Knowledge Spillovers and the Formation of Spatial Networks

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Abstract: This paper proposes a new approach to city formation – the network formation approach. The main driving force of population agglomeration is uncompensated knowledge spillovers. Because knowledge can be transmitted only when both parties are linked in the network sense, the network formation approach is a natural framework to define and examine the underlying spatial configuration of the equilibrium. While it is beneficial to be connected to take advantage of knowledge transmission from other locations, maintaining a link is costly. Depending on its roles, a location may become a core, serving as a knowledge aggregation and transmission node for other connected peripheral locations. We find that a spatial equilibrium may feature monocentric, multicentric, urban-rural, or multiple urban areas. We examine under which conditions a particular spatial configuration may emerge and perform comparative statics with respect to changes in knowledge spillover, link maintenance, urban land rent, rent gradient, and urban unemployment parameters.

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"If one man starts a new idea, it is taken up by others and combined with suggestions of their own; and thus it becomes the source of new ideas." (Marshall, A., 1895, *Principles of Economics*, Macmillan and Co., London, p. 352).

1 Introduction

The study of endogenous city formation and equilibrium spatial configuration has gained enormous attention over the past few decades. While there is a global trend of urbanization, the internal structures of different urban areas are very divergent. For example, McMillen and Smith (2003) document that the U.S. urban structures may feature monocentric (such as Austin and Salt Lake City), duocentric (such as Las Vegas and Pittsburgh), and core-periphery with multiple subcenters (such as New York and Los Angeles). One may inquire: what are the underlying determinants that lead to various urban structures? Almost four decades ago, Jacobs (1969) stresses that knowledge spillovers in forms of production externality are crucial for city formation, firm clustering, and geographical concentration of research activity. Despite her compelling arguments, a comprehensive theoretical analysis of knowledge-based, locally agglomerative activity and the resulting spatial structure still remains largely open. The present paper attempts to develop such an analysis. This task is important because the provision of a more thorough analysis of possible forces to explain the formation of cities of various structures can help guide urban-related policy concerning land, housing, transportation system, and local public amenities.

The main driving forces of spatial agglomeration in this paper are uncompensated knowledge spillovers. Such spillovers, possibly in forms of physical, human or research capital, have long been regarded as important sources for sustained economic growth (see pivotal works by Shell 1966, Romer 1996 and Lucas 1988). While there has been abundant empirical evidence supporting the role of geographical concentration in facilitating knowledge transmission (e.g., see Rauch 1993, Saxenian 1994, and many others cited in recent work by Carlino, Chatterjee, Hunt 2004), just how important are knowledge spillovers to inducing agglomeration? In supporting Jacobs's proposition, Jaffe, Trajtenberg and Henderson (1993, 1996) find that patents are more likely to cite previous patents from the same area, thereby providing a compelling argument for knowledge-based agglomeration of inventive activity. Moreover, Audretsch and Feldman (1996) show that innovative activity clusters more in industries where knowledge spillovers are more significant – this finding is robust even after controlling for the geographical concentration of production. Furthermore, knowledge spillovers are found, in reality, highly localized (Rosenthal and Strange 2003) and are hence of particular relevance for the primary purpose of our paper – to examine the internal structure of a local urban area.

In this paper, we propose a network formation approach to knowledge-based city formation and spatial agglomeration. Because knowledge can be transmitted only when the parties involved are linked in the network sense, the network formation approach concerning equilibrium link patterns is a natural framework within which spatial equilibrium can be defined and the associated spatial configurations can be examined.

We focus on modeling the transmission, aggregation and spillover of knowledge on a spatial network where links on the graph represent potential route of transportation, communication, or trade between locations. Each worker possesses one unit of knowledge, so the total amount of knowledge at a location simply equals the population of its employed workers. All workers at a particular location are regarded as a collective player of our network formation game and these locations in our economy are thereby the nodes of the spatial network. Locations are categorized into cores and peripheries, where the former serves as the knowledge aggregating device. All disconnected peripheries feature full employment, but cores and peripheries connected with cores suffer nontrivial urban unemployment. While knowledge can be transmitted once two locations are linked, the spillover of aggregated knowledge is generated only by cores in the benchmark setup. In terms of knowledge spillovers, each periphery, upon paying a link maintenance cost, is served by the closest cores. Via a series of links, knowledge is transmitted or spilled over with decay. Thus, one may view link costs in our model as "payments" to intermediate inputs, productive knowledge stocks, aggregated by the core. There are two major tradeoffs incorporated in our basic environment:

- the benefit of connection with an urban area to take advantage of knowledge transmission vs. the cost of maintaining such a network link;
- the benefit of forming a core city aggregating knowledge inflows and serving others peripheries vs. the cost of incurring higher land rent.

Workers are homogeneous can choose locations freely to maximize per capita income and hence per capita income is equalized across all locations in locational equilibrium.

In terms of network formation, we employ an equilibrium concept that is a combination of standard Nash equilibrium and pairwise stability commonly seen in network games, to suit our study of the formation of spatial networks. In particular, the spatial configuration can change in any possible ways via severing/establishing links, switching roles between cores and peripheries, or combinations of any of them. Agents in a location are allowed to sever links unilaterally, but the establishment of a new link or the service provision of a core to a periphery must require mutual consent.

The main findings of the paper are as follows. A spatial equilibrium may feature monocentric, multicentric, urban-rural, or multiple urban areas, or coexistence of two or more of these configurations (multiple equilibria). With strong knowledge spillovers and low link costs, a single core is sufficient to serve the entire local economy and the monocentric configuration arises in equilibrium. With sufficiently weak knowledge spillovers and high link costs, there does not exist a spatial equilibrium featuring population agglomeration and the spatial economy degenerates. In the intermediate ranges of knowledge decays and link costs, there may be multiple cores, featuring either multicentric (with all location connected) or multiple urban areas (with different urban areas disconnected). With moderately high knowledge spillovers but sufficiently high link costs, the local economy features a single core but the outskirt locations are disconnected, implying an urban-rural configuration. Furthermore, by performing comparative statics with respect to changes in urban land rent, rent gradient, and urban unemployment parameters, we find that an increase in the urban land rent or a lower urban unemployment rate leads to a flatter population distribution, whereas a flatter rent gradient makes the population distribution more concentrated.

Literature Review

By ways of modeling methodology, previous studies of city formation and spatial configuration can be roughly divided into three streams: (i) new economic geography models (established by Krugman 1993 and Fujita and Krugman 1995, and generalized by Fujita and Mori 1997 and Berliant and Kung 2006), (ii) matching models (developed by Helsley and Strange 1990 and Abdel-Rahman and Wang 1995, and generalized by Coulson, Laing and Wang 2001 and Brueckner and Yves 2003), and (iii) production externality models (pioneered by Ogawa and Fujita 1980 and Fujita and Ogawa 1980 and generalized by Berliant, Peng and Wang 2002 and Lucas and Rossi-Hansberg 2002). The present paper follows the third stream, by incorporating an explicit production externality rather than one with imperfect competition or frictional matching.

The two pioneers studying urban configurations with production externality serving as the main agglomeration force are Ogawa and Fujita (1980) and Fujita and Ogawa (1982). Using numerical simulations, they find equilibria with multiple symmetric centers where population and commuting cost are the key determinants of the number of city centers in equilibrium. In a general-equilibrium framework, Berliant, Peng and Wang (2002) allow the extent to which knowledge in forms of capital spills across firms depends on geographical factors, particularly the distance between the firms and an overall firm dispersion measure. They prove by contradiction that multicentric spatial configuration cannot arise in competitive equilibrium within a static, closed city framework, in contrast with Ogawa and Fujita and Fujita and Ogawa. By extending Berliant, Peng and Wang to allow continual increase of population into an urban area, Berliant and Wang (forthcoming) establish conditions under which subcenters (with less population than the core) may form as population grows. Finally, Lucas and Rossi-Hansberg (2002) also consider production externality, but the spillover is in forms of labor. They take the level of utility attained by city residents as exogenous and allow population inflows/outflows to/from the city. As a consequence, multiple symmetric centers are natural equilibrium outcomes, as in Ogawa and Fujita and Fujita and Ogawa. In contrast with this urban configuration literature, our network formation approach permits not only more types of urban configurations (e.g., the urban-rural type) but also more strategic behaviors concerning locational interactions such as severing/establishing links with other locations and switching roles between cores and peripheries.

Our paper is related to the recently developed social/economic network literature. Aumann and Myerson (1988) were the first to study network formation in a strategic context in a cooperative game. But only since about a decade ago, the study of social/economic network formation has received greater attention. In their pivotal works, Jackson and Wolinsky (1996) and Dutta and Mutuswami (1997) formulate the network equilibrium based on pairwise stability: a particular link structure (or graph) is stable if no linked agent wants sever the existing link to become unlinked and no unlinked agent wants to form a new link. Wang and Watts (2006) consider a modified (stronger) version of pairwise stability in that no linked agents wants to simultaneously sever the existing link and form a new link. Bala and Goyal (2000) examine the endogenous formation networks using the strongest equilibrium concept: in their game, any individual players can form and sever links and an equilibrium is attained when no individual agent deviates. While this latter equilibrium concept is rooted on standard Nash equilibrium in noncooperative games, pairwise stability (or its modified versions) is also useful for defining equilibrium in many social/economic networks (e.g., information-processing networks, collaboration networks, trade networks, etc.). In our paper, we need to combine these equilibrium concepts to suit the particular need in our paper, i.e., city formation. Specifically, while we allow agents in a location to take some actions unilaterally (such as sever links), we restrict some other actions to be reached by consensus from all locations involved (such as the establishment of a new link and the service provision of a core to a periphery). Moreover, we must allow for additional decision-making by players to links: agents in a location can decide whether their residence should serve as a core or a periphery. This additional decision-making is to some degree (in the technical sense) similar to sellers' discrete choice concerning whether to join a sellers' association in the trade network game considered by Wang and Watts (2006). Furthermore, another major difference between our network games and those in the literature is that we allow the size (population) of each node (location) to be endogenuously determined in equilibrium. Our model is designed to include these distinctive features, because they are essential in the locational equilibrium under our considerations.

Finally, our paper is also related to the small but growing literature attempting at establishing the microfoundations for knowledge-based agglomeration models. For example, Glaeser (1999) stresses that cities can serve to promote knowledge acquisition by younger, less skilled workers from older and more skilled workers. Helsley and Strange (2002) construct a matching model in which firms are permitted to earn monopoly rents by selling their new projects to the local market and such rents provide incentives for the development of new ideas. Berliant, Reed and Wang (2006) establish a matching framework where individuals with different types of knowledge exchange their ideas to enhance their productivity and, via a thick-matching effect, population agglomeration may facilitate better knowledge exchange and hence higher output. Berliant and Fujita (2007) model the process of knowledge creation via matching and find that equilibrium matching patterns may evolve over time in a way resembling square dancing. In contrast with this literature, we consider very different equilibrium concept in our modified network formation games and examine explicitly the internal urban structure in locational equilibrium under which matching patterns can be affected by various strategic behaviors in spatial link formation and locality role determination.

2 The Model

This paper studies the transmission, aggregation and spillover of knowledge on a spatial network. We model workers at each location, or simply called a *location*, as a "collective player" of a network formation game. Players have fixed positions on a given graph which represents their geographic relationships. There are preexisting links on the graph which represent potential route of transportation, communication, or knowledge trade between locations. Before going into formal definitions, we introduce the main features of the model as follows.

- Core and periphery locations. A location can choose to be a core player or a periphery player.
- Link maintenance. A location can choose to maintain its links to other locations or to sever links. The link cost between a core location and a periphery location is paid by the periphery. The link cost is equally shared if the two linked locations are both cores or both peripheries.
- Stock of knowledge. Each worker possesses one unit of knowledge. The total amount of knowledge at a location equals the population of its employed workers.
- Service by a core. Each periphery location is served by the closest cores. When there are more than one core with the same shortest distance, the periphery is served by all such cores.
- Knowledge transmission, aggregation and spillovers. The knowledge of a location is transmitted with decays along maintained links away from the home location until it reaches a core location. A core aggregates all knowledge transmitted to itself, including the knowledge possessed by local workers. Cores serve peripheries by providing aggregated knowledge along links with decays (knowledge spillovers).
- While all disconnected peripheries feature full employment, cores and peripheries connected with cores suffer nontrivial urban unemployment.
- Worker mobility. Workers are homogeneous and freely mobile, and they choose locations that maximize expected per capita income. Therefore, per capita income is equalized across all locations in equilibrium.

2.1 Core/Periphery Locations and Urban Areas

The set of all locations is denoted by I. Locations are nodes on a geographic graph g, which is connected[1]. A location $i \in I$ chooses whether to be a core (c) or a periphery (p), and whether to maintain or server links with other locations. Location i's choice of roles is denoted by $\sigma_i \in \{c, p\}$. Denote the binary choice by location i for whether to link with location j as l_{ij}, where $l_{ij} = 1$ if i wants to link with j and $l_{ij} = 0$, otherwise. Location i's choice of maintained links is denoted by $l_i \in 2^{\{ij|ij \in g\}}$ and its set of linked locations is by $L_i = \{j | l_{ij} \cdot l_{ji} = 1\}$. Let M denote the total number of links maintained in this local economy. Let $\sigma = (\sigma_i)_{i \in I}$ and $l = (l_i)_{i \in I}$. A pair (σ, l) constitute a *spatial configuration*. It then defines an economic graph $g(\sigma, l) = \cup_{i \in I} l_i$ which is composed of maintained links among locations. Let $C = \{i \in I \mid \sigma_i = c\}$ denote the set of core locations, and $P = \{i \in I \mid \sigma_i = p\}$ denote the set of periphery locations. Let $d(i, j)$ denote the shortest distance between connected players i and j on the economic graph $g(\sigma, l)$. When i and j are not connected, $d(i, j) = \infty$.

Let $c(j)$ denote the set of core locations serving a periphery location j:

$$c(j) = \{i \in C \mid d(i, j) \leq d(k, j) \text{ for all } k \in C\}.$$

This is usually unique unless there is a tie among core locations that have the same shortest distance to j. A periphery location can spill knowledge over to another location if this transmission is not blocked by a core location. Let $s(j)$ denote the set of periphery locations that can transmit knowledge to a location j not blocked by a core:

$$s(j) = \{k \in P \mid jk \in g(\sigma, l), \nexists\, i \in C \text{ s.t. } i \in v(jk)\}$$

where $v(jk)$ denotes the node (or vertices) on the shortest path linking i, k on $g(\sigma, l)$.

A location j is said to be in an *urban area* U if either $j \in C$ or $\exists\, i \in C$ s.t. $d(i, j) < \infty$; that is, j is either a core itself or linked to a core directly or indirectly.

2.2 Knowledge Transmission, Aggregation and Spillovers

Each worker chooses a location to reside and possesses one unit of knowledge. Let N_i denote the population of employed workers at location i, so the amount of local knowledge at location i is also N_i. Knowledge gets transmitted or spilled over to another location with decay; a

[1] All locaitons are on a connected graph g. Each location can chooose to maintain or sever the link.

location receives δ^t $(0 \leq \delta < 1)$ unit of knowledge if one unit of knowledge gets transmitted through t links.

Let K_i denote the amount of knowledge (called the knowledge stock) used in production at location i. The amount of raw knowledge at a core location i is $N_i + \sum_{j \in s(i)} \delta^{d(i,j)} N_j$, which is a sum of raw knowledge created by the core's own workers and knowledge transmitted from all peripheries linked with the core. Core i aggregates knowledge into a stock measured by

$$K_i = \kappa \left(N_i + \sum_{j \in s(i)} \delta^{d(i,j)} N_j \right) \tag{1}$$

where parameter $\kappa \geq 1$ is the local multiplier of knowledge creation by the core.

A periphery location j has its own local knowledge and receives spilled aggregate knowledge from core locations. Periphery j's knowledge stock is

$$K_j = N_j + \sum_{i \in c(j)} \delta^{d(j,i)} K_i \tag{2}$$

which adds up the periphery's own raw knowledge with knowledge spilled over from all cores serving the periphery.

In an alternative setup when a periphery is allowed to receive knowledge spillovers from other periphery locations as well, its knowledge stock becomes

$$K_j = N_j + \sum_{k \in s(j)} \delta^{d(j,k)} N_k + \sum_{i \in c(j)} \delta^{d(j,i)} K_i. \tag{3}$$

2.3 Per Capita Income

It takes a unit cost z to maintain a link. The total link cost paid by a location is equally shared by all workers in the location. Let $n_p(j) = \{k \mid k \in P, d(k,j) = 1\}$ denote the set of periphery locations linked with j. Let $n_c(j) = \{k \mid k \in C, d(k,j) = 1\}$ denote the set of core locations linked with j. A core location does not pay for its links. A periphery location shares link cost with adjacent periphery locations and pays the entire cost for maintaining direct links with cores. Thus, a periphery pays total link costs

$$Z_j = \frac{z \left| n_p(j) \right|}{2} + z \left| n_c(j) \right|. \tag{4}$$

where $|\bullet|$ denotes the number of elements of a given set.

Workers produce output with knowledge using a Cobb-Douglas production technology given by $Y_i = K_i^{\alpha} N_i^{1-\alpha}$, where the scaling factor is normalized to one by choice of units. An employed worker at location i gets per capita output

$$y_i = (K_i/N_i)^a . \tag{5}$$

Following Harris and Todaro (1970), we consider full employment in rural areas and nontrivial underemployment in urban areas. Given an urban employment rate $e \in (0,1)$, the expected income for a worker residing in an urban area $i \in U$ is ey_i. Note that N_i is the working population at location i and N_i/e is the total population including nonworkers. Total worker population in the economy is 1. Therefore, the population identity implies

$$\sum_{i \in U} N_i/e + \sum_{j \in I \setminus U} N_j = 1. \tag{6}$$

Denote by $Q_i(\sigma, l)$ the exogenous per capita land rent paid by employed workers in i under a spatial configuration (σ, l). Workers at a core location $i \in C$ are therefore expected to earn a net income

$$\bar{y}_i = e\left[(K_j/N_j)^a - Q_i(\sigma, l)\right] \tag{7}$$

while workers at a periphery location $j \in P$ are expected to earn a net income

$$\bar{y}_j = e\left[(K_j/N_j)^a - Z_j/N_j - Q_j(\sigma, l)\right]. \tag{8}$$

Given a configuration (σ, l), we assume that land rent collection in core is simply q and that land rent in a periphery location j served by core i takes the following form:

$$Q_j(\sigma, l) = b^{d(j,i)} q. \tag{9}$$

The coefficient b represents the rent gradient, where the rent schedule is downward-sloped and decreasing geometrically in the distance away from the core with which a peripheral location are connected.

2.4 Locational Equilibrium

Given a configuration (σ, l), workers move to seek maximal per capita income. Consequently, per capita income is equalized among locations. Worker equilibrium is obtained if there exists a solution $\left\{(N_i)_{i\in I}, \bar{y}\right\}$ to the following equations:

$$\bar{y} = e\left[(K_j/N_j)^a - Q_i(\sigma, l)\right], \forall i \in C, \tag{10}$$

$$\bar{y} = e\left[(K_j/N_j)^a - \frac{z\,|n_p(j)|}{2N_j} - \frac{z\,|n_c(j)|}{N_j} - Q_j(\sigma, l)\right] - \frac{z\,|n_p(j)|}{2} - z\,|n_c(j)|, \forall j \in P, \tag{11}$$

$$\sum_{i\in U} N_i/e + \sum_{j\in I\setminus U} N_j = 1. \tag{12}$$

Let $\bar{y}(\sigma, l)$ denote the (equalized) per capita income in a worker equilibrium for a given configuration (σ, l) and $\bar{y}_i\left(\sigma, l, (N_i)_{i\in I}\right)$ denote the per capita income of location i under a given configuration and a given population distribution (which may not be a worker equilibrium).

Location i can choose a strategy $(\sigma_i, l_i) \in A_i = \{c, p\} \times 2^{\{ij|ij\in g\}}$ seeking maximal per capita income. We employ an equilibrium concept that is a combination of standard Nash equilibrium and pairwise stability commonly seen in network games. This modification of the equilibrium concept is to capture more naturally the behavior of localities to suit our study of the formation of the internal structure of a given local economy. The main features our equilibrium concept are specified as follows.

- Locations can take some actions unilaterally, while there are other actions that require consent from all locations involved.
- Locations can deviate by changing the configuration in any possible ways in A_i via switching roles, sever or establish links, or combinations of all of them.
- Locations can sever links unilaterally. But it needs mutual consent to establish a new link – particularly, both locations involved must have higher per capita incomes than that without the link. When a location switches from a periphery to a core, it also needs consent from linked peripheries since the link costs are paid by periphery locations.

Therefore, a deviation is successful if (i) location i has a higher per capita income, (ii) when a new link is established, both locations have higher per capita income, and (iii) when a periphery location i switches role to be a core, all periphery locations immediately linked with i need to be better off. It is defined formally in the following.

Definition. A *spatial equilibrium* is a pair of spatial configuration and population distribution $\left((\sigma, l), (N_i^*)_{i\in I}\right)$ such that Equations (10) to (11) are satisfied, and there is no location

$i \in I$ and no $(\sigma_i', l_i') \in A_i$ such that

$$\begin{aligned} \bar{y}_i'\left(\sigma', l', (N_i^*)_{i\in I}\right) &> \bar{y}(\sigma, l), \\ \bar{y}_j'\left(\sigma', l', (N_i^*)_{i\in I}\right) &> \bar{y}(\sigma, l), \forall j \in J_1 \cup J_2, \end{aligned}$$

where

$$\begin{aligned} J_1 &= \{ij \in g \mid ij \in l_i' \backslash l_i\}, \\ J_2 &= \begin{cases} \{ij \in g \mid d(i,j) = 1, \sigma_j = p\} \text{ if } \sigma_i = p,\ \sigma_i' = c, \\ \emptyset, \text{ otherwise.} \end{cases} \end{aligned}$$

3 General Propositions

We are now ready to establish four fundamental theorems concerning the spatial equilibrium in our network formation game. For simplicity, the following results are for parameters $e = 1$, $q = 0$.

The first theorem is on the existence of spatial equilibrium with population agglomeration. By agglomeration, we mean a core emerges with links to other periphery locations. When knowledge decays not too fast and link costs are not too large, there is an equilibrium with one core that is connected, via direct or indirect links, to all other locations; moreover, any location can be such a core.

Theorem 1. (Existence & Indeterminacy) *In a local network economy with a finite number of locations on an arbitrary graph, a spatial equilibrium with any location as the single core connected to all other locations exists if knowledge decays and link maintenance costs are not too large.*

Proof. Suppose we have a network economy with a finite number of locations. Let (σ^*, l^*) denote the following configuration: Pick location c as the core. Link every location to the core by a path with the shortest distance. Thus, (σ^*, l^*) is a tree graph where the distance from the core to location i is $d(c, i)$.

(i) First, we show that there is a solution with positive populations that equalizes per capita income. For population distribution $(N_i)_{i\in I}$, which is on the simplex of $\sum_{i\in I} N_i = 1$, the stock of aggregated knowledge at the core is

$$K_c = \kappa\left(N_c + \sum_{i\in P} \delta^{d(c,i)} N_i\right).$$

Therefore, the core has per capita income

$$y_c\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right)=\left(\frac{K_c}{N_c}\right)^{\alpha}.$$

A periphery location $i\in P=I\backslash c$ has per capita income

$$y_i\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right)=\left(1+\delta^{d(c,i)}\frac{K_c}{N_i}\right)^{\alpha}-\frac{l_i\left(\sigma^*,l^*\right)z}{N_i}\text{ for }i\in P$$

where $l_i\left(\sigma^*,l^*\right)z/N_i$ denotes the per capita link costs of i on configuration (σ^*,l^*). Define the following functions:

$$\begin{aligned}\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right) &= \max\left[0,y_c\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right)-y_i\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right)\right]\text{ for }i\in P\\ \eta_c\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right) &= \max\left[0,\frac{1}{|P|}\sum_{i\in P}y_i\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right)-y_c\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right)\right].\end{aligned}$$

Construct map F as follows:

$$\begin{aligned}F_i\left(\left(N_i\right)_{i\in I}\right) &= \frac{N_i+\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right)}{1+\sum_{i\in I}\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right)}\text{ for }i\in P\\ F_c\left(\left(N_i\right)_{i\in I}\right) &= \frac{N_c+\eta_c\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right)}{1+\sum_{i\in I}\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i\right)_{i\in I}\right)}.\end{aligned}$$

It maps a population distribution from the simplex to itself continuously. By Brower's Fixed Point Theorem, there is a population distribution $(N_i^*)_{i\in I}$ such that $F_i\left((N_i^*)_{i\in I}\right)=N_i^*$. This means, for $i\in P$,

$$N_i^*=\frac{N_i^*+\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)}{1+\sum_{i\in I}\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)},$$

$$N_i^*\sum_{i\in I}\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)=\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right),$$

and, for c,

$$N_c^*=\frac{N_c^*+\eta_c\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)}{1+\sum_{i\in I}\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)},$$

$$N_c^*\sum_{i\in I}\eta_c\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)=\eta_c\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right).$$

Suppose for some $i\in I$, $\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)>0$. Then $\sum_{i\in I}\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)>0$ and this means $\eta_i\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)\geq 0$ for all $i\in I$ since $N_i^*\geq 0$. Then, we have

$$\sum_{i\in P}\left[y_c\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)-y_i\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)\right]+$$
$$\frac{1}{|P|}\sum_{i\in P}y_i\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)-y_c\left(\left(\sigma^*,l^*\right),\left(N_i^*\right)_{i\in I}\right)>0,$$

and

$$\begin{aligned}
y_c\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) - y_i\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) &\geq 0, \\
\frac{1}{|P|}\sum_{i\in P} y_i\left(\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right)\right) - y_c\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) &\geq 0.
\end{aligned}$$

This implies the following contradiction,

$$\sum_{i\in P}\left[y_c\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) - y_i\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right)\right] \\
+ \sum_{i\in P} y_i\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) - |P|\, y_c\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) > 0.$$

Therefore, $\eta_i\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) = 0$ for all i, which implies

$$\begin{aligned}
y_c\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) - y_i\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) &= 0, \text{ for } i \in P, \\
\frac{1}{|P|}\sum_{i\in P} y_i\left(\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right)\right) - y_c\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) &= 0.
\end{aligned}$$

Otherwise, suppose $y_c\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) - y_i\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) < 0$ for some $i \in P$; there is a contradiction:

$$\sum_{i\in P}\left[y_c\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) - y_i\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right)\right] \\
+ \sum_{i\in P} y_i\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) - |P|\, y_c\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) < 0.$$

Therefore, $\left(N_i^*\right)_{i\in I}$ is an equilibrium population distribution.

Next, we rule out zero population. Note that

$$\begin{aligned}
\lim_{N_c\to 0} y_c\left(\left(\sigma^*, l^*\right), \left(N_i\right)_{i\in I}\right) &= \infty, \\
\lim_{N_i\to 0} y_i\left(\left(\sigma^*, l^*\right), \left(N_i\right)_{i\in I}\right) &= -\infty.
\end{aligned}$$

The second limit follows from,

$$\lim_{N_i\to 0}\frac{\left(1+\delta^{d(c,i)}\frac{K_c}{N_i}\right)^{\alpha}}{\frac{l_i(\sigma^*,l^*)z}{N_i}} = \frac{\left(\left(N_i\right)^{1/\alpha} + \delta^{d(c,i)} K_c \left(N_i\right)^{(1-\alpha)/\alpha}\right)^{\alpha}}{l_i\left(\sigma^*, l^*\right) z} = 0.$$

Then, to have $y_c\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) - y_i\left(\left(\sigma^*, l^*\right), \left(N_i^*\right)_{i\in I}\right) = 0$, it must be the case that $N_i^* > 0$ and $N_c^* > 0$.

Finally, it is obvious that population distribution $(N_i^*)_{i \in I}$ yields positive income when z is not too large.

(ii) No periphery location wants to switch to be a core. Suppose location j has income

$$y_j\left((\sigma^*, l^*), (N_i)_{i\in I}\right) = \left(1 + \delta^{d(c,j)}\kappa \frac{N_c + \sum_{i\in P}\delta^{d(c,i)}N_i}{N_j}\right)^{\alpha} - \frac{l_j(\sigma^*, l^*)\, z}{N_j},$$

and switches to be a core. Suppose there is a configuration g' that links all other locations with j of the shortest distance which is denoted by $d'(c,i)$, except for location c. This is the most favorable configuration for j as a core where the per capita income is

$$y_j\left(g', (N_i)_{i\in I}\right) = \left(\kappa \frac{\sum_{i\in P\setminus j}\delta^{d'(c,i)}N_i}{N_j}\right)^{\alpha} \quad \text{for } i \in P.$$

Compare the above two expressions: the lost of knowledge in N_c dominates the saving in link costs if δ is large enough and z is small enough. Location j will not switch even if it is under the most favorable configuration g'. So, it will not switch in (σ^*, l^*).

(iii) No location wants to change links (and not better ways to link). By changing link, a location will not increase the knowledge received from the core since they are linked with the shortest distance. A location may sever a link to reduce the cost. Any link severed will reduce K_c since (σ^*, l^*) is a tree. However, if δ is large enough and z is small enough, the lost in knowledge dominates the save in link costs. ∎

The second theorem concerns the impossibility of spatial equilibrium due to high decay or high link costs.

Theorem 2. (Impossibility on Spatial Networks) *In a local network economy with a finite number of locations on an arbitrary graph, with sufficiently large knowledge decays and link maintenance costs, the spatial equilibrium degenerates with all locations disconnected and no population agglomeration regardless of the magnitude of the local multiplier of knowledge creation by the core.*

Proof. Suppose c is a core in a configuration (σ', l'). Any location j adjacent to a core has income

$$y_j\left((\sigma', l'), (N_i)_{i\in I}\right) = \left(1 + \delta\kappa \frac{N_c + \sum_{i\in s(c)}\delta^{d(c,i)}N_i}{N_j}\right)^{\alpha} - \frac{l_j(\sigma^*, l^*)\, z}{N_j}.$$

As δ decreases or z increases, this value decreases below 1 – the per capita income in isolation. In this case, no location will link together. ∎

We proceed to discuss more specific symmetric patterns which have odd numbers of locations on a line, single cores at the center, and declining populations from the center. This means the monocentric configuration where all locations are connected to the core and the urban-rural configration where some locations on the ends are not connected to the core.

Let $2D$ denote the number of peripheries connected to the core and label locations on the right side of the core from 1 to D. It is noted that, all connected peripheries, except for the locations 1 and D (which pay for 3/2 and 1/2 of the per capita link cost), pay for the the per capita link cost. We can thus establish:

Proposition 1. *In an monocentric configuration,*

(i) *except for $i = 1$ and D, the population declines in a rate fater than the knowledge decay rate, i.e., $N_{i+1}/N_i < \delta$;*

(ii) *per capita income declines with population if z small enough.*

Proof.

(i) The aggregated knowledge at the core is

$$K_c = \kappa \left(N_c + 2 \sum_{i=1}^{D} \delta^i N_i \right).$$

For $i = 2, ...D - 2$,

$$\begin{aligned} y_i &= \left(1 + \delta^i \frac{K_c}{N_i}\right)^\alpha - \frac{z}{N_i}, \\ y_{i+1} &= \left(1 + \delta^{i+1} \frac{K_c}{N_{i+1}}\right)^\alpha - \frac{z}{N_{i+1}}. \end{aligned}$$

Equalizing income, we have

$$\left(1 + \delta^{i+1} \frac{K_c}{N_{i+1}}\right)^\alpha - \left(1 + \delta^i \frac{K_c}{N_i}\right)^\alpha = \frac{z}{N_{i+1}} - \frac{z}{N_i}.$$

Since we have a decreasing population from the center, $N_i > N_{i+1}$,

$$\left(1 + \delta^{i+1} \frac{K_c}{N_{i+1}}\right)^\alpha - \left(1 + \delta^i \frac{K_c}{N_i}\right)^\alpha > 0.$$

This means

$$N_{i+1}/N_i < \delta.$$

(ii) Let $K_c^{-i} = K_c - \kappa\delta^i N_i$. Then,

$$\frac{d}{dN_i}\left[\left(1+\delta^i\frac{K_c}{N_i}\right)^\alpha - \frac{z}{N_i}\right] = -\alpha\left(1+\delta^i\frac{K_c}{N_i}\right)^\alpha \kappa\delta^i\frac{1}{(N_i)^2} + \frac{z}{(N_i)^2}$$

which is negative for small z. ∎

Similar properties apply to the urban-rural configuration where the only differences have to do with the aggregate knowledge at the core and the boundry of the MSA – neither would affect the arguments of the proof.

4 Spatial Configurations

In this section, we further investigate spatial equilibrium using simulation exercises to gain better insights toward understanding the emergence of spatial network formation and the changes in internal equilibrium urban structures in response to changes in knowledge spillover, link maintenance, urban land rent, rent gradient, and urban unemployment parameters.

For simplicity, we restrict our attention to a linear spatial structure with 5 locations, indexed from left to right by $i = 1, 2, 3, 4, 5$, where the five locations as a whole constitute a closed local economy. This simple structure is sufficient for generating four symmetric spatial configurations of particular interest.

(i) Monocentric configuration ($\theta = m$): Location 3 serves as the unique core while locations 1, 2, 4, and 5 are all peripheries.

(ii) Multicentric configuration, particularly duocentric configuration[2] ($\theta = d$): There are two patterns of duocentric configuration. The first has both locations 2 and 4 serve as cores; locations 1 and 5 are peripheries, served by cores 2 and 4, respectively; location 3 is a periphery served by both cores. The second has locations 1 and 5 as cores and 2, 3, and 4 as peripheries. With 5 locations, there is only one symmetric tricentric configuration where locations 1, 3, and 5 are cores and locations 2 and 4 are peripheries.

(iii) Urban-rural configuration ($\theta = u$): Locations 3 is the unique core, serving periphery locations 2 and 4; locations 1 and 5 are "rural" peripheries, disconnected from the urban area $U = \{2, 3, 4\}$.

[2]This includes the tricentric configuration as well. Since the range of existence and the comparative statics are similar to those of the duocentric case, we present only the duocentric configuration for simplicity.

(iv) Two-MSA configuration ($\theta = s$): either location 1 (resp. 5) is a core serving peripheral cities 2 (resp. 4), or location 2 (resp. 4) is a core serving peripheral cities 1 (resp. 5); locations 3 is completely disconnected, there are two separate urban areas (MSAs), $U = \{1, 2\} \cup \{4, 5\}$.

Recall that in the benchmark economy described in Section 2, periphery locations transmit knowledge to core locations and periphery locations only receive spillovers from the closest core. It is obvious that a location cannot play core alone; there must be a link to a periphery location. Moreover, two core locations will not link together because there is no benefit.

4.1 Benchmark Models

To describe configurations, we use letters "c" and "p" to indicate the choice of being a core or a periphery. For convenience, we use "-" in a configuration to indicate the presence of a link between two locations and ".." to represent disconnection. Note that no periphery locations will remain linked without a core since there is no gain but only costs. A core will not sever any link to another location while remaining a core, since link costs are paid by peripheries.

4.1.1 Monocentric configuration (p-p-c-p-p)

In this configuration, location 3 receives knowledge spillovers from all locations, and serves all periphery locations. The per capita incomes at all locations are as follows:

$$\bar{y}_1 = e\left[\left(\left(\delta^4\kappa + 1\right) + \delta^2\kappa\left(\delta N_2 + N_3 + \delta N_4 + \delta^2 N_5\right)/N_1\right)^{\alpha} - z/\left(2N_1\right) - b^2 q\right],$$
$$\bar{y}_2 = e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(\delta^2 N_1 + +N_3 + \delta N_4 + \delta^2 N_5\right)/N_2\right)^{\alpha} - 3z/\left(2N_2\right) - bq\right],$$
$$\bar{y}_3 = e\left[\left(\kappa + \delta\kappa\left(\delta N_1 + N_2 + N_4 + \delta N_5\right)/N_3\right)^{\alpha} - q\right],$$
$$\bar{y}_4 = e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(\delta^2 N_1 + \delta N_2 + N_3 + +\delta^2 N_5\right)/N_4\right)^{a} - 3z/\left(2N_4\right) - bq\right],$$
$$\bar{y}_5 = e\left[\left(\left(\delta^4\kappa + 1\right) + \delta^2\kappa\left(\delta^2 N_1 + \delta N_2 + N_3 + \delta N_4\right)/N_5\right)^{a} - z/\left(2N_5\right) - b^2 q\right].$$

Incomes are equalized in equilibrium; $\bar{y}_i = \bar{y}$. Since all of the worker population lives in urban areas, population feasibility requires $\frac{1}{e}\sum_{i=1}^{5} N_i = 1$.

Locations may deviate in the following ways: (i) A location can sever all links and stay alone. It then yields the rural per capita income, which is 1. Equilibrium requires

$$\bar{y} - 1 \geq 0.$$

(ii) Location 2 (similarly for location 4) can sever the link with location 1 (p..p-c-p-p). Location 2 pays only for one link but do not receive knowledge from location 1. Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(N_3 + \delta N_4 + \delta^2 N_5\right)/N_2\right)^\alpha - z/N_2 - bq\right] \geq 0.$$

(iii) Location 2 can also play core and link only with location 1 (p-c..c-p-p). This move should be mutually beneficial to location 1. Equilibrium requires

$$\bar{y} - e\left[(\kappa + \delta\kappa N_1/N_2)^\alpha - q\right] \geq 0, \text{ or}$$
$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa N_2/N_1\right)^\alpha - z/N_1 - bq\right] \geq 0.$$

(iv) Location 1 (similarly for location 5) can play core (c-p-c-p-p). Location 1 pays for no link but only receives knowledge from location 2. This move should be mutually beneficial to location 2 (similarly for location 4). Equilibrium requires

$$\bar{y} - e\left[(\kappa + \delta\kappa N_2/N_1)^a - q\right] \geq 0, \text{ or}$$
$$\bar{y} - e\left[\left(\left(2\delta^2\kappa + 1\right) + \delta\kappa\left(N_1 + N_3 + \delta N_4 + \delta^2 N_5\right)/N_2\right)^\alpha - 2z/N_2 - bq\right] \geq 0.$$

4.1.2 Multicentric configuration (p-c-p-c-p)

There are two types of multicentric configurations: (i) duocentric (p-c-p-c-p and c-p-p-p-c) and (ii) tricentric (c-p-c-p-c). In this subsection, we for brevity discuss only the first pattern of the duocentric configuration, p-c-p-c-p. We will relegate the second pattern of the duocentric configuration and the tricentric configuration to the Appendix.

We illustrate the case with locations 2 and 4 as cores. Location 2 (respectively, 4) as a core receives spillovers from locations 1 and 3 (respectively, 5 and 3) and serves back. The per capita incomes at all locations are as follows:

$$\bar{y}_1 = e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(N_2 + \delta N_3\right)/N_1\right)^\alpha - z/N_1 - bq\right],$$
$$\bar{y}_2 = e\left[\left(\kappa + \delta\kappa\left(N_1 + N_3\right)/N_2\right)^\alpha - q\right],$$
$$\bar{y}_3 = e\left[\left(\left(2\delta^2\kappa + 1\right) + \delta\kappa\left(\delta N_1 + N_2 + N_4 + \delta N_5\right)/N_3\right)^\alpha - 2z/N_3 - bq\right],$$
$$\bar{y}_4 = e\left[\left(\kappa + \delta\kappa\left(N_3 + N_5\right)/N_4\right)^\alpha - q\right],$$
$$\bar{y}_5 = e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(\delta N_3 + N_4\right)/N_5\right)^\alpha - z/N_5 - bq\right].$$

Incomes are equalized in equilibrium; $\bar{y}_i = \bar{y}$. Population feasibility requires $\frac{1}{e}\sum_{i=1}^{5} N_i = 1$.

Locations may deviate in the following ways: (i) Every location can sever links and stay alone; equilibrium requires

$$\bar{y} - 1 \geq 0.$$

(ii) Location 2 (similarly for location 4) can play periphery (p-p-p-c-p). It then receive knowledge from all locations but pays for one link. Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^4\kappa + 1\right) + \delta^2\kappa\left(\delta^3 N_1 + \delta N_3 + N_4 + \delta N_5\right)/N_2\right)^\alpha - z/N_2 - b^2 q\right] \geq 0.$$

(iii) Location 2 can play periphery and sever the link with location 1 (p..p-p-c-p). Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^4\kappa + 1\right) + \delta^2\kappa\left(\delta N_3 + N_4 + \delta N_5\right)/N_2\right)^\alpha - z/\left(2N_2\right) - b^2 q\right] \geq 0.$$

(iv) Locations 3 can sever one link (p-c-p..c-p). It saves link cost but loses knowledge from locations 4 and 5. Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(\delta N_1 + N_2\right)/N_3\right)^\alpha - z/N_3 - bq\right].$$

4.1.3 Urban-rural configuration (p..p-c-p..p)

Rural locations 1 and 5 yield unit per capita income. Location 3 receives spillovers from locations 2 and 4 and serves back. The per capita income at all urban locations are as follows:

$$\bar{y}_2 = e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(N_3 + \delta N_4\right)/N_2\right)^\alpha - z/N_2 - bq\right],$$
$$\bar{y}_3 = e\left[\left(\kappa + \delta\kappa\left(N_2 + N_4\right)/N_3\right)^\alpha - q\right],$$
$$\bar{y}_4 = e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(\delta N_2 + N_3\right)/N_4\right)^\alpha - z/N_4 - bq\right].$$

Incomes are equalized in equilibrium; $\bar{y}_i = \bar{y} = 1$. Population feasibility requires $N_1 + \frac{N_2+N_3+N_4}{e} + N_5 = 1$, and $N_1 = N_5$.

No location will deviate to stay alone since the equilibrium per capita income is 1. Locations may deviate in the following ways: (i) Location 2 (similarly for location 4) can link with location 1 (similarly for location 5) and form (p-p-c-p..p). This move needs to be mutually beneficial. They both pay more link cost but also receive more knowledge. Equilibrium requires

$$\begin{aligned}\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(\delta^2 e N_1 + N_3 + \delta N_4\right)/N_2\right)^\alpha - 3z/\left(2N_2\right) - bq\right] &\geq 0, \text{ or} \\ \bar{y} - e\left[\left(\left(\delta^4\kappa + 1\right) + \delta^2\kappa\left(\delta N_2 + N_3 + \delta N_4\right)/\left(eN_1\right)\right)^\alpha - z/\left(2eN_1\right) - b^2 q\right] &\geq 0.\end{aligned}$$

(ii) Location 2 can play core and link with location 1 (p-c..c-p..p). It saves link cost by being a core. This move needs to be mutually beneficial to location 1. Equilibrium requires

$$\bar{y} - e\left[(\kappa + \delta\kappa e N_1/N_2)^{\alpha} - q\right] \geq 0, \text{ or}$$
$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa N_2/\left(eN_1\right)\right)^{\alpha} - z/\left(eN_1\right) - bq\right].$$

(iii) Location 1 (similarly for location 5) can play core and link with location 2 (similarly for location 4) and form (c-p-c-p..p). This move needs to be mutually beneficial. Equilibrium requires

$$\bar{y} - e\left[(\kappa + \delta\kappa N_2/\left(eN_1\right))^{\alpha} - q\right] \geq 0, \text{ or}$$
$$\bar{y} - e\left[\left(\left(2\delta^2\kappa + 1\right) + \delta\left(\kappa e N_1 + N_3\right)/N_2\right)^{\alpha} - 2z/N_2 - bq\right].$$

4.1.4 Two-MSA configuration (c-p..p..p-c)

There are two cases of two-MSA systems, with locations 1 and 5 as cores (c-p..p..p-c) and with locations 2 and 4 as cores (p-c..p..c-p); location 3 is the rural place separating the two MSAs. We analyze the first case here as an example, and the second case can be found in the Appendix. Locations 1 and 5 receive knowledge from locations 2 and 4 respectively and serve back. Rural location 3 yields unit per capita income. The per capita incomes of all locations are

$$\bar{y}_1 = e\left[(\kappa + \delta\kappa N_2/N_1)^{\alpha} - q\right],$$
$$\bar{y}_2 = e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa N_1/N_2\right)^{\alpha} - z/N_2 - bq\right],$$
$$\bar{y}_3 = 1,$$
$$\bar{y}_4 = e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa N_5/N_4\right)^{\alpha} - z/N_4 - bq\right],$$
$$\bar{y}_5 = e\left[(\kappa + \delta\kappa N_4/N_5)^{\alpha} - q\right].$$

Incomes are equalized in equilibrium; $\bar{y}_i = \bar{y}$. Population feasibility requires $\left(\frac{N_1+N_2+N_4+N_5}{e}\right)$ $+\ N_3 = 1$.

Locations may deviate in the following ways: (i) Locations 2 and 3 (similarly for locations 4 and 3) can link together (c-p-p..p-c). They both pay more link cost but receive more knowledge. This move should be mutually beneficial. Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(N_1 + \delta^2 e N_3\right)/N_2\right)^{\alpha} - 3z/\left(2N_2\right) - bq\right] \geq 0, \text{ or}$$
$$\bar{y} - e\left[\left(\left(\delta^4\kappa + 1\right) + \delta^2\kappa\left(N_1 + \delta N_2\right)/\left(eN_3\right)\right)^{\alpha} - z/\left(2eN_3\right) - b^2 q\right] \geq 0.$$

(ii) Location 3 can link with locations 2 and 4 (c-p-p-p-c) and forms a duocentric configuration. This move should be beneficial to locations 2 and 4. Equilibrium requires

$$\bar{y} - e\left[\left(\left(2\delta^4\kappa+1\right)+\delta^2\kappa\left(N_1+\left(\delta+\delta^3\right)N_2+\left(\delta+\delta^3\right)N_4+N_5\right)/\left(eN_3\right)\right)^{\alpha}\text{-}z/\left(eN_3\right)\text{-}b^2q\right] \geq 0, \text{ or}$$
$$\bar{y} - e\left[\left(\left(\delta^2\kappa+1\right)+\delta\kappa\left(N_1+\delta^2eN_3+\delta^3N_4\right)/N_2\right)^{\alpha} - 3z/\left(2N_2\right) - bq\right] \geq 0, \text{ or}$$
$$\bar{y} - e\left[\left(\left(\delta^2\kappa+1\right)+\delta\kappa\left(\delta^3N_2+\delta^2eN_3+N_5\right)/N_4\right)^{\alpha} - 3z/\left(2N_4\right) - bq\right] \geq 0.$$

4.2 Alternative Setups

Besides the benchmark models, there are alternative ways to model the transition of knowledge. For example, a periphery location may pass knowledge to other peripheries. We will examine the monocentric configuration for this setup. Also, a periphery location may receive services from all cores instead of the closest one. We will examine the duocentric configuration for this setup. The equilibria in these alternative setups are contrasted with the benchmarks in the numerical analysis in Section 4.1.

4.2.1 Alternative Monocentric (p-p-c-p-p)

The monocentric configuration is examined when knowledge from a periphery location can spillover to another periphery location before it is stopped by a core. For example, the knowledge from location 1 shows up at location 2 once without the multiplier and once with the multiplier. The per capita incomes at all locations are as follows:

$$y_1 = \left(\left(\delta^4\kappa+1\right)+\left(\left(\delta^3\kappa+\delta\right)N_2+\delta^2\kappa N_3+\delta^3\kappa N_4+\delta^4\kappa N_5\right)/N_1\right)^{\alpha} - z/\left(2N_1\right) - b^2q,$$
$$y_2 = \left(\left(\delta^2\kappa+1\right)+\left(\left(\delta^3\kappa+\delta\right)N_1+\delta\kappa N_3+\delta^2\kappa N_4+\delta^3\kappa N_5\right)/N_2\right)^{\alpha} - 3z/\left(2N_2\right) - bq,$$
$$y_3 = \left(\kappa+\delta\kappa\left(\delta N_1+N_2+N_4+\delta N_5\right)/N_3\right)^{\alpha} - q,$$
$$y_4 = \left(\left(\delta^2\kappa+1\right)+\left(\delta^3\kappa N_1+\delta^2\kappa N_2+\delta\kappa N_3+\left(\delta^3\kappa+\delta\right)N_5\right)/N_4\right)^{\alpha} - 3z/\left(2N_4\right) - bq,$$
$$y_5 = \left(\left(\delta^4\kappa+1\right)+\left(\delta^4\kappa N_1+\delta^3\kappa N_2+\delta^2\kappa N_3+\left(\delta^3\kappa+\delta\right)N_4\right)/N_5\right)^{\alpha} - z/\left(2N_5\right) - b^2q.$$

Incomes are equalized in equilibrium; $\bar{y}_i = \bar{y}$. Population feasibility requires $\frac{1}{e}\sum_{i=1}^{5} N_i = 1$. The deviation conditions are the same as for the benchmark model in Section 3.1.

4.2.2 Alternative Duocentric (p-c-p-c-p)

The duocentric configuration, with locations 2 and 4 as cores, is examined when the cores serve all periphery locations. The per capita incomes at all locations are as follows:

$$\bar{y}_1 = \left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(N_2 + \left(\delta + \delta^3\right)N_3 + \delta^2 N_4 + \delta^3 N_5\right)/N_1\right)^\alpha - z/N_1 - bq,$$
$$\bar{y}_2 = \left(\kappa + \delta\kappa\left(N_1 + N_3\right)/N_2\right)^\alpha - q,$$
$$\bar{y}_3 = \left(\left(2\delta^2\kappa + 1\right) + \delta\kappa\left(\delta N_1 + N_2 + N_4 + \delta N_5\right)/N_3\right)^\alpha - 2z/N_3 - bq,$$
$$\bar{y}_4 = \left(\kappa + \delta\kappa\left(N_3 + \delta\kappa N_5\right)/N_4\right)^\alpha - q,$$
$$\bar{y}_5 = \left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(\delta^3 N_1 + \delta^2 N_2 + \left(\delta + \delta^3\right)N_3 + N_4\right)/N_5\right)^\alpha - z/N_5 - bq.$$

Incomes are equalized in equilibrium; $\bar{y}_i = \bar{y}$. Population feasibility requires $\frac{1}{e}\sum_{i=1}^{5} N_i = 1$. The deviation conditions are the same as for the benchmark model in Section 3.2.

5 Numerical Analysis

We next turn to establishing sets of parameters to support a particular spatial configuration. We then perform a comparative-static analysis with respect to changes in rent, link cost and knowledge transmission decay parameters.

5.1 Equilibrium Configuration

We begin by examining the benchmark model in which only cores create knowledge spillovers and peripheries are only served by the closest core(s). We focus on two key parameters z and δ that are crucial for determining the equilibrium configuration. The benchmark values of other parameters are given by: $a = 0.5$, $\kappa = 1.2$, $e = 0.9$, $b = 0.8$, and $q = 0.002$. The benchmark values of (z, δ) to support each of the three spatial configurations in equilibrium, as well as the equilibrium distribution of working populations and expected per capita net income, are given as follows:

z	δ	N_1	N_2	N_3	N_4	N_5	$\bar{y}$
(i) Monocentric benchmark							
0.02	0.7	0.1648	0.2057	0.2591	0.2057	0.1648	1.6286
(ii) Multicentric benchmark							
0.008	0.24	0.1227	0.2546	0.2454	0.2546	0.1227	1.1425
(iii) Urban-Rural benchmark							
0.04	0.2	0.0494	0.6079	0.0494	0.1466	0.1466	1.000
(iv) Two-MSA benchmark							
0.05	0.5	0.0261	0.4008	0.1626	0.4008	0.0261	1.000

We set the ranges for the two key parameters as: $z \in [0, 0.1]$ and $\delta \in [0, 1]$. Over this parameter space, we can pin down equilibrium configurations. For illustrative purposes, we only summarize the most representative cases in the table below, where we use "none" to represent cases with no agglomerative symmetric equilibrium. In terms of the population distribution and economic activity, this latter case is comparable to the completely mixed configuration in Ogawa and Fujita (1980), and Fujita and Ogawa (1982) and Berliant, Peng and Wang (2002).

z / δ	Low	Intermediate Low	Intermediate	Intermediate High	High
High	m	m	m, s	m, u, s	m, u
Intermediate-High	m, d	m, d	m, s	u, s	u
Intermediate	m, d, s	d	s	u, s	u
Intermediate-Low	d, s	d, u, s	u, s	u	None
Low	d, s	s	u	None	None

The complete characterization of all symmetric equilibrium configurations, including tricentric (with three cores) and pure rural (with no population agglomeration), are depicted in Figure 1. We also provide a list of equilibrium configurations, including some representative asymmetric cases, in the Appendix.

Thus, depending on the values of the two key parameters, an equilibrium with local population agglomeration may or may not exist. With sufficiently low knowledge spillovers (i.e., δ is sufficiently low) and sufficiently high link costs (i.e., z is sufficiently high), the environment approaches to one described by Starrett's Spatial Impossibility Theorem under which there does not exist a spatial equilibrium featuring population agglomeration. Thus, no core is ever formed and every location of the entire local economy has exactly the same measure of population – such a configuration can therefore be called as pure rural (p..p..p..p..p). When knowledge spillovers are strong enough or inter-location links are not too costly, an agglomerative symmetric equilibrium exists – it may feature a unique spatial configuration, or coexistence of more than one spatial configurations (multiple equilibria).

First, with sufficiently high values of δ, knowledge spillovers are strong enough for a single core to serve the entire local economy, provided that the link cost z is not too large. The monocentric configuration therefore emerges in equilibrium if the link cost is low; otherwise, the urban-rural configuration or the two-MSA configuration may arise. These three configurations can coexist when the link costs take an intermediate range (between 0.050 and 0.072).

Second, when δ is moderately high, the equilibrium configuration need not be concentrated even when the link cost is not extremely large. With a sufficiently low link cost, monocentric and duocentric configurations can coexist. As the link cost rises, those residing at location 3 are not willing to pay both link costs and hence the monocentric configuration becomes the sole equilibrium outcome. As the link cost continues to increase, those residing in the outskirts (locations 1 and 5) are not willing to maintain links with the urban area. As a consequence, the equilibrium features either the urban-rural configuration where the outskirt locations are disconnected, or the two-MSA configuration where the outskirts themselves become cores in two separated urban areas.

Third, when δ is moderately low, a single core cannot serve the entire local economy. The duocentric configuration emerges if the link cost is low. As the link cost rises (falling in the range between 0.046 and 0.056), the equilibrium features two MSAs. As the link cost continues to increase, it is too costly for the entire local economy to be linked; as a result, the outskirt locations sever links with the geographically centered urban area and the urban-rural configuration arises in equilibrium.

Fourth, when δ is sufficiently low, the area that a single core can serve becomes more limited. With a sufficiently low link cost, duocentric and urban-rural configurations can coexist. As the link cost rises, the urban-rural configuration becomes the sole equilibrium outcome. As the link cost continues to increase, there does not exist an equilibrium configuration with population agglomeration.

Remark 1. What happens if peripheries also generate knowledge spillovers? Let us illustrate the findings by focusing on the monocentric configuration. Under the benchmark parameter values, the equilibrium outcome is given by:

N_1	N_2	N_3	N_4	N_5	$\bar{y}$
0.1832	0.2070	0.2195	0.2070	0.1832	1.7447

With an overall greater gain from knowledge spillovers, those residing in locations 2 and 4 are more willing to maintain the link with the outskirts; thus, the monocentric configuration can now emerge even with higher link costs.

Remark 2. What happens if peripheries are served by all cores? This would only affect the outcomes with multiple cores, that is, the duocentric configuration in our consideration. Under the benchmark parameter values, the equilibrium outcome is as follows:

N_1	N_2	N_3	N_4	N_5	$\bar{y}$
0.1134	0.2770	0.2191	0.2770	0.1134	1.0746

When the outskirt peripheries (locations 1 and 5) are also served by both cores, the advantage of the middle location (location 3) reduces relative to the outskirt locations. As a consequence, those residing at location 3 are no longer willing to pay link costs to both cores, thereby decreases the parameter space for the duocentric configuration to arise.

5.2 Comparative Statics

The per capita income of a location is determined by three factors: own knowledge, spillovers, the costs (link cost and rent), and its own population. We will examine numerically the effects of parameters on population distribution. But first, we can see the effects of equilibrium population analytically in the following, which is instrumental in understanding the comparative statics.

Let $S_i^*\left(N_i^*\right)$ denote the knowledge spillovers received by location i in equilibrium; it is a function of equilibrium population N_i^*. Local knowledge and spilled knowledge are separated in the per capita income function as follows

$$\bar{y}_i = e\left(\lambda_i + S_i^*\left(N_i^*\right)/N_i^*\right)^\alpha - Z_i/N_i^* - Q_i. \tag{13}$$

where λ_i depends on each location (see Section 3 for details). Suppose per capital income at location i, $\eta_i\left(N_i^*, h\right)$, is determined by equilibrium population N_i^* and a parameter h. Denote $\varepsilon^S = \frac{N_i^* S_i^{*\prime}}{S^*}$ as the spillover elasticity. When h changes, we can apply Implicit Function Theorem to obtain:

$$\frac{dN_i^*}{dh} = -\frac{d\eta_i/dh}{d\eta_i/dN_i^*}. \tag{14}$$

where

$$\frac{d\eta_i}{dN_i^*} = \alpha e \frac{S_i^*\left(N_i^*\right)}{\left(N_i^*\right)^2}\left(\varepsilon^S - 1\right) / \left(\lambda + \frac{S_i^*\left(N_i^*\right)}{N_i^*}\right)^{1-\alpha} + \frac{Z_i}{\left(N_i^*\right)^2},$$

Tooled with these analytic insights, we are ready to perform comparative-static exercises for each equilibrium configurations, in the following four subsections, respectively. Throughout all configurations, we take the ranges of the land rent and rent gradient parameters as: $b \in [0.6, 1]$ and $q \in [0, 0.04]$. In all but the rural-urban configuration, we set the employment rate in the urban area at 90%, i.e., $e = 0.9$. For the urban-rural configuration, we will study the Harris-Todaro proposition of rural-urban migration by perturbing this employment rate parameter in the range of $e \in [0.88, 0.92]$. With regard to the two key parameters (z, δ), we will perturb them by $(0.01, 0.1)$, respectively, around their benchmark values (which varies with different spatial configurations).

5.2.1 Monocentric

In this spatial configuration, the benchmark values of (z, δ) are $(0.02, 0.70)$, so the ranges of perturbations for comparative statics are $z \in [0.01, 0.03]$, and $\delta \in [0.06, 0.08]$. We summarize the numerical results in Table 1.

Intuitively, under the benchmark parametrization, the spillover elasticity is not only less than one and it outweighs the link cost effect. Thus, a flatter land rent gradient (a higher b) reduces the relative disadvantage for residing in the core. As a result, the population distribution becomes more concentrated. An increase in the land rent (q), on the contrary, makes it more costly to reside in the core and hence leads to a flatter population distribution. With a higher link cost (z), the two locations connected to the core (locations 2 and 4) become most disadvantageous. Thus, the working populations of locations 2 and 4 fall whereas those of locations 1, 3, and 5 rise. Finally, in response to a stronger knowledge spillover (less decays δ), the disadvantage of outskirt locations reduces, so the working population distribution becomes flatter.

5.2.2 Multicentric

Since the comparative-static results in the tricentric case is parallel to those in the duocentric case, we will focus on the latter for the sake of brevity (with detailed configurations of all types plotted in Figure A in the Appendix). In the duocentric configuration, the benchmark values of (z, δ) are $(0.008, 0.24)$ and the ranges of perturbations are $z \in [0.00, 0.018]$, and $\delta \in [0.14, 0.34]$. The comparative-static results are presented in Table 2.

The intuition with regard to changes in the land rent gradient and the level of land rent is identical to that in the case of monocentric configuration. For an increase in the link cost, it is noted that location 3 is most disadvantageous (as it must pay both link costs with the two cores). Therefore, the working population in location 3 shrinks more than the two outskirt peripheries. On the contrary, as knowledge spillovers become stronger, location 3 benefits most (as it is served by both cores); its working population thus rises by more than the two outskirt peripheries. When the link cost becomes too high or the knowledge transmission becomes too weak, the two cores are not sufficient to serve the entire local economy. Indeed, under this parameter range, only a two-MSA configuration can arise in equilibrium. When the link cost continues to increase, an urban-rural configuration may emerge as an equilibrium outcome.

5.2.3 Urban-rural (p..p-c-p..p)

In this case, the benchmark values of (z, δ) are $(0.04, 0.20)$ and the ranges of perturbations are $z \in [0.03, 0.05]$, and $\delta \in [0.10, 0.30]$. Also recall that we perturb z in the range of $(0.88, 0.92)$. The comparative-static results are reported in Table 3.

Under the benchmark parametrization, the spillover elasticity need not outweigh the link cost effect. In response to a decrease in the land rent gradient, the core attracts more working population from the rural areas. When the level of land rent rises, the changes in working population is not monotone: the core shrinks, whereas the peripheries in the urban area expands more than proportionately than the rural areas. Thus, the urban area falls and the working population distribution within the urban area becomes flatter. As the link cost increases, the urban area gains more working population, indicating that the spillover elasticity effect is outweighed by the link cost effect. By similar arguments, a stronger knowledge spillover leads to a population reduction in the urban area. In this case, the working population distribution within the urban area is steeper. When knowledge transmissions become too weak, an urban-rural configuration is no longer stable. In this case, the only configuration that may arise is the non-agglomerative equilibrium where all locations are disconnected. Finally, in response to a higher employment opportunity in the urban area, the overall urban working population rises and the population distribution becomes more concentrated within the urban area. When urban employment opportunities become too good, the urban-rural configuration also collapse, as all workers desire to migrate to the core.

5.2.4 Two-MSA (c-p..p..p-c)

In this case, the benchmark values of (z, δ) are $(0.05, 0.5)$ and the ranges of perturbations are $z \in [0.04, 0.06]$, and $\delta \in [0.4, 0.6]$. The comparative-static results are given in Table 4.

As one can see, an increase in the land rent gradient encourages more working population to move into the disconnected rural area (location 3). Since a higher level of land rent hurts connected peripheral locations (locations 2 and 4) more than proportionately, workers migrate away from these locations to either become disconnected (residing in location 3) or join the cores (locations 1 and 5). In response to an increase in the link cost or a reduction in the knowledge transmission, the urban areas (both cores and connected peripheries) gain more working population from the disconnected rural area, due again to a strong link cost effect. When maintaining the link becomes too costly, this configuration can no longer arise in equilibrium, because the population in this closed economy is not large enough for the two MSAs to sustain to generate income no less than the rural income ($\bar{y} = 1$). Indeed, under this parameter range, the only equilibrium outcome is the urban-rural configuration.

6 Concluding Remarks

In this paper, we have developed a network formation approach to knowledge-based city formation and spatial agglomeration. The framework allows a thorough analysis of the transmission, aggregation and spillover of knowledge on a spatial network. By employing an equilibrium concept that suits the particular need of our study, we show that spatial equilibrium may feature monocentric, multicentric, urban-rural, or multiple urban areas, where multiple equilibria may arise. The stronger knowledge spillovers and the lower link costs are, the more likely the local economy features a monocentric configuration. As the strength of knowledge spillovers declines, more than one core may form. In response to an increase in the link cost, some locations may become disconnected, thereby generating multiple-urban-area and urban-rural configurations. When the urban land rent or unemployment rate falls or the rent gradient is flattened, the population distribution of the local economy becomes more concentrated.

Along these lines, the most natural extension is to examine how the spatial configuration of the local economy changes over time in response to a continual increase in population. Our model is complex enough, so the extension of this spatial network formation framework to permit dynamics seems implausible at the first glance. However, one may adopt a modeling strategy similar to one proposed by Berliant and Wang (forthcoming), where, with discrete locational choice and full depreciation of capital, the dynamic optimization problem with population change boils down to period-by-period optimization. In addition to these tactics, all we need to add is to further assume location-players are myopic in the network formation sense as defined by Jackson and Watts (2002). A preliminary analysis suggests that, in response to population growth, an initially monocentric economy may be transformed into multicentric or multiple-urban-area spatial structures. We also expect that if rising population makes urban employment disproportionately more difficult, then suburbanization may occur in which workers flow into linked peripheries within the urban areas.

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Table 1: Comparative Statics under Monocentric Configuration

		N_1	N_2	N_3	N_4	N_5	$\bar{y}$
	0.6	0.1648	0.2057	0.2590	0.2057	0.1648	1.6289
	0.7	0.1648	0.2057	0.2590	0.2057	0.1648	1.6288
b	**0.8**	**0.1648**	**0.2057**	**0.2591**	**0.2057**	**0.1648**	**1.6286**
	0.9	0.1648	0.2057	0.2591	0.2057	0.1648	1.6285
	1.0	0.1647	0.2057	0.2592	0.2057	0.1647	1.6283
	0.000	0.1647	0.2057	0.2592	0.2057	0.1647	1.6301
	0.001	0.1648	0.2057	0.2591	0.2057	0.1648	1.6294
q	**0.002**	**0.1648**	**0.2057**	**0.2591**	**0.2057**	**0.1648**	**1.6286**
	0.003	0.1648	0.2057	0.2590	0.2057	0.1648	1.6279
	0.004	0.1649	0.2057	0.2589	0.2057	0.1649	1.6271
	0.010	0.1610	0.2163	0.2455	0.2163	0.1610	1.6703
	0.015	0.1628	0.2112	0.2521	0.2112	0.1628	1.6496
z	**0.020**	**0.1648**	**0.2057**	**0.2591**	**0.2057**	**0.1648**	**1.6286**
	0.025	0.1670	0.1998	0.2664	0.1998	0.1670	1.6074
	0.030	0.1696	0.1933	0.2742	0.1933	0.1696	1.5858
	0.60	0.1418	0.2220	0.2725	0.2220	0.1418	1.5102
	0.65	0.1516	0.2191	0.2585	0.2191	0.1516	1.5872
δ	**0.70**	**0.1610**	**0.2163**	**0.2455**	**0.2163**	**0.1610**	**1.6703**
	0.75	0.1700	0.2134	0.2333	0.2134	0.1700	1.7596
	0.80	0.1786	0.2105	0.2218	0.2105	0.1786	1.8555

Table 2: Comparative Statics under Duocentric Configuration

		N_1	N_2	N_3	N_4	N_5	$\bar{y}$
	0.6	0.1228	0.2544	0.2456	0.2544	0.1228	1.1426
	0.7	0.1227	0.2545	0.2455	0.2545	0.1227	1.1425
b	**0.8**	**0.1227**	**0.2546**	**0.2454**	**0.2546**	**0.1227**	**1.1425**
	0.9	0.1227	0.2546	0.2454	0.2546	0.1227	1.1424
	1.0	0.1226	0.2547	0.2453	0.2547	0.1226	1.1423
	0.000	0.1226	0.2547	0.2453	0.2547	0.1226	1.1441
	0.001	0.1227	0.2546	0.2454	0.2546	0.1227	1.1433
q	**0.002**	**0.1227**	**0.2546**	**0.2454**	**0.2546**	**0.1227**	**1.1425**
	0.003	0.1227	0.2545	0.2455	0.2545	0.1227	1.1416
	0.004	0.1228	0.2544	0.2456	0.2544	0.1228	1.1408
	0.000	0.1345	0.2310	0.2690	0.2310	0.1345	1.1727
	0.003	0.1303	0.2393	0.2607	0.2393	0.1303	1.1614
z	**0.008**	**0.1227**	**0.2546**	**0.2454**	**0.2546**	**0.1227**	**1.1425**
	0.013	N/A	N/A	N/A	N/A	N/A	N/A
	0.018	N/A	N/A	N/A	N/A	N/A	N/A
	0.14	N/A	N/A	N/A	N/A	N/A	N/A
	0.19	0.1133	0.2733	0.2267	0.2733	0.1133	1.0944
δ	**0.24**	**0.1227**	**0.2546**	**0.2454**	**0.2546**	**0.1227**	**1.1425**
	0.29	0.1297	0.2406	0.2594	0.2406	0.1297	1.1931
	0.34	0.1353	0.2293	0.2707	0.2293	0.1353	1.2460

Table 3: Comparative Statics under Urban-Rural Configuration

		N_1	N_2	N_3	N_4	N_5	$\bar{y}$
	0.6	0.1468	0.0494	0.6077	0.0494	0.1468	1
	0.7	0.1467	0.0494	0.6078	0.0494	0.1467	1
b	**0.8**	**0.1466**	**0.0494**	**0.6079**	**0.0494**	**0.1466**	**1**
	0.9	0.1465	0.0494	0.6081	0.0494	0.1465	1
	1.0	0.1465	0.0494	0.6082	0.0494	0.1465	1
	0.000	0.1446	0.0447	0.6213	0.0447	0.1446	1
	0.001	0.1458	0.0471	0.6142	0.0471	0.1458	1
q	**0.002**	**0.1466**	**0.0494**	**0.6079**	**0.0494**	**0.1466**	**1**
	0.003	0.1470	0.0518	0.6024	0.0518	0.1470	1
	0.004	0.1471	0.0541	0.5976	0.0541	0.1471	1
	0.030	0.2350	0.0371	0.4560	0.0371	0.2350	1
	0.035	0.1908	0.0432	0.5319	0.0432	0.1908	1
z	**0.040**	**0.1466**	**0.0494**	**0.6079**	**0.0494**	**0.1466**	**1**
	0.045	0.1024	0.0556	0.6839	0.0556	0.1024	1
	0.050	0.0583	0.0618	0.7599	0.0618	0.0583	1
	0.10	N/A	N/A	N/A	N/A	N/A	N/A
	0.15	0.0336	0.0831	0.7666	0.0831	0.0336	1
δ	**0.20**	**0.1466**	**0.0494**	**0.6079**	**0.0494**	**0.1466**	**1**
	0.25	0.2002	0.0345	0.5306	0.0345	0.2002	1
	0.30	0.2312	0.0263	0.4851	0.0263	0.2312	1
	0.88	0.0785	0.1203	0.6023	0.1203	0.0785	1
	0.89	0.1274	0.0813	0.5826	0.0813	0.1274	1
e	**0.90**	**0.1466**	**0.0494**	**0.6079**	**0.0494**	**0.1466**	**1**
	0.91	0.0744	0.0202	0.8106	0.0202	0.0744	1
	0.92	0	0	0	0	0	0

Table 4: Comparative Statics under Two-MSA Configuration

		N_1	N_2	N_3	N_4	N_5	$\bar{y}$
	0.6	0.0261	0.4007	0.1628	0.4007	0.0261	1
	0.7	0.0261	0.4007	0.1627	0.4007	0.0261	1
b	**0.8**	**0.0261**	**0.4008**	**0.1626**	**0.4008**	**0.0261**	**1**
	0.9	0.0261	0.4008	0.1625	0.4008	0.0261	1
	1.0	0.0261	0.4008	0.1624	0.4008	0.0261	1
	0.000	0.0240	0.4172	0.1307	0.4172	0.0240	1
	0.001	0.0251	0.4086	0.1475	0.4086	0.0251	1
q	**0.002**	**0.0261**	**0.4008**	**0.1626**	**0.4008**	**0.0261**	**1**
	0.003	0.0271	0.3936	0.1763	0.3936	0.0271	1
	0.004	0.0280	0.3871	0.1887	0.3871	0.0280	1
	0.040	0.0208	0.3206	0.3523	0.3206	0.0208	1
	0.045	0.0235	0.3607	0.2575	0.3607	0.0235	1
z	**0.050**	**0.0261**	**0.4008**	**0.1626**	**0.4008**	**0.0261**	**1**
	0.055	0.0287	0.4408	0.0678	0.4408	0.0287	1
	0.060	N/A	N/A	N/A	N/A	N/A	N/A
	0.40	0.0358	0.4406	0.0525	0.4406	0.0358	1
	0.45	0.0302	0.4180	0.1152	0.4180	0.0302	1
δ	**0.50**	**0.0261**	**0.4008**	**0.1626**	**0.4008**	**0.0261**	**1**
	0.55	0.0229	0.3873	0.1997	0.3873	0.0229	1
	0.60	0.0204	0.3764	0.2294	0.3764	0.0204	1

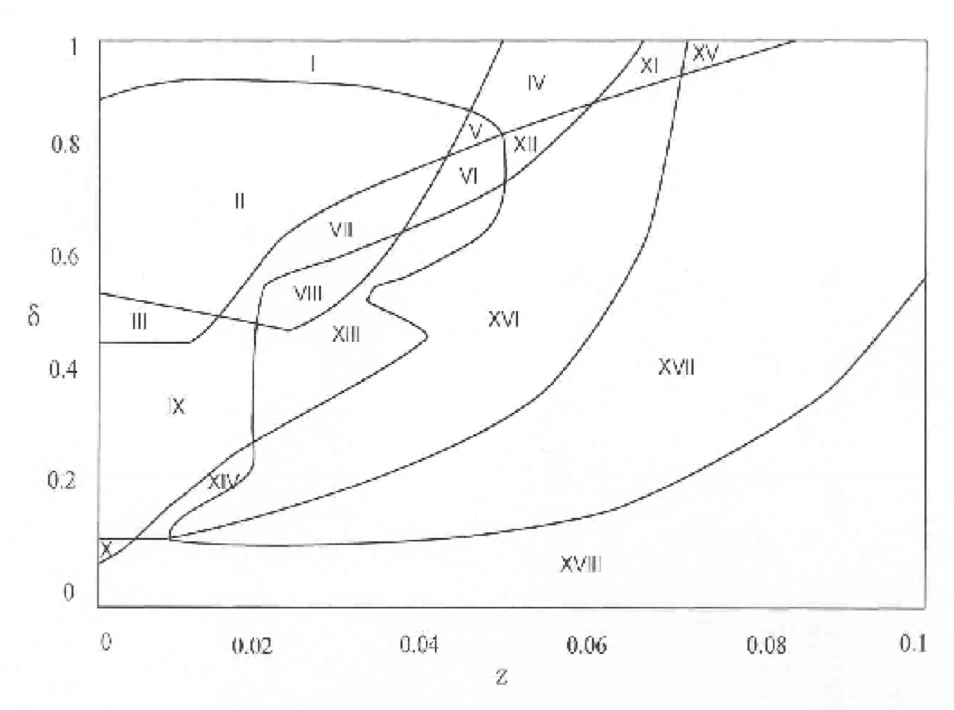


Figure 1: Equilibrium Configurations

Unique Configuration	Zone	Two Coexistent Configurations	Zone	Three Coexistent Configurations	Zone
m	I	m, d	II	m, d, s	III, V
d	VII, X	m, u	XV	m, u, s	XI
u	XVII	m, s	IV	d, u, s	XIII
s	XII, XIV	d, u	VIII		
		d, s	VI, IX		
		u, s	XVI		

Appendix

In this appendix, we provide a fuller analysis of all possible spatial configurations. More specifically, in addition to the spatial configurations of greatest interest (monocentric, benchmark duocentric, urban-rural and benchmark two-MSA configurations), there are four additional symmetric configurations:

(A) Duocentric configuration 2 ($\theta = d2$): Locations 1 and 5 as cores and locations 2, 3, and 4 as peripheries.

(B) Tricentric configuration ($\theta = t$): Locations 1, 3 and 5 serve as cores; locations 2 and 4 are peripheries, each served by two cores, $c(2) = \{1, 3\}$ and $c(4) = \{3, 5\}$.

(C) Two-MSA configuration 2 ($\theta = s2$): Locations 2 and 4 are cores, serving peripheral cities 1 and 5, respectively; locations 3 is completely disconnected, so there are two MSAs, $U = \{1, 2\} \cup \{4, 5\}$.

(D) All-rural configuration ($\theta = r$): All locations are rural, disconnected with each other.

A. Duocentric configuration 2 (c-p-p-p-c)

In this configuration, locations 1 and 5 receive knowledge from locations 2 and 3, and 3 and 4 respectively. They then serve back. The per capita incomes at all locations are as follow

$$\begin{aligned}
\bar{y}_1 &= e\left[\left(\kappa + \delta\kappa\left(N_2 + \delta N_3 + \delta^2 N_4\right)/N_1\right)^{\alpha} - q\right], \\
\bar{y}_2 &= e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(N_1 + \delta^2 N_3 + \delta^3 N_4\right)/N_2\right)^{\alpha} - 3z/\left(2N_2\right) - bq\right], \\
\bar{y}_3 &= e\left[\left(\left(2\delta^4\kappa + 1\right) + \delta^2\kappa\left(N_1 + \left(\delta + \delta^3\right)N_2 + \left(\delta + \delta^3\right)N_4 + N_5\right)/N_3\right)^{\alpha} - z/N_3 - b^2 q\right], \\
\bar{y}_4 &= e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(\delta^3 N_2 + \delta^2 N_3 + N_5\right)/N_4\right)^{\alpha} - 3z/\left(2N_4\right) - bq\right], \\
\bar{y}_5 &= e\left[\left(\kappa + \delta\kappa\left(\delta^2 N_2 + \delta N_3 + N_4\right)/N_5\right)^{\alpha} - q\right].
\end{aligned}$$

Incomes are equalized in equilibrium; $\bar{y}_i = \bar{y}$. Population feasibility requires $(N_1 + N_2 + N_3 + N_4 + N_5)/e = 1$.

Locations may deviate in the following ways: (i) Every location can sever links and stay alone; equilibrium requires

$$\bar{y} - 1 \geq 0.$$

(ii) Location 1 (similarly for location 5) can play periphery (p-p-p-p-c). It then receive knowledge from all locations but pays for one link. Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^8\kappa + 1\right) + \delta^4\kappa\left(\delta^3 N_2 + \delta^2 N_3 + \delta N_4 + N_5\right)/N_1\right)^{\alpha} - z/\left(2N_1\right) - b^4 q\right] \geq 0.$$

(iii) Location 2 (similarly for location 4) can sever the link to location 3 (c-p..p-p-c). It saves half of a link cost but loses knowledge from location 3. Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa N_1/N_2\right)^{\alpha} - z/N_2 - bq\right] \geq 0.$$

(iv) Location 2 (similarly for location 4) can sever the link to location 1 (c..p-p-p-c). It saves link cost but loses service from location 1; it is served by location 5 instead. Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^6\kappa + 1\right) + \delta^3\kappa\left(\delta^2 N_3 + \delta N_4 + N_5\right)/N_2\right)^\alpha - z/\left(2N_2\right) - b^3 q\right] \geq 0.$$

(v) Location 2 (similarly for location 4) can play core (c..c-p-p-c) and have location 3 pay for link cost. This move should be mutually beneficial to location 3. Equilibrium requires

$$\begin{aligned} \bar{y} - e\left[\left(\kappa + \delta\kappa\left(N_3 + \delta N_4\right)/N_2\right)^\alpha - q\right] &\geq 0, \text{ or} \\ \bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(N_2 + \delta^2 N_4\right)/N_3\right)^\alpha - 3z/\left(2N_3\right) - bq\right] &\geq 0. \end{aligned}$$

(vi) Location 3 can sever the link to location 4 (c-p-p..p-c). It saves half of a link cost but loses serve from location 5. Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^4\kappa + 1\right) + \delta^2\kappa\left(N_1 + \delta N_2\right)/N_3\right)^\alpha - z/\left(2N_3\right) - b^2 q\right] \geq 0.$$

(vii) Locations 3 can play core (c-p-c-p-c) and have locations 2 and 4 pay for link costs. This move should be mutually beneficial to locations 2 and 4. Equilibrium requires

$$\begin{aligned} \bar{y} - e\left[\left(\kappa + \delta\kappa\left(N_2 + N_4\right)/N_3\right)^\alpha - q\right] &\geq 0, \text{ or} \\ \bar{y} - e\left[\left(\left(2\delta^2\kappa + 1\right) + \delta\kappa\left(N_1 + N_3 + \delta N_4\right)/N_2\right)^\alpha - 2z/N_2 - bq\right] &\geq 0, \text{ or} \\ \bar{y} - e\left[\left(\left(2\delta^2\kappa + 1\right) + \delta\kappa\left(\delta N_2 + N_3 + N_5\right)/N_4\right)^\alpha - 2z/N_4 - bq\right] &\geq 0. \end{aligned}$$

(viii) Locations 3 can play core and sever the link to location 4 (c-p-c..p-c). This move should be mutually beneficial to locations 2. Equilibrium requires

$$\begin{aligned} \bar{y} - e\left[\left(\kappa + \delta\kappa N_2/N_3\right)^\alpha - q\right] &\geq 0, \text{ or} \\ \bar{y} - e\left[\left(\left(2\delta^2\kappa + 1\right) + \delta\kappa\left(N_1 + N_3\right)/N_2\right)^\alpha - 2z/N_2 - bq\right] &\geq 0. \end{aligned}$$

B. Tricentric Configuration (c-p-c-p-c)

The per capita income at each location is

$$\begin{aligned} \bar{y}_1 &= e\left[\left(\kappa + \delta\kappa N_2/N_1\right)^\alpha - q\right], \\ \bar{y}_2 &= e\left[\left(\left(2\delta^2\kappa + 1\right) + \delta\kappa\left(N_1 + N_3 + \delta N_4\right)/N_2\right)^\alpha - 2z/N_2 - bq\right], \\ \bar{y}_3 &= e\left[\left(\kappa + \delta\kappa\left(N_2 + N_4\right)/N_3\right)^\alpha - q\right], \\ \bar{y}_4 &= e\left[\left(\left(2\delta^2\kappa + 1\right) + \delta\kappa\left(\delta N_2 + N_3 + N_5\right)/N_4\right)^\alpha - 2z/N_4 - bq\right], \\ \bar{y}_5 &= e\left[\left(\kappa + \delta\kappa N_4/N_5\right)^a - q\right]. \end{aligned}$$

In equilibrium $\bar{y}_i = \bar{y}$. Population feasibility requires $\left(N_1 + N_2 + N_3 + N_4 + N_5\right)/e = 1$.

Locations may deviate in the following ways: (i) Any location can server all links and stay alone. Equilibrium requires

$$\bar{y} - 1 \geq 0.$$

(ii) Location 1 (similarly for location 5) can play periphery (p-p-c-p-c). Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^4\kappa + 1\right) + \delta^2\kappa\left(\delta N_2 + N_3 + \delta N_4\right)/N_1\right)^\alpha - z/\left(2N_1\right) - b^2 q\right] \geq 0.$$

(iii) Location 2 (similarly for location 4) can sever the link to location 3 (c-p..c-p-c). Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa N_1/N_2\right)^\alpha - z/N_2 - bq\right] \geq 0.$$

(iv) Location 2 (similarly for location 4) can sever the link to location 1 (c..p-c-p-c). Equilibrium requires

$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(N_3 + \delta N_4\right)/N_2\right)^\alpha - z/N_2 - bq\right] \geq 0.$$

(v) Location 3 can play periphery (c-p-p-p-c). Equilibrium requires

$$\bar{y} - e\left[\left(\left(2\delta^4 + 1\right) + \left(\delta^2\kappa N_1 + \left(\delta^3 + \delta^5\right)\kappa N_2 + \left(\delta^3 + \delta^5\right)\kappa N_4 + \delta^2\kappa N_5\right)/N_3\right)^\alpha - z/N_3 - b^2 q\right].$$

C. Two-MSA configuration 2 (p-c..p..c-p)

This configuration has locations 2 and 4 as cores. They receive knowledge from locations 1 and 5 respectively and serve back. Rural location 3 is disconnected from others and yields unit per capita income. The per capita incomes of all locations are

$$\begin{aligned}
\bar{y}_1 &= e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa N_2/N_1\right)^\alpha - z/N_1 - bq\right], \\
\bar{y}_2 &= e\left[\left(\kappa + \delta\kappa N_1/N_2\right)^\alpha - q\right], \\
\bar{y}_3 &= 1, \\
\bar{y}_4 &= e\left[\left(\kappa + \delta\kappa N_5/N_4\right)^\alpha - q\right], \\
\bar{y}_5 &= e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa N_4/N_5\right)^\alpha - z/N_5 - bq\right].
\end{aligned}$$

In equilibrium $\bar{y}_i = \bar{y}$. Population feasibility requires $\left(N_1 + N_2 + N_4 + N_5\right)/e + N_3 = 1$.

Locations may deviate in the following ways: (i) Locations 2 (similarly for location 4) can link together with location 3 (p-c-p..c-p). Location 3 pays link cost and receives knowledge spillovers. This move should be mutually beneficial. Equilibrium requires

$$\begin{aligned}
\bar{y} - e\left[\left(\kappa + \delta\kappa\left(N_1 + eN_3\right)/N_2\right)^\alpha - q\right] &\geq 0, \text{ or} \\
\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(\delta N_1 + N_2\right)/\left(eN_3\right)\right)^\alpha - z/\left(eN_3\right) - bq\right] &\geq 0.
\end{aligned}$$

(ii) Location 2 can switch to link with location 3 (p..c-p..c-p). This move should be beneficial to location 3. Equilibrium requires

$$\begin{aligned}
\bar{y} - e\left[\left(\kappa + \delta\kappa e N_3/N_2\right)^\alpha - q\right] &\geq 0, \text{ or} \\
\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa N_2/\left(eN_3\right)\right)^\alpha - z/\left(eN_3\right) - bq\right] &\geq 0.
\end{aligned}$$

(iii) Location 3 can link with both locations 2 and 4 (p-c-p-c-p) and forms a duocentric configuration. This move should be beneficial to locations 2 and 4. Equilibrium requires

$$\begin{gathered}
\bar{y} - e\left[\left(\left(2\delta^2\kappa + 1\right) + \delta\kappa\left(\delta N_1 + N_2 + N_4 + \delta N_5\right)/\left(eN_3\right)\right)^\alpha - 2z/\left(eN_3\right) - bq\right] \geq 0, \text{ or} \\
\bar{y} - e\left[\left(\kappa + \delta\kappa\left(N_1 + eN_3\right)/N_2\right)^\alpha - q\right] \geq 0, \text{ or} \\
\bar{y} - e\left[\left(\kappa + \delta\kappa\left(eN_3 + N_5\right)/N_4\right)^\alpha - q\right] \geq 0.
\end{gathered}$$

D. All-Rural Configuration (p..p..p..p..p)

This means all locations are disconnected. To minimize the incentives for establishing a link, we let population equally distributed among 5 locations.

$$\bar{y}_i = 1, N_i = 1/5 \ \forall i = 1, ..., 5.$$

They may deviate in the following ways: (i) Any one city can play core and link with another (c-p). Equilibrium requires (for example, for core location 1 and periphery location 2)

$$\bar{y} - e\left[(\kappa + \delta\kappa N_2/N_1)^{\alpha} - q\right] \geq 0, \text{ or}$$
$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa N_1/N_2\right)^{a} - z/\left(eN_2\right) - bq\right] \geq 0.$$

(ii) Any middle city can play core and link with two sides (p-c-p). Equilibrium requires (for example, for core location 2 and periphery location 1 and 3)

$$\bar{y} - e\left[(\kappa + \delta\kappa\left(N_1 + N_3\right)/N_2)^{\alpha} - q\right] \geq 0, \text{ or}$$
$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(N_2 + \delta N_3\right)/N_1\right)^{a} - z/\left(eN_1\right) - bq\right] \geq 0,$$
$$\bar{y} - e\left[\left(\left(\delta^2\kappa + 1\right) + \delta\kappa\left(\delta N_1 + N_2\right)/N_3\right)^{a} - z/\left(eN_3\right) - bq\right] \geq 0.$$

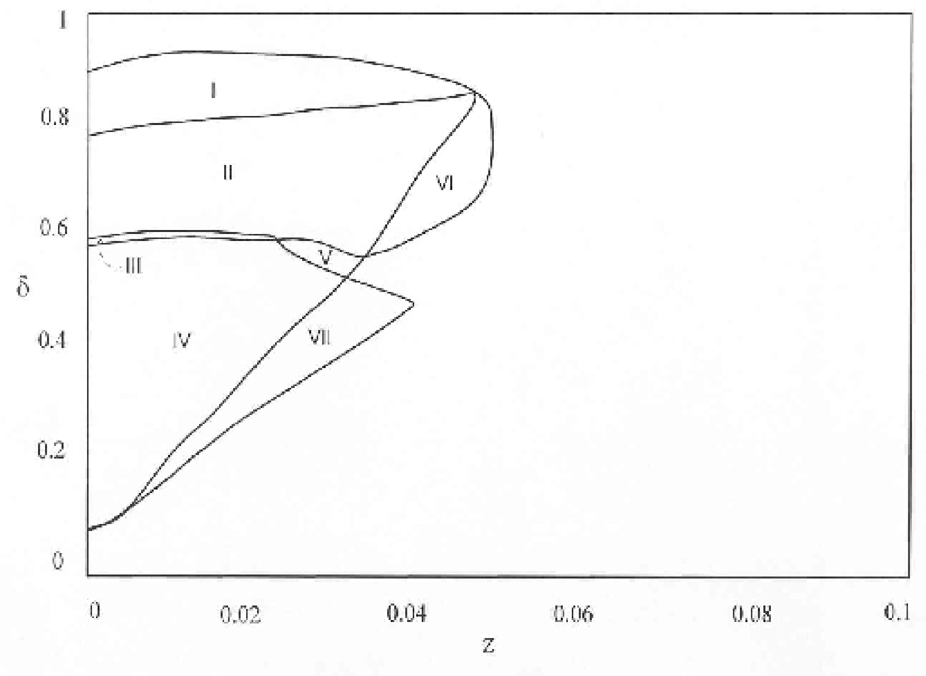
E. Complete List of Configurations (Inclusive of Asymmetric Cases)

For completeness, we include both symmetric and asymmetric configurations and label the case without population agglomeration as the "non-agglomerative" configuration. For brevity, we will only list representative symmetric cases – for example, we will not list the asymmetric duocentric configuration p-p-c-p-c because it is isomorphic to configuration c-p-c-p-p.

Configuration		Symmetric Patterns	Asymmetric Patterns
Monocentric		p-p-c-p-p	p-c-p-p-p c-p-p-p-p
Multicentric	Duocentric	p-c-p-c-p c-p-p-p-c	c-p-c-p-p p-c-p-p-c
	Tricentric	c-p-c-p-c	N/A
Urban-Rural		p..p-c-p..p	p..p..p-c-p p..p-p-c..p p..p..p-p-c p-p-c..p..p
Two-MSA		c-p..p..p-c p-c..p..c-p	p-c-p..c-p p-c-p..p-c p-c..p..p-c
Non-agglomerative		p..p..p..p..p	N/A

F. Detailed Classification of Multicentric Configurations

We provide detailed diagrammatic illustration of the three symmetric multicentric configurations:



(i) duocentric configuration d1 (the benchmark duocentric configuration, p-c-p-c-p),

(ii) duocentric configuration d2 (c-p-p-p-c),

(iii) tricentric configuration t (c-p-c-p-c).

Unique Configuration	Zone	Two Coexistent Configurations	Zone	Three Coexistent Configurations	Zone
d1	I, V	d1, d2	II	d1, d2, t	III
d2	VI	d1, t	IV		
t	VII				