National electorates and international environmental agreements.

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January 22, 2008

Abstract

The dispute about the aftermath of the Kyoto protocol aimed at reducing global warming proves the difficulty to reach international agreements about environmental issues. In this paper we relate this difficulty to the differences in the structural characteristics of national electorates. We set up a political economy model of a two-country world economy, where an international environmental agreement on environmental taxes has to be bargained by the two national elected leaders; the two national electorates decide whether to allow their leader to participate to the bargaining process through majority voting. In the case of disagreement, no bargaining takes place and each country deals with the environment non-cooperatively. In our set-up, the generalized median voter theorem applies. The outcome of the political process therefore depends on the national income distributions and impacts of environmental quality of individual welfare, on the type of agreement being bargained and the type of bargaining process. We prove that any agreement involves higher taxes in both countries than in the case of no-agreement. Depending on the type of agreement which is at stake, an agreement may or may not be reached, in particular if the political process involves a constraint on tax rates. If feasible, an IEA may generate losers in either countries. However there is no unambiguous relationships between the extent of inequality and the relative gains for the median voters.

1 Introduction

IEA.

- The importance of international environmental negative externalities. The green house effect, the ozone layer, biological diversity, to name a few.
- But international negotiations and bargaining sometimes fail. No international order on the environment, no worldwide authority. Witness the Kyoto protocol.
- Why these failures? Our answer: Internal political considerations. Electorates may not be in favor of an IEA.
- The literature: neglecting the political side of the problem. Coalition theory and bargaining theory applied to IEA. Kempf and Rossignol (2007) on distributive issues and the environment in a closed economy.
- Here a political economy perspective of international environmental agreements (IEA), relating the outcome of an international bargaining process on IEA to the pressures of national electorates.
- Distributive considerations cannot be neglected in the framing of an IEA as any agreement involves taxes and public spending. They play a role through the political process. An IEA involves differentiated consequences on agents because of different resources. The role of inequality.

What do we do here:

- set up a simple economic model with environment. Two countries sharing an international environment. Unequal income distributions in the two countries. National public policies effective in dealing with the environment. However no international authority.
- provide a political process over the negotiation of an IEA involving direct elections in the two countries. Here an IEA is negotiated between delegates from the two countries, selected by their electorate. Hence an IEA requires unanimous consent from the two national electorates.
- study three types of IEA, involving different sets of constraints on the policies to be implemented.

What do we get:

- study autarky (non cooperative national environmental policies) as a benchmark against which evaluation of any IEA.
- define the gains from an IEA for any agent and prove that they depend on the relative income of this agent.

- prove that the generalized median voter theorem applies to the political process.
- prove that any IEA involves higher taxes in both countries than autarky.
- prove that there may exist losers to an IEA in each country.
- prove that the political process may fail to establish an IEA, depending on the items of the bargaining.
- prove that there is no unambiguous relationship between the wealths of the country median voters and their gains.
- compare the tax rates obtained under different constraints on the bargaining stage.

Plan of the paper:

2 A two-country economy with international environment

2.1 A general model

We consider a two-country world economy. In each country j there is an odd number N_j of agents. There are $N = N_1 + N_2$ individuals, agent i belongs to country 1 for $i \in \mathbf{C}_1$, where $\mathbf{C}_1 = [1; N_1]$, and to country 2 for $i \in \mathbf{C}_2$, where $\mathbf{C}_2 = [N_1+1; N_1+N_2]$.

In any country, the distribution of endowments is unequal. Without loss of generality, we assume that in each country the agents are ranked according to their endowment level, i.e.:

$$\begin{aligned} \text{if } i &> i', \text{ and } i, i' \in \mathbf{C}_1, \text{ then } y_i > y_{i'} \\ \text{if } i &> i', \text{ and } i, i' \in \mathbf{C}_2, \text{ then } y_i > y_{i'} \end{aligned}$$

The endowment distributions may differ across countries. We do not make any assumption on this point. Remark that we do not assume that all agents in Country 1 are richer (or poorer) than any agent in country 2.

The median individual's wealth in country j is denoted by y_{m_j}.

The individual utility of an agent i depends both on her consumption of a private good, c_i and on the quality of the environment E. The environmental quality is shared by both countries.

$$W_i = W(c_i, E) \tag{1}$$

Agents are taxed so as to finance public expenditures. These expenditures aim at protecting the quality of the environment, that is:

$$E = E(\tau_1, \tau_2) \tag{2}$$

where τ_j denotes the tax rate applied in country j. E is an increasing function of its two arguments. Both countries have a symmetric effect on the environment. Hence we write:

$$E = H(\tau_1) + H(\tau_2). \tag{3}$$

$H(\tau)$ is assumed to be an increasing concave function of τ.

Given the previous equations, the welfare of agent i in country j can be rewritten as:

$$\begin{aligned} W(y_i, \tau_j, \tau_{-j}) &= U\left(y_i(1-\tau_j)\right) + H(\tau_j) + H(\tau_{-j}) & (4) \\ &= F(y_i, \tau_j) + H(\tau_{-j}) & (5) \end{aligned}$$

where U is a utility function, and we set

$$\begin{aligned} f(y, \tau) &= U\left(y(1-\tau)\right) & (6) \\ F(y, \tau) &= f(y, \tau) + H(\tau) \end{aligned}$$

We make the following assumptions on these functions:

A1 U is a CRRA utility function, of relative risk aversion λ,
i.e. $U(x) = \frac{x^{1-\lambda}}{1-\lambda}$ if $\lambda \neq 1$, and $U(x) = \ln x$ if $\lambda = 1$.

A2 H is an increasing concave function of τ

Taxation is proportional to income, hence we get:

$$c_i = (1 - \tau_j) y_i \tag{7}$$

The present model is minimal but is sufficient to emphasize the key ingredients of a political approach to IEAs: each individual faces a trade-off private consumption and fiscal contribution to a public environmental good, called here “environmental quality”; in each country there is inequality so the various voters do have different views on this trade-off; finally, the environment is shared by both countries which provides the rationale for an IEA.

2.2 The non-cooperation solution.

The logic of an IEA between two sovereign countries is cooperating so as to improve the situation of any involved policymaker, compared to the situation of no-cooperation. Hence we first have to characterize the no-cooperation situation as it is the cornerstone of an IEA.

Each country sets its tax rate in isolation, non-cooperatively through a direct majority voting rule, taking as given the other country's policy decision.

Given the assumed welfare function in (4), the optimum tax rate wished by agent i when non-cooperating with country j, when the prevailing tax rate in country $-j$ is τ_{-j}, is

$$\tau^*(y_i) = \arg\max_{\tau_i} W(y_i, \tau_i, \tau_{-j}) = \arg\max_{\tau_i} F(y_i, \tau_i)$$

This optimum $\tau^*(y_i)$ does not depend on the tax rate τ_{-j} in the other country. This dominant strategy result evidently comes from the separability property of the function E: the marginal impact of τ_j on j's welfare does not depend on τ_{-j}. [1]

Without loss of generality, we assume that $\tau_2^* \leq \tau_1^*$, i.e. country 1 spends more for the environment than country 2.

We can prove the following

Proposition 1 *Under A1-A2, the preferences are single peaked and the median voter theorem applies.*

Proof See Appendix A. □

Each agent in any country wants to tax and spend for the environment because her individual welfare is directly affected by the quality of environment: the lower this quality the lower her welfare. Her response depends on her endowment, as the tax rate generates spending for environment protection. For $\lambda > 1$, the higher the value of y_i, the more agent i is willing to be taxed so as to protect the environment. The reverse is true if $\lambda < 1$.

This lemma simply states that in each country, the agent endowed with the median endowment is the Condorcet winner and the tax rate is fixed according to her own preference.

Given this lemma and the properties of the individual welfare functions, it is then immediate to state the following

Proposition 2 *There exists a unique non-cooperative political equilibrium* (τ_1^*, τ_2^*) *where* τ_j^* *refers to the tax rate wished by the median voter in Country* j *and is equal to:*

$$\tau_j^* = \arg\max_{\tau_j} F(y_{m_j}, \tau_j). \tag{8}$$

- If $\lambda > 1$, τ_j^* is an increasing function of y_{m_j}.
- If $\lambda < 1$, τ_j^* is a decreasing function of y_{m_j}.
- If $\lambda = 1$, τ_j^* does not depend on y_{m_j}.

Note that $\tau_j^* = \tau^*(y_{m_j})$ is independent from τ_{m-j}^* due to the assumption of additivity of E.

This solution will be used as a benchmark for the assessment of IEAs. Indeed, the status quo in the bargaining is naturally considered to be the non-cooperative equilibrium.

[1] This assumption could be relaxed at the expense of simplicity. As we are interested in the impact of an IEA relative to the case of non-cooperation, more than about the properties of non-cooperative policies with respect to the environment, this assumption suffices.

3 IEA

Given the non-cooperation solution (τ_1^*, τ_2^*) and the implied level of environmental quality enjoyed by any agent in the case of no agreement, there is scope for an IEA insofar as it improves the lot of the policymakers involved in setting it up. As there is no political jurisdiction whose limits correspond to the extent of the environmental public good, there must be unanimity between them to enforce an IEA. If one policymaker opposes a proposed international scheme for dealing with the environment, it cannot be enforced.

The process.

1. The election stage. First, each national constituency decides on who will participate on the negotiation of the IEA on its behalf. Special case: This decision may be taken by direct majority voting rule. Other special cases may be studied.

2. The bargaining stage. The two negotiators are characterized by $\widetilde{y}_1$ and $\widetilde{y}_2$. When the negociators meet, they bargain over the tax rates to apply in each country. They bargain according to a given rule which is exogenously given. **If there is no agreement, the non-cooperative tax rates (decided by the median voters) are implemented**

Regarding bargaining rules, we consider 3 types of agreements satisfying different sets of constraining rules.[2]

R1 Nash-bargaining: the two delegates bargain a pair of tax rates (τ_1, τ_2) using a Nash-bargaining rule.

R2 N-B + equal tax rate rule: An agreement must simultaneously satisfy the feasibility rule and the constraint that a common tax rate $(\tau_1 = \tau_2 = \tau)$ be applied by each country.

R3 N-B + equal gains rule: an agreement must satisfy the feasibility rule with the additional constraint that each decisionmaker gets an equal gain, i.e.

$$(\tau_1, \tau_2) \, |\Gamma_1(\widetilde{y}_1, \tau_1, \tau_2, \tau_1^*, \tau_2^*) = \Gamma_2(\widetilde{y}_2, \tau_1, \tau_2, \tau_1^*, \tau_2^*)$$

[Actual IEAs are generally international treaties negotiated by governmental executives and then ratified by Parliaments. This procedure is likely the result of uncertainty on preferences and the exact situation of the environmental issue at

[2] Given the assumed equality in power and the absence of size effects, we concentrate on equal rules. Our results generalize to other linear rules at the expense of clarity.

stake. Here there is no uncertainty: everything is known. As agents are assumed to be rational agents, it is therefore straightforward to consider that the negotiators are just representatives of the national electorates. Hence the first stage in each country is the decision on whether or not sending representatives and if so, who? As there is no uncertainty and perfect rationality, voters perfectly calculate the final outcome of their decision.

Remark that in each country, the median voter is decisive in choosing the national negotiator. Equivalently, the procedure is such that the two median voters negotiate the IEA according their own preferences.

A median voter is willing to negotiate an IEA insofar as it improves her situation with respect to the autarky equilibrium. It follows that the gains to be taken into consideration in the bargaining process are related to the median voters.]

For an agent i, the gain derived from an IEA, setting a pair of tax rates (τ_1, τ_2), with respect to **non-cooperation** is:

$$\Gamma_j(y_i, \tau_1, \tau_2, \tau_1^*, \tau_2^*) = W(y_i, \tau_j, \tau_{-j}) - W(y_i, \tau_j^*, \tau_{-j}^*).$$

Given the structural characteristics of the two constituencies and obviously the differing exposures to the environment, whether an IEA can be attained depends on the political process through which such an IEA is investigated. In particular, the set of rules contains a series of instructions on how to bargain, which must be followed by the negotiators. Here we do not investigate the justifications or the origins of these instructions as we take them for granted. But as will be clear later, we want to investigate the impact of differents sets of rules on the obtention and the characteristics of an IEA. In particular, this set of instructions may include constraints on the instruments and/or the objectives of the bargaining process under which negotiators look for an agreement. It also specifies the bargaining process to be followed by negotiators. In the following, we shall investigate the impact of a Nash-bargaining process so as to be able to assess the impact of inequality on the resulting IEA.

When two negotiators meet at the negotiation table, they are subject to a series of constraints which define the type of agreement they discuss upon. These constraints may or may not hamper their capacity to reach an agreement.

3.1 Bargaining

We state the following:

Definition 1 *(i) The set of feasible agreements under rule R1 is*

$$\mathcal{T}^1(\widetilde{y}_1, \widetilde{y}_2) = \{(\tau_1, \tau_2);\ \Gamma_1(\widetilde{y}_1, \tau_1, \tau_2, \tau_1^*, \tau_2^*) \geq 0 \ \textit{and}\ \Gamma_2(\widetilde{y}_2, \tau_1, \tau_2, \tau_1^*, \tau_2^*) \geq 0\}.$$

(ii) The set of feasible agreements under R2 is

$$\mathcal{T}^2(\widetilde{y}_1, \widetilde{y}_2) = \mathcal{T}^1(\widetilde{y}_1, \widetilde{y}_2) \cap \Delta.$$

where $\Delta = \{(\tau_1, \tau_2)\,|\tau_1 = \tau_2\}$
(iii) The set of feasible agreements under R3 is

$$\mathcal{T}^3(\widetilde{y}_1, \widetilde{y}_2) = \mathcal{T}^1(\widetilde{y}_1, \widetilde{y}_2) \cap D.$$

where $D = \{(\tau_1, \tau_2)\,|\Gamma_1(\widetilde{y}_1, \tau_1, \tau_2, \tau_1^*, \tau_2^*) = \;\Gamma_2(\widetilde{y}_2, \tau_1, \tau_2, \tau_1^*, \tau_2^*)\}$

Definition 2 *The sets of strongly feasible agreements are*

$$\begin{aligned}
\mathcal{T}_+^1(\widetilde{y}_1, \widetilde{y}_2) &= \{(\tau_1, \tau_2);\; \Gamma_1(\widetilde{y}_1, \tau_1, \tau_2, \tau_1^*, \tau_2^*) > 0 \text{ and } \Gamma_2(\widetilde{y}_2, \tau_1, \tau_2, \tau_1^*, \tau_2^*) > 0\} \\
\mathcal{T}_+^2(\widetilde{y}_1, \widetilde{y}_2) &= \mathcal{T}_+^1(\widetilde{y}_1, \widetilde{y}_2) \cap \Delta \\
\mathcal{T}_+^3(y_{m_1}, \widetilde{y}_2) &= \mathcal{T}_+^1(\widetilde{y}_1, \widetilde{y}_2) \cap D.
\end{aligned}$$

Proposition 3 *For any* $(\widetilde{y}_1, \widetilde{y}_2)$*,*
(i) $\mathcal{T}^1(\widetilde{y}_1, \widetilde{y}_2)$ *is a convex subset of* $[0;1] \times [0;1]$ *with* $(\tau_1^*, \tau_2^*) \in \mathcal{T}^1(\widetilde{y}_1, \widetilde{y}_2)$*.*
$\mathcal{T}_+^1(\widetilde{y}_1, \widetilde{y}_2)$ *is non-empty (if...)*
(ii) for R2, the set of feasible IEAs $\mathcal{T}^2(\widetilde{y}_1, \widetilde{y}_2)$ *may be empty. For any given* $\widetilde{y}_1 > 0$ *:*
- *If* $\lambda > 1$*, there exists a critical value* $\overline{y} < \widetilde{y}_2$ *such that:*
  - *if* $\widetilde{y}_2 \in [\overline{y}; \widetilde{y}_1]$*, there exists an equal rate agreement;*
  - *if* $\widetilde{y}_2 < \overline{y}$*, then the country 2 delegate rejects any equal rate agreement.*
- *If* $\lambda < 1$*, there exists a critical value* $\overline{y} > \widetilde{y}_2$ *such that:*
  - *if* $\widetilde{y}_2 \in [\widetilde{y}_1; \overline{y}]$*, there exists an equal rate agreement;*
  - *if* $\widetilde{y}_2 > \overline{y}$*, then the country 2 delegate rejects any equal rate agreement.*

(iii) for R3, the set of strongly feasible IEA $\mathcal{T}_+^3(\widetilde{y}_1, \widetilde{y}_2)$ *is non-empty (if...)*

Proof See Appendix C. □

Remark: This can be enlarged to any bargaining process with equal tax / equal gain.

With respect to the occurrence of losers, we can prove the following:

Proposition 4 *For any given strongly feasible agreement* (τ_1, τ_2)*:*
(i) If $\lambda > 1$*, then there exist* $\widehat{y}_1 \in]0, y_{m_1}[$ *and* $\widehat{y}_2 \in]0, y_{m_2}[$*, such that*

$$\text{for } i \in \mathbf{C}_j, \text{ we have } y_i > \widehat{y}_j \Leftrightarrow \Gamma_j(y_i, \tau_1, \tau_2, \tau_1^*, \tau_2^*) > 0 \tag{9}$$

(ii) If $\lambda < 1$*, then there exist* $\widehat{y}_1 \in]y_{m_1}, +\infty[$ *and* $\widehat{y}_2 \in]y_{m_2}, +\infty[$*, such that*

$$\text{for } i \in \mathbf{C}_j, \text{ we have } y_i < \widehat{y}_j \Leftrightarrow \Gamma_j(y_i, \tau_1, \tau_2, \tau_1^*, \tau_2^*) > 0 \tag{10}$$

(iii) For every $\lambda > 0$*, the threshold value* $\widehat{y}_j$ *is a decreasing function of* y_{m_j}*, and an increasing function of* $y_{m_{-j}}$*.*

Proof See Appendix D. □

3.1.1 The Nash-bargaining IEA

Proposition 5 *A Nash-bargained IEA is such that:*

- *If* $\lambda > 1$*, then* τ_j^f *is an increasing function of* $\widetilde{y}_j$ *and a decreasing function of* $\widetilde{y}_{-j}$
- *If* $\lambda < 1$*, then* τ_j^f *is a decreasing function of* $\widetilde{y}_j$ *and an increasing function of* $\widetilde{y}_{-j}$

Proof See Appendix E. □

3.1.2 The equal tax rate rule

We suppose that in the case of an equal rate IEA, the feasibility condition ($y_{m_2} > \overline{y}$ if $\lambda > 1$ and $y_{m_2} < \overline{y}$ if $\lambda < 1$) is satisfied, so that there exists a rate τ^r which is unanimously agree. Then we get the following properties with respect to this agreement, assuming that $\tau_2^* \leq \tau_1^*$:

Proposition 6 *A Nash-bargained equal rate IEA is such that, for any given* $\widetilde{y}_1 > 0$*:*
τ^r *is a increasing function of* $\widetilde{y}_2$ *on* $\widetilde{y}_2 \in]0; \widetilde{y}_1[$ *if* $\lambda > 1$*,*
τ^r *is a decreasing function of* $\widetilde{y}_2$ *on* $\widetilde{y}_2 \in]\widetilde{y}_1; +\infty[$ *if* $\lambda < 1$*,*
and τ^r *is an ambiguous function of* $\widetilde{y}_1$*.*

Proof See Appendix F. □

3.1.3 The equal gain rule

We show that:

Proposition 7 *A Nash-bargained equal gain IEA* (τ_1^g, τ_2^g) *is unique and such that:*

- *If* $\lambda > 1$*, then* τ_j^g *is an increasing function of* $\widetilde{y}_j$*, and a decreasing function of* $\widetilde{y}_{-j}$, $\forall j = 1, 2$.
- *If* $\lambda < 1$*, then* τ_j^g *is a decreasing function of* $\widetilde{y}_j$*, and an increasing function of* $\widetilde{y}_{-j}$, $\forall j = 1, 2$.

Proof See Appendix G. □

3.1.4 Comparing IEAs.

We prove the following:

Proposition 8 *For a given pair* $(\widetilde{y}_1, \widetilde{y}_2)$, *the tax rates* τ^r, τ_1^f, τ_2^f *are such that*
(i) $\tau_2^f = \tau_2^g = \tau_1^g = \tau_1^f = \tau^r$, *if* $\widetilde{y}_1 = \widetilde{y}_2$,
(ii) $\tau_2^f < \tau^r < \tau_1^f$, *if* $\tau^*(\widetilde{y}_1) > \tau^*(\widetilde{y}_2)$,
(iii) $\tau_2^f < \tau_2^g < \tau_1^g < \tau_1^f$,*if* $\tau^*(\widetilde{y}_1) > \tau^*(\widetilde{y}_2)$
where $\tau^*(\widetilde{y}_j)$ *denotes the preferred non cooperative tax rate of the delegate* $\widetilde{y}_j$

Proof See Appendix A. □

3.2 Election of delegates.

Up to now, the various propositions corresponding to Stages 2 and 3 are valid for any choice of delegates !

3.2.1 Direct election

Who is the Condorcet winner?

Proposition 9 *Under direct election, for any rule, in each country, the median voter is chosen as the country delegate to the bargaining stage.*

Proof See Appendix I:
Single crossing property.
Ca va pour les règles R1 et R3. Pas clair pour R2. □
No strategic delegation.
The median voter is the decisive voter and chooses to be her country's negotiator.
This is (likely) due to the particular utility function we use. Additively separable.

4 Conclusion

The impact of redistributive considerations, internal to sovereign countries, on the characteristics and existence of IEA. Interaction between the political processes, specific to nations, the constraints put on the IEA, and the properties of IEA.

- An IEA implies more effort for environmental protection than no agreement (non cooperation)
- There always exist agreements which are "feasible", that is that improve the lot of both policymakers.
- An IEA involves differentiated gains and losses to different individuals in different countries. An IEA is necessarily linked to distributive issues.

- An IEA may not be reached, if the constraints on the bargaining process are seemed too heavy for one national policymaker.
- There may be winners and losers to an IEA in any or both countries.
- The terms of the agreement reflect the distribution schedules in both countries. Higher discrepancies, less taxation, less spending.
- On the whole, IEA depend on the political process, including the bargaining process which takes place between the two countries AND the income distribution, as it cannot be analysed independently of distributive issues, as well as on technical analysis focusing on interdependence.

This complexity and multidimensionality are what makes IEAs so difficult to achieve.

Appendix

A Proof of Lemma 1

$f(y,\tau) = U(y(1-\tau))$ and $F(y,\tau) = f(y,\tau) + H(\tau)$ thus

$$\frac{\partial^2 F}{\partial y \partial \tau} = \frac{\partial^2 f}{\partial y \partial \tau} = -U'(y(1-\tau)) - y(1-\tau)U"(y(1-\tau)) = (\lambda - 1)U'(y(1-\tau)) \quad (11)$$

where λ is the relative risk aversion. Then $\frac{\partial^2 F}{\partial y \partial \tau}$ and $\frac{\partial^2 f}{\partial y \partial \tau}$ have the sign of $(\lambda - 1)$.
- Assume first that $\lambda > 1$. Since $\frac{\partial^2 F}{\partial y \partial \tau} = \frac{\partial^2 f}{\partial y \partial \tau} > 0$, then $y \mapsto \frac{\partial F}{\partial \tau}$ is an increasing function, i.e. $\frac{\partial F}{\partial \tau}(y_i,\tau) > \frac{\partial F}{\partial \tau}(y_{i'},\tau)$, for any τ, if $y_i > y_{i'}$
Let $L(\tau) = [F(y_i,\tau) - F(y_i,\tau')] - [F(y_{i'},\tau) - F(y_{i'},\tau')]$ for τ' fixed, and $y_i > y_{i'}$ fixed.
We have $L(\tau') = 0$ and $L'(\tau) = \frac{\partial F}{\partial \tau}(y_i,\tau) - \frac{\partial F}{\partial \tau}(y_{i'},\tau) > 0$
thus $L(\tau) > 0$ for all τ such that $\tau > \tau'$.
$\tau > \tau'$ and $y_i > y_{i'} \Rightarrow F(y_i,\tau) - F(y_i,\tau') > F(y_{i'},\tau) - F(y_{i'},\tau')$
i.e. the single crossing property of Gans and Smart is satisfied.
Thus the median voter theorem applies (see Persson-Tabellini (2000))
- A similar proof is valid if $\lambda < 1$, i.e. if $\frac{\partial^2 F}{\partial y \partial \tau}$ and $\frac{\partial^2 f}{\partial y \partial \tau}$ are negative. $\square$

B Proof of Proposition 2

1) Here we assume that $\lambda > 1$, where λ is the relative risk aversion.
- First, we show that $y \mapsto \Gamma_j(y,\tau_1,\tau_2,\tau_1^*,\tau_2^*)$ is increasing if $\tau_j > \tau_j^*$, decreasing if $\tau_j < \tau_j^*$, and constant if $\tau_j = \tau_j^*$.
We have $\Gamma_j(y,\tau_1,\tau_2,\tau_1^*,\tau_2^*) = F(y,\tau_j) + H(\tau_{-j}) - F(y,\tau_j^*) - H(\tau_{-j}^*)$
$\frac{\partial}{\partial y}\Gamma_j(y,\tau_1,\tau_2,\tau_1^*,\tau_2^*) = \frac{\partial}{\partial y}\left[F(y,\tau_j) - F(y,\tau_j^*)\right] = \frac{\partial^2 F}{\partial \tau \partial y}(y,\tau_j^b)(\tau_j - \tau_j^*)$ for a given $\tau_j^b \in]\tau_j^*;\tau_j[$ according to the Taylor-Lagrange formula.
Then $\frac{\partial}{\partial y}\Gamma_j(y,\tau_1,\tau_2,\tau_1^*,\tau_2^*)$ is of the sign of $(\tau_j - \tau_j^*)$
- To simplify, we set $\Gamma_j(y) = \Gamma_j(y,\tau_1,\tau_2,\tau_1^*,\tau_2^*)$.
If $\tau_j > \tau_j^*$, a majority of agents i in country j satisfy $\Gamma_j(y) \geq 0$ if and only if: there exists $\widehat{y}_j$ such that $y > \widehat{y}_j \Leftrightarrow \Gamma_j(y) > 0$ and those y are a majority in country j. This is true iff $\Gamma(y_{m_j}) \geq 0$, and we have then $y \geq y_{m_j} \Rightarrow \Gamma_j(y) \geq 0$.
If $\tau_j < \tau_j^*$, the agreement is ratified if and only if: there exists $\widehat{y}_j$ such that $y < \widehat{y}_j \Leftrightarrow \Gamma_j(y) > 0$ and those y are a majority in country j. This is true iff $\Gamma(y_{m_j}) \geq 0$, and we have then $y \leq y_{m_j} \Rightarrow \Gamma(y) \geq 0$.
2) If the relative risk aversion λ is lower than 1, $\lambda < 1$, then the same proof is valid replacing increasing by decreasing, sign of $(\tau_j - \tau_j^*)$ by sign of $-(\tau_j - \tau_j^*)$ etc...
thus Proposition 2 is proven. $\square$

C Proof of Proposition 3

Proof of proposition 3(i) - If $(\tau_1, \tau_2) \in \mathcal{T}^1(y_{m_1}, y_{m_2})$, then $\Gamma_1 \geq 0$, where:

$$\begin{aligned}\Gamma_1 &= \Gamma_1(y_{m_1}, \tau_1, \tau_2, \tau_1^*, \tau_2^*) \\ &= [W(y_{m_1}, \tau_1, \tau_2) - W(y_{m_1}, \tau_1^*, \tau_2)] + [W(y_{m_1}, \tau_1^*, \tau_2) - W(y_{m_1}, \tau_1^*, \tau_2^*)]\,.\end{aligned}$$

The first bracket is always negative since $\tau_1^* = \arg\max_{\tau_1} W(y_{m_1}, \tau_1, \tau_2)$ for all τ_2, and the sum of the 2 brackets is positive since $\Gamma_1 \geq 0$, thus the second bracket must be positive, i.e. $F(y_{m_1}, \tau_1^*) + H(\tau_2) \geq F(y_{m_1}, \tau_1^*) + H(\tau_2^*)$, i.e. $H(\tau_2) \geq H(\tau_2^*)$, which gives $\tau_2 \geq \tau_2^*$ because H is an increasing function. Similarly one can find $\tau_1 \geq \tau_1^*$.

$\Gamma_1 = \Gamma_1(y_{m_1}, \tau_1, \tau_2, \tau_1^*, \tau_2^*) = F(y_{m_1}, \tau_1) + H(\tau_2) - F(y_{m_1}, \tau_1^*) - H(\tau_2^*)$

$\Gamma_1 = 0 \Leftrightarrow H(\tau_2) = F(y_{m_1}, \tau_1^*) + H(\tau_2^*) - F(y_{m_1}, \tau_1)$

i.e.

$\Gamma_1 = 0 \Leftrightarrow \tau_2 = H^{-1}(\widetilde{F}(\tau_1))$, where $\widetilde{F}(\tau_1) = F(y_{m_1}, \tau_1^*) + H(\tau_2^*) - F(y_{m_1}, \tau_1)$ is an increasing convex function on $\tau_1 \in [\tau_1^*; 1[$.

H^{-1} is an increasing convex function, then $\psi_1(\tau_1) = H^{-1}(\widetilde{F}(\tau_1))$ defines an increasing convex function on $[\tau_1^*; 1[$.

$\psi_1(\tau_1^*) = \tau_2^*$ and $\psi_1'(\tau_1^*) = \frac{\widetilde{F}'(\tau_1)}{H'(H^{-1}(\tau_1^*))} = 0$

Similarly, we show that $\Gamma_2 = 0 \Leftrightarrow \tau_2 = \psi_2(\tau_1)$, where ψ_2 is an increasing concave function on $[\tau_1^*; 1[$, and $\psi_2(\tau_1^*) = \tau_2^*$, $\psi_2'(\tau_1^*) = \infty$

$\psi_2 - \psi_1$ is a concave function, with $(\psi_2 - \psi_1)(\tau_1^*) = 0$ and $(\psi_2' - \psi_1')(\tau_1^*) = \infty$, so that there exists a unique $\widetilde{\tau}_1 \in]\tau_1^*; 1[$ with $(\psi_2 - \psi_1)(\widetilde{\tau}_1) = 0$ and $\psi_1(\tau_1) < \psi_2(\tau_1) \Leftrightarrow \tau_1 < \widetilde{\tau}_1$

We set $\widetilde{\tau}_2 = \psi_1(\widetilde{\tau}_1) = \psi_2(\widetilde{\tau}_1)$

- Now we want to show that $\mathcal{T}_+^1(y_{m_1}, y_{m_2}) \neq \varnothing$.

Let $(\tau_1, \tau_2) \in [\tau_1^*; 1] \times [\tau_2^*; 1]$. We set $h_i = \tau_i - \tau_i^*$, for $i = 1; 2$.

$\Gamma_1 = \Gamma_1(y_{m_1}, \tau_1, \tau_2) = F(y_{m_1}, \tau_1) + H(\tau_2) - F(y_{m_1}, \tau_1^*) - H(\tau_2^*)$

$= F(y_{m_1}, \tau_1) - F(y_{m_1}, \tau_1^*) + H(\tau_2) - H(\tau_2^*)$

$= \frac{\partial F}{\partial \tau}(y_{m_1}, \tau_1^*)h_1 + o(h_1) + H'(\tau_2^*)h_2 + o(h_2)$ as $h_1 \to 0$ and $h_2 \to 0$.

$= H'(\tau_2^*)h_2 + o(h_1) + o(h_2)$ since $\tau_1^* = \arg\max_{\tau_1} F(y_{m_1}, \tau_1)$

$= H'(\tau_2^*)h + o(h)$ as $h \to 0$ for $h = h_1 = h_2$

$\Gamma_1 = H'(\tau_2^*)h + o(h)$ with $H'(\tau_2^*) > 0$ thus $\Gamma_1 > 0$ for $h > 0$ close to 0.

We can write the same proof for $\Gamma_2 > 0$.

Thus $\Gamma_1 > 0$ and $\Gamma_2 > 0$ for $h > 0$ close to 0, i.e. $\mathcal{T} \neq \varnothing$.

Proof of proposition 3(ii) - Let $\gamma_2(y_{m_2}, \tau) = \Gamma_2(\tau, \tau)$

$\exists ? \widetilde{y}$ such that $y_{m_2} > \widetilde{y} \Rightarrow \gamma_2(y_{m_2}, \tau) < 0$, $\forall \tau > \tau_2^*$?

$\frac{\partial \gamma_2}{\partial \tau} = \frac{\partial F}{\partial \tau}(y_{m_2}, \tau) + H'(\tau) = \frac{\partial F_{2H}}{\partial \tau}(y_{m_2}, \tau)$

where $F_{2H}(y_{m_2}, \tau) = f(y_{m_2}, \tau) + 2H(\tau)$

and we set $\tau_2^{**} = \arg\max_\tau F_{2H}(y_{m_2}, \tau)$.

Then $\frac{\partial F_{2H}(y_{m_2}, \tau)}{\partial \tau} < 0$ iff $\tau > \tau_2^{**}$

- We have

$(\gamma_2(y_{m_2}, \tau) < 0, \forall \tau > \tau_2^*) \Leftrightarrow \gamma_2(y_{m_2}, \tau_2^{**}) < 0$

$(\gamma_2(y_{m_2}, \tau) < 0, \forall \tau > \tau_2^*) \Leftrightarrow F_{2H}(y_{m_2}, \tau_2^{**}) < F(y_{m_2}, \tau_2^*) + H(\tau_1^*)$
Let $B(y_{m_2}) = F(y_{m_2}, \tau_2^*(y_{m_2})) + H(\tau_1^*) - F_{2H}(y_{m_2}, \tau_2^{**}(y_{m_2}))$
We know that $(\gamma_2(y_{m_2}, \tau) < 0, \forall \tau > \tau_2^*) \Leftrightarrow B(y_{m_2}) > 0$
By the enveloppe theorem:

$$B'(y_{m_2}) = \frac{\partial F}{\partial y_{m_2}}(y_{m_2}, \tau_2^*) + \frac{\partial F}{\partial \tau}(y_{m_2}, \tau_2^*)\frac{d\tau_2^*}{dy_{m_2}} - \frac{\partial F_{2H}}{\partial y_{m_2}}(y_{m_2}, \tau_2^{**}) - \frac{\partial F_{2H}}{\partial \tau}(y_{m_2}, \tau_2^{**})\frac{d\tau_2^{**}}{dy_{m_2}}$$

where $\frac{\partial F}{\partial \tau}(y_{m_2}, \tau_2^*) = \frac{\partial F_{2H}}{\partial \tau}(y_{m_2}, \tau_2^{**}) = 0$

$$B'(y_{m_2}) = \frac{\partial F}{\partial y_{m_2}}(y_{m_2}, \tau_2^*) - \frac{\partial F_{2H}}{\partial y_{m_2}}(y_{m_2}, \tau_2^{**})$$

$$B'(y_{m_2}) = \frac{\partial f}{\partial y_{m_2}}(y_{m_2}, \tau_2^*) - \frac{\partial f}{\partial y_{m_2}}(y_{m_2}, \tau_2^{**}) < 0 \text{ (assuming } \frac{\partial^2 f}{\partial y \partial \tau} > 0)$$

$y_{m_2} \mapsto B(y_{m_2})$ is a decreasing function on $]0; +\infty[$

$$B(y_{m_1}) = F(y_{m_1}, \tau_1^*) + H(\tau_1^*) - F_{2H}(y_{m_1}, \tau_1^{**})$$

$$B(y_{m_1}) = F(y_{m_1}, \tau_1^*) - F_{2H}(y_{m_1}, \tau_1^{**}) < 0$$

thus for $y_{m_1} = y_{m_2}$, $\exists \tau$ such that $\gamma_2(\tau) \geq 0$, i.e. an equal rate agreement is possible.

$$B(y_{m_2}) = F(y_{m_2}, \tau_2^*) + H(\tau_1^*) - F(y_{m_2}, \tau_2^{**}) - H(\tau_2^{**})$$
$$= [F(y_{m_2}, \tau_2^*) - F(y_{m_2}, \tau_2^{**})] + [H(\tau_1^*) - H(\tau_2^{**})]$$

where the first bracket is positive (by definition of τ_2^*), and the second bracket is positive iff $\tau_2^{**} < \tau_1^*$ (i.e. for y_{m_2} small enough).

Thus the existence of $\widetilde{y}$ such that $B(y_{m_2}) < 0$ for $y_{m_2} < \widetilde{y}$ and $B(y_{m_2}) > 0$ for $y_{m_2} > \widetilde{y}$.

- We must check that there is an equal rate agreement iff country 2 wants it.

It is sufficient to show that $\gamma_1 \geq \gamma_2$, when $y_{m_1} \geq y_{m_2}$

$$\gamma_1(y_{m_1}, \tau) = f(y_{m_1}, \tau) + 2H(\tau) - f(y_{m_1}, \tau_1^*) - H(\tau_1^*) - H(\tau_2^*)$$
$$\gamma_2(y_{m_2}, \tau) = f(y_{m_2}, \tau) + 2H(\tau) - f(y_{m_2}, \tau_2^*) - H(\tau_1^*) - H(\tau_2^*)$$
$$(\gamma_1 - \gamma_2)(\tau) = [f(y_{m_1}, \tau) - f(y_{m_1}, \tau_1^*)] - [f(y_{m_2}, \tau) - f(y_{m_2}, \tau_2^*)]$$

$(\gamma_1 - \gamma_2)(\tau_1^*) = 0 - [f(y_{m_2}, \tau_1^*) - f(y_{m_2}, \tau_2^*)] > 0$ since $f(y, \tau)$ is a decreasing function of τ

$$(\gamma_1 - \gamma_2)(\tau_2^*) = [f(y_{m_1}, \tau_2^*) - f(y_{m_1}, \tau_1^*)] > 0$$

$(\gamma_1 - \gamma_2)'(\tau) = \frac{\partial f}{\partial \tau}(y_{m_1}, \tau) - \frac{\partial f}{\partial \tau}(y_{m_2}, \tau) > 0$ because $\frac{\partial^2 f}{\partial \tau \partial y} > 0$ and $y_{m_1} > y_{m_2}$

$\gamma_1' > \gamma_2'$ and $\gamma_1(\tau_1^*) \geq \gamma_2(\tau_1^*)$

thus $\gamma_1(\tau) \geq \gamma_2(\tau)$, for any $\tau \geq \tau_1^*$

Proof of proposition 3(iii) (τ_1^g, τ_2^g) is an equal gain agreement iff $\Gamma_1(y_{m_1}, \tau_1^g, \tau_2^g) = \Gamma_2(y_{m_2}, \tau_2^g, \tau_1^g) > 0$

In that case, we necessarily have $\tau_1^g > \tau_1^*$ and $\tau_2^g > \tau_2^*$.

We know that $\Gamma_1(y_{m_1}, \tau_1^*, \tau_2^*) = \Gamma_2(y_{m_2}, \tau_2^*, \tau_1^*) = 0$

Assume that τ_1, τ_2 are such that $\tau_1 \geq \tau_1^*$ and $\tau_2 \geq \tau_2^*$.

We have $\frac{\partial(\Gamma_2 - \Gamma_1)}{\partial \tau_2} = \frac{\partial F}{\partial \tau_2}(y_{m_2}, \tau_2) - H'(\tau_2) < 0$ since $\frac{\partial F}{\partial \tau_2}(y_{m_2}, \tau_2) \leq 0$ for $\tau_2 \geq \tau_2^*$

Thus the implicit functions theorem can be applied: there exists a function ψ, defined at least in a neighborhood $]\tau_1^* - \varepsilon; \tau_1^* + \varepsilon[$of τ_1^*, such that $(\Gamma_2 - \Gamma_1)(\tau_1, \tau_2) = 0 \Leftrightarrow \tau_2 = \psi(\tau_1)$

This implies that $(\tau_1, \psi(\tau_1))$ is an equal gain agreement, for every $\tau_1 \in]\tau_1^*; \tau_1^* + \varepsilon[$

□

D Proof of Proposition 4

Let (τ_1, τ_2) be a strongly feasible agreement, i.e. such that $\Gamma_1(y_{m_1}, \tau_1, \tau_2) > 0$ and $\Gamma_2(y_{m_2}, \tau_2, \tau_1) > 0$. We assume for the moment that $\lambda > 1$.

- First, we want to show that there exists $\widehat{y}_1 \in [0; y_{m_1}]$ such that for $i \in \mathbf{C}_1$, we have $y_i < \widehat{y}_1 \Leftrightarrow \Gamma_1(y_i, \tau_1, \tau_2) < 0$.

It is sufficient to show that $y \mapsto \Gamma_1(y, \tau_1, \tau_2)$ is an increasing function.

$\Gamma_1(y, \tau_1, \tau_2) = W(y, \tau_1, \tau_2) - W(y, \tau_1^*, \tau_2^*) = f(y, \tau_1) - f(y, \tau_1^*) + H(\tau_1) + H(\tau_2) - H(\tau_1^*) - H(\tau_2^*)$

Let $d(y) = f(y, \tau_1) - f(y, \tau_1^*)$

$d'(y) = \frac{\partial f}{\partial y}(y, \tau_1) - \frac{\partial f}{\partial y}(y, \tau_1^*) = \frac{\partial^2 f}{\partial y \partial \tau}(y, \tau_d)(\tau_1 - \tau_1^*)$ for a $\tau_d \in]\tau_1^*, \tau_1[$according to the Taylor-Lagrange formula.

Since $\frac{\partial^2 f}{\partial y \partial \tau} > 0$ and $\tau_1 > \tau_1^*$, then $d'(y) > 0$. Hence $y \mapsto d(y)$ and $y \mapsto \Gamma_1(y, \tau_1, \tau_2)$ are increasing functions.

Note that $\widehat{y}_1 = 0$ means that $\Gamma_1(y, \tau_1, \tau_2) \geq 0$ for every $y \geq 0$

- Now, if we assume moreover that $\frac{\partial^2 f}{\partial y \partial \tau} \geq \frac{c}{y^a} > 0$ uniformly, with $c > 0$ and $a > 1$, then $d'(y) \geq \frac{c(\tau_1 - \tau_1^*)}{y^a}$ for all $y < y_{m_1}$.

$d(y_{m_1}) - d(y) = \int_y^{y_{m_1}} d'(t)dt \geq \int_y^{y_{m_1}} \frac{c}{t^a}(\tau_1 - \tau_1^*)dt \to +\infty$ as $y \to 0$, since $a > 1$.

Thus $d(y) \to -\infty$ as $y \to 0$, i.e. $\Gamma_1(y, \tau_1, \tau_2) < 0$ for y small enough.

- Similar proofs are valid for Γ_2 and $\widehat{y}_2$.

- Now we prove the end of Proposition 4

We have $\Gamma_1(\widehat{y}_1, \tau_1, \tau_2, \tau_1^*, \tau_2^*) = 0$

- First we show that $\widehat{y}_1$ is a decreasing function of y_{m_1}.

$\frac{d\widehat{y}_1}{dy_{m_1}} = -\frac{\frac{\partial \Gamma_1}{\partial y_{m_1}}}{\frac{\partial \Gamma_1}{\partial \widehat{y}_1}}$ has the sign of $-\frac{\partial \Gamma_1}{\partial y_{m_1}}$ (since $\frac{\partial \Gamma_1}{\partial \widehat{y}_1} > 0$)

$-sign(\frac{\partial \Gamma_1}{\partial y_{m_1}}) = sign(\frac{\partial W}{\partial y_{m_1}}(\widehat{y}_1, \tau_1^*, \tau_2^*)) = sign(\frac{\partial W}{\partial \tau_1^*}(\widehat{y}_1, \tau_1^*, \tau_2^*)) = sign(\frac{\partial F}{\partial \tau_1^*}(\widehat{y}_1, \tau_1^*))$

$\frac{\partial F}{\partial \tau_1^*}(y_{m_1}, \tau_1^*) = 0$

$\frac{\partial F}{\partial \tau_1^*}(\widehat{y}_1, \tau_1^*) - \frac{\partial F}{\partial \tau_1^*}(y_{m_1}, \tau_1^*) = \frac{\partial^2 F}{\partial y \partial \tau_1}(\widetilde{\widetilde{y}}, \tau_1^*).(\widehat{y}_1 - y_{m_1}) < 0$ for a $\widetilde{\widetilde{y}} \in]\widehat{y}_1; y_{m_1}[$

thus $\frac{d\widehat{y}_1}{dy_{m_1}} < 0$

- Now we show that $\widehat{y}_1$ is an increasing function of y_{m_2}.

$\frac{d\widehat{y}_1}{dy_{m_2}} = -\frac{\frac{\partial \Gamma_1}{\partial y_{m_2}}}{\frac{\partial \Gamma_1}{\partial \widehat{y}_1}}$ has the sign of $-\frac{\partial \Gamma_1}{\partial y_{m_2}}$ (since $\frac{\partial \Gamma_1}{\partial \widehat{y}_1} > 0$)

$-sign(\frac{\partial \Gamma_1}{\partial y_{m_2}}) = sign(\frac{\partial W}{\partial y_{m_2}}(\widehat{y}_1, \tau_1^*, \tau_2^*)) = sign(\frac{\partial W}{\partial \tau_2^*}(\widehat{y}_1, \tau_1^*, \tau_2^*)) = sign(H'(\tau_2^*))$ positive

thus $\frac{d\widehat{y}_1}{dy_{m_2}} > 0$ □

E Proof of Proposition 5

We set $\Gamma = \Gamma_1 \Gamma_2$, and $A_1 = \frac{\partial \Gamma}{\partial \tau_1}$, $A_2 = \frac{\partial \Gamma}{\partial \tau_2}$

(i) The first order conditions are: $A_1 = 0$ and $A_2 = 0$
i.e. in $(\tau_1, \tau_2) = (\tau_1^f, \tau_2^f)$

$A_1 = \frac{\partial F}{\partial \tau_1}(y_{m_1}; \tau_1)\Gamma_2 + H'(\tau_1)\Gamma_1 = 0$

$A_2 = \frac{\partial F}{\partial \tau_2}(y_{m_2}; \tau_2)\Gamma_1 + H'(\tau_2)\Gamma_2 = 0$

Applying the implicit function theorem, we have $\frac{d\tau_1^f}{dy_{m_1}} = \frac{\frac{\partial A_1}{\partial y_{m_1}}}{-\frac{\partial A_1}{\partial \tau_1}}$, where:

- the denominator is positive, because of the concavity of $\tau_1 \mapsto \Gamma(\tau_1, \tau_2)$, for τ_2 fixed. Let us prove this concavity:

$\frac{\partial A_1}{\partial \tau_1} = \frac{\partial^2 \Gamma_1}{\partial \tau_1^2} = \Gamma_1"\Gamma_2 + 2\Gamma_1'\Gamma_2' + \Gamma_1\Gamma_2"$

Here $\Gamma_1" < 0$, $\Gamma_2" < 0$, and Γ_1', Γ_2' have opposite signs (because $A_1 = 0$).

Thus $\frac{\partial^2 \Gamma_1}{\partial \tau_1^2} < 0$

- Now we study $\frac{\partial A_1}{\partial y_{m_1}}$

$\frac{\partial A_1}{\partial y_{m_1}} = \frac{\partial^2 F}{\partial y_{m_1}\partial \tau_1}(y_{m_1}, \tau_1)\Gamma_2 + \frac{\partial F}{\partial \tau_1}(y_{m_1}, \tau_1)\frac{\partial \Gamma_2}{\partial y_{m_1}} + H'(\tau_1)\frac{\partial \Gamma_1}{\partial y_{m_1}} > 0$

because: $\Gamma_2 > 0$, $\frac{\partial^2 F}{\partial y_{m_1}\partial \tau_1} > 0$ by hypothesis ,$\frac{\partial F}{\partial \tau_1}(y_{m_1}, \tau_1) < 0$ for $\tau_1 > \tau_1^*$, $\frac{\partial \Gamma_2}{\partial y_{m_1}} < 0$ since τ_1^* is an increasing function of y_{m_1}, $H'(\tau_1) > 0$ and $\frac{\partial \Gamma_1}{\partial y_{m_1}} > 0$ (proof below).

Let us prove that $\frac{\partial \Gamma_1}{\partial y_{m_1}} > 0$.

$\frac{\partial \Gamma_1}{\partial y_{m_1}} = \frac{\partial}{\partial y_{m_1}}\left[F(y_{m_1}, \tau_1) - F(y_{m_1}, \tau_1^*(y_{m_1}))\right]$

$\frac{\partial \Gamma_1}{\partial y_{m_1}} = \frac{\partial F}{\partial y_{m_1}}(y_{m_1}, \tau_1^f) - \frac{\partial F}{\partial y_{m_1}}(y_{m_1}, \tau_1^*) - \frac{\partial F}{\partial \tau_1}(y_{m_1}, \tau_1^*)\frac{d\tau_1^*}{dy_{m_1}}$

$\frac{\partial \Gamma_1}{\partial y_{m_1}} = \frac{\partial F}{\partial y_{m_1}}(y_{m_1}, \tau_1^f) - \frac{\partial F}{\partial y_{m_1}}(y_{m_1}, \tau_1^*)$ since $\frac{\partial F}{\partial \tau_1}(y_{m_1}, \tau_1^*) = 0$

thus $\frac{\partial \Gamma_1}{\partial y_{m_1}} > 0$ since $\tau_1^f > \tau_1^*$ and $\frac{\partial^2 F}{\partial y_{m_1}\partial \tau_1} > 0$

As a consequence, we have $\frac{\partial A_1}{\partial y_{m_1}} > 0$ and $\frac{d\tau_1^f}{dy_{m_1}} > 0$, i.e. τ_1^f is an increasing function of y_{m_1}.

- Moreover, $\frac{d\tau_1^f}{dy_{m_2}} = \frac{\frac{\partial A_1}{\partial y_{m_2}}}{-\frac{\partial A_1}{\partial \tau_1}}$, where the denominator is positive

and $\frac{\partial A_1}{\partial y_{m_2}} = \frac{\partial^2 F}{\partial y_{m_2}\partial \tau_1}(y_{m_1}, \tau_1)\Gamma_2 + \frac{\partial F}{\partial \tau_1}(y_{m_1}, \tau_1)\frac{\partial \Gamma_2}{\partial y_{m_2}} + H'(\tau_1)\frac{\partial \Gamma_1}{\partial y_{m_2}} < 0$

because: $\frac{\partial^2 F}{\partial y_{m_2}\partial \tau_1} = 0$, $\frac{\partial F}{\partial \tau_1}(y_{m_1}, \tau_1) < 0$, $\frac{\partial \Gamma_2}{\partial y_{m_2}} > 0$, $H'(\tau_1) > 0$, $\frac{\partial \Gamma_1}{\partial y_{m_2}} < 0$

thus $\frac{d\tau_1^f}{dy_{m_2}} < 0$

Hence τ_1^f is an increasing function of y_{m_1} and a decreasing function of y_{m_2}.

Similarly, we can prove that τ_2^f is a decreasing function of y_{m_2} and an increasing function of y_{m_1}.

(ii) is immediate.

(iii). To be completed.

F Proof of Proposition 6

(i) We must show that τ^r is an increasing function of y_{m_2}.

$\frac{d\tau^r}{dy_{m_2}} = \frac{\frac{\partial \Omega}{\partial y_{m_2}}}{-\frac{\partial \Omega}{\partial \tau}}$ where:

- the denominator is positive, by concavity of $\tau \mapsto \Gamma(\tau,\tau) = \gamma(\tau) = \gamma_1(\tau)\gamma_2(\tau) = \Gamma_1(\tau,\tau)\Gamma_2(\tau,\tau)$. Let us prove this concavity. We must show that $\Omega'(\tau^r) < 0$, where $\Omega(\tau) = \gamma'(\tau) = \gamma_1'(\tau)\gamma_2(\tau) + \gamma_1(\tau)\gamma_2'(\tau)$

$\Omega'(\tau^r) = \frac{\partial^2 F_{2H}}{\partial \tau^2}(y_{m_1},\tau^r)\Gamma_2 + 2\frac{\partial F_{2H}}{\partial \tau}(y_{m_1},\tau^r)\frac{\partial F_{2H}}{\partial \tau}(y_{m_2},\tau^r) + \Gamma_1 \frac{\partial^2 F_{2H}}{\partial \tau^2}(y_{m_2},\tau^r) < 0$

because $\frac{\partial^2 F_{2H}}{\partial \tau^2}(y_{m_1},\tau^r) < 0$, $\frac{\partial F_{2H}}{\partial \tau}(y_{m_1},\tau^r)$ and $\frac{\partial F_{2H}}{\partial \tau}(y_{m_2},\tau^r)$ have opposite signs, $\frac{\partial^2 F_{2H}}{\partial \tau^2}(y_{m_2},\tau^r) < 0$.

- Now, we study $\frac{\partial \Omega}{\partial y_{m_2}}$

$\frac{\partial \Omega}{\partial y_{m_2}} = \frac{\partial^2 F_{2H}}{\partial y_{m_2}\partial \tau}(y_{m_2},\tau)\Gamma_1 + \frac{\partial F_{2H}}{\partial \tau}(y_{m_2},\tau)\frac{\partial \Gamma_1}{\partial y_{m_2}} + \frac{\partial F_{2H}}{\partial \tau}(y_{m_1},\tau)\frac{\partial \Gamma_2}{\partial y_{m_2}} > 0$

because: $\frac{\partial^2 F_{2H}}{\partial y_{m_2}\partial \tau}(y_{m_2},\tau) > 0$, $\frac{\partial F_{2H}}{\partial \tau}(y_{m_2},\tau) < 0$ (since $\tau^r > \tau_2^{**}$), $\frac{\partial \Gamma_1}{\partial y_{m_2}} < 0$, $\frac{\partial F_{2H}}{\partial \tau}(y_{m_1},\tau) > 0$ (since $\tau^r < \tau_1^{**}$), $\frac{\partial \Gamma_2}{\partial y_{m_2}} > 0$

$\frac{\partial \Omega}{\partial y_{m_2}} > 0$ thus $\frac{d\tau_1^r}{dy_{m_2}} > 0$

Hence τ^r is an increasing function of y_{m_2}.

G Proof of Proposition 7

$(i/)$ To maximize $\Gamma_1(y_{m_1},\tau_1,\tau_2)\cdot\Gamma_2(y_{m_2},\tau_2,\tau_1)$ for $(\tau_1,\tau_2) \in \mathcal{T}$ with $\Gamma_1(y_{m_1},\tau_1,\tau_2) = \Gamma_2(y_{m_2},\tau_2,\tau_1)$ is equivalent to maximize $\Gamma_1(y_{m_1},\tau_1,\tau_2)$ for $(\tau_1,\tau_2) \in \mathcal{T}$ with $\Gamma_1(y_{m_1},\tau_1,\tau_2) = \Gamma_2(y_{m_2},\tau_2,\tau_1)$ i.e. to maximize $\Gamma_1(y_{m_1},\tau_1,\psi(\tau_1))$ for $\tau_1 > \tau_1^*$

Let $g(\tau_1) = \Gamma_1(y_{m_1},\tau_1,\psi(\tau_1))$ for $\tau_1 > \tau_1^*$

$g'(\tau_1) = \frac{\partial \Gamma_1}{\partial \tau_1} + \psi'(\tau_1)\frac{\partial \Gamma_1}{\partial \tau_2} = \frac{\partial F}{\partial \tau_1}(y_{m_1},\tau_1) - \frac{\frac{\partial f}{\partial \tau_1}(y_{m_1},\tau_1)}{\frac{\partial f}{\partial \tau_2}(y_{m_2},\tau_2)}H'(\tau_2)$

since $\psi'(\tau_1) = \frac{\frac{\partial(\Gamma_2-\Gamma_1)}{\partial \tau_1}}{-\frac{\partial(\Gamma_2-\Gamma_1)}{\partial \tau_2}} = \frac{H'(\tau_1) - \frac{\partial F}{\partial \tau_1}(y_{m_1},\tau_1)}{-\left[\frac{\partial F}{\partial \tau_2}(y_{m_2},\tau_2) - H'(\tau_2)\right]} = -\frac{\frac{\partial f}{\partial \tau_1}(y_{m_1},\tau_1)}{\frac{\partial f}{\partial \tau_2}(y_{m_2},\tau_2)}$

Thus $g'(\tau_1) = \frac{\partial f}{\partial \tau_1}(y_{m_1},\tau_1) + H'(\tau_1) - \frac{\frac{\partial f}{\partial \tau_1}(y_{m_1},\tau_1)}{\frac{\partial f}{\partial \tau_2}(y_{m_2},\tau_2)}H'(\tau_2) = \frac{\partial f}{\partial \tau_1}(y_{m_1},\tau_1)\left[1 - \frac{H'(\tau_2)}{\frac{\partial f}{\partial \tau_2}(y_{m_2},\tau_2)}\right] + H'(\tau_1)$

We have $g'(\tau_1^*) = \frac{\partial f}{\partial \tau_1}(y_{m_1},\tau_1^*)\left[1 - \frac{H'(\tau_2)}{\frac{\partial f}{\partial \tau_2}(y_{m_2},\tau_2)}\right] + H'(\tau_1^*) = H'(\tau_1^*) > 0$

and $g"(\tau_1^*) = \frac{\partial^2 f}{\partial \tau_1^2}(y_{m_1},\tau_1^*)\left[1 - \frac{H'(\tau_2)}{\frac{\partial f}{\partial \tau_2}(y_{m_2},\tau_2)}\right] + H"(\tau_1^*) < 0$ by concavity of H and of $\tau \mapsto f(y_{m_1},\tau)$

g is concave on $\tau_1 > \tau_1^*$ and $g'(\tau_1^*) > 0$ thus there always exists a unique $\tau_1^g > \tau_1^*$ maximizing g.

Finally (τ_1^g, τ_2^g) is the unique equal gain agreement maximizing $\Gamma_1(y_{m_1},\tau_1,\tau_2) \cdot \Gamma_2(y_{m_2},\tau_2,\tau_1)$, where $\tau_2^g = \psi(\tau_1^g)$.

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$(ii/)$..................

H Proof of Proposition 8

1 - First, we want to prove that $\tau_2^f < \tau_2^{**} < \tau_1^{**} < \tau_1^f$

According to Proposition , if $y_{m_1} = y_{m_2}$, then $\tau_1^{f,eq} = \tau_2^{f,eq}$, but $\frac{d\tau_2^f}{dy_{m_2}} < 0 < \frac{d\tau_1^f}{dy_{m_2}}$.

If y_{m_2} is increasing (y_{m_1} fixed), then $\tau_2^f < \tau_2^{f,eq} = \tau_1^{f,eq} < \tau_1^f$, i.e. $\tau_2^f < \tau_1^f$

• $A_1 = 0$ gives $\frac{\partial F}{\partial \tau_1}(y_{m_1};\tau_1)\Gamma_2 + H'(\tau_1)\Gamma_1 = 0$

$\frac{\Gamma_1}{\Gamma_2} = \frac{-\frac{\partial F}{\partial \tau_1}(y_{m_1};\tau_1)}{H'(\tau_1)}$

If $\Gamma_1 > \Gamma_2$, thus $-\frac{\partial F}{\partial \tau_1}(y_{m_1};\tau_1) > H'(\tau_1)$

i.e. $0 > \frac{\partial F}{\partial \tau_1}(y_{m_1};\tau_1) + H'(\tau_1)$

i.e. $0 > \frac{\partial F_{2H}}{\partial \tau_1}(y_{m_1};\tau_1)$, which means that $\tau_1^f > \tau_1^{**}$

• $A_2 = 0$ gives $\frac{\Gamma_1}{\Gamma_2} = \frac{H'(\tau_2)}{-\frac{\partial F}{\partial \tau_2}(y_{m_2};\tau_2)}$

If $\Gamma_1 > \Gamma_2$, thus $H'(\tau_2) > -\frac{\partial F}{\partial \tau_2}(y_{m_2};\tau_2)$

i.e. $\frac{\partial F_{2H}}{\partial \tau_2}(y_{m_2};\tau_2) > 0$, which means that $\tau_2^f < \tau_2^{**}$

As we know that $\tau_2^{**} < \tau_1^{**}$, we conclude that $\tau_2^f < \tau_2^{**} < \tau_1^{**} < \tau_1^f$.

2 - Now, we must show that $\tau_2^{**} < \tau^r < \tau_1^{**}$. We set $\Omega = \frac{\partial \Gamma}{\partial \tau} = \frac{\partial \Gamma_1}{\partial \tau}\Gamma_2 + \Gamma_1\frac{\partial \Gamma_2}{\partial \tau}$

where $\Gamma_1 = \Gamma_1(\tau,\tau) = F_{2H}(y_{m_1},\tau) - F(y_{m_1},\tau_1^*) - H(\tau_2^*)$, and $\Gamma_2 = \Gamma_2(\tau,\tau) = F_{2H}(y_{m_2},\tau) - F(y_{m_2},\tau_2^*) - H(\tau_1^*)$.

Thus $\Omega = \frac{\partial F_{2H}}{\partial \tau}(y_{m_1},\tau)\Gamma_2 + \Gamma_1\frac{\partial F_{2H}}{\partial \tau}(y_{m_2},\tau)$. τ^r is solution of the first order condition $\Omega = 0$. Hence we deduce: $\frac{\partial F_{2H}}{\partial \tau}(y_{m_i},\tau) < 0 \Leftrightarrow \tau^r > \tau_i^{**}$, but $y_{m_1} < y_{m_2}$ implies that $\tau_2^{**} < \tau_1^{**}$, thus $\tau_2^{**} < \tau^r < \tau_1^{**}$.