

# Crowding out of private-sector work by workfare

## - An optimal tax analysis

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### Abstract

A usual objection against work obligations in return for transfer payments is that they can crowd out private-sector work. The present paper argues that this crowding out can indeed happen, but that it is second-best. A discrete version of the standard Mirrlees (1971) model is analyzed. The workfare productivity emerges to be the crucial determinant. We find two individual thresholds of workfare productivity. For any work obligation that exhibits a productivity beyond the first threshold the respective type should be on workfare. Moreover, if workfare productivity exceeds a second threshold then the type should not be regularly employed anymore. It turns out that these two thresholds are in fact the same. The crowding out occurs entirely in the sense that workfare and private-sector employment are mutually exclusive. The welfare increase is due to the fact that workfare provides an additional instrument besides the distortion of the individual labor-leisure decision. Workfare enhances redistribution and lowers the extent of distortionary taxation.

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# 1 Introduction

Workfare for the unemployed has become a widely used instrument in many Western welfare states such as the United States, the United Kingdom or Denmark. Besley and Coate argue that <<the cost of using workfare ... is that public-sector work "crowds out" private-sector work>> (Besley and Coate, 1992, p. 260) in the sense that a person being regularly employed in a world without work requirements will be unemployed but on workfare once the social policy instrument of workfare is available.

Indeed, there is evidence e.g. from Germany that private-sector employment of a low qualification level is substituted by so called "1-euro-workers" as a kind of unemployed people being on workfare (Kettner and Rebien, 2007, p. 61). Such crowding out seems to occur particularly in East Germany (Hohendanner, 2007, p. 23).

The present paper analyzes this alleged disadvantage of workfare within an optimal tax framework (Mirrlees, 1971). The theoretical literature has dealt to some extent with the problem of crowding out. The sketched substitution is generally seen as a central objection against the use of workfare. In an extension of Besley and Coate (1992) to a dynamic environment Schroyen and Torsvik (2005) show that workfare crowds out the poor's market income. Crowding out necessitates larger transfers to the poor in order to guarantee them an income above the poverty line. This runs counter to the authors' aim to minimize fiscal costs of a poverty alleviation program. Similar results are derived by Oliveira and Côrte-Real (2006). In Leblanc (2004) the government, which again minimizes budgetary costs of a poverty alleviation program, is given the ability to include

private labor supply as part of its policy. Such a framing avoids crowding out by assumption. However, if the government is not assumed to be able to set the labor supply, then individuals are free to choose their utility maximizing working time and workfare can crowd out this private supply. Therefore, workfare entails an additional cost as a policy option.

Our results are as follows. The workfare productivity emerges to be the crucial determinant. We find two individual thresholds of workfare productivity for the least productive type being employed. For any work obligation that exhibits a productivity beyond the first threshold the respective type should be on workfare. Moreover, if workfare productivity exceeds a second threshold then the type should not be regularly employed anymore. It turns out that these two thresholds are in fact the same. The crowding out occurs entirely in the sense that workfare and private-sector employment are mutually exclusive. Crowding out takes place gradually as it first affects people with low productivity. However, for each productivity type a crowding out threshold can be specified. The crowding out is second-best since overall welfare increases. This is due to the fact that workfare provides an additional instrument besides the distortion of the individual labor-leisure decision. Workfare enhances redistribution and lowers the extent of distortionary taxation.

The remainder of the paper is organized as follows. Section 2 presents the model and section 3 specifies the problems that are addressed within the model. The analysis in section 4 leads to the main results. Some illustrative examples are provided in section 5, and section 6 summarizes the findings and concludes.

## 2 The Model

A finite variant of the standard optimal non-linear taxation model (Homburg, 2001) - the "standard model" - is considered. There are several types  $h = 1, 2, \dots, H$  ( $H > 1$ ), whose exogenous productivities and fractions are denoted as  $w^h$  and  $f^h$ , respectively. Productivities correspond to wage rates in the competitive labor market, with  $0 < w^1 < \dots < w^H$ . To keep the model as general as possible, it is assumed that  $v^h$  hours of observable workfare can be imposed on each individual, even if mandatory work seems to be unsuitable for high income classes. The generated output of workfare per hour is  $\pi \geq 0$ . This workfare productivity is assumed to be lower than market productivity, i.e.,  $\pi < w^1$  (Lohse, 2007). Due to their uniform dimension 'time' an additive connection of  $l$  and  $v$  is assumed. A person consuming  $c^h$ , and earning gross labor income  $y^h$  enjoys utility  $u(c^h, l^h + v^h)$ , where  $l^h = y^h/w^h$ . An individual  $h+1$  who imitates  $h$  has utility  $u(c^h, y^h/w^{h+1} + v^h) =: \hat{u}^{h+1}$ . The utility function  $u$  satisfies the usual properties (strictly monotonically increasing in  $c$ , strictly monotonically decreasing in  $l$  and  $v$ ; and  $Hess\ u$  is negative definite). For convenience, partial derivatives are denoted with subscripts that refer to the respective arguments, i.e.  $u_l = u_v =: u_2$  and  $u_c =: u_1$ . Moreover, its cross derivative vanishes and  $u_1 \rightarrow \infty$  for  $c \rightarrow 0$  at  $l$  and  $v$  constant.

The marginal rate of substitution of a person with productivity  $w^h$  is defined as  $mrs^h := -u_2(c^h, l^h + v^h)/u_1(c^h, l^h + v^h)$ . We assume  $mrs^h \rightarrow \infty$  for  $l^h + v^h \rightarrow l^{\max}$  with  $l^{\max}$  being an upper time limit.

The social planner's problem reads:

$$\begin{aligned}
\max_{(c^h, l^h, v^h)_{h=1 \dots H}} EU &= \sum_{h=1}^H u(c^h, l^h + v^h) f^h \\
\text{s.t. } (BC) \quad g &\leq \sum_{h=1}^H (y^h - c^h + \pi v^h) f^h \\
(IC) \quad u(c^k, l^k + v^k) &\geq u\left(c^h, \frac{y^h}{w^k} + v^h\right) \quad \forall h, k \text{ and } \frac{y^h}{w^k} \leq l^{\max}.
\end{aligned} \tag{1}$$

The government maximizes expected utility of a person choosing a tax-transfer scheme from behind a veil of ignorance subject to a budget constraint (*BC*) with an exogenous revenue requirement  $g \geq 0$  and subject to incentive compatibility constraints (*IC*).

Optimal taxes and transfers follow as  $T^h = y^h - c^h$ . The discrete marginal tax rates  $m^h$  are defined for any  $y^h \neq y^{h-1}$  as

$$m^h = \frac{T^h - T^{h-1}}{y^h - y^{h-1}}. \tag{2}$$

The discrete marginal tax rate is the relevant marginal rate for the purpose of redistribution. To stick with the usual notation, we will denote the local (or implicit) marginal tax rates by  $T'^h$ . They are given by

$$T'^h = 1 - \frac{mrs^h}{w^h}. \tag{3}$$

These are the marginal tax rates crucial for incentive considerations.

### 3 Specification of crowding out

Obviously, welfare is increasing in workfare productivity  $\pi$  since a higher output per hour in work requirement only has a positive output effect but neither a

utility nor an incentive cost. However, the restriction of  $\pi < w^1$  puts an upper limit on the workfare productivity.

First, consider a setting  $\pi = 0$ . The resulting optimum is characterized by  $v^h = 0$  for all  $h$  (Homburg, 2003a). In any such second-best allocation, type  $\underline{k}$  denotes the least productive individual being regularly employed whereas his left-hand neighbor is the most productive individual being unemployed.<sup>1</sup> The figure 1 illustrates this separation of types with respect to their employment status.

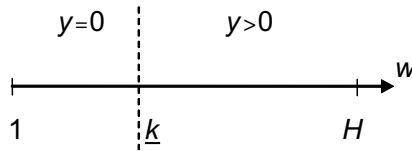


Figure 1: Separation of types

Work requirements that exhibit a certain productivity  $\pi > 0$  can be optimal for the unemployed, i.e. for types smaller than  $\underline{k}$  (Lohse, 2007). The questions addressed here are the following: What happens if  $\underline{k}$  is obliged to carry out some workfare  $v^{\underline{k}} > 0$  in addition to his regular working time  $l^{\underline{k}} > 0$ ? Is there any feasible workfare productivity  $\pi$  with  $0 < \pi < w^1$  such that this work obligation for  $\underline{k}$  is welfare increasing? If yes, which conclusions have to be drawn with regard to his regular employment? And, finally, can the results be generalized to  $k + 1, k + 2, \dots$ , i.e. to more productive types?

<sup>1</sup>In the absence of unemployment, it is  $\underline{k} = 1$ .

## 4 The role of the workfare productivity

A second-best optimum possesses the following two features:

$$y^h \geq y^{h-1} \text{ and } c^h \geq c^{h-1} \text{ and } u(c^h, l^h + v^h) = u(c^{h-1}, y^{h-1}/w^h + v^h) \forall h. \quad (4)$$

The two inequalities state that income and consumption increase weakly in productivity (monotonicity property). The right-hand equality states that all downward adjacent self-selection constraints are binding at an optimum. Together with the monotonicity, this so-called chain property implies that all remaining self-selection constraints are automatically satisfied and can thus be neglected. The government's budget constraint also holds with equality. Since the gradients of the constraints are linearly independent, a Lagrangian approach can be formulated with non-negative multipliers  $\lambda$  and  $\mu^h$  for the *BC* and the downward adjacent incentive compatibility constraints (*DIC*), respectively (Bertsekas, 1999; Homburg, 2003b).

$$\begin{aligned} \mathcal{L} = & \sum_{h=1}^H u(c^h, l^h + v^h) f^h + \lambda \left( \sum_{h=1}^H (y^h - c^h + v^h \pi) f^h - g \right) \\ & + \sum_{h=1}^{H-1} \mu^{h+1} \left( u(c^{h+1}, l^{h+1} + v^{h+1}) - u(c^h, \frac{y^h}{w^{h+1}} + v^h) \right) \end{aligned} \quad (5)$$

Differentiating (5) with respect to  $v^k$  reveals the different effects that a work obligation entails:

$$u_2^k f^k + \mu^k u_2^k - \mu^{k+1} \widehat{u}_2^{k+1} + \lambda \pi f^k. \quad (6)$$

The first term is a utility effect which is clearly negative since workfare causes

disutility. The two following terms denote an incentive effect, where  $v^{\underline{k}}$  tightens the *DIC* with respect to  $\underline{k} - 1$ , but relaxes the *DIC* of  $\pi_v^{\underline{k}} + 1$  with respect to  $\underline{k}$ . The last term specifies the positive output effect since workfare is productive. In the further analysis the partial derivatives of (5) will become useful. The one with respect to  $c^h$  can be rearranged to

$$\mu^{h+1} = \mu^h + \frac{u_1^h - \lambda}{u_1^h} f^h, \quad (7)$$

where  $\lambda$  turns out to be the harmonic mean of the marginal utility of consumption, and the one with respect to  $l^h$  reads

$$\lambda w^h f^h = \mu^{h+1} \widehat{u}_2^{h+1} \frac{w^h}{w^{h+1}} - (\mu^h + f^h) u_2^h. \quad (8)$$

The question whether the implementation of a work obligation for person  $\underline{k}$  is welfare increasing, is answered by the following Proposition.

**Proposition 1** *For any  $\pi$  being smaller than  $w^1$ , but larger than  $\pi_v^{\underline{k}}$  with*

$$\pi_v^{\underline{k}} = \frac{-(\mu^{\underline{k}} + f^{\underline{k}})(u_2^{\underline{k}} - \widehat{u}_2^{\underline{k}+1}) \sum_{h=1}^H \frac{f^h}{u_1^h}}{f^{\underline{k}}} + \widehat{mrs}^{\underline{k}+1}, \quad (9)$$

*the implementation of a work obligation for the individual  $\underline{k}$  in addition to its regular employment  $l^{\underline{k}}$  is welfare increasing.*

**Proof.** See appendix. ■

Rearranging equation (9) to

$$f^{\underline{k}}(u_2^{\underline{k}} - \widehat{u}_2^{\underline{k}+1}) + \lambda f^{\underline{k}}(\pi_v^{\underline{k}} - \widehat{mrs}^{\underline{k}}) + \mu^{\underline{k}}(u_2^{\underline{k}} - \widehat{u}_2^{\underline{k}+1}) = 0 \quad (10)$$



simplifies the interpretation of the threshold  $\pi^{\underline{k}}$ . This threshold is characterized by the fact that the utility effect at size  $f^{\underline{k}}$  plus the output effect at size  $f^{\underline{k}}$  and measured by  $\lambda$  plus the incentive effect measured by  $\mu^{\underline{k}}$  add up to zero. For any  $\pi > \pi^{\underline{k}}$  the sum is strictly positive according to Proposition 1.

Now, facing an optimal implementation of workfare for type  $\underline{k}$  if  $\pi > \pi^{\underline{k}}$ , the question arises whether his level of employment  $l^{\underline{k}} > 0$  is in turn still optimal. To put it differently, what are the consequences of optimal workfare on regular employment? This question is addressed in the subsequent Proposition.

**Proposition 2** *For any  $\pi$  being smaller than  $w^1$ , but larger than  $\pi^{\underline{k}}$  with*

$$\pi_l^{\underline{k}} = \left( \frac{\hat{u}_2^{\underline{k}+1}}{u_1^{\underline{k}}} + w^{\underline{k}+1} \right) \frac{\hat{u}_2^{\underline{k}+1} - u_2^{\underline{k}}}{\hat{u}_2^{\underline{k}+1} - u_2^{\underline{k}}(w^{\underline{k}+1}/w^{\underline{k}})} + \widehat{mrs}^{\underline{k}+1} \quad (11)$$

*the abolition of regular employment for the individual  $\underline{k}$  is welfare increasing.*

**Proof.** See appendix. ■

Rearranging equation (11) yields

$$\frac{\pi_l^{\underline{k}} - \widehat{mrs}^{\underline{k}+1}}{\hat{u}_2^{\underline{k}+1} - u_2^{\underline{k}}} = \frac{w^{\underline{k}+1} - \widehat{mrs}^{\underline{k}+1}}{\hat{u}_2^{\underline{k}+1} - u_2^{\underline{k}}(w^{\underline{k}+1}/w^{\underline{k}})}. \quad (12)$$

The workfare productivity threshold  $\pi_l^{\underline{k}}$  beyond which regular employment should optimally be decreased, is characterized by (12): The left-hand side displays the difference between the workfare productivity and the marginal rate of substitution of type  $\underline{k} + 1$  imitating  $\underline{k}$  which is measured by the difference in marginal disutility of work of  $\underline{k} + 1$  imitating  $\underline{k}$  and type  $\underline{k}$  himself. This fraction equals the difference between  $\underline{k} + 1$ 's wage rate and the marginal rate of substitution of type  $\underline{k} + 1$  imitating  $\underline{k}$  which is measured by the difference in marginal disutility of work of  $\underline{k} + 1$  imitating  $\underline{k}$  and type  $\underline{k}$  himself where the minuend is multiplied

by the fraction of the two wage rates as the inner derivative with respect to  $l^k$ . If  $\pi$  exceeds the threshold  $\pi_l^k$ , then in (12) the left-hand side will be larger than the right-hand side implying that  $l^k$ 's regular employment should be decreased. To sum up, Proposition 1 states that for any  $\pi$  larger than  $\pi_v^k$  the least productive regularly employed type  $k$  should be additionally on workfare. Moreover, for any  $\pi$  exceeding  $\pi_l^k$  his regular employment should optimally be decreased. The remaining question is what the relationship is between the two thresholds of implementation of workfare  $\pi_v^k$  and abolition of regular employment  $\pi_l^k$ ?

**Proposition 3** *For any  $\pi$  being smaller than  $w^1$ , but larger than  $\pi^k$  with  $\pi^k = \pi_v^k \equiv \pi_l^k$  the implementation of workfare and simultaneously the reduction of regular employment is welfare increasing.*

**Proof.** See appendix. ■

Since in a neighborhood of  $\pi^k$ , the change in welfare due to workfare denoted as  $dM(v^k)/dv^k$  is strictly monotonically increasing in  $\pi$  (see equation (16) in the appendix), whereas the change in welfare due to a change in regular working time denoted as  $dM(\alpha^k)^v/d\alpha^k$  is strictly monotonically decreasing in  $\pi$  (see equation (24) in the appendix), we can establish the following result:

**Proposition 4** *If  $\pi^k < \pi$ , then an increase in the workfare productivity  $\pi$  leads to an optimal and entire crowding out of private-sector employment through workfare for  $k$  such that  $k$  is unemployed but on workfare.*

Figure 2 illustrates Proposition 4. On the horizontal axis, the workfare productivity  $\pi$  is denoted, and on the vertical axis the change in the welfare through a change in working time and workfare, respectively, is given.

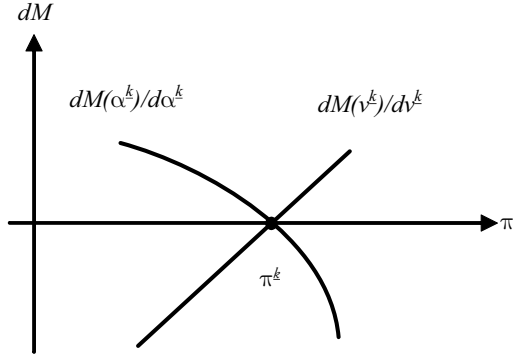


Figure 2: Crowding out for type  $\underline{k}$

In case of a rather low workfare productivity with  $\pi < \pi^{\underline{k}}$ , the implementation of a work obligation for person  $\underline{k}$  would be welfare decreasing; the corresponding straight line  $dM(v^{\underline{k}})/dv^{\underline{k}}$  runs in the negative quadrant. By contrast, if  $\pi > \pi^{\underline{k}}$ , then person  $\underline{k}$  should be on workfare since the straight line  $dM(v^{\underline{k}})/dv^{\underline{k}}$  runs in the positive quadrant. At the same time,  $dM(\alpha^{\underline{k}})^v/d\alpha^{\underline{k}}$  does not run in the positive, but in the negative quadrant. Consequently, the private-sector working time of the individual  $\underline{k}$  should optimally be reduced. As both graphs are strictly monotonic, an entire crowding out of the private-sector work occurs. Although the analysis has focused on the individual  $\underline{k}$  being the least productive type being employed, the calculation of a crowding out threshold according to Proposition 4 can be generalized. It can be carried out for each type  $h < H$  since workfare can gradually crowd out one type by another. Only the most productive one,  $H$ , is never subject to workfare. This becomes clear when comparing the model enlarged by workfare with the standard model without workfare. In the standard model, the only channel of preventing the more

productive from claiming benefits payments is distortionary taxation. In the context examined here, workfare becomes a second instrument to reach this goal. The extension to which workfare is used, crucially depends on the workfare productivity. The more productive it is, the more it should be used. Thus, from an optimal tax perspective, the displacement of private-sector work through workfare is second-best since it pays to accept an enlargement of the public sector at the cost of the private one. Of course, it should be kept in mind that the initial requirement of  $\pi < w^1$  still has to hold.

The analysis reveals a further insight that is socially important.

**Corollary 1** *Private-sector employment and workfare are mutually exclusive.*

Already a small income from private-sector employment is sufficient for not being subject to workfare. Whether the employed person faces a positive tax or a negative one, i.e., additionally receives some transfer, is not decisive for the validity of the Corollary. This Corollary is also interesting from a technical point of view. It implies that the introduction of workfare as a third dimension in the Mirrlees model does not create problems of multidimensional screening that are rather hard to deal with (e.g. Matthews and Moore, 1987; Basov, 2005).

## 5 Illustrations

To illustrate the findings, some simulations of second-best tax-transfer schemes with workfare are provided. They clarify the welfare effects of workfare and show how the different assumptions about the workfare productivity affect second-best work obligations.

First, consider an economy with four different productivity types. Assume a utility function  $u(c^h, l^h) = 1,000[\ln(c^h) + \ln(500 - l^h) - 12]$  with 500 being the maximum time per month. The per-capita revenue is  $g = 1000$ , i.e., the tax system also serves to finance the provision of a public good. Table 1 depicts the standard second-best optimum.

Table 1: Standard optimum

<b>h</b>	<b>w</b>	<b>f</b>	<b>c</b>	<b>y</b>	<i>mrs</i>	$\pi^N$	<b>T</b>	<b>m</b>	<b>T'</b>	<b>u</b>
1	2	10%	548	0	1.1	1.1	-548	-	45%	521
2	4	10%	644	298	1.51	1.88	-346	68%	62%	521
3	8	55%	1362	2250	6.22	8.58	888	63%	23%	604
4	16	25%	2798	5202	16.00	-	2404	51%	0%	1101

$$\mathbf{EU} = \mathbf{712}$$

The individual with wage rate  $w^1 = 2$  is unemployed ( $y^1 = 0$ ) whereas its right-hand neighbor with  $w^2 = 4$  is the least productive type being employed ( $y^2 = 298$ ). The seventh column displays the workfare productivity threshold  $\pi^k$  for all people that are potentially subject to workfare. The threshold for type 3 with wage rate  $w^3 = 8$  is beyond the upper limit of  $\pi$ , i.e.  $\pi^3 = 8.58 > 2 = w^1$ . However, individuals with wage rate  $w^2 = 4$  have a threshold of just  $\pi^2 = 1.8$ . Therefore, if work obligations are implemented that exhibit a productivity between 1.8 and 2, then person 2 will optimally quit her private-sector employment and be on workfare. This would lead to a welfare increase. Table 2 displays a second-best optimum of the same economy in which workfare

has been introduced at a uniform productivity of  $\pi = 1.9$ . The sixth column gives the individual work requirement.<sup>2</sup>

Table 2: Optimum with workfare at  $\pi = 1.9$

<b>h</b>	<b>w</b>	<b>f</b>	<b>c</b>	<b>y</b>	<b>v</b>	<i>mrs</i>	$\pi^N$	<b>T</b>	<b>m</b>	<b>T'</b>	<b>u</b>
1	2	10%	747	0	107	1.9	—	−747	—	5%	589
2	4	10%	747	0	107	1.9	—	−747	—	53%	589
3	8	55%	1354	2266	0	6.24	8.49	912	73%	22%	589
4	16	25%	2788	5216	0	16	—	2428	51%	0%	1092

$$\mathbf{EU} = \mathbf{715}$$

A comparison of table 2 with table 1 reveals that the use of workfare is optimal not only for people with  $w^1 = 2$ , but also with  $w^2 = 4$ . For the latter, an entire crowding out of private-sector work has occurred. Hence, instead of using distortionary taxes for them, workfare has become the welfare maximizing instrument to avoid mimicking. It is interesting to notice that people on workfare are now better off whereas the upper part of the society is worse off. This reveals the two channels through which workfare increases welfare. Firstly, it reduces the extent of distortionary taxation and secondly it facilitates a higher degree of redistribution (Lohse, 2007).

<sup>2</sup>For an intensive analysis of a second-best optimum with workfare see Lohse (2007).

## 6 Conclusion

This paper has dealt with productive work requirements as they have become standard in several Western welfare states. The crucial issue was the crowding out of private-sector work through workfare. It is a widely held belief that such a crowding out serves as a strong argument against the use of workfare. The present paper confutes this opinion from an optimal tax perspective. Productive workfare can be second-best. The crowding out is a necessary consequence to reach the welfare optimum when workfare exceeds a specific threshold, where the threshold still lies below the individual wage rate. In such a situation it is economically cheaper to use work obligations (for the unemployed) instead of distortionary taxes (for the employed) as a means to prevent the rich from mimicking the poor.

In evaluating the model, two caveats come to mind. Firstly, a competitive labor market was assumed throughout the analysis. This assumption is known from the standard model and tends to weaken the policy implications. Secondly, the present model does not take account of moonlighting. With this extension, workfare might be optimal even when being unproductive.

## 7 Appendix

### 7.1 Proposition 1

**Proof.** Starting from an optimum with  $\pi = 0$  and  $v^h = 0$  for all  $h$  a work obligation  $v^{\underline{k}}$  with  $l^{\underline{k}} + v^{\underline{k}} \leq l^{\max}$  is introduced for type  $\underline{k} > 0$ . Denoting  $\bar{c}$  and  $\bar{l}$  as the vector of consumption and work, respectively, and  $v^{\underline{k}}$  as the introduced

parameter of workfare which will be kept constant while maximizing, the social planner's maximization problem reads in short notation: Maximize the objective function  $EU(\bar{c}, \bar{l}, v^k)$  subject to  $BC(\bar{c}, \bar{l}, v^k) = g$ , and to  $DIC(\bar{c}, \bar{l}, v^k) = 0$  for  $h = 1, \dots, H-1$ . Let  $\lambda$  and  $\mu^h$  with  $h = 1, \dots, H$ , with  $h = 0, \dots, H$  be the respective Lagrangian multipliers. And let  $(c^{0*} \dots c^{H*}, l^{0*} \dots l^{H*}, \lambda^*, \mu^{2*} \dots \mu^{H*}, v^{k*})$  be the solution of the optimization problem. The value function is

$$M(v^{k*}) = EU(c^{0*} \dots c^{H*}, l^{0*} \dots l^{H*}, \lambda^*, \mu^{2*} \dots \mu^{H*}, v^{k*}), \quad (13)$$

where  $M(0)$  denotes the maximum value of the standard optimum. Let

$$\begin{aligned} \mathcal{L}^v &= \sum_{h=1}^{k-1} u(c^h, l^h) f^h + u(c^k, l^k + v^k) f^k + \sum_{h=k+1}^H u(c^h, l^h) f^h \\ &+ \lambda \left( \sum_{h=1}^H (y^h - c^h) f^h + \pi v^k f^k - g \right) \\ &+ \sum_{h=1}^{k-2} \mu^{h+1} \left( u(c^{h+1}, l^{h+1}) - u(c^h, \frac{y^h}{w^{h+1}}) \right) \\ &+ \mu^k \left( u(c^k, l^k + v^k) - u(c^{k-1}, \frac{y^{k-1}}{w^k}) \right) \\ &+ \mu^{k+1} \left( u(c^{k+1}, l^{k+1}) - u(c^k, \frac{y^k}{w^{k+1}} + v^k) \right) \\ &+ \sum_{h=k+1}^{H-1} \mu^{h+1} \left( u(c^{h+1}, l^{h+1}) - u(c^h, \frac{y^h}{w^{h+1}}) \right) \end{aligned} \quad (14)$$

be the associated Lagrangian. The change in the maximum value through the implementation of  $v^k$  is

$$\frac{dM(v^k)}{dv^k} = \left. \frac{\partial \mathcal{L}^v}{\partial v^k} \right|_{\text{standard optimum}} = u_2^k(f^k + \mu^k) - \hat{u}_2^{k+1} \mu^{k+1} + \lambda \pi f^k, \quad (15)$$

where  $u_2^k := \partial u(c^k, l^k + v^k) / \partial v^k$  and  $\hat{u}_2^{k+1} := \partial u(c^k, y^k / w^{k+1} + v^k) / \partial v^k$ .<sup>3</sup> The sign of  $dM(v^k) / dv^k$  is crucial. A positive sign would indicate the introduction

<sup>3</sup>Since a person  $H + 1$  does not exist, it holds  $\mu^0 = \mu^{H+1} = 0$ .



of workfare as being welfare increasing. Rearranging terms and substituting in from equation (7) yields

$$\frac{dM(v^{\underline{k}})}{dv^{\underline{k}}} = (\mu^{\underline{k}} + f^{\underline{k}})(u_2^{\underline{k}} - \widehat{u}_2^{k+1}) + \frac{f^{\underline{k}}}{\sum_{h=1}^H \frac{f^h}{u_1^h}} (\pi - \widehat{mrs}^{k+1}), \quad (16)$$

where  $\widehat{mrs}^{k+1} := -\widehat{u}_2^{k+1}/u_1^k$  is the marginal rate of substitution of the imitator. The first term in brackets in (16) is strictly positive, and the second term is strictly negative since the imitator has a smaller disutility from work. The first term in the second summand is the harmonic mean of the marginal utility of consumption times the population size of type  $\underline{k}$ , and is hence strictly positive. Consequently, the sign of  $dM(v^{\underline{k}})/dv^{\underline{k}}$  is determined by the difference between the weakly positive workfare productivity and the strictly positive marginal rate of substitution of the imitator as given by the last term in brackets. What remains is the question if a productive work obligation might also be welfare increasing if person  $\underline{k}$  is privately employed ( $l^{\underline{k}} > 0$ ). Obviously,  $dM(v^{\underline{k}})/dv^{\underline{k}}$  is linear and strictly monotonically increasing in the workfare productivity  $\pi$ . Moreover,  $dM(v^{\underline{k}})/dv^{\underline{k}}$  has a zero at

$$\pi_v^{\underline{k}} = \frac{-(\mu^{\underline{k}} + f^{\underline{k}})(u_2^{\underline{k}} - \widehat{u}_2^{k+1}) \sum_{h=1}^H \frac{f^h}{u_1^h}}{f^{\underline{k}}} + \widehat{mrs}^{k+1}, \quad (17)$$

where the subscript  $v$  refers to the maximum variation due to workfare and the superscript  $\underline{k}$  indicates the workfare productivity at which the variation of the maximum value for type  $k$  is zero. In case of a workfare productivity  $\pi$  with  $\pi < \pi_v^{\underline{k}}$ , any work obligation for person  $\underline{k}$  would be welfare decreasing. By contrast, if workfare is sufficiently productive, i.e. if  $\pi > \pi_v^{\underline{k}}$  obtains, then even the employed person  $\underline{k}$  should optimally be on workfare. ■

## 7.2 Proposition 2

**Proof.** Let  $\alpha^k$  denote the parameter for the variation of type  $k$ 's working time  $l^k$ . In an optimum with  $\pi = 0$  and  $v^h = 0$  for all  $h$ , it is  $\alpha^k = 1$ . The objective function  $EU(\bar{c}, \bar{l}, \alpha^k)$  has to be maximized subject to  $BC(\bar{c}, \bar{l}, \alpha^k) = g$ , to  $DIC(\bar{c}, \bar{l}, \alpha^k) = 0$  for  $h = 1, \dots, H-1$ . Let  $\lambda$  and  $\mu^h$  with  $h = 1 \dots H$  be the respective Lagrangian multipliers. And let  $(c^{0*} \dots c^{H*}, l^{0*} \dots l^{H*}, \lambda^*, \mu^{2*} \dots \mu^{H*}, a^{k*})$  be the solution of the optimization problem. The value function reads

$$M(\alpha^{k*}) = EU(c^{0*} \dots c^{H*}, l^{0*} \dots l^{H*}, \lambda^*, \mu^{2*} \dots \mu^{H*}, a^{k*}). \quad (18)$$

Then the associated Lagrangian is:

$$\begin{aligned} \mathcal{L}^\alpha &= \sum_{h=1}^{k-1} u(c^h, l^h) f^h + u(c^k, \alpha^k l^k + v^k) f^k + \sum_{h=k+1}^H u(c^h, l^h) f^h \\ &+ \lambda \left( \sum_{h=1}^{k-1} (y^h - c^h) f^h + (\alpha^k w^k l^k - c^k) f^k + \sum_{h=k+1}^H (y^h - c^h) f^h - g \right) \\ &+ \sum_{h=1}^{k-2} \mu^{h+1} \left( u(c^{h+1}, l^{h+1}) - u(c^h, \frac{y^h}{w^{h+1}}) \right) \\ &+ \mu^k \left( u(c^k, \alpha^k l^k) - u(c^{k-1}, \frac{y^{k-1}}{w^k}) \right) + \mu^{k+1} \left( u(c^{k+1}, l^{k+1}) - u(c^k, \frac{\alpha^k w^k l^k}{w^{k+1}}) \right) \\ &+ \sum_{h=k+1}^{H-1} \mu^{h+1} \left( u(c^{h+1}, l^{h+1}) - u(c^h, \frac{y^h}{w^{h+1}}) \right). \end{aligned} \quad (19)$$

The variation of the maximum value by a change in  $\alpha^k$  is given by

$$\frac{dM(\alpha^k)}{d\alpha^k} = \left. \frac{\partial \mathcal{L}^\alpha}{\partial \alpha^k} \right|_{\substack{\text{standard} \\ \text{optimum}}} = u_2^k l^k f^k + \lambda w^k l^k f^k + \mu^k u_2^k l^k - \mu^{k+1} \widehat{u}_2^{k+1} \frac{w^k l^k}{w^{k+1}}. \quad (20)$$

Rearranging terms and substituting in equation (7) yields

$$\frac{dM(\alpha^k)}{d\alpha^k} = l^k (\mu^k + f^k) \left( u_2^k - \widehat{u}_2^{k+1} \frac{w^k}{w^{k+1}} \right) + \frac{w^k l^k f^k}{w^{k+1}} \frac{1}{\sum_{h=1}^H \frac{f^h}{u_1^h}} (w^{k+1} - \widehat{mrs}^{k+1}). \quad (21)$$

The first summand is strictly negative because the disutility from labor is higher for type  $\underline{k}$ . However, the second summand is strictly positive for any person  $\underline{k}+1 < H$  since only the highest type  $H$  will be taxed in an undistortionary way (Sadka, 1976; Seade, 1977).<sup>5</sup> To determine the sign of  $dM(\alpha^{\underline{k}})/d\alpha^{\underline{k}}$  equation (17) is rearranged to

$$\frac{1}{\sum_{h=1}^H \frac{f^h}{u_1^h}} = \frac{-(\mu^{\underline{k}} + f^{\underline{k}})(u_2^{\underline{k}} - \widehat{u}_2^{\underline{k}+1})}{(\pi - \widehat{mrs}^{\underline{k}+1})f^{\underline{k}}} \quad (22)$$

and inserted in equation (21). Denoting  $dM(\alpha^{\underline{k}})^v/d\alpha^{\underline{k}} := dM(\alpha^{\underline{k}})/d\alpha^{\underline{k}}|_{\pi^{\underline{k}}}$ , and rearranging terms yields

$$\begin{aligned} \frac{dM(\alpha^{\underline{k}})^v}{d\alpha^{\underline{k}}} &= \frac{w^{\underline{k}}l^{\underline{k}}}{w^{\underline{k}+1}}(\mu^{\underline{k}} + f^{\underline{k}}) \left[ u_2^{\underline{k}} \left( \frac{w^{\underline{k}+1}}{w^{\underline{k}}} - \frac{w^{\underline{k}+1} - \widehat{mrs}^{\underline{k}+1}}{\pi - \widehat{mrs}^{\underline{k}+1}} \right) \right. \\ &\quad \left. - \widehat{u}_2^{\underline{k}+1} \left( 1 - \frac{w - \widehat{mrs}^{\underline{k}+1}}{\pi - \widehat{mrs}^{\underline{k}+1}} \right) \right]. \end{aligned} \quad (23)$$

This maximum value is strictly monotonically decreasing in the workfare productivity since

$$\frac{d(dM(\alpha^{\underline{k}})^v/d\alpha^{\underline{k}})}{d\pi} = \frac{w^{\underline{k}+1} - \widehat{mrs}^{\underline{k}+1}}{(\pi - \widehat{mrs}^{\underline{k}+1})^2} \frac{w^{\underline{k}}l^{\underline{k}}}{w^{\underline{k}+1}}(\mu^{\underline{k}} + f^{\underline{k}})(u_2^{\underline{k}} - \widehat{u}_2^{\underline{k}+1}) < 0 \quad (24)$$

and is strictly concave:

$$\frac{d^2(dM(\alpha^{\underline{k}})^v/d\alpha^{\underline{k}})}{d\pi^2} = -\frac{w^{\underline{k}+1} - \widehat{mrs}^{\underline{k}+1}}{(\pi - \widehat{mrs}^{\underline{k}+1})^3} \frac{w^{\underline{k}}l^{\underline{k}}}{w^{\underline{k}+1}}(\mu^{\underline{k}} + f^{\underline{k}})(u_2^{\underline{k}} - \widehat{u}_2^{\underline{k}+1}) > 0. \quad (25)$$

Finally,  $dM(\alpha^{\underline{k}})^v/d\alpha^{\underline{k}}$  has a single zero at

$$\pi_l^{\underline{k}} = \left( \frac{\widehat{u}_2^{\underline{k}+1}}{u_1^{\underline{k}}} + w^{\underline{k}+1} \right) \frac{\widehat{u}_2^{\underline{k}+1} - u_2^{\underline{k}}}{\widehat{u}_2^{\underline{k}+1} - u_2^{\underline{k}}(w^{\underline{k}+1}/w^{\underline{k}})} + \widehat{mrs}^{\underline{k}+1} \quad (26)$$

which is the claimed threshold. ■

<sup>5</sup>Since the labor-leisure choice of all types except  $H$  is distorted, i.e.  $T^h > 0$  for all  $h < H$ , it obtains  $1 - \frac{mrs^h}{w^h} > 0 \iff w^h > mrs^h$ . Thus, for person  $k$  holds  $w^{k+1} > w^k > mrs^k > \widehat{mrs}^{k+1}$ .

### 7.3 Proposition 3

**Proof.** Using (7) equation (8) can be rearranged:

$$\lambda w^k f^k = \left( \mu^k + \frac{u_1^k - \lambda}{u_1^k} \right) \widehat{u}_2^{k+1} \frac{w^k}{w^{k+1}} - (\mu^k + f^k) u_2^k \quad (27)$$

$$= (\mu^k + f^k) \widehat{u}_2^{k+1} \frac{w^k}{w^{k+1}} - \frac{\lambda}{u_1^k} f^k \widehat{u}_2^{k+1} \frac{w^k}{w^{k+1}} - (\mu^k + f^k) u_2^k \quad (28)$$

$$= (\mu^k + f^k) \left( \frac{\widehat{u}_2^{k+1}}{w^{k+1}} - \frac{u_2^k}{w^k} \right) w^k + \lambda f^k \widehat{mrs}^{k+1} \frac{w^k}{w^{k+1}} \quad (29)$$

which finally yields

$$w^{k+1} - \widehat{mrs}^{k+1} = \left( \widehat{u}_2^{k+1} - u_2^k \frac{w^{k+1}}{w^k} \right) \frac{(\mu^k + f^k)}{\lambda f^k}. \quad (30)$$

Substituting (30) into (11) yields

$$\pi_l^k = \left( \widehat{u}_2^{k+1} - u_2^k \frac{w^{k+1}}{w^k} \right) \frac{(\mu^k + f^k)}{\lambda f^k} \frac{\widehat{u}_2^{k+1} - u_2^k}{\widehat{u}_2^{k+1} - u_2^k (w^{k+1}/w^k)} + \widehat{mrs}^{k+1}. \quad (31)$$

Rearranging and substituting in for  $\lambda$  the harmonic mean of the marginal utility of consumption gives

$$\pi_l^k = \frac{-(\mu^k + f^k)(u_2^k - \widehat{u}_2^{k+1}) \sum_{h=1}^H \frac{f^h}{u_1^h}}{f^k} + \widehat{mrs}^{k+1} \quad (32)$$

which establishes the Proposition. ■

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