing, the Principle of Maximum Differentiation will never be satisfied.*

By assuming location is determined prior to quality in a one-dimensional model, Ferreira and Thisse (1996) derive the Max-Min and Min-Max result. Moreover, letting location be endogenously determined, Economides (1989) finds that firms locate as far apart as possible. However, by reversing the temporal ordering of location and quality decisions in a two-dimensional model, we show that the Principle of Minimum Differentiation can be valid if the degree of vertical differentiation is sufficiently high. By contrast, spatial dispersion emerges as the degree of vertical differentiation becomes sufficiently low. However, the Principle of Maximum Differentiation will never be satisfied.

We now examine the impact of the transport rate on the critical values of the determination of central agglomeration and spatial dispersion. Differentiating Δ_3^A and Δ_3^D with respect to t, we obtain:

$$\partial\Delta_3^A / \partial t = 3/2(2-\overline{\theta})(1+\overline{\theta}) > 0, \tag{18.1}$$

$$\partial \Delta_3^D / \partial t = 4/(2-\overline{\theta})(5+2\overline{\theta}) > 0. \quad (18.2)$$

Equations (18.1) demonstrate that other things equal, a rise in the transport rate increases the critical value of the degree of vertical differentiation for firms to remain agglomerating at market center. This arises because the competition effect gets to be stronger as the transport rate is higher. In order to balance this stronger separating effect, the critical value of the degree of vertical differentiation has to be higher to keep firms agglomerate at the market center. We see from (18.2) that a rise in the transport rate increase the critical value of the degree of vertical differentiation for serving the whole market. This happens because the delivered prices are increased, which reduces the demand of the remote endpoint, as the transport rate is higher. In order to keep two firms competing at the remote endpoint, the critical value of the degree of vertical differentiation has to be higher to balance the strengthened competition effect. Accordingly, we yield the following Lemma:

Lemma 1. *Other things being equal, the critical values of the degree of vertical differentiation for firms to keep agglomerate at the market center as well as to serve the entire market get to be higher, as the transport rate is higher.*

Manipulating (13.2), we can derive the distance (degree of horizontal differentiation) between the two firms' location equilibria under the case of spatial dispersion as follows:

$$x_2^D - x_1^D = 1-(2/3t)(\alpha_2 - \alpha_1)(2-\bar{\theta})(\bar{\theta}+1). \tag{19}$$

Differentiating the distance with respect to the degree of vertical differentiation as well as the transport rate, we have:

$$\partial(x_2^D - x_1^D)/\partial(\alpha_2 - \alpha_1) = -(2-\bar{\theta})(\bar{\theta}+1)/3t < 0, \tag{20.1}$$

$$\partial(x_2^D - x_1^D)/\partial t = 2(\alpha_2 - \alpha_1)(2-\bar{\theta})(\bar{\theta}+1)/3t^2 > 0. \tag{20.2}$$

We see from (20) that the distance, between the two firms, decreases as the degree of vertical differentiation rises, while it increases as the transport rate increases. Intuitively, the products become more differentiated leading to a weaker competition effect as the degree of the vertical differentiation is higher. The upshot is that the two firms approach each other spatially. On the other hand, the competition effect strengthens due to higher delivered prices as the transport rate rises. This induces firms to move further apart, while they charge higher prices and earn higher profits.

Accordingly, we can establish:

Proposition 3. *Assuming that firms engage in discriminatory pricing, firms locate further apart in the dispersion equilibrium, when the degree of vertical differentiation is lower or the transport rate is higher.*

3. The Extended Model with Uniform Delivered and Mill Pricing

In this section, we will examine firms' location equilibria as firms use uniform delivered and mill pricing in the commodity market. We first discuss the case of uniform delivered pricing. Firms charge the same delivered price at each point of the Hotelling line, respectively, in this case. Thus, the indirect utility of a consumer, who purchases from firm *i*, can be expressed as:

$$u = k + \theta\alpha_i - p_i^{\,u}, i = 1,2, \tag{21}$$

where the superscript "*u*" denotes the variables associated with the case of uniform delivered pricing.

The marginal consumer, who is indifferent between buying one unit of product from either firm, following (21), acts so as to satisfy:

$$\hat{\theta}^u = (p_2^{\,u} - p_1^{\,u})/(\alpha_2 - \alpha_1). \tag{22}$$

Firms' demand functions under the case of uniform delivered pricing are therefore equal to:

$$q_1^{\,u} = [(p_2^{\,u} - p_1^{\,u})/(\alpha_2 - \alpha_1)] - (\overline{\theta} - 1), \tag{23.1}$$

$$q_2^{\,u} = \overline{\theta} - [(p_2^{\,u} - p_1^{\,u})/(\alpha_2 - \alpha_1)]. \tag{23.2}$$

Firm's aggregate operating profits can be obtained by integrating its operating profit of each point along the Hotelling line, and can be expressed as:

$$\Pi_i{}^u = \int_0^1 (p_i{}^u - t|x - x_i|)q_i(x)dx = q_i{}^u[p_i{}^u - t(x_i{}^2 - x_i + \frac{1}{2})], i = 1,2. \tag{24}$$

In order to save space, we now omit the same steps as those in the case of discriminatory pricing, and jump directly to the profit-maximizing conditions for location. These conditions are as follows:[18]

$$\partial\Pi_i{}^u / \partial x_i = 2t(\alpha_2 - \alpha_1)q_1{}^u(1 - 2x_i) = 0, i = 1,2. \tag{25}$$

We see from (25) that the term on the right-hand side is denoted as a cost-saving effect, while the competition effect vanishes. This arises because firms charge the same price for every point along the Hotelling line, which leads to the result that firms are unable to increase price and profits by locating further away from each other. Thus, the competition effect disappears. The only effect left is the cost-saving effect, where firms will locate at the center of the Hotelling line to minimize transport costs. This result can be supported by solving (25) while considering all three constraints; viz. the second-order, the stability and the market serving conditions. We may write the firms' optimal locations as follows:[19]

$$x_i{}^u = \frac{1}{2}, i = 1,2. \tag{26}$$

Accordingly, we have the following proposition:

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[18] The second-order conditions are:
$\partial^2\Pi_1{}^u / \partial x_1{}^2 \le 0$, if $(\alpha_2 - \alpha_1) \ge [t/2(2-\overline{\theta})][6x_1(x_1 - 1) - 2x_2(x_2 - 1) + 1] \equiv \Delta_1{}^u$.
$\partial^2\Pi_2{}^u / \partial x_2{}^2 \le 0$, if $(\alpha_2 - \alpha_1) \ge [t/2(1+\overline{\theta})][6x_2(x_2 - 1) - 2x_1(x_1 - 1) + 1] \equiv \Delta_2{}^u$.

[19] Substituting $x_1{}^u = x_2{}^u = 1/2$ into the second-order conditions, we can calculate that the critical values $\Delta_1{}^u = \Delta_2{}^u = 0$. Moreover, the stability condition is definitely greater than zero and the market-serving is also satisfied.

Proposition 4. *Assuming that firms adopt uniform delivered pricing, central agglomeration is the unique location equilibrium if the degree of vertical differentiation is greater than zero.*

This result is sharply different from that derived in the case of discriminatory pricing, in which spatial dispersion happens as the degree of vertical differentiation is sufficiently low.

Next, we turn to examine the case of mill pricing. The indirect utility of a consumer residing at site x can be rewritten as:

$$u(x) = k + \theta\alpha_i - p_i^f - t|x - x_i|, \tag{27}$$

where the superscript "*f*" denotes the variables associated with the case of mill pricing.

The marginal consumer's choice satisfies:

$$\hat{\theta}^f(x) = [p_2^f - p_1^f + t(|x - x_2| - |x - x_1|)]/(\alpha_2 - \alpha_1). \tag{28}$$

Firms' demand functions under the case of mill pricing can be expressed as:

$$q_1^f(x) = [(p_2^f - p_1^f) + t(|x - x_2| - |x - x_1|)/(\alpha_2 - \alpha_1)] - (\overline{\theta} - 1), \tag{29.1}$$

$$q_2^f(x) = \overline{\theta} - [(p_2^f - p_1^f) + t(|x - x_2| - |x - x_1|)/(\alpha_2 - \alpha_1)]. \tag{29.2}$$

Firms' aggregate operating profits functions under the case of mill pricing can similarly be written as:

$$\Pi_1{}^f = \int_0^1 q_1{}^f(x) p_1{}^f dx$$
$$= p_1{}^f[(p_2{}^f - p_1{}^f) + t(x_2 - x_1)(x_2 + x_1 - 1) - (\overline{\theta} - 1)(\alpha_2 - \alpha_1)/(\alpha_2 - \alpha_1)], \tag{30.1}$$

$$\Pi_2{}^f = \int_0^1 q_2{}^f(x) p_2{}^f dx$$
$$= p_2{}^f[\overline{\theta}(\alpha_2 - \alpha_1) - (p_2{}^f - p_1{}^f) - t(x_2 - x_1)(x_2 + x_1 - 1)/(\alpha_2 - \alpha_1)]. \tag{30.2}$$

As before, we omit all intervening steps as with in the case of discriminatory pricing, and jump directly to the profit-maximizing conditions of location. These conditions are as follows:

$$\partial \Pi_i{}^f / \partial x_i = 2t(\alpha_2 - \alpha_1) q_1{}^u (1 - 2x_i) = 0, i = 1,2. \tag{31}$$

Likewise, solving (31) with due consideration of all three constraints, yields the firms' optimal locations as follows:

$$x_i{}^f = \frac{1}{2}, i = 1,2. \tag{32}$$

We see from (30) that central agglomeration is the unique location equilibrium. This occurs because the competition effect is no longer present due to charging the same mill price at each point of the Hotelling line. Consequently, we can establish:

Proposition 5. *Assuming that firms charge mill pricing, central agglomeration is the unique location equilibrium if the degree of vertical differentiation is greater than zero.*

4. Concluding Remarks

This paper has constructed a two-dimensional framework to take account of both the features of horizontal and vertical differentiation. It has employed a two-stage game to examine location configurations, where firms first simultaneously decide optimal locations and then engage in Bertrand price competition with three pricing policies, taking the degree of vertical differentiation as given. We have shown that firms' location decisions depend on two countervailing forces: the centrifugal competition effect and the centripetal cost-saving effect. The focus of this paper is on the impact of vertical differentiation to firms' location decision via changes in the competition effect. We have argued that the higher the degree of vertical differentiation, the weaker the competition effect will be. This weakens the centrifugal competition effect in the determination of firms' locations and creates the possibility for satisfaction of the Principle of Minimum Differentiation. Several striking results are derived as follows.

First of all, assuming that firms engage in discriminatory pricing in the commodity market, firms agglomerate at the market center when the degree of vertical differentiation is sufficiently high, while they move apart when it lies in between $\Delta_3^D \le (\alpha_2 - \alpha_1) \le \Delta_3^A$. Moreover, with respect to dispersion equilibrium, firms locate further apart when the degree of vertical differentiation is lower, as well as when the transport rate is higher. However, the Principle of Maximum Differentiation can never emerge.

Secondly, firms locate at the market center as long as the degree of vertical differentiation is greater than zero, in both cases where firms conduct uniform delivered and mill pricing.

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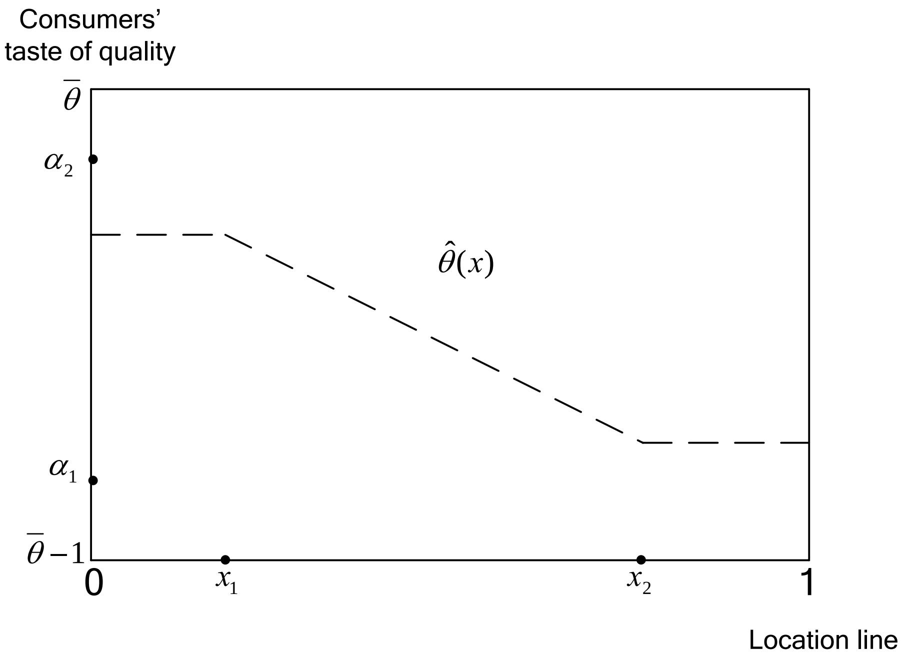

Fig. 1. The two-dimensional framework

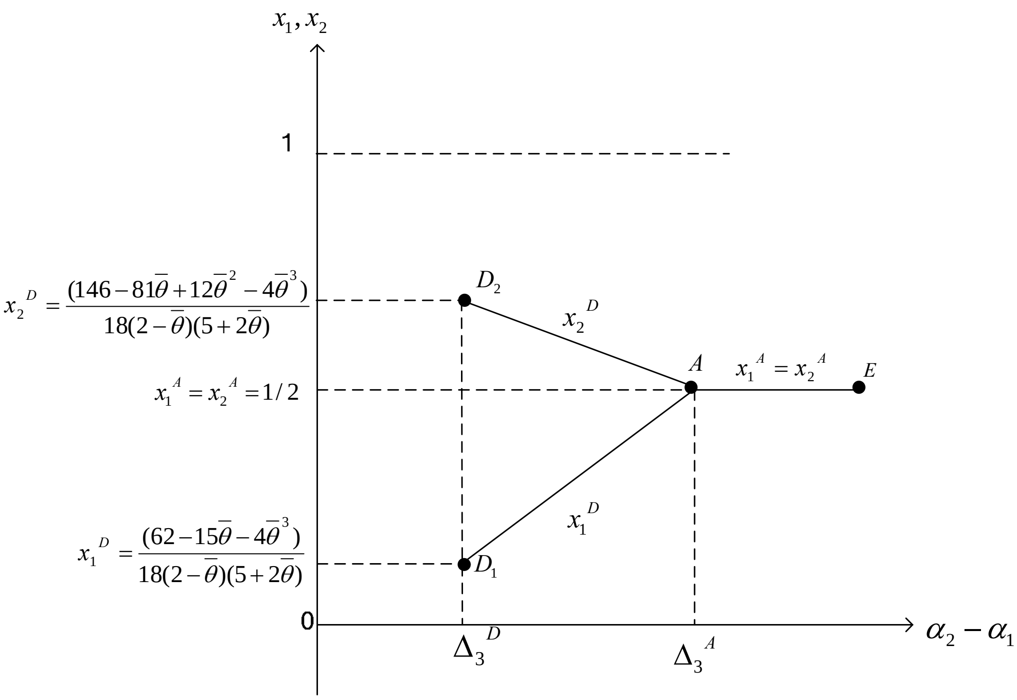

Fig.2. The relationship between firms' location equilibrium and the degree of vertical differentiation