Economic Geography with Multiple Manufacturing Sectors: a Synthesis of the FC and the FE Model

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Abstract: This paper synthesizes a modified model to incorporate both footloose capital and footloose entrepreneur manufacturing industries in the economy. The examination is based on a tradable analytical structure between two identical economies, and it shows that the footloose capital and the footloose entrepreneur manufacturing industries will interact each other and yield more fruitful and realistic spatial distributions which are driven by the demand-linked forces and cost-linked forces within industry and between industries, and also it yields two different kinds of pitchfork bifurcation in the footloose entrepreneur manufacturing sector, '*subcritical*' and '*supercritical*', and the determination of the bifurcation is depending on an interesting exogenous parameters. More importantly, it features the emergence of partial agglomerations of both industries, and yields five types of equilibrium configuration which is depending on the form of bifurcation, the break point, and the sustain point of various trade freeness on both sectors. It is also shown that the alternative configuration is determined by the exogenous parameters associated with the consumer expenditure share as well as the elasticities of substitution within these two manufacturing sectors, and accordingly the emergence of the different bifurcation represents the different evolutionary trajectories of equilibrium configurations. This finding of the bifurcation switching is absent in the conventional literature of new economy geography, in which only one mobile production factor is incorporated in the one or two IRS manufacturing industry.

JEL Classification: F12, F15, F21, R12.

Keywords**:** Multiple industries, Footloose capital, Footloose entrepreneur, Partial agglomeration, Bifurcation.

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1 Introduction

Ever since Krugman (1991) developed a frontier model that shows how a country can endogenously determine an industrialized '*core*' and an agricultural '*periphery*' for *ex ante* two identical economy, then many studies have examined a few interesting issues of the economic geography based on Krugman's core-periphery (CP) model. While Krugman (1991) and the followers only considered one kind of mobile production factor across regions for manufacturing industry, for example, Martin and Rogers' (1995) footloose capital (FC) model incorporated mobile capital into manufacturing industry and assumed that labor can not move across regions but can switch between constant-return-to-scale agricultural and increasing-return-to-scale (IRS) manufacturing industry. Krugman and Venables (1995) assumed that all manufacturing goods can be consumed by the households for final consumption or used by production inputs by the other manufacturing firms, and then they assumed the manufacturing goods (either employed by production inputs or final consumption) is tradable across countries, while the production input of labor is immobile. With similar specification, Venables (1996) specify manufacturing industry into two sectors: upstream and downstream. The only input of intermediate goods for the downstream industry is tradeable across countries, while the input of the labors for both upstream firms and downstream firms are immobile across countries. And Forslid and Ottaviano (2003) develop a footloose entrepreneur (FE) model to consider the mobile skilled worker and immobile unskilled worker in the manufacturing industry. In the alternative model on the examination of new economic geography, Ottaviano *et al* (2002) also only consider one kind of mobile worker in the model. Furthermore, Tabuchi and Thisse (2006) consider the model based on the Ottaviano *et al* (2002) to incorporate two industries to employ the mobile worker only and produce two different goods that interact in consumers' demands, however they assume the extreme case in which one good is perfectly mobile whereas the other goods is nontradeable. All the literatures mentioned above viewed only one production input in the manufacturing industry is mobile across regions. Even though NEG focuses on one mobile input, there is no reason to believe that what is true for two mobile inputs employed by the two different industry and interaction each other through the production sector associated with one common production input and the consumers' utility function still holds in the new trade theory.[1]

No matter how they set the fixed cost of production in their models, most of these conventional studies, only consider one mobile production input in the IRS sector and result in only two stable equilibrium outcomes, full agglomeration or completely dispersed of the industry configuration. The existence of partial agglomerations in real world never be proved as one of possible configurations except Tabuchi's (1998) and Pflüger's (2004) works. Tabuchi (1998) involved the intra-city congestions into Krugman's model and re-examine the spatial configuration of economic activities. He also considered only one IRS industry. Pflüger (2004) exhibited partial agglomerations based on a '*supercritical pitchfork bifurcation*' with quasi-linear utility function rather than the '*subcritical pitchfork bifurcation*' of the conventional models with Cobb-Douglas setting. Pflüger (2004) concluded that this feature may be a better description for some of the agglomerative processes that are initiated by economic integration (decreasing transport costs) than the '*catastrophic*' emergence of complete agglomeration predicted by the CP model. However, Pflüger (2004) also involved only one IRS sector driven by human capital (skilled labor) as the fixed cost and documented only one kind of pitchfork bifurcation.

Regarding the literature associated with multiple IRS sectors, Fujita *et al.* (1999a) is the first paper to investigate this issue. They used Cobb-Douglas utility function to develop a general spatial-equilibrium model associated with multiple cities and multiple manufacturing industries to resurrect the central place theory. The fixed cost employed by their manufacturing industries is one kind of homogenous labor. It implies that the labor can migrate across different regions and manufacturing industries freely as specified by Krugman (1991). By the contrast, Tabuchi and Thisse (2006) used quasi-linear utility function to study the location of two industries and to investigate whether the results valid for one industry holds true in the case of two industries. However, they still assumed only one kind of freely mobile labor across the manufacturing industries and countries. Zeng (2006) incorporated more labor heterogeneity

[1] Even though Fujita *et al* (1999), Venable (1999), and Tabuchi and Thisse (2006) specify a mutiple-industry model, while only one mobile input in production function of manufacturing industry is considered.

into his multiple-sector model. Setting quasi-linear utility function and using skilled labor as the fixed costs of his manufacturing sectors, Zeng (2006) is not only classified labors into skilled and unskilled as shown in Forslid and Ottaviano (2003), but also differentiated his manufacturing productions from each other by employing different kinds of skilled labor. In other words, the skilled labor in Zeng (2006) is heterogeneous, thus the skilled labor can not switch between different manufacturing industries but they can only migrate across regions. In addition, the setting of the transportation cost for the manufacturing goods in these three papers is not identical. Fujita *et al.* (1999a) and Tabuchi and Thisse (2006) specify a different transportation costs for various manufacturing goods while Zeng (2006) set the identical transportation cost for every kind of manufacturing good. All these literature demonstrated the necessity to model multiple IRS sectors so as to comprehend the real economy better. And how heterogeneities of manufacturing production factors and of transportation costs impact industrial equilibrium configurations should be studied further.

Therefore, the aim of this paper is to develop a model associated with two IRS industries as well as two mobile production inputs with Cobb-Douglas utility function, in which these two IRS sectors interact each other through the common production input (unskilled labor) and utility function and drive the demand-linked forces and cost-linked forces of agglomeration. The two IRS industries are specify by a FE manufacturing sector and a FC manufacturing sector respectively. Therefore, this paper investigates the industrial configurations based on a tradable analytical structure between two identical economies. Thus, this paper builds up a synthesis of Martin and Rogers' (1995) FC model and Forslid and Ottaviano's (2003) FE model to see the interactions of two IRS industries associated with two mobile production inputs to re-examine the industry configuration by the concerning in the new trade theory.

Through the examination based on a tradable analytical structure between two identical economies as shown in the literature, this paper not only shows that the FC manufacturing sector and the FE manufacturing sector will interact each other and yield more complicated and realistic spatial distributions driven by the demand-linked forces and cost-linked forces within industry and between industries, but also finds out the emergence of two different kinds of pitchfork bifurcation in the FE manufacturing sector, '*subcritical*' and '*supercritical*', and this bifurcation is depending on the combination of a few interesting exogenous parameters. Furthermore, it features the existence of partial agglomerations of both industries, and it exists five types of equilibrium configuration, in which would be classified by the form of bifurcation, the break point, and the sustain point of various trade freeness on both industries. This paper also documents that the alternative specifications of the exogenous parameters associated with the consumer expenditure share as well as the elasticities of substitution within these two manufacturing sectors will yield the shifting between the bifurcation types of the FE manufacturing sector, and the emergence of the different bifurcation represents different evolutionary trajectories of equilibrium configurations. This finding of the bifurcation switching is absent in the conventional literature of new economy geography, in which only one mobile production factor is incorporated in the one or two IRS manufacturing industry.

The remainder of the paper is organized as follows. The model is specified in section 2, whereas the global analysis is characterized in section 3. In section 4, we examine the equilibrium and stability. Section 5 features the equilibrium configurations. In section 6, we investigate the economic institution of the determination of the types of different pitchfork bifurcation with simulation scenarios set by given exogenous parameters. And Section 7 gives the concluding remarks.

2 The model

Consider the economic space formed by two regions, denoted by i and j, and involves three sectors, the agricultural sector ($\mathbb{A}$-sector), the footloose capital manufacturing sector ($\mathbb{FC}$-sector) and the footloose entrepreneur manufacturing sector ($\mathbb{FE}$-sector). Total endowments of skilled and unskilled labor are H and L respectively, the skilled worker can be thought of as self-employed entrepreneurs and can migrate freely between regions while unskilled workers, thus $H_i + H_j = H$ and $L_i + L_j = L$, where H_i and L_i is the employment of the skilled and unskilled labor in region i. Each worker supplies one unit of labor inelasticity, and also each of them regardless of skilled or unskilled has an identical K units of capital endowment in each region. With respect to the endowment of their capital, each worker can choose her own capital to invest in domestic or foreign industries. Accordingly, we specify "unskilled labor" as the immobile production factor across regions, and the "skilled labor" as well as the "capital" the mobile production factors. And also we assume that the unskilled labor can work on the all these three sectors, while the skilled worker (capital) is only employed in the $\mathbb{FE}$-sector ($\mathbb{FC}$-sector). Hence, we allow the unskilled worker to freely move between the sectors within the region while immobile across regions. In addition, we also consider a continuum of footloose firms in the both $\mathbb{FE}$-sector and $\mathbb{FC}$-sector.

There are three goods in the economy, and each consumer must consume all these three goods. The first good is homogenous and produced by the $\mathbb{A}$-sector with constant return to scale, and it is produced by the unskilled labor only. The second good is a horizontally differentiated good which is produced by the $\mathbb{FC}$-sector, we call the $\mathbb{FC}$-good, and employ both the capital and unskilled worker as production factors. The third good is also the horizontally differentiated good, and is provided by the $\mathbb{FE}$-sector, denoted by $\mathbb{FE}$-good, and used the skilled and unskilled worker as production factors. Both capital and skilled labor specified as a fixed production inputs, and the unskilled worker a variable production inputs. There is no scope economies so that, due to increasing returns to scale in both $\mathbb{FE}$-sector and $\mathbb{FC}$-sector, there is a one-to-one relationship between firms and differentiated goods in these two industries. Since each firm sells a differentiated good in both sectors, the firm faces a downward-sloping demand under increasing return to scale and imperfect competition in both manufacturing sectors.

2.1 Household consumer behaviors

Household preferences are identical between workers as well as regions, and the utility function in region i is captured by the following:

$$U_i = A_i^{\alpha}\ M_i^{\beta}\ X_i^{\gamma}, \quad \alpha + \beta + \gamma = 1, \qquad i = r, s \tag{1}$$

where A_i is the consumption of agricultural products ($\mathbb{A}$-good), M_i the consumption of $\mathbb{FC}$-good, X_i consumption of $\mathbb{FE}$-good, α, β, and γ is the expenditure share of $\mathbb{A}$-good, $\mathbb{FC}$-good, and $\mathbb{FE}$-good, respectively, where $0 < \alpha < 1$, $0 < \beta < 1$, and $0 < \gamma < 1$.

Each individual in region i maximizes her utility subject to the income constraint Y_i:

$$P_{A_i}\ A_i + P_{M_i}\ M_i + P_{X_i}\ X_i = Y_i \tag{2}$$

where P_{A_i} is the price of $\mathbb{A}$-goods in region i, P_{M_i} is the composite price index of $\mathbb{FC}$-good in region i, P_{X_i} is the composite price index of $\mathbb{FE}$-good in region i. On the other hand, the income of the represent worker in region i sources from the wage income of both skilled (w_{H_i}) and unskilled labors (w_{L_i}) as well as the capital return from the capital investment in domestic and foreign countries, respectively, that is

$$Y_i = w_{L_i}\ \frac{L}{2} + w_{H_i}\ H_i + r_i\ K(\frac{L}{2} + H_i)k_i + r_j\ K(\frac{L}{2} + H_i)(1 - k_i),\ i, j = r, s,\ i \neq j \tag{3}$$

where k_i is the share of capital investment in domestic manufacturing industry with respect to region i, and thus $1 - k_i$ is the share in foreign manufacturing industry. For simplicity, we assume that both skilled and unskilled labor in the same region have identical behavior on capital investment.

Since the $\mathbb{A}$-sector is a constant return to scale industry by employing the unskilled labor only, and therefore the $\mathbb{A}$-good is chosen as numeraire, thus $P_{A_i} = w_{L_i} = 1$. Then, with the utility maximization and budget constraint, we have the indirect utility function in region i, i.e. V_i, as following:

$$V_i = \alpha^\alpha\ \beta^\beta\ \gamma^\gamma\ Y_i\,(P_{M_i})^{-\beta}\,(P_{X_i})^{-\gamma} \tag{4}$$

2.1.1 Demand for footloose capital manufacturing sector

Standard utility maximization of () subject to the budget constrain () yields demand of each variety of $\mathbb{FC}$-good by households in region i for a variety produced in the region i and j, respectively, as

$$m_{ii} = \beta\ \ (P_{M_i})^{\sigma_M - 1}\,(p_{M_{ii}})^{-\sigma_M}\,Y_i \tag{5}$$

$$m_{ji} = \beta\ \ (P_{M_i})^{\sigma_M - 1}\left(\tau_M\ p_{M_{jj}}\right)^{-\sigma_M} Y_i \tag{6}$$

where m_{ii} is the demand of each $\mathbb{FC}$-good produced and consumed in region i, m_{ji} is the demand of each $\mathbb{FC}$-good produced in region j and consumed in region i, $p_{M_{ii}}$ $(_{M_{jj}})$

is the unit price of $\mathbb{FC}$-good produced and consumed in region i (j), and σ_M denotes the demand elasticity and the elasticity of substitution of the $\mathbb{FC}$-goods, τ_M denotes the transport cost of $\mathbb{FC}$-good between country i and j, and P_{M_i} is the price index of $\mathbb{FC}$-goods in region i, and denoted by

$$P_{M_i} = \left[n_{M_i} \left(p_{M_{ii}} \right)^{1-\sigma_M} + n_{M_j} \left(p_{M_{ji}} \right)^{1-\sigma_M} \right]^{\frac{1}{1-\sigma_M}} \tag{7}$$

where n_{M_i} (n_{M_j}) is the number of $\mathbb{FC}$-firms in region i (j), $p_{M_{ji}}$ is the unit price of $\mathbb{FC}$-good produced in region j and consumed in region i.

2.1.2 Demand for footloose entrepreneur manufacturing sector

Similar utility maximization processes, we have the demand of each variety of $\mathbb{FE}$-good in region i, and produced in the region i and j, respectively, as follows:

$$x_{ii} = \gamma \ \left(P_{X_i} \right)^{\sigma_X - 1} \left(p_{X_{ii}} \right)^{-\sigma_X} Y_i \tag{8}$$

$$x_{ji} = \gamma \ \left(P_{X_i} \right)^{\sigma_X - 1} \left(\tau_X \ p_{X_{jj}} \right)^{-\sigma_X} Y_i \tag{9}$$

where x_{ii} is the consumption of each $\mathbb{FE}$-good produced and consumed in region i, x_{ji} is the consumption of each $\mathbb{FE}$-good produced in region j and consumed in region i, $p_{X_{ii}}$ $(p_{X_{jj}})$ is the unit price of $\mathbb{FE}$-good produced and consumed in region i (j), σ_X denotes both the elasticity of demand of any variety and the elasticity of substitution between any two varieties of $\mathbb{FE}$-goods, τ_X the transport cost of $\mathbb{FE}$-good between country i and j, and P_{X_i} represents the CES price index of $\mathbb{FE}$-good in region i, and denoted by

$$P_{X_i} = \left[n_{X_i} \left(p_{X_{ii}} \right)^{1-\sigma_X} + n_{X_j} \left(p_{X_{ji}} \right)^{1-\sigma_X} \right]^{\frac{1}{1-\sigma_X}} \tag{10}$$

where n_{X_i} (n_{X_j}) is the number of $\mathbb{FE}$-firms in region i (j), and $p_{X_{ji}}$ is the unit price of $\mathbb{FE}$-good produced in region j and consumed in region i.

2.2 The production

Now, we move to the examination of the production side. Firms in $\mathbb{FC}$-sector and $\mathbb{FE}$-sector are monopolistically competitive and employ different fixed production factors under increasing return to scale. In the economy, the firm in the $\mathbb{FC}$-sector uses capital as fixed input and unskilled labor as variable inputs, while the firm in the $\mathbb{FE}$-sector employs skilled labor as fixed input and unskilled labor as variable inputs. Baldwin *et al.* (2003) documents such two kinds of the industry setting as 'the footloose entrepreneur' and 'the footloose capital' model, respectively. Both $\mathbb{FE}$-good and $\mathbb{FC}$-good are horizontally differentiation, and ensures a one-to-one relationship between firms and varieties. And please notice that both sectors as well as the agricultural sector employ the common production factor, i.e., unskilled worker. Therefore, all these three sectors will compete each other on the employment of unskilled labor.

2.2.1 Production of footloose capital manufacturing sector

From the equilibrium between supply and demand sides and the utility maximization, we have the quantity of $\mathbb{FC}$-good production for each firm in region i, i.e. q_{M_i}, as the following

$$q_{M_i} = \beta \left[Y_i \left(P_{M_i}\right)^{\sigma_M - 1} \left(p_{M_{ii}}\right)^{-\sigma_M} + Y_j \left(P_{M_j}\right)^{\sigma_M - 1} \left(p_{M_{ii}}\right)^{-\sigma_M} \left(\tau_M\right)^{1-\sigma_M} \right] \tag{11}$$

In order to provide q_M units of $\mathbb{FC}$-good, a firm incurs a fixed input requirement of F_M units of capital and a marginal input requirement of c_M units of unskilled labor. Therefore, a typical $\mathbb{FC}$-firm located in region i maximizes profit:

$$\begin{aligned} \pi_{M_i} &= p_{M_{ii}}\, m_{ii} + p_{M_{ij}}\, m_{ij} - F_M\, r_i - c_M \left(m_{ii} + \tau_M\, m_{ij}\right) w_{L_i} \\ &= p_{M_{ii}}\, q_{M_i} - F_M\, r_i - c_M\, q_{M_i} \end{aligned} \tag{12}$$

since the firms in the agricultural sector produce a homogenous good under perfect competition and constant return to scale and employ only unskilled labor. Without loss of generality, the price of agricultural good is chosen as numeraire, i.e., $P_A = 1$, so that the wage of the unskilled labor w_{L_i} is equal to 1. And $\tau_M\, m_{ij}$ represents total supply to the demand for the distant region j, in which the of the iceberg transport costs is associated. And $q_{M_i} = m_{ii} + \tau_M \ m_{ij}$ is the total production by a typical $\mathbb{FC}$-firm in region i. Therefore, the first order condition for profit maximization yields

$$p_{M_{ii}} = \frac{\sigma_M \ c_M}{\sigma_M - 1} \tag{13}$$

$$p_{M_{ij}} = \frac{\tau_M \ \sigma_M \ c_M}{\sigma_M - 1} \tag{14}$$

Substituting (13) and (14) into (**??**) gives the composite price index of $\mathbb{FC}$-good in region i as:

$$P_{M_i} = \frac{\sigma_M \ c_M}{\sigma_M - 1} \left[n_{M_i} + n_{M_j} \ \phi_M\right]^{\frac{1}{1-\sigma_M}} \tag{15}$$

where $\phi_M \equiv (\tau_M)^{1-\sigma_M} \in (0,1)$ is a measure of the degree of the freeness of trade on $\mathbb{FC}$-good. Furthermore, substituting (13), (14) and (15) into (11), we have:

$$q_{M_i} = \frac{\beta (\sigma_M - 1)}{\sigma_M \ c_M} \left[\frac{Y_i}{n_{M_i} + n_{M_j} \ \phi_M} + \frac{Y_j \ \phi_M}{n_{M_i} \ \phi_M + n_{M_j}}\right] \tag{16}$$

Similarly, under free entry assumption of monopolistic competition in $\mathbb{FC}$-sector, the profit of each $\mathbb{FC}$-firm in region i equals 0 ($\pi_{M_i} = 0$). Using (13), (14) and (16), we have the price of capital in region i as:

$$r_i = \frac{\beta}{F_M \ \sigma_M} \left[\frac{Y_i}{n_{M_i} + n_{M_j} \ \phi_M} + \frac{Y_j \ \phi_M}{n_{M_i} \ \phi_M + n_{M_j}}\right] \tag{17}$$

And given the fixed input requirement F_M, capital market clearing condition implies that the number of $\mathbb{FC}$-firms in equilibrium is determined by:

$$n_{M_i} = \frac{K\left(\frac{L}{2} + H_i\right) k_i + K\left(\frac{L}{2} + H_j\right)(1 - k_j)}{F_M} \tag{18}$$

so that the number of active $\mathbb{FC}$-firms in a region is proportional to the amount of total capital employment, and this total capital inputs incorporate the investments from both home and foreign households.

2.2.2 Production of footloose entrepreneur manufacturing sector

Now, we move to the examination of the production of $\mathbb{FE}$-good, we have the quantity of $\mathbb{FE}$-good production for each firm in region i, i.e. q_{X_i}, as the following:

$$q_{X_i} = \gamma \left[Y_i \left(P_{X_i}\right)^{\sigma_X - 1} \left(p_{X_{ii}}\right)^{-\sigma_X} + Y_j \left(P_{X_j}\right)^{\sigma_X - 1} \left(p_{X_{ii}}\right)^{-\sigma_X} \left(\tau_X\right)^{1-\sigma_X} \right] \tag{19}$$

In order to provide q_X units of $\mathbb{FE}$-good, a firm incurs a fixed input requirement of F_X units of skilled labor and a marginal input requirement of c_X units of unskilled labor. Therefore, a typical $\mathbb{FE}$-firm located in region i maximizes its profit as:

$$\begin{aligned} \pi_{X_i} &= p_{X_{ii}}\ x_{ii} + p_{X_{ij}}\ x_{ij} - F_X\ w_{H_i} - c_X \left(x_{ii} + \tau_X\ x_{ij}\right) w_{L_i} \\ &= p_{X_{ii}}\ q_{X_i} - F_X\ w_{H_i} - c_X\ q_{X_i} \end{aligned} \tag{20}$$

where $\tau_X\ x_{ij}$ implies the total supply to the distant region j, and the household only obtain x_{ij} units since the setting of iceberg transport costs. And $q_{X_i} = x_{ii} + \tau_X\ x_{ij}$ is the total production by a typical $\mathbb{FE}$-frim in region i. The first order condition for maximization gives:

$$p_{X_{ii}} = \frac{\sigma_X\ c_X}{\sigma_X - 1} \tag{21}$$

$$p_{X_{ij}} = \frac{\tau_X\ \sigma_X\ c_X}{\sigma_X - 1} \tag{22}$$

Substituting (21) and (22) into (**??**) yields:

$$P_{X_i} = \frac{\sigma_X\ c_X}{\sigma_X - 1} \left[n_{X_i} + n_{X_j}\ \phi_X\right]^{\frac{1}{1-\sigma_X}} \tag{23}$$

where $\phi_X \equiv \left(\tau_X\right)^{1-\sigma_X} \in (0,1)$. It denotes the degree of freeness of trade on $\mathbb{FE}$-good. Further substituting (21), (22),and (23) into (19), we obtain:

$$q_{X_i} = \frac{\gamma\left(\sigma_X - 1\right)}{\sigma_X\ c_X} \left[\frac{Y_i}{n_{X_i} + n_{X_j}\ \phi_X} + \frac{Y_j\ \phi_X}{n_{X_i}\ \phi_X + n_{X_j}}\right] \tag{24}$$

Under free entry assumption of monopolistic competition, the profit of each $\mathbb{FE}$-firm in each region equals 0 ($\pi_{X_i} = \pi_{X_j} = 0$). Using (21), (22), and (24), we can obtain the nominal wage of skilled labor in region i as:

$$w_{H_i} = \frac{\gamma}{F_X\ \sigma_X} \left[\frac{Y_i}{n_{X_i} + n_{X_j}\ \phi_X} + \frac{Y_j\ \phi_X}{n_{X_i}\ \phi_X + n_{X_j}}\right] \tag{25}$$

Given the fixed input requirement F_X, skilled labor market clearing condition implies that in equilibrium the number of $\mathbb{FE}$-firms is determined by:

$$n_{X_i} = \frac{H_i}{F_X} \tag{26}$$

so that the number of active $\mathbb{FE}$-firms in a region is proportional to the number of its skilled workers in that region.

3 Global Analyses

We assume that the capital endowment as well as the capital investment behavior of all residents in this economy is identical regardless of skilled and unskilled in each region, and since the mobile of capital is costless, therefore they will invest in the region associated with the higher capital return, thus we denote $k_i = 1 - k_j = k$, $i \neq j = 1, 2$, as the share of the capital endowment of the household in region i and investment in region i, i.e. $k_i = k$, while each worker in region j will also put k share of her capital endowment to invest in region i, that is $1 - k_j = k$. It implies that every worker (or consumer) has identical investment portfolio no matter where she lives as well what the skilled she has. In other words, k can be regarded as the share of the worldwide total capital endowment employed in region i. Moreover, we define $h \equiv H_i/H$ as the share of skilled workers that reside in region i, then the system consisting of a few equations determines the endogenous variables n_{X_i}, n_{X_j}, n_{M_i}, n_{M_j}, Y_i, Y_j, w_{H_1}, w_{H_2}, r_i, and r_j for a given allocation of skilled labor h and capital k. Solving the equation system simultaneously, we can get solutions of w_{Hi},and r_i (see the Appendix 1 for the details)

Define $w \equiv w_{H_i}/w_{H_j}$ (see the Appendix 1 for the details), and then differentiating w with respect to h shows that the region i with more skilled workers offers a higher (lower) wage whenever ϕ_X is larger (smaller) than the threshold ϕ_{Xw}:

$$\phi_{Xw} = \frac{(H + L)\,\sigma_M\,(\sigma_X - \gamma) - H\beta\sigma_X}{(H + L)\,\sigma_M\,(\sigma_X + \gamma) + H\beta\sigma_X} \tag{27}$$

where $\phi_{Xw} \in (0, 1)$. Similar interpretation with that of Forslid and Ottaviano (2003), the ϕ_{Xw} is the result of a trade-off between two opposing forces, ‘market crowding effect’ and ‘market size effect’.

Furthermore, define $r \equiv r_i/r_j$ as the related unit capital return (or price) for region i to j, then differentiating r with respect to h and k respectively shows that (see the Appendix 2 for the details)

$$\frac{\partial r}{\partial h} > 0, \frac{\partial r}{\partial k} < 0$$

The intuition with $\frac{\partial r}{\partial h} > 0$ represents that the agglomeration force sourced from 'market size effect' is dominant, and it implies that $\mathbb{FC}$-firms in the region associated with more skilled workers (i.e., more $\mathbb{FE}$-firms) are willing to pay higher rewards to attract capital. And the outcome with $\frac{\partial r}{\partial k} < 0$ implies that the dispersion force with 'market crowding effect' is dominant, and it documents that $\mathbb{FC}$-firms in the region i associated with more capital (i.e., more $\mathbb{FC}$-firms) will pay lower price for the capital investment because of capital input competition. The trade-off of both effects will determine the mobility of capital, in turn the distribution of $\mathbb{FC}$-firms.

Differentiating P_{X_i} with respect to h and k, respectively, we have (see the Appendix 2 for the details)

$$\frac{\partial P_{X_i}}{\partial h} < 0, \frac{\partial P_{M_i}}{\partial k} < 0$$

which reveals the agglomeration force associated with 'cost-of-living effect'. They implies that the region associated with more skilled workers has a lower local $\mathbb{FE}$-good price index since more varieties of $\mathbb{FE}$-good produced in this region. Similarly, the region i associated with more capital has a lower local $\mathbb{FC}$-good price index. In turn, other things be equal, either increasing skilled labors or increasing the capital will induce the cost of living decreasing. Since we assume that every worker must consume both $\mathbb{FC}$-good and $\mathbb{FE}$-good, therefore, the lower cost of living index will increase the utility, in turn, it would attract more skilled workers to move in this region.

It should be noted that the shifting of skilled labor h yields the expenditure shifting of both sectors. In $\mathbb{FE}$-sector, the agglomeration force triggered by increasing skilled labor h is self-reinforcing. This feature is called circular causality (Baldwin *et al.*, 2003) caused by two different cycles. One links to demand effect and the other links to costs effect. Therefore, not only expenditure shifting leads to production shifting of $\mathbb{FE}$-sector, but also production shifting of $\mathbb{FE}$-sector leads to expenditure shifting. That is, the agglomeration of $\mathbb{FE}$-sector causes lower local $\mathbb{FE}$-good price index, and in turn encourages more skilled labor to migrate. However, the agglomeration force caused by increasing employed capital k features neither demand-linked nor cost-linked circular causality. The demand-linkage is ruled out since all capital income is repatriated. It means that expenditure shifting can lead to production shifting of $\mathbb{FC}$-sector, but production shifting of $\mathbb{FC}$-sector does not lead to expenditure shifting since there is no labor migrate. The cost-linkage is ruled out since physical capital is attracted by rewards defined in terms of the numeraire rather than rewards deflated by the local price index. Thus, although the production shifting of $\mathbb{FC}$-sector does have a cost-of-living effect, this does not in turn encourage further production shifting of $\mathbb{FC}$-sector but rather the move of footloose entrepreneurs (i.e. $\mathbb{FE}$-sector) inter-industrially. Consequently, both of the cost-of-living effects caused by production shifting of each sector attracts more skilled labors (i.e. $\mathbb{FE}$-sector) to agglomerate further. Figure 1 illustrates the forces at work in the model.

4 Equilibrium and stability

4.1 Footloose capital manufacturing sector $r_i - r_j$

To analyze the location choice of capital investment and skilled workers, the nominal interest rates r_i (capital price) and real wages (including the sum of the nominal wage income and the return on capital investment deflated by cost-of-living index) of skilled labor ω_{H_i} are the two key variables to determine the location of capital investment and the working site of skilled labor. Namely, the capital will invest to the region with higher return (nominal price), and under the utility maximization, the skilled labor will migrate to work in the region associated with higher real wage (i.e., utility). And in equilibrium, if both regions have $\mathbb{FC}$-firms, then the capital prices in both regions must be identical, also if both regions exist $\mathbb{FE}$-firms, the real wage of the skilled workers between two regions must be equalize.

With the examination of the solutions r_i and r_j, we can express the difference of capital price between two regions $r_i - r_j$ as the function of H, L, β, γ, σ_X, σ_M, K, h, k, ϕ_M, and ϕ_X. The inspection of $r_i - r_j$ reveals that (see the Appendix 3 for the details).

$$r_i - r_j \gtreqless 0, \quad \text{if} \;\; G(h, k, \phi_M, \phi_X) \lesseqgtr 0$$

That is, all interior equilibria are determined by the solutions of the function $G(h, k, \phi_M, \phi_X) = 0$. And furthermore, we have

$$\begin{aligned} G_k(\cdot) &\equiv \frac{\partial G(h, k, \phi_M, \phi_X)}{\partial k} \\ &= G_0 \cdot \left[\sigma_X \left(h + (1-h)\,\phi_X\right)(1 - h + h\;\phi_X) - \gamma\; h\,(1-h)\left(1 - \phi_X^2\right)\right] \geq 0 \end{aligned} \tag{28}$$

where $G_0 \equiv 2(H+L)\sigma_M(1-\phi_M)^2 \geq 0$. The slope of $G(h, k, \phi_M, \phi_X)$ with respect to k is determined by the exogenous variables H, L, ϕ_M, σ_M, ϕ_X, σ_X, and the endogenous variable h. For $\phi_M \neq 1$, we can see that $G(h, k, \phi_M, \phi_X) = 0$ has only one solution of k for $0 < k < 1$, and the value of the solution k depends on h. It means the interior equilibrium of $G(h, k, \phi_M, \phi_X) = 0$ is not necessary symmetric, but shifts according to the function of h:

$$k = f(h) = \frac{(1-2h)\,H\;\beta\;\Psi - (H+L)\,\sigma_M\left[\Psi - \Phi + (1-2h)\,(1-\phi_M)\,\phi_X\right]}{2\,(H+L)\,\sigma_M\left[\Phi - \Psi\,(1-\phi_M)\right]} \tag{29}$$

where $\Psi \equiv \sigma_X\,(1-\phi_M)\,(h + (1-h)\,\phi_X)\,(1 - h + h\;\phi_X)$,

and $\Phi \equiv \gamma\;h\,(1-h)\,(1-\phi_M)\left(1-\phi_X^2\right)$

Plus $\frac{\partial f(h)}{\partial h} > 0$ and $k = f(1/2) = 1/2$, we assure that $r_i - r_j = 0$ has only one solution for k, and k is greater than 1/2 if h is greater than 1/2. It implies that the solution k of $G(h, k, \phi_M, \phi_X) = 0$ is a stable equilibrium whenever $G_k(\cdot) \geq 0$. The property $G_k(\cdot) \geq 0$ always holds, and when $\phi_M = 1$, we know that $G_k(\cdot) = 0$. Therefore, the interior equilibrium is always stable. The 'break point' of the $\mathbb{FC}$-sector (ϕ_{MB}) equals 1. There is not any balck-hole condition with respect to the distribution of employed capital ($\mathbb{FC}$-firms).

In an extreme case that $\phi_M = 1$, the trading $\mathbb{FC}$-goods is costless between regions ($\tau_M = 1$), r_i and r_j are always identical (i.e., $G(h, k, \phi_M, \phi_X) = 0$) no matter how the capital investment distribute between two regions. In turn, it means that any $k \in (0, 1)$ can be a stable equilibrium.

Furthermore, solving the equation $G(h,k,\phi_M,\phi_X)=0$, it yields $\phi_M=1$. Therefore, the sustain point of the $\mathbb{FC}$-sector (ϕ_{MX}) also equals 1. However, when $\phi_M<1$, the value of the solution k depends on h. It implies that the stable interior equilibrium of r_i-r_j is not necessary symmetric, but determined by the function of h.

$\mathbb{FC}$-firms does not have black-hole condition. Exploring its possible core-peripheral agglomeration, we have to check the value of k when h equals 1 (or 0), the black-hole condition or core-peripheral agglomeration of $\mathbb{FE}$-firms.

$$k=f(1)=\frac{1}{2}+\frac{1}{2}\frac{(1+\phi_M)\left[H\ \beta\ \sigma_X+\gamma\ \sigma_M\,(H+L)\right]}{(H+L)\,\sigma_M\ \sigma_X\,(1-\phi_M)} \tag{30}$$

Based on the previous formation, we obtain that the threshold of ϕ_M, termed as ϕ_{MC}, determines whether k can reach 1 when $h=1$.

Under the condition that $h=1$ ($h=0$), if $\phi_M\geq\phi_{MC}$, then $k=1$ ($k=0$); otherwise, $k<1$ ($k>0$). And

$$\phi_{MC}=\frac{(H+L)\,(\sigma_X-\gamma)\,\sigma_M-H\ \beta\ \sigma_X}{(H+L)\,(\sigma_X+\gamma)\,\sigma_M+H\ \beta\ \sigma_X} \tag{31}$$

4.2 Footloose entrepreneur manufacturing sector $\omega_{H_i}-\omega_{H_j}$

Since the total income of skilled workers involves the wage income and capital return from the investment, note that $P_A=1$, thus the real wages of skilled labor can be specified as :

$$\omega_{H_i}=\frac{w_{H_i}+r_i\ K\ k_i+r_j\ K\ \ (1-k_i)}{P_{M_i}^{\beta}\ P_{X_i}^{\gamma}} \tag{32}$$

After using (26), (18), (A.11), (A.12), (A.13), and (32), we can express $\omega_{H_i}-\omega_{H_j}$ as the function of K, F_M, c_M, F_X, c_X, H, L, β, γ, σ_X, σ_M, h, k, ϕ_M, and ϕ_X. The inspection of $\omega_{H_i}-\omega_{H_j}$ reveals that (see the Appendix 4 for the details)

$$\omega_{H_i}-\omega_{H_j}\gtreqless 0,\ \text{if}\ \ V(h,k,\phi_M,\phi_X)\lesseqgtr 0.$$

It implies that the determination of equilibrium locations of skilled labor depends on $V(h,k,\phi_M,\phi_X)$. In particular, all interior equilibria are the solutions of $V(h,k,\phi_M,\phi_X)=0$ .While fully $\mathbb{FE}$-firm-agglomerated configurations, $h=0$ or $h=1$, are equilibria if and only if $V(0, f(0), \phi_M, \phi_X) > 0$ or $V(1, f(1), \phi_M, \phi_X) < 0$ respectively. A corner configuration ($h = 0$ or $h = 1$) is always stable when it is an equilibrium, while an interior equilibrium ($0 < h < 1$) is stable if and only if the slope of $\omega_{H_i} - \omega_{H_j}$ with respect to h is non-positive in its neighborhood, the condition that the slope of $V(h, k, \phi_M, \phi_X)$ with respect to h is positive in its neighborhood.

It can be proved that $V(h, k, \phi_M, \phi_X) = 0$ has at most three solutions for $0 < h < 1$. It is readily verified that one solution exists for any values of parameters. This is the symmetric outcome $h = 1/2$, which entails a symmetric distribution of skilled workers and $\mathbb{FE}$-firms. This solution is stable whenever $V_h(\cdot)|_{h=1/2} \equiv \frac{\partial V(h,k,\phi_M,\phi_X)}{\partial h}_{h=1/2} \geq 0$.

Because $k = f(1/2) = 1/2$,

$$V_h(\cdot)|_{h=1/2} = V_h(\cdot)|_{h=1/2,k=1/2} = \left(\frac{1}{2} + \frac{1}{2}\phi_M\right)^{\frac{\beta}{\sigma_M - 1}} \left(\frac{1}{2} + \frac{1}{2}\phi_X\right)^{\frac{\gamma}{\sigma_X - 1} - 1} \frac{\gamma\left(1 - \phi_X^2\right)}{\sigma_X - 1} \cdot \Omega \geq 0 \tag{33}$$

where

$$\begin{aligned} \Omega \;\equiv\; & (H + L)\,\sigma_M\,(\sigma_X - 1 - \gamma)\,(\sigma_X - \gamma) - H\;\beta\;\sigma_X\,(2\sigma_X - 1 - \gamma) \\ & - \left[(H + L)\,\sigma_M\,(\sigma_X - 1 + \gamma)\,(\sigma_X + \gamma) + H\;\beta\;\sigma_X\,(2\sigma_X - 1 + \gamma)\right]\phi_X \end{aligned} \tag{34}$$

This simply says that this equilibrium is stable whenever $\Omega \geq 0$. Therefore, we can get the 'break point' of the $\mathbb{FE}$-sector as

$$\phi_{XB} \equiv \frac{(H + L)\,\sigma_M\,(\sigma_X - 1 - \gamma)\,(\sigma_X - \gamma) - H\;\beta\;\sigma_X\,(2\sigma_X - 1 - \gamma)}{(H + L)\,\sigma_M\,(\sigma_X - 1 + \gamma)\,(\sigma_X + \gamma) + H\;\beta\;\sigma_X\,(2\sigma_X - 1 + \gamma)} \tag{35}$$

While fully $\mathbb{FE}$-firm-agglomerated configurations $h = 0$ or $h = 1$ are equilibria if and only if $V(0, k, \phi_M, \phi_X) > 0$ or $V(1, k, \phi_M, \phi_X) < 0$ respectively. The solution ϕ_{XS} to $V(0, k, \phi_M, \phi_X) = -V(1, k, \phi_M, \phi_X) = 0$ is what Fujita *et al.*(1999b) call the 'sustain point' $T(S)$. Based on the analysis of corner solutions of k in section 4.1, we know that under the condition that $h = 1$ ($h = 0$), if $\phi_M \geq \phi_{MC}$, then $k = 1$ ($k = 0$); otherwise, $k = f(1) < 1$ ($k = f(0) > 0$). Therefore, calculating ϕ_{XS} needs to consider the two conditions (see the Appendix 5 for the details):

(i), if $\phi_M < \phi_{MC}$, then ϕ_{XS} is the solution to $V(1, f(1), \phi_M, \phi_X) = 0$, and it is a constant determined by exogenous variables H, L, σ_M, σ_X, β, and γ.

(ii), If $\phi_M \geq \phi_{MC}$, then ϕ_{XS} is the solution to $V(1, 1, \phi_M, \phi_X) = 0$. The value of ϕ_{XS} in this case is determined by not only H, L, σ_M, σ_X, β, and γ, but also ϕ_M, the trade freeness of $\mathbb{FC}$-sector. In other words, the value of ϕ_{XS} changes along with the change of ϕ_M under the condition that $\phi_M \geq \phi_{MC}$.

Apart from $h = 1/2$, $V(h, k, \phi_M, \phi_X)$ has at most two other interior equilibria that are symmetrically placed around it. As soon as the equilibrium $h = 1/2$ changes its stability, two additional steady states appear in its neighborhood. Due to the symmetry of the model such steady states are symmetric around it. However, $V(h, k, \phi_M, \phi_X)$ has two kinds of bifurcation form. If $V_{hhh}(h, k, \phi_M, \phi_{XB})|_{h=1/2,\ k=1/2} < 0$, the bifurcation is '*subcritical*': as ϕ_X falls below ϕ_{XB} the persistent steady state $h = 1/2$ gains stability giving rise to two unstable symmetric steady states in its neighborhood (Guckenheimer and Holmes, 1990; Forsid and Ottaviano, 2003). The ϕ_{XB} represents the 'break point' called by Fujita *et al.* (1999b). Oppositely, if $V_{hhh}(h, k, \phi_M, \phi_{XB})|_{h=1/2,\ k=1/2} > 0$, the bifurcation is '*supercritical*'[2]: as ϕ_X rises above ϕ_{XB} the persistent steady state $h = 1/2$ loses stability giving rise to two stable symmetric steady states in its neighborhood. The inspection of $V_{hhh}(h, k, \phi_M, \phi_{XB})|_{h=1/2,\ k=1/2}$ shows that (see the Appendix 6 for the details)

$$V_{hhh}(h, k, \phi_M, \phi_{XB})|_{h=1/2,\ k=1/2} \gtreqless 0, \text{ if } \Delta \gtreqless 0$$

where

$$\begin{aligned}
\Delta \;\equiv\;& \beta\,\gamma\,(1-\gamma)\,(1+\gamma)\,[\xi + \zeta\,(\eta+\xi)]^2 + \;\sigma_X\,\{-\beta\,\gamma\,[5\zeta\,\eta + 2\,(1+\zeta)\,\xi]\,[\xi + \zeta\,(\eta+\xi)] + \\
& \sigma_X[2\zeta\,(1+\zeta)\,\eta + (2\gamma\,\zeta\,\eta + (1+\zeta)\,\xi)\,(2\beta\,\zeta\,\eta - 1 - \zeta) \\
& + 2\,(1+\zeta)\,\gamma^2\,(\xi + \zeta(3\eta+\xi)) + (1+\zeta)\,\sigma_X\,(2\zeta\,\eta - (1+\zeta)\,\xi)\,(\sigma_X - 2)]\} \qquad (36)
\end{aligned}$$

and $\xi \equiv \frac{\gamma}{\beta}, \eta \equiv \frac{\sigma_X}{\sigma_M}, \zeta \equiv \frac{H}{L}$.

If the bifurcation of $\mathbb{FE}$-sector is '*subcritical*', then its 'black-hole condition' ($\phi_{XB} < 0$) shows that $(H+L)\,\sigma_M\,(\sigma_X - 1 - \gamma)\,(\sigma_X - \gamma) - H\;\beta\;\sigma_X\,(2\sigma_X - 1 - \gamma) < 0$ and its 'no-black-hole condition' ($\phi_{XB} \geq 0$) shows that $(H+L)\,\sigma_M\,(\sigma_X - 1 - \gamma)\,(\sigma_X - \gamma) -$

[2] In Forslid and Ottaviano's (2003) model that considers one footloose enetreprenur manufacturing industry only, the form of bifurcation is always subcritical under the no-balck-hole condition. That is why there does not exist any stable partial equilibrium in their modeled economy.

$H\ \beta\ \sigma_X\,(2\sigma_X - 1 - \gamma) \geq 0$. Substituting $\gamma \equiv \xi\ \beta$, $\sigma_X \equiv \eta\ \sigma_M$, and $H \equiv \zeta\ L$ into γ, σ_X, and H respectively simplifies the 'black-hole condition' as $\beta\ \zeta\ \eta(1 + \beta\ \xi - 2\eta\ \sigma_M) + (1+\zeta)(\beta\ \xi - \eta\ \sigma_M)(1 + \beta\ \xi - \eta\ \sigma_M) < 0$ and the 'no-black-hole condition' as $\beta\ \zeta\ \eta(1 + \beta\ \xi - 2\eta\ \sigma_M) + (1+\zeta)(\beta\ \xi - \eta\ \sigma_M)(1 + \beta\ \xi - \eta\ \sigma_M) \geq 0$.

If the bifurcation of $\mathbb{FE}$-sector is '*supercritical*', then its 'black-hole condition' ($\phi_{XS} < 0$) and its 'no-black-hole condition' ($\phi_{XS} \geq 0$) need numerical methods to verify. However, the 'no-black-hole condition' under the '*subcritical*' bifurcation sufficiently satisfies the 'no-black-hole condition' under the '*supercritical*' bifurcation because $\phi_{XB} < \phi_{XS}$ holds when the bifurcation is '*supercritical*'.

5 Equilibrium Configurations and Characteristics

The equilibrium and stability properties are analyzed in the previous section, we move to examine the effect of various interesting parameters on the configurations of industries in the spatial economy. In particular, it may emerge partial agglomerations on both manufacturing industries in some given parameter sets. First, we investigate the $\mathbb{FC}$-sector, the partial agglomeration of capital allocation (i.e., the partial agglomeration of $\mathbb{FC}$-firms) appears, because the allocation of capital investment is affected by the distribution of skilled labor, and there is a threshold ϕ_{MC} for trade freeness of $\mathbb{FC}$-sector (ϕ_M) to yield the extent of the influence. Once the ϕ_M is less than ϕ_{MC}, the agglomeration degree of capital (i.e. the $\mathbb{FC}$-firms) will not reflect the full magnitude of skilled labor to agglomerate. Therefore, there at least exists a stable partial agglomeration of $\mathbb{FC}$-sector, in which the $\mathbb{FC}$-firms don't locate in only one region while skilled workers (i.e. $\mathbb{FE}$-sector) may have full agglomeration. The bifurcation map of the $\mathbb{FC}$-sector is shown in Figure 2. Please notice that this kind of partial agglomeration of $\mathbb{FC}$-firms never emerges in the FC model developed by Martin and Rogers (1995), in which only the footloose capital firms be examined in the framework of economic geography.

Next, about the spatial distribution of the $\mathbb{FE}$-sector, there are two different forms of bifurcation, and it would depend on $\Delta \gtreqless 0$ (i.e. $V_{hhh}(h, k, \phi_M, \phi_{XB})|_{h=1/2,\ k=1/2} \gtreqless 0$). The difference of real wage function of skilled labors between both regions would depending on two different forms: *subcritical* and *supercritical* bifurcation. In the former case, we have the identical result of the equilibrium configurations as those documented by Foslid and Ottaviano (2003), as shown in Figure 3. In which, only the full agglomeration and completely dispersion of $\mathbb{FE}$-firms exist, and the multiple equilibria emerge in the intermediate trade freeness. However, in the later case, stable partial agglomerations appear when the trade freeness on $\mathbb{FE}$-good (ϕ_X) falls between the sustain point (ϕ_{XS}) and break point (ϕ_{XB}), as shown in Figure 4. It reveals that full agglomeration, completely dispersion, and partial agglomeration appear depending on the range of ϕ_X, and please notices that it does not emerge the multiple equilibria configurations. The outcome of the later case is also never emerge in the finding of Foslid and Ottaviano (2003), in which only the footloose entrepreneur firms be examined in the discussion of industry distribution.

Therefore, excluding black-hole condition of ϕ_X and $\phi_M = 1$, all the alternatives of stable equilibria based on the interaction of the two manufacturing sectors can be categorized into five types of equilibrium configuration associated with the parameter space of ϕ_X and ϕ_M, the combination of both trade freeness of the two sectors. These configurations are termed as X^AM^A, X^AM^P, X^PM^A, X^PM^P, and X^SM^S, which are summarized in Table 1. The first two capital alphabets 'X' and 'M' denote $\mathbb{FE}$-sector and $\mathbb{FC}$-sector as mention previously, and their superscripts 'A' , 'P' and 'S' represents 'full agglomeration', 'partial agglomeration', and 'symmetric distribution (completely dispersion)' respectively.

Table 1 Five kinds of equilibrium configurations.

Symbol	Equilibria	Conditions
X^AM^A	$h = 1, k = 1$	$\{\phi_X \geq \phi_{XS}\}$ and $\{\phi_M \geq \phi_{MC}\}$
X^AM^P	$h = 1, \frac{1}{2} < k < 1$	$\{\phi_X \geq \phi_{XS}\}$ and $\{\phi_M < \phi_{MC}\}$
X^PM^A	$\frac{1}{2} < h < 1, k = 1$	$\{supercritical : \phi_{XB} < \phi_X < \phi_{XS}\}$ and $\{\phi_M \geq \phi_{MC}\}$
X^PM^P	$\frac{1}{2} < h < 1, \frac{1}{2} < k < 1$	$\{supercritical : \phi_{XB} < \phi_X < \phi_{XS}\}$ and $\{\phi_M < \phi_{MC}\}$
X^SM^S	$h = \frac{1}{2}, k = \frac{1}{2}$	$\{\phi_X \leq \phi_{XB}\}$ and $\{\phi_M \in (0, 1)\}$

where ϕ_{MC} represents a threshold of ϕ_M to determine whether k can reach 1 incorporated with $h = 1$; ϕ_{XS} is the 'sustain point' for the $\mathbb{FE}$-sector, and its value needs numerical methods to approach; ϕ_{XB} is the 'break point' for the $\mathbb{FE}$-sector.

Putting these five equilibrium configurations into a diagram based on the parameter space of ϕ_X and ϕ_M, we have Figure 5 and Figure 6 respectively. In Figure 5, when the bifurcation form of $\mathbb{FE}$-sector is *subcritical*, the sustain point and the break point of the freeness of trade of $\mathbb{FE}$-sector and the threshold ϕ_{MC} for trade freeness of $\mathbb{FC}$-sector define three types of equilibrium configuration: $X^A M^A$, $X^A M^P$, and $X^S M^S$. It is worth noticing that when the trade freeness of $\mathbb{FE}$-sector (ϕ_X) falls between the sustain point (ϕ_{XS}) and break point (ϕ_{XB}) as well as the trade freeness of $\mathbb{FC}$-sector (ϕ_M) rises above ϕ_{MC}, then the spaces of configurations $X^A M^A$ and $X^S M^S$ co-exist. It implies the existence of multiple equilibria in this parameter sets. Similarly, the spaces of configurations $X^A M^P$ and $X^S M^S$ overlap if the trade freeness of $\mathbb{FC}$-sector (ϕ_M) falls below ϕ_{MC}. In Figure 6, when the bifurcation form of $\mathbb{FE}$-sector is *supercritical*, the related magnitude of the sustain point as well as the break point of the trade freeness of $\mathbb{FE}$-sector and the threshold ϕ_{MC} for trade freeness of $\mathbb{FC}$-sector will yield all five types of equilibrium configurations. Besides the spatial distribution of $X^A M^A$, $X^A M^P$, and $X^S M^S$ as mentioned above, $X^P M^A$ and $X^P M^P$ show up under the condition that the trade freeness of $\mathbb{FE}$-sector (ϕ_X) falls between the sustain point (ϕ_{XS}) and break point (ϕ_{XB}).

About the configuration $X^S M^S$, the threshold ϕ_{MC} does not work as the key value to separate 'full agglomeration' and 'partial agglomeration' configurations for $\mathbb{FC}$-sector. It is because once the stable symmetric distribution of $\mathbb{FE}$-sector appears, i.e. $h = 1/2$, the stable equilibrium of $\mathbb{FC}$-sector must be symmetric too, i.e. $k = 1/2$. Therefore, configuration such as $X^S M^A$ or $X^S M^P$ never be an equilibrium configuration with $\phi_X < \phi_{XB}$.

As it turns out, Figure 4 and Figure 6 reveal the findings that both manufacturing industries are full agglomeration (completely dispersion) when ϕ_M as well as ϕ_X are high (low) enough, and between them, then the partial agglomeration of either manufacturing industry may emerge.

6 Discussions

We turn to discuss the economic intuition of the impact of Δ on the industry configuration, in which its sign determines the form of pitchfork bifurcation, we simulate the fluctuations according to some given sets of exogenous parameters, in order to simplify the analysis, we denote $\xi \equiv \frac{\gamma}{\beta}$, $\eta \equiv \frac{\sigma_X}{\sigma_M}$, and $\zeta \equiv \frac{H}{L}$. While the scenario 1 examines the effect of change ξ (the ratio of the expenditure share of both manufacturing sectors) on the sign of Δ, the scenario 2 tries to figure out the relation between the sign of Δ and η (the ratio of the elasticities of substitution of the two manufacturing sectors).

Scenario 1: let $\zeta \equiv \frac{H}{L} = 0.2$, $\eta \equiv \frac{\sigma_X}{\sigma_M} = 1$, and $\sigma_M = 3$

Figure 7 illustrates the fluctuations of Δ with respect to ξ when $\beta = 0.25$, $\beta = 0.5$, $\beta = 0.75$, and $\beta = 0.99$. Because $0 < \beta < 1$, $0 < \gamma < 1$, and $0 < \beta + \gamma < 1$, the reasonable solutions to $\Delta = 0$ are 0.335683 and 0.343192 when $\beta = 0.25$ and $\beta = 0.5$, respectively; there is no reasonable solution to $\Delta = 0$ when $\beta = 0.75$ or $\beta = 0.99$[3]. It shows that under the condition given by these exogenous parameters, the less share of the $\mathbb{FE}$-good on the household consumption, the more likely to be positive the sign of Δ is. It means the bifurcation of the $\mathbb{FE}$-sector becomes '*supercritical*' when the expenditure share of the $\mathbb{FE}$-sector is small enough.

Scenario 2: let $\zeta \equiv \frac{H}{L} = 0.2$, $\xi \equiv \frac{\gamma}{\beta} = 1$, and $\beta = 0.4$

Figure 8 illustrates the fluctuations of Δ with respect to η under $\sigma_M = 1.1$, $\sigma_M = 3$, $\sigma_M = 5$, and $\sigma_M = 10$. Because $\sigma_M > 1$ and $\sigma_X > 1$, the reasonable range of the solutions to $\Delta = 0$ for these four cases are $\{1.71058, 2.46693\}$, $\{0.493594, 2.95975\}$, $\{0.288834, 2.98631\}$, and $\{0.142075, 2.9967\}$ respectively. It shows that under the condition given by these exogenous parameters, Δ changes three times from positive to negative and then finally to positive again. The most obvious tendency we can conclude is that the sign of Δ is likely to become positive (the bifurcation of the $\mathbb{FE}$-sector becomes '*supercritical*') when the elasticity of substitution of the $\mathbb{FE}$-sector is much larger than that of the $\mathbb{FC}$-sector.

The conclusive relations based on the simulated condition might be partial and parameter-specific. However, it can be asserted that the differences between '*super-*

[3] Although there are also multiple solutions to $\Delta = 0$ when $\beta = 0.75$ (e.x. 0.357578) or $\beta = 0.99$ (e.x. 0.382203), all these solutions do not meet the requirements of $0 < \gamma < 1$ and $0 < \beta + \gamma < 1$.

critical' and '*subcritical*' bifurcation types not only mean the different composition of stable equilibria, but also imply the different kind of shifting process between equilibria, gradually or catastrophically. Therefore, after the analysis in this section, we can conclude that the relations between the consumer expenditure share as well as the elasticities of substitution of these two manufacturing sectors, $\mathbb{FE}$-sector and $\mathbb{FC}$-sector, will yield the shifting between the bifurcation types of the $\mathbb{FE}$-sector and represent different evolutionary trajectories of equilibrium configurations.

7 Conclusions

This paper incorporates a footloose entrepreneur manufacturing industry and a footloose capital manufacturing industry with IRS in a new economy geography model to re-examine the equilibrium configurations. Through the examination based on a tradable analytical structure between two identical economies, this paper shows that the FC manufacturing sector and the FE manufacturing sector will interact each other and yield more fruitful and realistic spatial distributions driven by the demand-linked forces and cost-linked forces within industry and between industries, and also finds out the emergence of two different kinds of pitchfork bifurcation in the FE manufacturing sector, '*subcritical*' and '*supercritical*', in which the bifurcation will depend on the interesting parameters set. What is particularly nice about these results feature the existence of partial agglomerations of both sectors and yield five types of equilibrium configuration depending on the form of bifurcation, the break point, and the sustain point of trade freeness on both sectors. This paper also documents that the relations between the consumer expenditure share as well as the elasticities of substitution of these two manufacturing sectors will yield the different bifurcation types of the FE manufacturing sector, and more importantly, the different bifurcation type represents different evolutionary trajectories of equilibrium configurations. This finding of the bifurcation switching is absent in the conventional literature of new economy geography which modeled only one IRS manufacturing industry with Cobb-Douglas utility function.

Several extensions are worth studying. First, instead of the $\mathbb{FE}$-sector in consumption, we can incorporate the $\mathbb{FE}$-sector into the production side, with the format as Venables (1996), to model the $\mathbb{FE}$-goods as the required inputs of manufacturing industry, and to re-examine the possibility of the configurations with partial agglomeration of up-stream and down-stream industries. Second, assume the behavior (or preference) of capital investment between the skilled and unskilled workers is different, for example, risk-lover for the skilled labor, and investigate the impact on the equilibrium configurations. Third, impose the different tariff (or the other tax scheme) on the $\mathbb{FE}$-goods and $\mathbb{FC}$-goods, and discuss the impact of the tariff (or tax) policy on the equilibrium configurations.

Appendix 1: Solution of equations in the model

The system consisting of the following equations determines the endogenous variables n_{X_i}, n_{X_j}, n_{M_i}, n_{M_j}, Y_i, Y_j, w_{H_i}, w_{H_j}, r_i,and r_j for a given allocation of skilled labor h and capital k.

$$\begin{aligned}
Y_i &= \frac{L}{2} + w_{H_i}\ H_i + r_i\ K\ (\frac{L}{2} + H_i)\ k_i + r_j\ K\ (\frac{L}{2} + H_i)\ (1 - k_i) && \text{(A.1)} \\
Y_j &= \frac{L}{2} + w_{H_j}\ H_j + r_j\ K\ (\frac{L}{2} + H_j)\ (1 - k_j) + r_j\ K\ (\frac{L}{2} + H_j)\ k_j && \text{(A.2)} \\
n_{X_i} &= \frac{H_i}{F_X} && \text{(A.3)} \\
n_{X_j} &= \frac{H_j}{F_X} && \text{(A.4)} \\
n_{M_i} &= \frac{K\left(\frac{L}{2} + H_i\right) k_i + K\left(\frac{L}{2} + H_j\right)(1 - k_j)}{F_M} && \text{(A.5)} \\
n_{M_j} &= \frac{K\left(\frac{L}{2} + H_i\right)(1 - k_i) + K\left(\frac{L}{2} + H_j\right) k_j}{F_M} && \text{(A.6)} \\
w_{H_i} &= \frac{\gamma}{F_X\ \sigma_X}\left[\frac{Y_i}{n_{X_i} + n_{X_j}\ \phi_X} + \frac{Y_j\ \phi_X}{n_{X_i}\ \phi_X + n_{X_j}}\right] && \text{(A.7)} \\
w_{H_j} &= \frac{\gamma}{F_X\ \sigma_X}\left[\frac{Y_i\ \phi_X}{n_{X_i} + n_{X_j}\ \phi_X} + \frac{Y_j}{n_{X_i}\ \phi_X + n_{X_j}}\right] && \text{(A.8)} \\
r_i &= \frac{\beta}{F_M\ \sigma_M}\left[\frac{Y_i}{n_{M_i} + n_{M_j}\ \phi_M} + \frac{Y_j\ \phi_M}{n_{M_i}\ \phi_M + n_{M_j}}\right] && \text{(A.9)} \\
r_j &= \frac{\beta}{F_M\ \sigma_M}\left[\frac{Y_i\ \phi_M}{n_{M_i} + n_{M_j}\ \phi_M} + \frac{Y_j}{n_{M_i}\ \phi_M + n_{M_j}}\right] && \text{(A.10)} \\
k_i &= 1 - k_2 = k && \text{(A.11)} \\
H_i &= h\ H && \text{(A.12)} \\
H_j &= (1 - h)\ H && \text{(A.13)}
\end{aligned}$$

Solving the joint equations, we can get solutions of w_{Hi},and r_i :

$$w_{H_i} = \frac{L\,\gamma\,\{(1-h)(-1+2h)\,\Theta_1\;\Theta_2+\Theta_3\,[(1-h)\,\Theta_4+2h\;\sigma_X\;\phi_X]\}}{2H\,(H+L)\,(\sigma_X\;\Theta_5-\gamma\;\Theta_2\;\Theta_6)\,\Theta_{10}} \tag{A.14}$$

$$w_{H_j} = \frac{L\,\gamma\,\{h\,(1-2h)\,\Theta_1\;\Theta_2+\Theta_3\,[h\;\Theta_4+2\,(1-h)\;\;\sigma_X\;\phi_X]\}}{2H\,(H+L)\,(\sigma_X\;\Theta_5-\gamma\;\Theta_2\;\Theta_6)\,\Theta_{10}} \tag{A.15}$$

$$r_i = \frac{L\,\beta\,\sigma_X\left[\begin{array}{c}(-1+2h)\,(1-k)\,\Theta_1\Theta_5\;\Theta_8\\ +\Theta_3\left[\begin{array}{c}\sigma_X\,[(1-k+k\;\phi_M)+\phi_M\,(k+(1-k)\,\phi_M)]\,\Theta_5\\ -\gamma\,[(1-k)\,\Theta_5\;\Theta_9+2k\;\phi_M\;\Theta_2\;\Theta_6]\end{array}\right]\end{array}\right]}{2K\,(H+L)^2\,\sigma_M\;\Theta_7\,(\sigma_X\;\Theta_5-\gamma\;\Theta_2\;\Theta_6)\,\Theta_{10}} \tag{A.16}$$

$$r_j = \frac{L\,\beta\,\sigma_X\left[\begin{array}{c}(1-2h)\,k\;\Theta_1\Theta_5\;\Theta_8\\ +\Theta_3\left[\begin{array}{c}\sigma_X\,[(k+(1-k)\,\phi_M)+\phi_M\,((1-k)+k\;\phi_M)]\,\Theta_5\\ -\gamma\,[k\;\Theta_5\;\Theta_9+2\,(1-k)\;\;\phi_M\;\Theta_2\;\Theta_6]\end{array}\right]\end{array}\right]}{2K\,(H+L)^2\,\sigma_M\;\Theta_7\,(\sigma_X\;\Theta_5-\gamma\;\Theta_2\;\Theta_6)\,\Theta_{10}} \tag{A.17}$$

where $\Theta_1 \equiv H\;\beta\;\sigma_X$, $\Theta_2 \equiv \left(1-\phi_X^2\right)$, $\Theta_3 \equiv (H+L)\,\sigma_M$, $\Theta_4 \equiv (\sigma_X-\gamma)+(\sigma_X+\gamma)\,\phi_X^2$, $\Theta_5 \equiv (h+(1-h)\,\phi_X)\,(1-h+h\;\phi_X)$, $\Theta_6 \equiv (1-h)\,h$, $\Theta_7 \equiv (k+(1-k)\,\phi_M)*(1-k+k\;\phi_M)$, $\Theta_8 \equiv \left(1-\phi_M^2\right)$, $\Theta_9 \equiv \left(\phi_M^2+1\right)$, and $\Theta_{10} \equiv \sigma_M\,(\sigma_X-\gamma)-\beta\;\sigma_X$.

Define $w \equiv w_{H_i}/w_{H_j}$and $r \equiv r_i/r_j$,

$$w \equiv \frac{w_{H_i}}{w_{H_j}} = \frac{(1-h)\,(-1+2h)\,\Theta_1\;\Theta_2+\Theta_3\,[(1-h)\,\Theta_4+2h\;\sigma_X\;\phi_X]}{h\,(1-2h)\,\Theta_1\;\Theta_2+\Theta_3\,[h\;\Theta_4+2\,(1-h)\;\;\sigma_X\;\phi_X]} \tag{A.18}$$

$$r \equiv \frac{r_i}{r_j} = \frac{\begin{array}{c}(-1+2h)\,(1-k)\,\Theta_1\Theta_5\;\Theta_8\\ +\Theta_3\left[\begin{array}{c}\sigma_X\,[(1-k+k\;\phi_M)+\phi_M\,(k+(1-k)\,\phi_M)]\,\Theta_5\\ -\gamma\,[(1-k)\,\Theta_5\;\Theta_9+2k\;\phi_M\;\Theta_2\;\Theta_6]\end{array}\right]\end{array}}{\begin{array}{c}(1-2h)\,k\;\Theta_1\Theta_5\;\Theta_8\\ +\Theta_3\left[\begin{array}{c}\sigma_X\,[(k+(1-k)\,\phi_M)+\phi_M\,((1-k)+k\;\phi_M)]\,\Theta_5\\ -\gamma\,[k\;\Theta_5\;\Theta_9+2\,(1-k)\;\;\phi_M\;\Theta_2\;\Theta_6]\end{array}\right]\end{array}} \tag{A.19}$$

Appendix 2: Proof of $\partial r/\partial h > 0$, $\partial r/\partial k < 0$, $\partial P_{X_i}/\partial h < 0$, $\partial P_{M_i}/\partial k < 0$

Differentiating r with respect to h shows whether the region with more skilled workers offers a higher capital return:

$$\frac{\partial r}{\partial h} = \frac{2\Theta_3\ \Theta_7\ \Theta_8 \left\{ \begin{array}{c} 2\Theta_1\ \Theta_6 \left(\phi_X^2 + 1\right) \left[\Theta_6 \left(\Theta_4 - 4\sigma_X\ \phi_X\right) + 2\sigma_X\ \phi_X\right] \\ +\gamma\ \phi_X\ \Theta_3 \left[\Theta_4 - 2\Theta_6 \left(\Theta_4 - 2\sigma_X\ \phi_X\right)\right] + 2\sigma_X\ \phi_X^2\ \Theta_1 \left(1 - 2\Theta_6\right)^2 \\ +\gamma\ \phi_X\ \Theta_1\ \Theta_2 \left[-1 + 2\left(h\left(1 + h\left(1 - 2\left(2 - h\right)h\right)\right)\right)\right] \end{array} \right\}}{\left[\begin{array}{c} \left(1 - 2h\right) k\ \Theta_1 \Theta_5\ \Theta_8 \\ +\Theta_3 \left[\begin{array}{c} \sigma_X \left[\left(k + \left(1 - k\right)\phi_M\right) + \phi_M \left(\left(1 - k\right) + k\ \phi_M\right)\right] \Theta_5 \\ -\gamma \left[k\ \Theta_5\ \Theta_9 + 2\left(1 - k\right)\ \phi_M\ \Theta_2\ \Theta_6\right] \end{array} \right] \end{array} \right]^2} > 0 \tag{A.20}$$

where $\Theta_1 \equiv H\ \beta\ \sigma_X$, $\Theta_2 \equiv \left(1 - \phi_X^2\right)$, $\Theta_3 \equiv \left(H + L\right)\sigma_M$, $\Theta_4 \equiv \left(\sigma_X - \gamma\right) + \left(\sigma_X + \gamma\right)\phi_X^2$, $\Theta_5 \equiv \left(h + \left(1 - h\right)\phi_X\right)\left(1 - h + h\ \phi_X\right)$, $\Theta_6 \equiv \left(1 - h\right)h$, $\Theta_7 \equiv \left(k + \left(1 - k\right)\phi_M\right) * \left(1 - k + k\ \phi_M\right)$, $\Theta_8 \equiv \left(1 - \phi_M^2\right)$, and $\Theta_9 \equiv \left(\phi_M^2 + 1\right)$.

Differentiating r with respect to k shows whether the region receiving more capital offers a higher capital return:

$$\frac{\partial r}{\partial k} = \frac{\begin{array}{c} -\Theta_5\ \left(\Theta_8\right)^2 \\ \left\{\left(1 - h\right)\left[\Theta_3\left(\sigma_X - \gamma\right) - \Theta_1\left(1 - 2h\right)\right] + h\ \phi_X \left[\Theta_3\left(\sigma_X + \gamma\right) - \Theta_1\left(1 - 2h\right)\right]\right\} \\ \left\{h\left[\Theta_1\left(1 - 2h\right) + \Theta_3\left(\sigma_X - \gamma\right)\right] + \left(1 - h\right)\phi_X \left[\Theta_1\left(1 - 2h\right) + \Theta_3\left(\sigma_X + \gamma\right)\right]\right\} \end{array}}{\left[\begin{array}{c} \left(1 - 2h\right) k\ \Theta_1 \Theta_5\ \Theta_8 \\ +\Theta_3 \left[\begin{array}{c} \sigma_X \left[\left(k + \left(1 - k\right)\phi_M\right) + \phi_M \left(\left(1 - k\right) + k\ \phi_M\right)\right] \Theta_5 \\ -\gamma \left[k\ \Theta_5\ \Theta_9 + 2\left(1 - k\right)\ \phi_M\ \Theta_2\ \Theta_6\right] \end{array} \right] \end{array} \right]^2} < 0 \tag{A.21}$$

Differentiating P_{X_1} with respect to h shows

$$\frac{\partial P_{X_i}}{\partial h} = \frac{-c_X\ \sigma_X \left(1-\phi_X\right) \left[H\left(h+\left(1-h\right)\phi_X\right)/F_X\right]^{\frac{1}{1-\sigma_X}}}{\left(1-\sigma_X\right)^2 \left(h+\left(1-h\right)\phi_X\right)} < 0 \tag{A.22}$$

$$\frac{\partial P_{M_i}}{\partial k} = \frac{-K\ \left(H+L\right) c_M\ \sigma_M \left(1-\phi_M\right) \left[K\left(H+L\right)\left(k+\left(1-k\right)\phi_M\right)/F_M\right]^{\frac{1}{1-\sigma_M}-1}}{F_M\left(1-\sigma_M\right)^2} < 0 \tag{A.23}$$

Appendix 3: Inspection of $r_i - r_j$

Let $\Theta_1 \equiv H\,\beta\,\sigma_X$, $\Theta_2 \equiv \left(1-\phi_X^2\right)$, $\Theta_3 \equiv \left(H+L\right)\sigma_M$, $\Theta_5 \equiv \left(h+\left(1-h\right)\phi_X\right)\left(1-h+h\ \phi_X\right)$, $\Theta_6 \equiv \left(1-h\right)h$, $\Theta_7 \equiv \left(k+\left(1-k\right)\phi_M\right) * \left(1-k+k\ \phi_M\right)$, $\Theta_8 \equiv \left(1-\phi_M^2\right)$, and $\Theta_{10} \equiv \sigma_M\left(\sigma_X-\gamma\right)-\beta\ \sigma_X$ We can express $r_1 - r_2$ as the following:

$$r_i - r_j = \frac{\frac{-L\ \beta\ \sigma_X}{\Theta_{10}} \cdot G(h,k,\phi_M,\phi_X)}{2K\left(H+L\right)\Theta_3\ \Theta_7\left[\sigma_X\ \Theta_5-\gamma\ \Theta_2\ \Theta_6\right]} \tag{A.24}$$

where

$$\begin{aligned} G(h,k,\phi_M,\phi_X) &= \Theta_5\left[\left(1-2h\right)\Theta_1\ \Theta_8-\sigma_X\ \Theta_3\ \left(1-2k\right)\left(1-\phi_M\right)^2\right] \\ &\quad +\gamma\ \Theta_3\left[\left(1-2k\right)\left(1-\phi_M\right)^2\Theta_2\ \Theta_6+\left(1-2h\right)\phi_X\ \Theta_8\right] \end{aligned} \tag{A.25}$$

Because the following conditions always hold,

$$\frac{-L\ \beta\ \sigma_X}{\Theta_{10}} < 0 \tag{A.26}$$

$$2K\left(H+L\right)\Theta_3\ \Theta_7\left[\sigma_X\ \Theta_5-\gamma\ \Theta_2\ \Theta_6\right] > 0 \tag{A.27}$$

we can assure that

$$r_i - r_j \gtreqless 0, \quad \text{if} \quad G(h,k,\phi_M,\phi_X) \lesseqgtr 0 \tag{A.28}$$

Appendix 4: Inspection of $\omega_{H_i} - \omega_{H_j}$

The real wages of skilled labor can be calculated by the following formula:

$$\omega_{H_i} = \frac{w_{H_i} + r_i\ K\ k_i + r_j\ K\ (1 - k_i)}{P_{A_i}^{\alpha}\ P_{M_i}^{\beta}\ P_{X_i}^{\gamma}} \tag{A.29}$$

After using (23), (26), (15), (18), (A.11), (A.12), and (A.13), we can express $\omega_{H_1} - \omega_{H_2}$ as the following:

$$\begin{aligned} \omega_{H_i} - \omega_{H_j} \quad = \quad & \frac{(-1+\sigma_X)^{\gamma} (c_M\ \sigma_M)^{-\beta} (c_X\ \sigma_X)^{1-\gamma} (\sigma_M - 1)^{\beta} (K (H+L))^{\frac{-\beta}{1-\sigma_M}}}{-2H (H+L) c_X\ \sigma_X\ \Theta_{10} [\sigma_X\ \Theta_5 - \gamma\ \Theta_2\ \Theta_6]} * \\ & L\ F_M^{\frac{\beta}{1-\sigma_M}} F_X^{\frac{\gamma}{1-\sigma_X}} H^{\frac{-\gamma}{1-\sigma_X}} * V(h,k,\phi_M,\phi_X) \end{aligned} \tag{A.30}$$

where $\Theta_1 \equiv H\ \beta\ \sigma_X$, $\Theta_2 \equiv \left(1 - \phi_X^2\right)$, $\Theta_3 \equiv (H+L)\,\sigma_M$, $\Theta_4 \equiv (\sigma_X - \gamma) + (\sigma_X + \gamma)\,\phi_X^2$, $\Theta_5 \equiv (h + (1-h)\,\phi_X)\,(1 - h + h\ \phi_X)$, $\Theta_6 \equiv (1-h)\,h$, $\Theta_{10} \equiv \sigma_M\,(\sigma_X - \gamma) - \beta\ \sigma_X$, and

$$\begin{aligned} V(h,k,\phi_M,\phi_X) \quad = \quad & (k + (1-k)\,\phi_M)^{\frac{-\beta}{1-\sigma_M}} (h + (1-h)\,\phi_X)^{\frac{-\gamma}{1-\sigma_X}} * \\ & \left\{ \begin{array}{c} \Theta_1 [(1-h)\,\gamma\ \Theta_2 - 2\sigma_X\ \Theta_5] \\ -\gamma\ \Theta_3 [(1-h)\,\Theta_4 + 2h\ \sigma_X\ \phi_X] \end{array} \right\} \\ & - (1 - k + k\phi_M)^{\frac{-\beta}{1-\sigma_M}} (1 - h + h\phi_X)^{\frac{-\gamma}{1-\sigma_X}} * \\ & \left\{ \begin{array}{c} \Theta_1 [h\ \gamma\ \Theta_2 - 2\sigma_X\ \Theta_5] \\ -\gamma\ \Theta_3 [\sigma_X (h + \phi_X (2 - 2h + h\phi_X)) - h\ \gamma\ \Theta_2] \end{array} \right\} \end{aligned} \tag{A.31}$$

Because the following conditions always hold,

$$-2H (H+L) c_X\ \sigma_X\ \Theta_{10} [\sigma_X\ \Theta_5 - \gamma\ \Theta_2\ \Theta_6] \quad < \quad 0 \tag{A.32}$$

$$L\ F_M^{\frac{\beta}{1-\sigma_M}} F_X^{\frac{\gamma}{1-\sigma_X}} H^{\frac{-\gamma}{1-\sigma_X}} \quad > \quad 0 \tag{A.33}$$

$$(-1+\sigma_X)^{\gamma} (c_M\ \sigma_M)^{-\beta} (c_X\ \sigma_X)^{1-\gamma} (\sigma_M - 1)^{\beta} (K (H+L))^{\frac{-\beta}{1-\sigma_M}} \quad > \quad 0 \tag{A.34}$$

the inspection of $\omega_{H_1} - \omega_{H_2}$ reveals that

$$\omega_{H_i} - \omega_{H_j} \gtreqless 0, \text{ if } V(h,k,\phi_M,\phi_X) \lesseqgtr 0. \tag{A.35}$$

Appendix 5: Derivation of ϕ_{XS}

The solution ϕ_{XS} to $V(0,k,\phi_M,\phi_X) = -V(1,k,\phi_M,\phi_X) = 0$ is what Fujita *et al.*(1999b) call the 'sustain point' $T(S)$. The analysis of corner solutions of k knows that under the condition that $h = 1$ ($h = 0$), if $\phi_M \geq \phi_{MC}$, then $k = 1$ ($k = 0$); otherwise, $k = f(1) < 1$ ($k = f(0) > 0$). Therefore, calculating ϕ_{XS} needs to consider the two conditions:

If $\phi_M < \phi_{MC}$, then ϕ_{XS} is the solution to $V(1, f(1), \phi_M, \phi_X) = 0$. After simplifying, the equation becomes

$$\begin{aligned} & 2\sigma_X \left[\Theta_3\ \gamma + \Theta_1\right] \left[\frac{\Theta_3 \left(\sigma_X + \gamma\right) + \Theta_1}{\Theta_3 \left(\sigma_X - \gamma\right) - \Theta_1}\right]^{\frac{\beta}{\sigma_M - 1}} \\ = \ & \phi_{XS}^{\frac{\gamma}{\sigma_X - 1} - 1} \left\{\Theta_3\ \gamma\ \Theta_4 - \Theta_1 \left[\gamma\ \Theta_2 - 2\sigma_X\ \phi_{XS}\right]\right\} \end{aligned} \tag{A.36}$$

where $\Theta_1 \equiv H\ \beta\ \sigma_X$, $\Theta_2 \equiv \left(1 - \phi_{XS}^2\right)$, $\Theta_3 \equiv (H+L)\,\sigma_M$, and $\Theta_4 \equiv (\sigma_X - \gamma) + (\sigma_X + \gamma)\,\phi_{XS}^2$. The value of ϕ_{XS} is independent from ϕ_M.

If $\phi_M \geq \phi_{MC}$, then ϕ_{XS} is the solution to $V(1,1,\phi_M,\phi_X) = 0$. After simplifying, the equation becomes

$$2\sigma_X \left[\Theta_1 + \Theta_3\ \gamma\right] + \phi_M^{\frac{\beta}{\sigma_M - 1}} \phi_{XS}^{\frac{\gamma}{\sigma_X - 1} - 1} \left\{\Theta_1 \left[\gamma\ \Theta_2 - 2\sigma_X\ \phi_{XS}\right] - \Theta_3\ \gamma\ \Theta_4\right\} = 0 \tag{A.37}$$

where $\Theta_1 \equiv H\ \beta\ \sigma_X$, $\Theta_2 \equiv \left(1 - \phi_{XS}^2\right)$, $\Theta_3 \equiv (H+L)\,\sigma_M$, and $\Theta_4 \equiv (\sigma_X - \gamma) + (\sigma_X + \gamma)\,\phi_{XS}^2$. The value of ϕ_{XS} depends on ϕ_M.

Appendix 6: Inspection of $V_{hhh}(h,k,\phi_M,\phi_{XB})|_{h=1/2,\ k=1/2}$

Differentiating $V(h,k,\phi_M,\phi_{XB})$ with respect to h three times and substituting $h = 1/2$ and $k = 1/2$ into the result shows that

$$
\begin{aligned}
V_{hhh}(h,k,\phi_M,\phi_{XB})|_{h=1/2,\ k=1/2} \;=\;& \frac{1}{(\sigma_X-1)^3} 2^{2-\frac{\gamma}{\sigma_X-1}} (1+\phi_{XB})^{2-\frac{\gamma}{\sigma_X-1}} (\gamma)(1-\phi_{XB})^3 * \\
& \left[(k+(1-k)\,\phi_M)^{\frac{\beta}{\sigma_M-1}} + (1-k+k\ \phi_M)^{\frac{\beta}{\sigma_M-1}}\right] * \\
& \left[\frac{4\,(\sigma_X-1)}{\Theta_3\,(\sigma_X-1+\gamma)\,(\gamma+\sigma_X)+\Theta_1\,(2\sigma_X-1+\gamma)}\right] * \\
& Q\,(H,L,\sigma_M,\sigma_X,\beta,\gamma) \qquad\qquad (\text{A.38})
\end{aligned}
$$

where $\Theta_1 \equiv H\ \beta\ \sigma_X$, $\Theta_3 \equiv (H+L)\,\sigma_M$, and

$$
\begin{aligned}
Q\,(H,L,\sigma_M,\sigma_X,\beta,\gamma) \;=\;& \Theta_3^2\ \gamma\,(\sigma_X-\gamma)\,(\gamma+\sigma_X-1)\,(\sigma_X+\gamma)\,(\gamma-\sigma_X+1) + \\
& \Theta_1^2\ \gamma\left[1-\gamma^2+\sigma_X\,(4\sigma_X-5)\right] + \\
& \Theta_1\ \Theta_3 \left[\begin{array}{c} 2\gamma^2\,(1-\gamma^2) + \\ \sigma_X\,(2\sigma_X\,(1+3\gamma^2+\sigma_X(\sigma_X-2))-7\gamma^2) \end{array}\right] \qquad (\text{A.39})
\end{aligned}
$$

Whether $V_{hhh}(h,k,\phi_M,\phi_{XB})|_{h=1/2,\ k=1/2} > 0$ or $V_{hhh}(h,k,\phi_M,\phi_{XB})|_{h=1/2,\ k=1/2} < 0$ depends upon $Q\,(H,L,\sigma_M,\sigma_X,\beta,\gamma) > 0$ or $Q\,(H,L,\sigma_M,\sigma_X,\beta,\gamma) < 0$. Let $\xi \equiv \frac{\gamma}{\beta}$, $\eta \equiv \frac{\sigma_X}{\sigma_M}$, and $\zeta \equiv \frac{H}{L}$, $Q\,(H,L,\sigma_M,\sigma_X,\beta,\gamma)$ becomes $L^2\beta\ \sigma_M^2 * \Delta$, where

$$
\begin{aligned}
\Delta \;\equiv\;& \beta\ \gamma\,(1-\gamma)\,(1+\gamma)\,[\xi+\zeta\,(\eta+\xi)]^2 +\ \sigma_X\,\{-\beta\ \gamma\,[5\zeta\ \eta+2\,(1+\zeta)\,\xi]\,[\xi+\zeta\,(\eta+\xi)] + \\
& \sigma_X[2\zeta\,(1+\zeta)\,\eta+(2\gamma\ \zeta\ \eta+(1+\zeta)\,\xi)\,(2\beta\ \zeta\ \eta-1-\zeta) \\
& +2\,(1+\zeta)\,\gamma^2\,(\xi+\zeta(3\eta+\xi))+(1+\zeta)\,\sigma_X\,(2\zeta\ \eta-(1+\zeta)\,\xi)\,(\sigma_X-2)]\} \qquad (\text{A.40})
\end{aligned}
$$

Therefore, inspection of $V_{hhh}(h,k,\phi_M,\phi_{XB})|_{h=1/2,\ k=1/2}$ shows that

$$
V_{hhh}(h,k,\phi_M,\phi_{XB})|_{h=1/2,\ k=1/2} \gtreqless \ \text{if}\ \Delta \gtreqless 0 \qquad (\text{A.41})
$$

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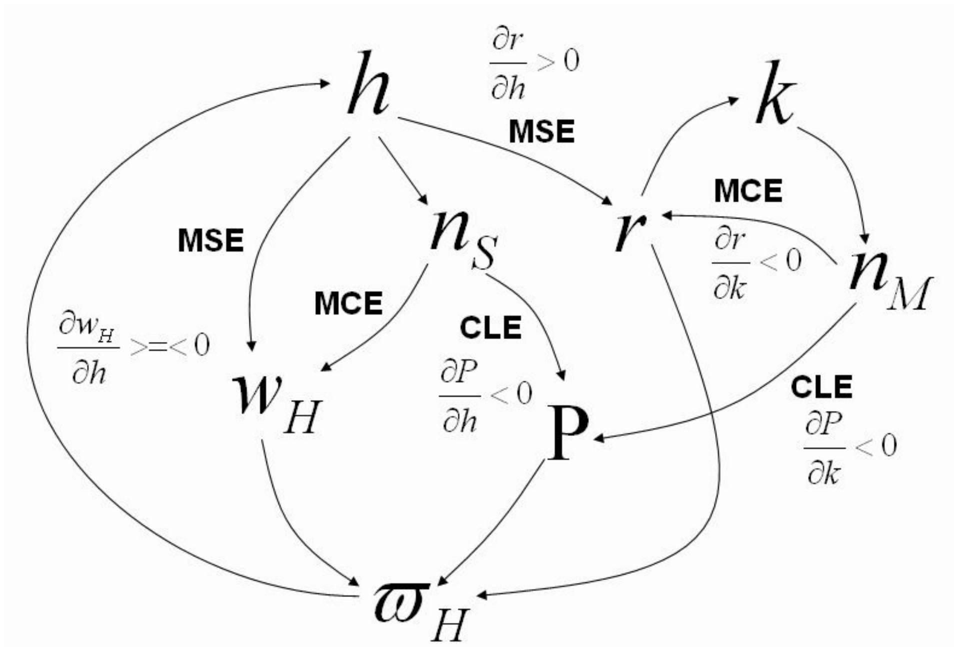

Figure 1: Forces at work in the economy

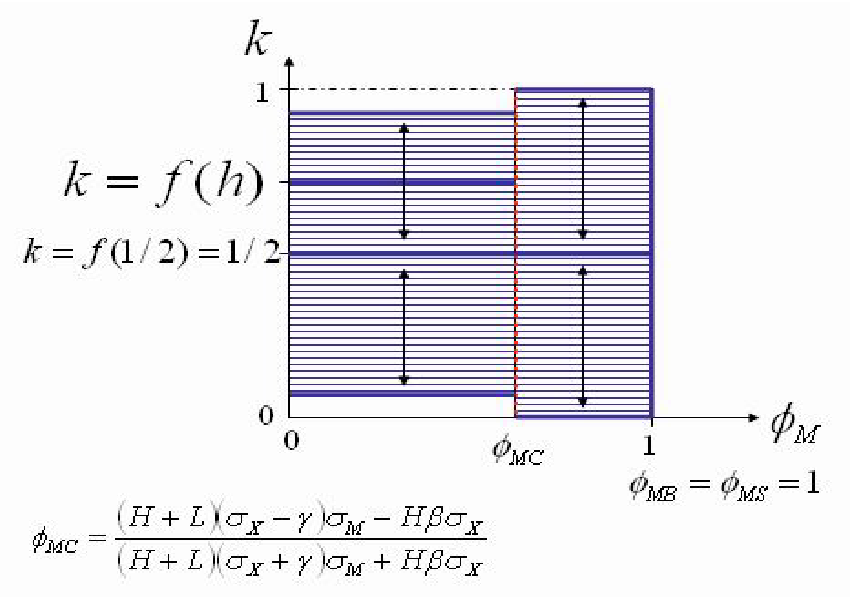

Figure 2: Bifurcation map of the $\mathbb{FC}$-sector

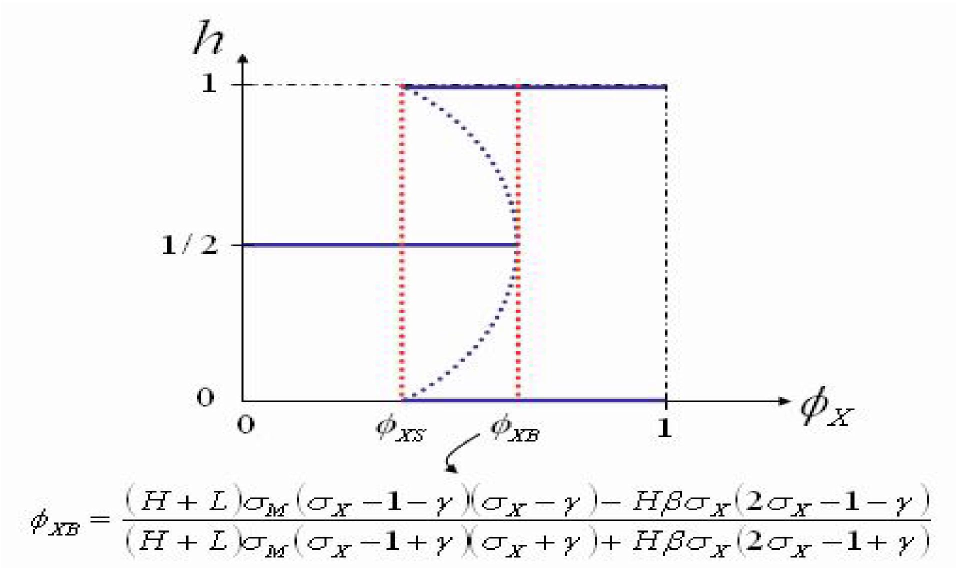

Figure 3: Subcritical bifurcation map of the $\mathbb{FE}$-sector

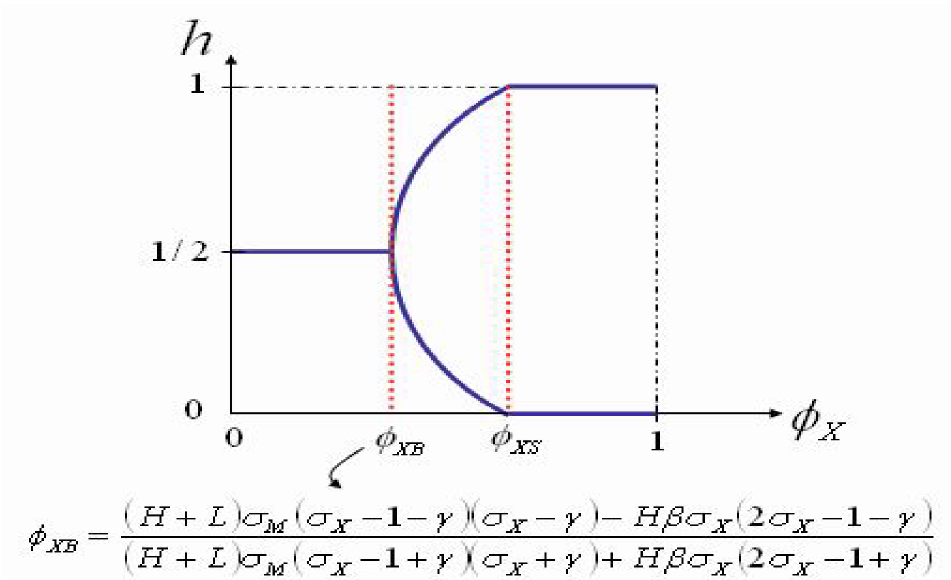

Figure 4: Supercritical bifurcation map of the $\mathbb{FE}$-sector

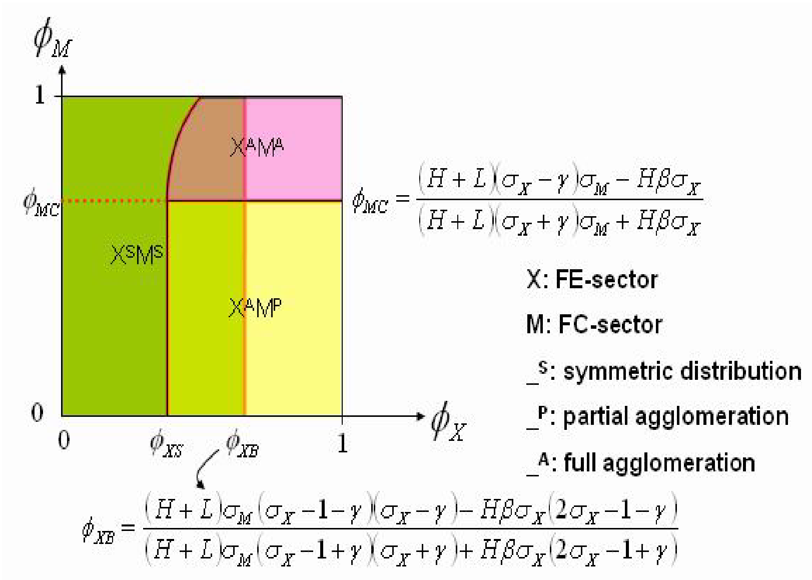

Figure 5: Equilibrium configurations (if $\mathbb{FE}$-sector is '*subcritical*')

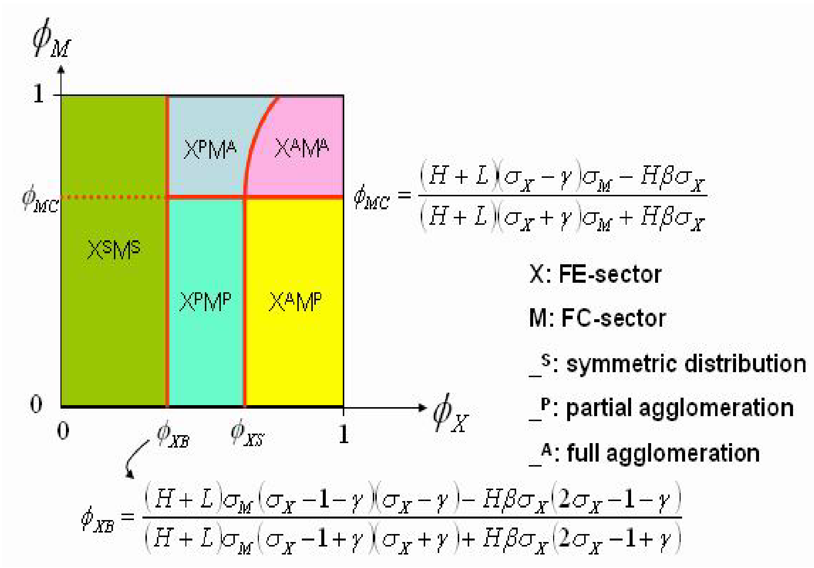

Figure 6: Equilibrium configuratons (if $\mathbb{FE}$-sector is '*supercritical*')

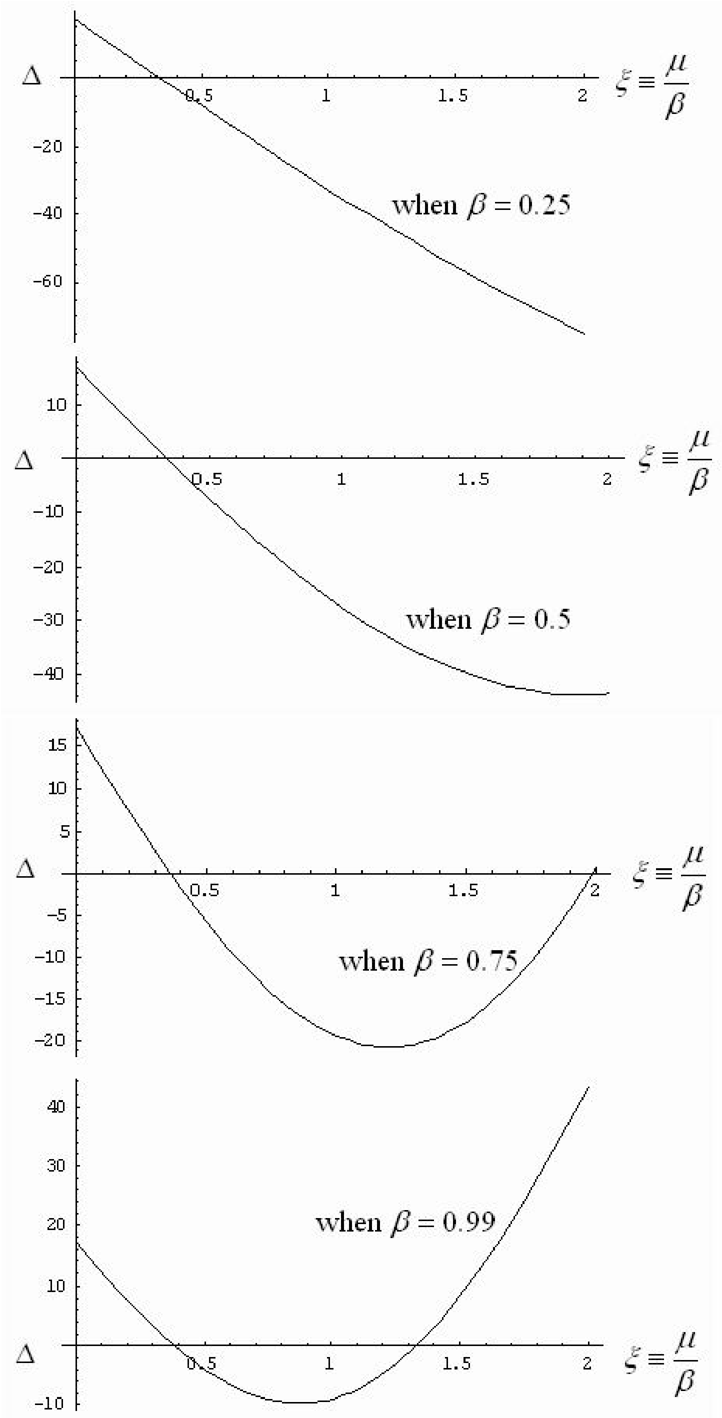

Figure 7: The fluctuations of Δ with respect to ξ when $\beta = 0.25$, $\beta = 0.5$, $\beta = 0.75$, and $\beta = 0.99$.

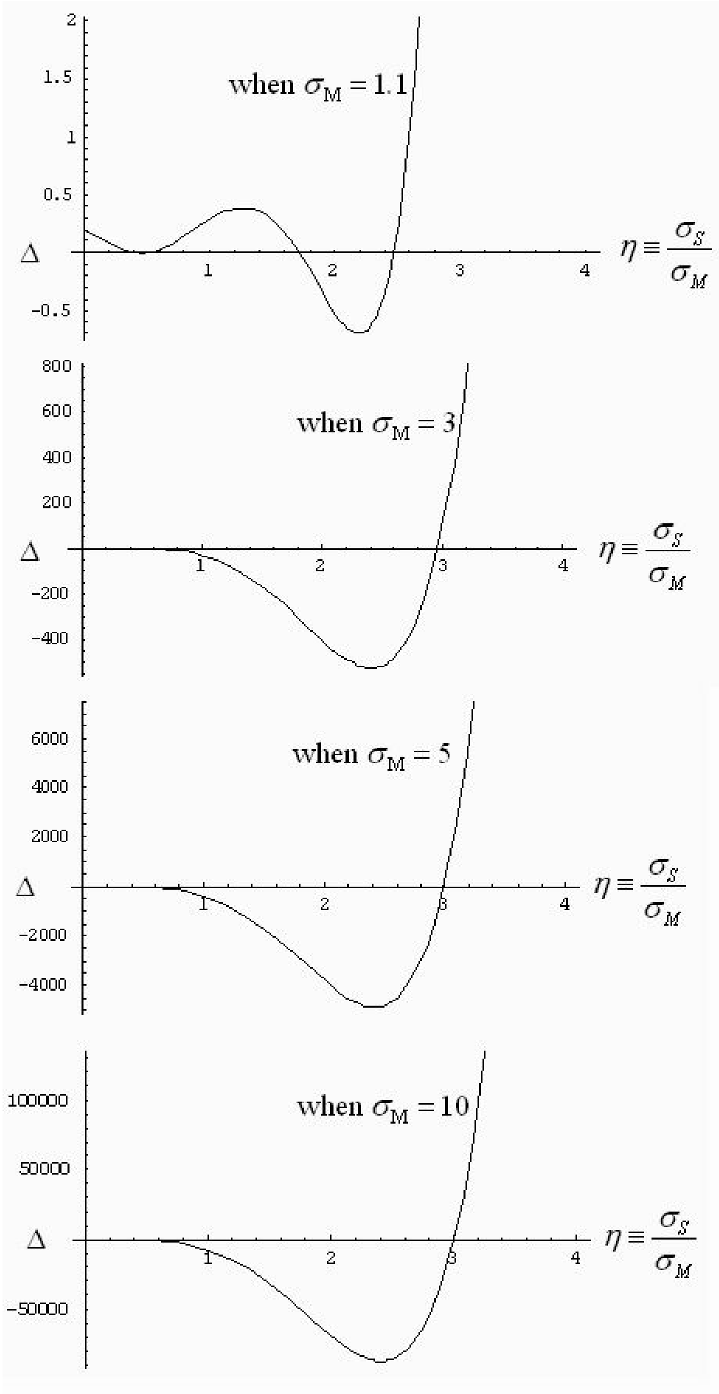

Figure 8: The fluctuations of Δ with respect to η when $\sigma_M = 1.1$, $\sigma_M = 3$, $\sigma_M = 5$, and $\sigma_M = 10$.