Bandwagon, underdog, and political competition:

The uni-dimensional case

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Abstract

The present paper studies the Hotelling-Downs and the Wittman-Roemer models of two-party competition when voter conformism is present and the policy space is uni-dimensional. We consider two types of voter conformism, bandwagon and underdog, and study their effects on political equilibrium of the two models. Even if voter conformism is present, the Hotelling-Downs parties propose an identical policy at the equilibrium, which is equal to the strict Condorcet winner. Thus voter conformism, both bandwagon and underdog, has no effect on the Hotelling-Downs political equilibrium. In the Wittman-Roemer model, the two parties propose differentiated equilibrium policies, and the extent of such policy differentiation depends on the degree of voter conformism. In general, the stronger the bandwagon effect is, the more differentiated the equilibrium policies are. The opposite holds when the underdog effect is present; an increasing underdog effect mitigates the policy differentiation of the two parties, although the effect is not large. We also find multiple Wittman-Roemer equilibria when the bandwagon effect is sufficiently strong.

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1. Introduction

In almost all democracies, pre-election opinion polls have become an integral part of national elections. Public opinion polls provide important information to the public about the views of their fellow citizens. By providing information about voting intentions, opinion polls may sometimes influence the behavior of voters and even election outcomes. Various theories about how this happens can be split into three groups: 'voter conformism,' 'strategic voting,' and 'participation/abstention.'

A well known example of voter conformism is a *bandwagon effect*. A bandwagon effect occurs when the poll prompts voters to back the candidate shown to be winning in the poll, thus increasing his/her chances of being on the 'winner's side' in the end. The idea that voters are susceptible to such an effect is old, and has remained persistent in spite of much debate on its empirical existence. Bartels (1985, 1988), for instance, shows that voters are motivated in part by a desire to vote for the winning candidate. The opposite of the bandwagon effect is an *underdog effect*; this occurs when people vote, out of sympathy, for the candidate perceived to be 'losing' the elections. In a meta-study of research on this topic, Irwin and van Holsteyn (2002) show that from the 1980s onward a bandwagon effect is found more often by empirical researchers, while it finds less empirical evidence for the existence of an underdog effect than that for the existence of a bandwagon effect. [1]

[1] There have been at least two explanations for the existence of voter conformism. The first consists in assuming that polls may exert a normative influence over voters; when voters perceive the existence of a *social norm* – defined by a majority preference expressed in polls in the case of a bandwagon effect – they may feel compelled to abandon their views and comply with such norms, to avoid perhaps cognitive dissonance. The second, which seems more compelling, consists in assuming that individuals may be influenced by polls because they use revealed public

In the era of computer-processed predictions which have attained a remarkable degree of accuracy in forecasting the outcomes of numerous elections, the possibility of bandwagon or underdog effects has aroused some concerns among people, most prominently in the US, where poll results are broadcast in the eastern parts while polls are still open in the west.[2]

The second category of theories on the effect of polls on voting concerns strategic voting. These theories are based on the idea that voters will sometimes not choose the candidate they prefer the most, but another, less-preferred, candidate from strategic considerations. An example can be found in the UK general election of 1997. Then Cabinet Minister, Michael Portillo's constituency of Enfield was believed to be a safe seat, but opinion polls showed the Labour candidate Stephen Twigg was steadily gaining support, which may have prompted undecided voters or supporters of other parties to support Twigg in order to remove Portillo.

The third category of theories concerns voter participation/abstention. It is often suggested that supporters of the candidate shown to be significantly lagging behind may give up casting their ballots, resulting in a landslide victory of another candidate. In the South Korean presidential election of 2007, for instance, when a conservative candidate, M.B. Lee, achieved a landslide victory over a liberal candidate, D.Y. Chung, with the vote shares of 48.7% versus 26.1%, it was widely believed that supporters of Chung had abstained significantly, concluding from several pre-election polls that Chung would have no chance of winning even if they would cast votes for

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preferences as *information* about the correct option to take. Considering they have strong incentives to minimize the costs of acquiring the information necessary to make right choices (Downs, 1957), voters may rely upon 'information shortcuts,' such as group references, party identification, or knowledge about where other voters stand on issues.

[2] In 1980, NBC News declared Ronald Reagan to be the winner of the presidential race on the basis of the exit polls several hours before the voting booths closed in the west. In France and South Korea, concern about the relationship between polls and the election outcome has led to a ban for publication of pre-election polls in the week prior to the election.

him. (Indeed the voting rate was 63%, the lowest one since 1987.) But the opposite of this phenomenon may happen as well. A well-known example is a *boomerang effect* where the likely supporters of the candidate shown to be winning feel that they are 'home and dry' and that their vote is not required, thus allowing another candidate to win.

Economists have long studied the effect of conformism on economic behavior; see Akerlof (1997), Banerjee (1992), Bernheim (1994), Birkchandani et al. (1992), Leibenstein (1950), and Schelling (1974). Not many political models, however, have been developed incorporating the effect of opinion polls on voter conformism and its consequence for political competition. The current paper aims at filling this gap in the literature. In the present paper, we are particularly interested in the effect of voter conformism, in the form of bandwagon or underdog effects, on equilibrium policies. For this, we adapt two well-known models of two-party political competition for voter conformism, and study its effect on the equilibria of the two models. One is the Hotelling-Downs model in which parties maximize their probabilities of victory, and the other is the Wittman-Roemer model in which parties maximize the expected utilities of their key constituents. Because we study models with two candidates, the issue of strategic voting is not our concern. Also the potential issue of voter participation/abstention is not explicitly modeled.[3]

It is often suggested that with the loosening of the bonds between parties and the electorate and the decline of social class as bases of electoral politics, more voters are potentially available for influence by opinion polls, which in turn mitigates the policy differentiation between parties (McAllister and Studlar, 1991). But evidence for this remains somewhat mixed,

[3] The issue of abstention in the context of political competition is carefully studied by Llavador (2000).

particularly when first order elections, such legislative and presidential elections – is concerned. The present paper tests this claim theoretically.

In defining voter conformism, we follow Simon's (1954) concise delineation of the bandwagon and underdog phenomena. Simon (1954) holds that the voting behavior is a function of voters' expectations of the electoral outcome, and published poll data influence these expectations. There is a bandwagon effect if voters are more likely to vote for a candidate when they expect him to win than when they expect him to lose; if the opposite holds, there is an underdog effect. [4]

We briefly remark on the difference between political conformism and economic conformism. Economic models are of individual, not collective, choice. Thus economic agents consume their choices regardless of the choices of others, while voters must consume the group choice independent of their own choices. Thus political actors must take into account not only the choices before them but also how their choices will influence the choices of others following them.

Section 2 presents the Hotelling-Downs and the Wittman Roemer models of political competition with voter conformism in a unified framework. Section 3 then studies the effect of voter conformism on the Hotelling-Downs political equilibrium. We prove that voter conformism has no effect on equilibrium policies, for the unique equilibrium in this model is both parties' choosing the same policy, which is a strict Condorcet winner. In section 4, we study the Wittman-Roemer model with voter conformism. Section 5 concludes.

[4] In like manner, social psychologists define conformism by behavioral patterns among people such that the probability of any individual adopting it depends upon the proportion of people who have already done so (Colman, 2003).

2. The model

Throughout the paper, we will maintain that there are two parties (or candidates representing them), L and R, and that the policy space, $T \subset [0,1]$, is one-dimensional. A generic element of T will be denoted by t, which we call a tax rate, or simply a policy. We assume that the party that wins the election has the full power of implementing its announced policy.[5]

There is a continuum of voters; we are modeling an election in large polities, where no individual voter is noticeable. Voters are endowed with one-dimensional characteristic, $w \in R_+$, whose distribution is given by a continuous distribution function $F(.)$ with its associated probability measure $P(.)$. We call w an income. The mean of w is denoted by μ. We assume that the set of voters indifferent between t and t', where $t \neq t'$, has *P*-measure zero.

Let $(t_L, t_R) \in T \times T$ be an arbitrary pair of policy positions of the two candidates and $x \in [0,1]$ an expected fraction of voters who prefer L to R, a number estimated perhaps through pre-election polls. Given (t_L, t_R, x), we assume that voter preferences are given by:

$$\begin{cases} (1-t_L)w + \alpha h(t_L\mu) + \theta\phi(x) & \text{from candidate L} \\ (1-t_R)w + \alpha h(t_R\mu) + \theta\phi(1-x) & \text{from candidate R} \end{cases}, \tag{1}$$

where $h(.)$ is an increasing and concave function, $\phi(.)$ is an increasing and convex function, and α is a positive constant.

Some remarks are in order regarding voter preferences.

[5] Ortuno-Ortin (1997) studies a model of political competition under proportional representation, in which the influence of the groups in favor of a certain policy is proportional to the percentage votes favoring that policy.

First, we are assuming that facing an election, voters care not only about the policy positions of candidates but also the vote shares the candidates would receive in the election. Note that voter utility functions consist of two parts: the quasi-linear function representing the economic interests of voters, $(1-t_j)w+\alpha h(t_j\mu)$, and a utility bonus/penalty from supporting the winning candidate, $\theta\phi(x_j)$, where $j=L,R$. Because $\phi(.)$ is increasing, we have $\phi(x)>\phi(1-x)$ if $x>\frac{1}{2}$. Thus a bandwagon effect is captured by the assumption that θ is positive; other things being equal, voters prefer a candidate whose reported vote share in a poll is greater than ½. The presence of an underdog effect would be equivalent to assuming that θ is negative. Finally, if $\theta=0$, there is no voter conformism.

Second, if we interpret $t\mu$ as the per capita amount of public goods, then α measure the extent to which voters value the consumption of public goods. The parameter θ, on the other hand, measures the relative salience of voter conformism. By allowing α and θ to vary across voters, one might model a voter endowed with three characteristics: (w,α,θ). For the sake of simplicity, we will maintain that parameter values of α and θ are identical for all voters; voters differ only in the level of incomes that they hold. Of course, this is a great simplification. If some voters are vulnerable to bandwagon effects, others may be susceptible to underdog effects; still others may receive no influence at all. Because we do not know who are more susceptible to which effect, we study each case separately by assuming that all voters are susceptible to the same effect.

Given (t_L,t_R,x) and voter preferences defined in equation (1), the set of voters who (strictly) prefer L to R is

$$\Omega(t_L,t_R,x)=\begin{cases}\{w\in R_+ \mid w<\alpha\dfrac{h(t_L\mu)-h(t_R\mu)}{t_L-t_R}+\theta\dfrac{\phi(x)-\phi(1-x)}{t_L-t_R}\} & \text{if } t_L>t_R\\ \{w\in R_+ \mid w>\alpha\dfrac{h(t_L\mu)-h(t_R\mu)}{t_L-t_R}+\theta\dfrac{\phi(x)-\phi(1-x)}{t_L-t_R}\} & \text{if } t_L<t_R\\ R_+ & \text{if } t_L=t_R \text{ and } x>\frac{1}{2}\\ \varnothing & \text{if } t_L=t_R \text{ and } x<\frac{1}{2}\\ \varnothing & \text{if } t_L=t_R \text{ and } x=\frac{1}{2}\end{cases} \tag{2}$$

(One needs to note a difference between the case in which $t_L = t_R$ and $x < \frac{1}{2}$ and the case where $t_L = t_R$ and $x = \frac{1}{2}$. In the former case, all voters strictly prefer R to L, while in the latter case, voters are indifferent between them.)

Thus, facing (t_L, t_R, x), the actual fraction of voters who prefer L to R is given by

$$x' = P(\Omega(t_L,t_R,x)), \tag{3}$$

where $$P(\Omega(t_L,t_R,x))=\begin{cases}F\left(\alpha\dfrac{h(t_L\mu)-h(t_R\mu)}{t_L-t_R}+\theta\dfrac{\phi(x)-\phi(1-x)}{t_L-t_R}\right) & \text{if } t_L>t_R\\ 1-F\left(\alpha\dfrac{h(t_L\mu)-h(t_R\mu)}{t_L-t_R}+\theta\dfrac{\phi(x)-\phi(1-x)}{t_L-t_R}\right) & \text{if } t_L<t_R\\ 1 & \text{if } t_L=t_R \text{ and } x>\frac{1}{2}\\ 0 & \text{if } t_L=t_R \text{ and } x<\frac{1}{2}\\ \frac{1}{2} & \text{if } t_L=t_R \text{ and } x=\frac{1}{2}.\end{cases}$$

We now introduce electoral uncertainty. Were there no uncertainty in elections, $P(\Omega(t_L, t_R, x))$, the fraction of voters who prefer L to R, would be identical to candidate L's actual vote share in the election. In reality, however, the outcome of elections is uncertain, because not everyone votes or some other unexpected random shocks (e.g., scandals of candidates) may occur. The party manifestoes, or the party conventions, also typically take place months before the elections, when uncertainty may be substantial. To capture this uncertainty in a simple way, we suppose the actual vote share for L is $P(\Omega(t_L, t_R, x)) + \varepsilon$, where ε is a random shock distributed by a symmetric distribution function $G(.)$ such that $G(0) = \frac{1}{2}$. Then the estimated probability of victory for L is

$$\pi(t_L, t_R; x) = \Pr(P(\Omega(t_L, t_R, x)) + \varepsilon > 0.5) = 1 - G\left(0.5 - P(\Omega(t_L, t_R, x))\right), \tag{4}$$

and the probability that R wins is $1 - \pi(t_L, t_R; x)$.

We now define the Hotelling-Downs equilibrium with voter conformism:

Definition 1: A Hotelling-Downs political equilibrium with voter conformism is a triple (t_L^*, t_R^*, x^*) such that:

(1) $t_L^* \in \arg\max \pi(t_L, t_R^*; x^*)$;

(2) $t_R^* \in \arg\max(1 - \pi(t_L^*, t_R; x^*))$;

and

(3) $x^* = P(\Omega(t_L^*, t_R^*, x^*))$.

The first two conditions require that (t_L^*, t_R^*) be mutual best responses of the two Hotelling-Downs parties at x^*.The third condition implies that polls (or expectations estimated from polls) are accurate; given (t_L^*, t_R^*), the actual fraction of voters who prefer L to R must be identical to the expected fraction of voters.

In the standard Hotelling-Downs model, where voter conformism is not present, an equilibrium condition requires only that (t_L^*, t_R^*) be mutual best responses of the two

Hotelling-Downs parties; the fraction of voters who prefer L to R is then automatically derived from (t_L^*, t_R^*). In contrast, for (t_L^*, t_R^*, x^*) to be a Hotelling-Downs equilibrium with voter conformism, the following two conditions must be simultaneously met: given x^*, (t_L^*, t_R^*) must be mutual best responses of the two Hotelling-Downs parties, and (t_L^*, t_R^*) must predict precisely the fraction of voters x^*. (Alternatively, given (t_L^*, t_R^*), the fraction must be x^* at which the mutual best responses of the two Hotelling-Downs parties are precisely (t_L^*, t_R^*).) Put it mathematically, Definition 1 requires that (t_L^*, t_R^*, x^*) be a fixed point of $\beta_L(t_R, x) \times \beta_R(t_L, x) \times P(\Omega(t_L, t_R, x))$, where $\beta_i(t_j, x)$ is the best-response correspondence of party i.

Condition (3) of Definition 1 exhibits another way of presenting bandwagon and underdog effects. Condition (3) requires that the equilibrium fraction of voters who prefer L to R be a fixed point of the map $P(\Omega(t_L^*, t_R^*, x))$ for given (t_L^*, t_R^*). The situation is illustrated in Figure 1. The horizontal axis measures the expected fraction of voters who prefer L to R and the vertical axis measures the actual fraction of voters who prefer L to R. Two possible shapes of the map $P(\Omega(t_L^*, t_R^*, x))$ are drawn for the case in which $t_L^* > t_R^*$. The presence of a bandwagon effect implies that the map is upward-sloping, while an underdog effect corresponds to the case in which the map is downward-sloping. The intersection of the map with the 45 degree line is the equilibrium fraction of voters who prefer L to R.

[Figure 1 about here]

Indeed there always exists such a fixed point. If $t_L^* > t_R^*$, there is at least one fixed point when $\theta > 0$, and only one fixed point when $\theta \leq 0$. If $t_L^* = t_R^*$, on the other hand, there are three fixed points: $0,\ \frac{1}{2},\ \text{and } 1$. (Drawing the map $P(\Omega(t_L^*, t_R^*, x))$ in this case is straightforward from equation (3).) (One needs to note, however, that not all these fixed points are equilibrium fractions; we repeat that for the fraction of voters as a fixed point to be an equilibrium fraction, the fixed point calculated at (t_L^*, t_R^*) must confirm the pair of policies as mutual-best responses at the fixed point.)

We now move on to the Wittman-Roemer political equilibrium with voter conformism. In this model, party membership will be determined endogenously. We assume a *perfectly representative democracy* where: (1) every citizen belongs to one and only one party; (2) each party member receives equal weight in the determination of the party's von Neumann-Morgenstern preferences; and (3) each citizen votes for the party of which he/he is a member.

To model party membership, suppose the polity is partitioned into two sets, H_L and H_R, such that $H_L \cup H_R = R_+$ and $H_L \cap H_R = \varnothing$. We assume that the party's von Neumann-Morgenstern utility function is the average of its members' utility functions representing economic interests. Thus for an arbitrary policy $t \in T$, they are:

$$V(t; H_L) = \begin{cases} \dfrac{1}{P(H_L)} \displaystyle\int_{w \in H_L} ((1-t)w + \alpha h(t\mu)) dP(w) & \text{if } P(H_L) \neq 0 \\ 0 & \text{if } P(H_L) = 0 \end{cases}, \tag{5}$$

and

$$V(t;H_R)=\begin{cases}\dfrac{1}{P(H_R)}\displaystyle\int_{w\in H_R}((1-t)w+\alpha h(t\mu))dP(w) & \text{if } P(H_R)\neq 0\\ 0 & \text{if } P(H_R)=0\end{cases}. \tag{6}$$

Because the utility function representing the economic interests is quasi-linear, the two parties' von Neumann-Morgenstern utility functions, defined by the average of its members' utility functions of the part representing economic interests, is identical to the utility function of a voter whose income equals the average of its members' incomes; for

$$\frac{1}{P(H_L)}\int_{w\in H_L}((1-t)w+\alpha h(t\mu))dP(w)=(1-t)w_L+\alpha h(t\mu), \tag{7}$$

and

$$\frac{1}{P(H_R)}\int_{w\in H_R}((1-t)w+\alpha h(t\mu))dP(w)=(1-t)w_R+\alpha h(t\mu), \tag{8}$$

where $w_L=\dfrac{1}{P(H_L)}\int_{w\in H_L}wdP(w)$ and $w_R=\dfrac{1}{P(H_R)}\int_{w\in H_R}wdP(w)$.

In the Wittman-Roemer model of political competition, each party maximizes the expected utility of its members. Thus the payoff function of the Left is:

$$\Pi(t_L,t_R;x,H_L)=\pi(t_L,t_R;x)V(t_L;H_L)+(1-\pi(t_L,t_R;x))V(t_R;H_L), \tag{9}$$

and the payoff function of the Right is:

$$\Pi(t_L,t_R;x,H_R)=\pi(t_L,t_R;x)V(t_L;H_R)+(1-\pi(t_L,t_R;x))V(t_R;H_R). \tag{10}$$

We now define:

Definition 2: A Wittman-Roemer political equilibrium with voter conformism is a partition of the constituency into H_L^* and H_R^* and a triple (t_L^*,t_R^*,x^*) such that:

(1) $\quad t_L^* \in \arg\max \pi(t_L, t_R^*; x^*)V(t_L; H_L^*) + (1-\pi(t_L, t_R^*; x^*))V(t_R^*; H_L^*)$

(2) $\quad t_R^* \in \arg\max \pi(t_L^*, t_R; x^*)V(t_L^*; H_R^*) + (1-\pi(t_L^*, t_R; x^*))V(t_R; H_R^*)$;

(3)
$$w \in H_L^* \Rightarrow (1-t_L^*)w + \alpha h(t_L^*\mu) + \theta\phi(x^*) \geq (1-t_R^*)w + \alpha h(t_R^*\mu) + \theta\phi(x^*),$$
$$w \in H_R^* \Rightarrow (1-t_R^*)w + \alpha h(t_R^*\mu) + \theta\phi(1-x^*) \geq (1-t_L^*)w + \alpha h(t_L^*\mu) + \theta\phi(1-x^*);$$

and

(4) $\quad x^* = P(H_L^*)$.

The first two conditions require that (t_L^*, t_R^*) be a Nash equilibrium, given (x^*, H_L^*, H_R^*). Each party/candidate plays a best response to the opponent at (x^*, H_L^*, H_R^*).

The third condition endogenizes party membership; it states that each citizen will vote for the party that accepts him as a constituent. In other words, it states that the party memberships (constituencies) are *stable*, in the sense that no member of either party is better represented by the other party. The idea here – that malcontents ‘vote with their feet’ by defecting to the other party – follows Caplin and Nalebuff (1997), and Baron (1993) first uses it in the context of political parties. We follow the formulation given by Roemer (2001). Condition (3) defines the partition of constituency, H_L^* and H_R^*, as functions of (t_L^*, t_R^*, x^*).

The fourth condition describes the political equilibrium by one of rational-expectation equilibria. It states that the equilibrium party memberships must be identical to expected party memberships at the equilibrium party platforms.

We presented the Wittman-Roemer equilibrium as a Nash equilibrium between two parties each of which is a unitary actor; each party maximizes its expected payoff. We can demonstrate that the Wittman-Roemer equilibrium can be viewed as an equilibrium between parties with factions that Nash-bargain with one another.

Think of two parties, each composed of two factions: opportunists whose objective is to maximize the probability of the party's victory in the election and militants who wish to announce a policy as close as possible to the ideal policy of the constituents of their party. Thus party L's opportunists have preferences represented by $\pi(t_L, t_R; x)$ and its militants have preferences represented by $V(t; H_L)$.

To model the Wittman-Roemer equilibrium as a Nash bargaining outcome between the factions, we need to specify the impasse payoffs, the payoffs of the factions should party L fails to come to an agreement. If party L's factions fail to come to an agreement, party R wins the election by default; hence the probability of victory for party L is zero and party R's policy will be implemented. Thus facing t_R^*, the Nash bargaining solution between the two factions of party L is the policy t_L that maximizes the Nash product:

$$(\pi(t_L, t_R^*; x^*) - 0)(V(t_L; H_L^*) - V(t_R^*; H_L^*)). \tag{11}$$

The solution to this problem maximizes $\pi(t_L, t_R^*; x^*)V(t_L; H_L^*) + (1 - \pi(t_L, t_R^*; x^*))V(t_R^*; H_L^*)$.

Similarly, facing a policy t_L^* from party L, party R's factions Nash-bargain to a policy t_R that maximizes:

$$(1 - \pi(t_L^*, t_R; x^*) - 0)(V(t_R; H_R^*) - V(t_L^*; H_R^*)). \tag{12}$$

The solution to this problem also maximizes $\pi(t_L^*, t_R; x^*)V(t_L^*; H_R^*) + (1 - \pi(t_L^*, t_R; x^*))V(t_R; H_R^*)$.

Thus we conclude that an unweighted Nash-bargaining equilibrium is precisely a Wittman-Roemer equilibrium.

Our interpretation of a Wittman-Roemer equilibrium as a Nash-bargaining solution between the two factions provides useful formulae in a differentiable environment.

The first order condition for the maximization of equation (11) is:

$$\lambda_L \frac{\partial V(t_L^*; H_L^*)}{\partial t_L} = -\frac{\partial \pi(t_L^*, t_R^*; x^*)}{\partial t_L}, \tag{13}$$

where $\lambda_L = \dfrac{\pi(t_L^*, t_R^*; x^*)}{V(t_L^*; H_L^*) - V(t_R^*; H_L^*)} > 0$. Thus if a move from t_L^* increases the payoff of party L's militants, then it must decrease the payoff of party L's opportunists.

Likewise, the first order condition for the maximization of equation (12) is:

$$\lambda_R \frac{\partial V(t_R^*; H_R^*)}{\partial t_R} = \frac{\partial \pi(t_L^*, t_R^*; x^*)}{\partial t_R}, \tag{14}$$

where $\lambda_R = \dfrac{1 - \pi(t_L^*, t_R^*; x^*)}{V(t_R^*; H_R^*) - V(t_L^*; H_R^*)} > 0$. Therefore, if a move from t_R^* increases the payoff of party R's militants, then it must decrease the payoff of party R's opportunists.

Although we presented the Wittman-Roemer equilibrium as a static concept, it is possible to interpret it as a stationary point of the following dynamic process. (A similar dynamic interpretation may be given to the Hotelling-Downs model as well.) Suppose there is a sequence of decision making over time (until the election time). In each period, parties revise their policies by maximizing their payoffs while taking as given the policies and the party memberships of the last period, and voters revise their party membership based upon the policies and the party memberships of the last period. A stationary point of this dynamic process is exactly identical to the Wittman-Roemer equilibrium defined in Definition 2.

3. Voter conformism and the Hotelling-Downs model of political competition

We first study the political equilibrium of the Hotelling-Downs model when voter conformism is present.

Theorem 1: The unique Hotelling-Downs equilibrium with voter conformism is $(t_L^*, t_R^*, x^*) = (t^*, t^*, \frac{1}{2})$, where t^* is strict Condorcet winner.

Proof: We first prove that $(t_L^*, t_R^*, x^*) = (t^*, t^*, \frac{1}{2})$ is an equilibrium. At $x^* = \frac{1}{2}$, equation (1) becomes

$$\begin{cases}(1-t_L)w + \alpha h(t_L\mu) + \theta\phi(\frac{1}{2}) & \text{from candidate L} \\ (1-t_R)w + \alpha h(t_R\mu) + \theta\phi(\frac{1}{2}) & \text{from candidate R}\end{cases}.$$

Because the same constant $\theta\phi(\frac{1}{2})$ appears in both lines, voters care only about the policy positions of the two parties. Thus the standard theorem on the Hotelling-Downs model applies. We thus have $t_L^*\big|_{x^*=\frac{1}{2}} = t_R^*\big|_{x^*=\frac{1}{2}} = t^*$, which in turn yields the vote share of $\frac{1}{2} = x^*$. Thus $(t_L^*, t_R^*, x^*) = (t^*, t^*, \frac{1}{2})$ is an equilibrium.

It remains to prove that there are no other equilibria than this.

First, the above argument also proves that the case in which $t_L^* = t_R^* \neq t^*$ and $x^* = \frac{1}{2}$ and the case where $t_L^* \neq t_R^*$ and $x^* = \frac{1}{2}$ do not constitute an equilibrium.

Second, we prove that the case in which $t_L^* = t_R^*$ and $x^* \neq \frac{1}{2}$ does not constitute an equilibrium. Note that if $t_L = t_R$, the equilibrium vote share which is not equal to $\frac{1}{2}$ must be either 0 or 1. Choosing $t_L^* = t_R^*$ is party L's best response to t_R^* only when $x^* = 1$, while choosing $t_R^* = t_L^*$ is party R's best response to t_L^* only when $x^* = 0$. Thus at $x^* \in \{0,1\}$, $t_L^* = t_R^*$ does not constitute a pair of mutual best responses.

Finally, we prove that any case in which $t_L^* \neq t_R^*$ and $x^* \neq \frac{1}{2}$ cannot be an equilibrium. Consider $t_L^* \neq t_R^*$ at which $x^* < \frac{1}{2}$. This case does not constitute an equilibrium because if party R chooses a policy equal to t_L^*, it could increase the fraction of voters who prefer R to L to 1, making its probability of victory at x^* almost equal to 1. Likewise, the case in which $t_L^* \neq t_R^*$ and $x^* > \frac{1}{2}$ does not constitute an equilibrium; party L could increase the fraction of voters who prefer L to R to 1 if it choose a policy equal to t_R^*. ∎

Thus if real politics is a Hotelling-Downs kind, voter conformism has no influence on party platforms.

4. Voter conformism and the Wittman-Roemer model of political competition

We now study the effect of voter conformism on the Wittman-Roemer political equilibrium. In some cases, there are trivial Wittman-Roemer equilibria at which at least one party's equilibrium policy is in a corner of the policy space. This usually happens when one party's probability of victory is either 1 or 0. (See examples in Roemer (2001).) However, we are more interested in non-trivial, interior, equilibria.

In this section, we will calculate nontrivial, interior, Wittman-Roemer political equilibria with voter conformism. A general characterization of such equilibria is somewhat difficult to obtain. We will thus rely upon numerical methods to find such equilibria. In the numerical computation, we chose for F a lognormal distribution derived from a normal distribution with mean *m* and standard deviation *s*. For G, we chose a normal distribution with mean *a* and standard deviation *b*. Finally, we chose $h(t\mu) = \sqrt{t\mu}$ and $\phi(x) = x^k$, where $k \geq 1$.

To estimate the parameter values of the lognormal distribution, we use the 2004 US Census Bureau data. According to the US Census Bureau 2004 Economic Survey, the mean household income in the United States was $60,528, and the Gini coefficient for household incomes in that year was 0.469. Our estimated parameters are then $m = 1.408$ and $s = 0.886$. For the parameters of the normal distribution, we simply set $a = 0$ and $b = 0.05$. Estimated lognormal and normal distributions are drawn in Figure 2. Finally, we chose $\alpha = 1.8$ and $k = 1.5$, and varied the values of θ from -1.5 to 1.5.

[Figure 2 about here]

There exists a Wittman-Roemer equilibrium for all values of θ, and in some cases we observe multiple equilibria, as Figure 3 illustrates.

[Figure 3 about here]

Our numerical calculation is based on first order conditions, and it remains to show that t_L^* and t_R^* are indeed maxima of the conditional payoff functions of the two parties: $\Pi(t, t_R^*; x^*, H_L^*)$

and $\Pi(t_L^*, t; x^*, H_R^*)$. We examined the conditional payoff functions and they are all maxima of the payoff functions. Figure 4 shows the conditional payoff functions when $\theta = 0.6$, at which three Wittman-Roemer equilibria exist.

[Figures 4 about here]

When multiple equilibria exist, we call them type-A, type-B, and type-C equilibria. Type-A equilibrium is the one in which x^* is significantly less than 0.5, type-B equilibrium is that in which x^* is significantly greater than 0.5, and type-C equilibrium is the one in which x^* is between them, which is nearby 0.5. We plot the three types of equilibria separately in Figure 5.

[Figure 5 about here]

We first note that the underdog effect has almost no significance on the Wittman-Roemer equilibrium, although it has a minor effect of mitigating the policy differentiation. We varied the value of θ from -0.01 to -1.5, but the tax rates and the vote shares are almost constant.

The bandwagon effect has, on the other hand, significantly different implications on the Wittman-Roemer equilibrium, in at least two ways.

First, there emerge multiple equilibria when the bandwagon effect is sufficiently strong. In our numerical calculations, the branching point is at $\theta = 0.586$. If θ is less than it, there exists a unique equilibrium. At $\theta = 0.586$, there emerge two equilibria. After that, one of the two equilibria branches into two separate equilibria. Thus, if $\theta > 0.586$, there always exist three equilibria.

Second, in type-A and type B equilibria, political parties diverge more as the bandwagon effect becomes larger. In type-C equilibrium, on the other hand, the difference in party platforms between the two parties is almost constant. When we examine the type-C equilibrium carefully, we note that the difference of party platforms, $t_L^* - t_R^*$, keeps decreasing from 0.1383 at $\theta = 0.59$ to 0.1362 at $\theta = 0.98$, and afterwards increases up to 0.1363 at $\theta = 1.5$.

Not all of these equilibria are, however, reasonable. Although there is no general procedure of selection among multiple equilibria, one might wish to check stability/instability of those equilibria in a dynamic context; recall that we gave a dynamic interpretation for the Wittman-Roemer equilibrium. One way of checking it is to examine the slope of the map $P(H_L(t_L^*, t_R^*, x))$ at its fixed point. If the absolute value of the slope is less than 1, it would be (locally) stable in a dynamic context; otherwise it would be unstable.

We find that all type-C equilibria would be unstable in a dynamic context, for the map $P(H_L(t_L^*, t_R^*, x))$ cuts the 45 degree line from below in all those equilibria. Thus more meaningful equilibria in a dynamic context seem those of type-A and type-B.

If we take a dynamic context into account, however, some of type-A and type-B equilibria would be unstable as well, in particular if the bandwagon effect is sufficiently strong. For instance, type-A equilibrium would be unstable if $\theta > 0.83$ and type-B equilibrium would be unstable if $\theta > 0.88$. Thus a meaningful range of parameter values for θ in a dynamic context would be between 0.586 and 0.83.

A sufficiently strong underdog effect would also make the equilibrium unstable in a dynamic context. When the values of θ are between -1.5 and -0.59, the slope of the map

$P(H_L(t_L^*, t_R^*, x))$ is greater than 1 in its absolute value. If θ lies between -0.59 and 0.586, on the other hand, the slope of the map is always less than 1 in its absolute value.

We thus observe that in reasonable equilibria, i.e., those which would be dynamically stable, an increasing bandwagon effect exacerbates the policy differentiation of the two parties while an increasing underdog effect mitigates it.

We now provide an intuition behind the results. The interpretation of the Wittman-Roemer equilibrium as a Nash-bargaining solution between the factions is useful for understanding the results.

Suppose party R is winning in pre-election polls, as in type-A equilibrium. Were there no bandwagon effects, a proposal of R-militants to change the current R policy to the direction of the ideal tax rate of its key constituents would not be achieved because it would require a sacrifice of R-opportunists. The presence of the bandwagon effect, on the other hand, gives a windfall gain to R-opportunists, due to to voter conformism. Thus R-militants can propose a policy change slightly to the direction of the ideal tax rate of the key constituents of party R without scarifying R-opportunists. Thus party R can decrease the tax rate. But a reduction of the tax rate of party R gives some extra benefits to L-opportunists, which makes it possible for L-militants to demand a policy change slightly to the direction of the ideal tax rate of party L's key constituents.

An explanation for policy differentiation at type-B equilibrium is similar. Suppose L is winning in pre-election polls, as in type-B equilibrium. The presence of the bandwagon effect in this situation gives a windfall gain to L-opportunists; thus L-militants can call for a higher tax rate without scarifying L-opportunists. An increase in party L's tax rate, in turn, implies some extra benefit to R-opportunists, which makes it possible for R-militants to propose a slightly lower tax rate.

Explanations for the mitigation of policy differences at the presence of underdog effects can be provided in similar ways.

The above explanations based on the bargaining perspective also provide an intuition for why neither bandwagon nor underdog effect has much effect on the policy differences when the vote share is nearby 0.5. If the vote share is nearby 0.5, windfall gains to opportunists of the winning party is very small; there is not much room for policy change.

5. Conclusion

In this paper, we studied the potential effect that voter conformism might have on political equilibria using two well-known models of political competition.

We find that voter conformism, both bandwagon and underdog, has no effect on the Hotelling-Downs political equilibrium. Even if voter conformism is present, the Hotelling-Downs parties propose an identical policy at the equilibrium, which is equal to the strict Condorcet winner. In the Wittman-Roemer world of politics, on the other hand, two parties propose differentiated equilibrium policies, and the extent of such policy differentiation depends on voter conformism. In general, the stronger the bandwagon effect is, the more differentiated the equilibrium policies are. The opposite holds when the underdog effect is present; an increasing underdog effect mitigates the policy differentiation of the two parties, although that effect is not large.

The present paper studies the effect of voter conformism on political equilibrium in a uni-dimensional policy space. It is well known that both the Hotelling-Downs and the Wittman-Roemer models of political competition do not possess generic equilibria when the policy space is multi-dimensional. There are models that possess generic equilibria in a multi-dimensional policy space, such as the probabilistic voting model of Lindbeck and Weibull (1987), or the party unanimity Nash equilibrium model of Roemer (1999, 2001). We leave the study of the effect of voter conformism on these models for future research.

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Figure 1: Bandwagon and underdog effects

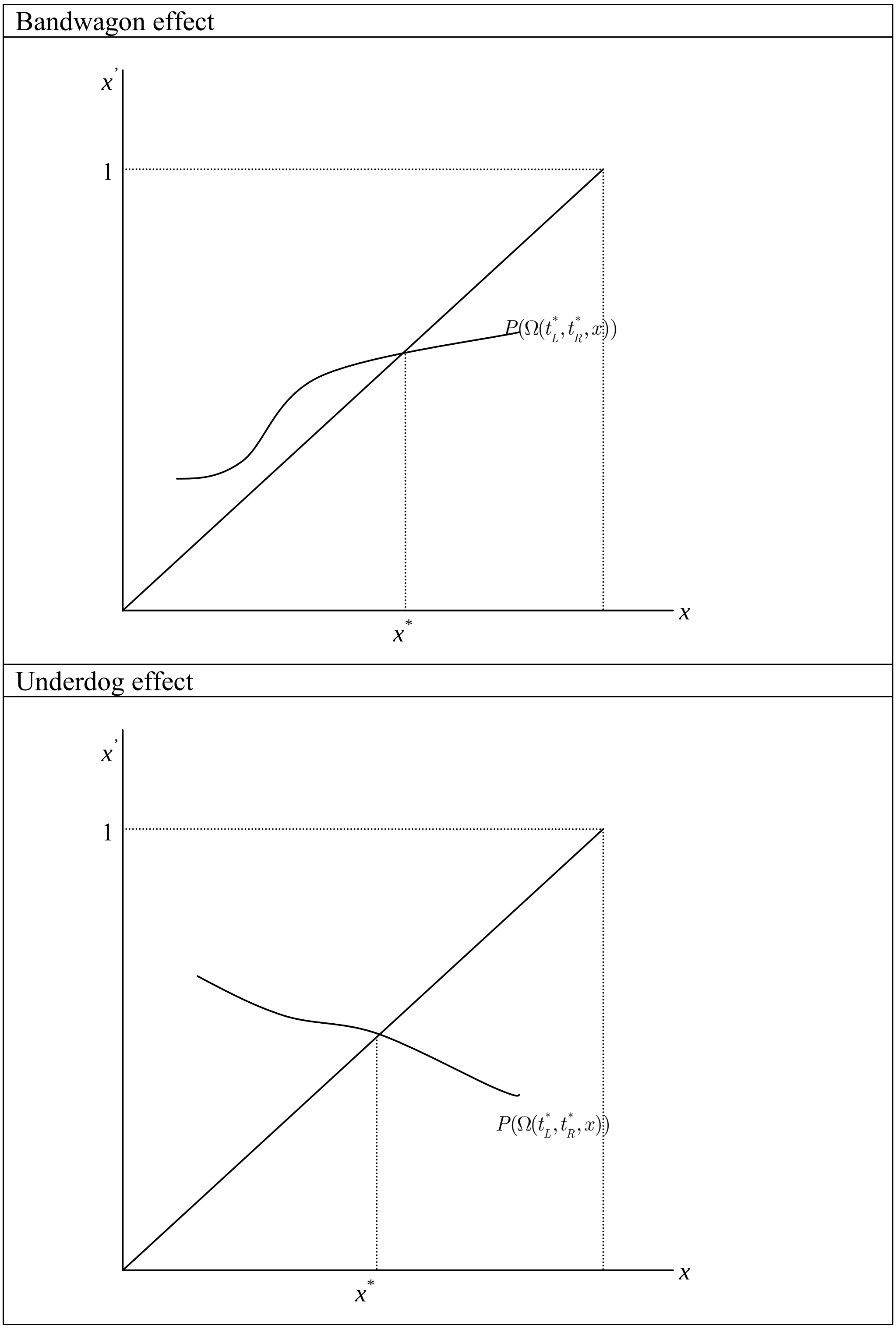
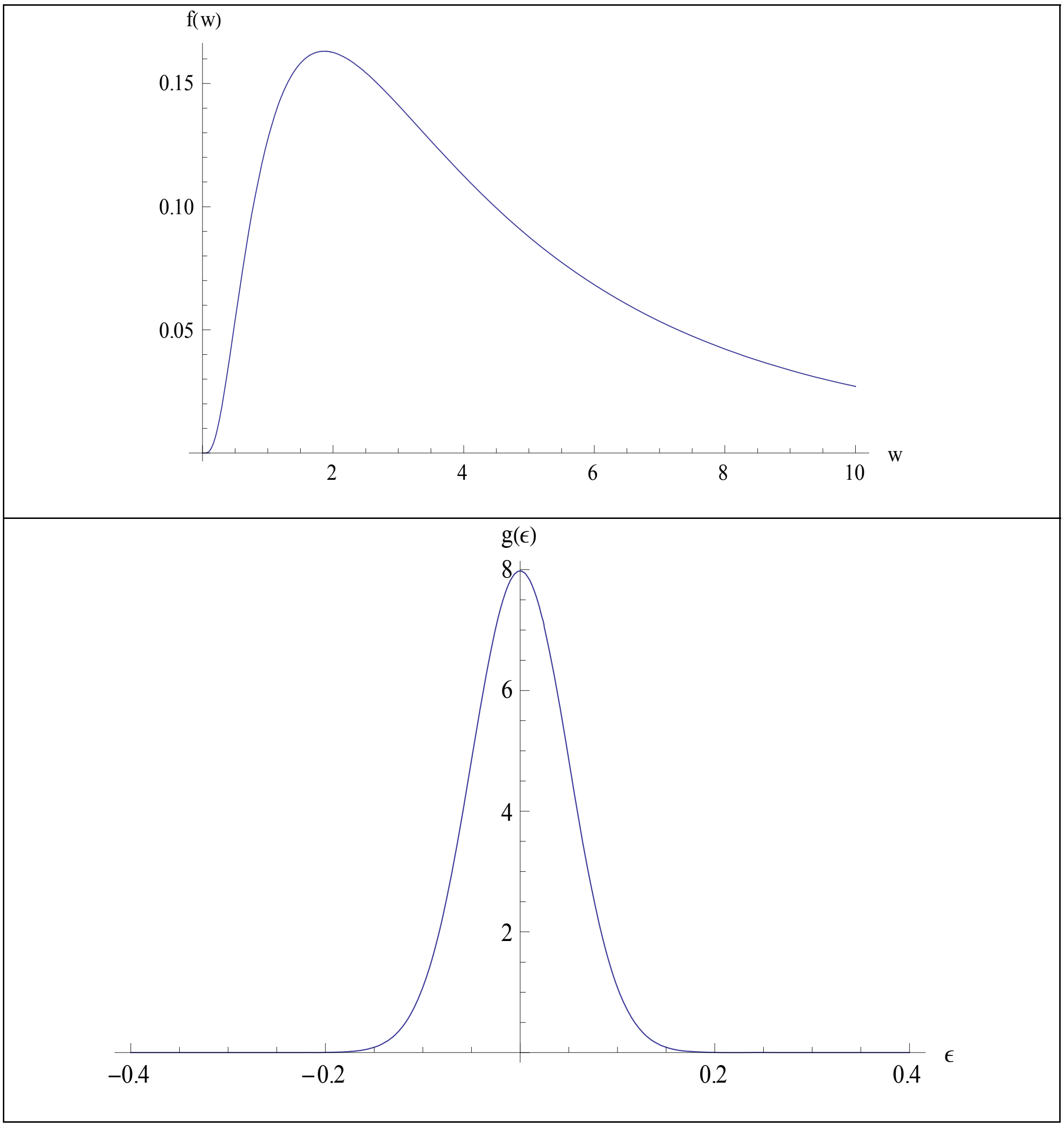
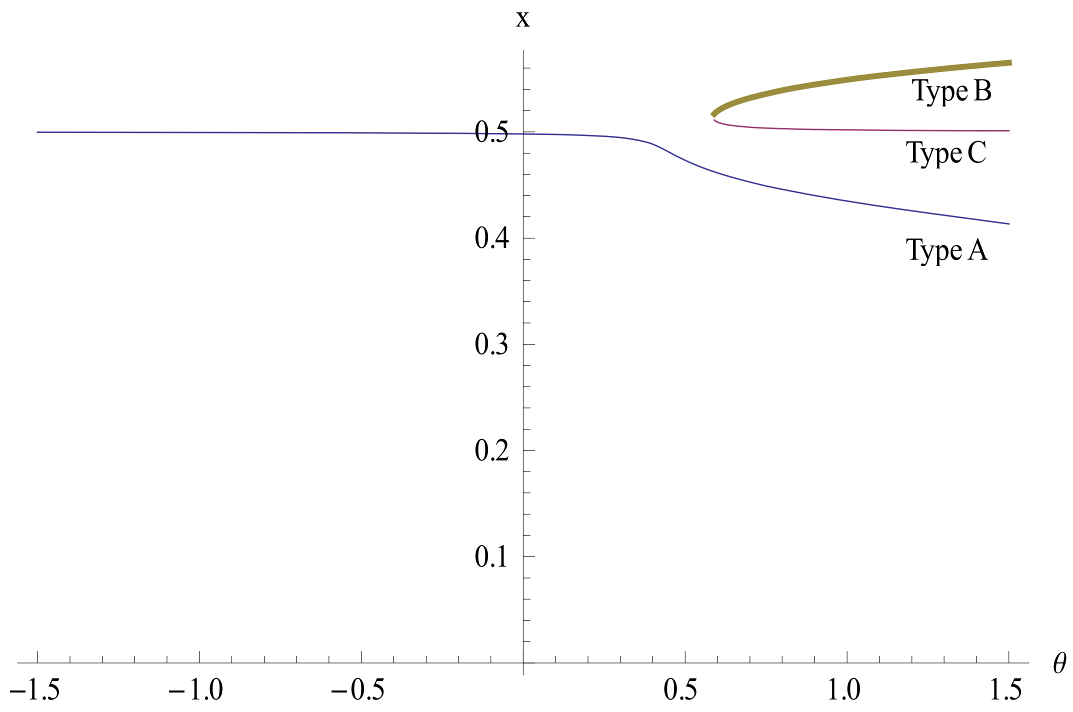


Figure 2: Density functions of w and ε



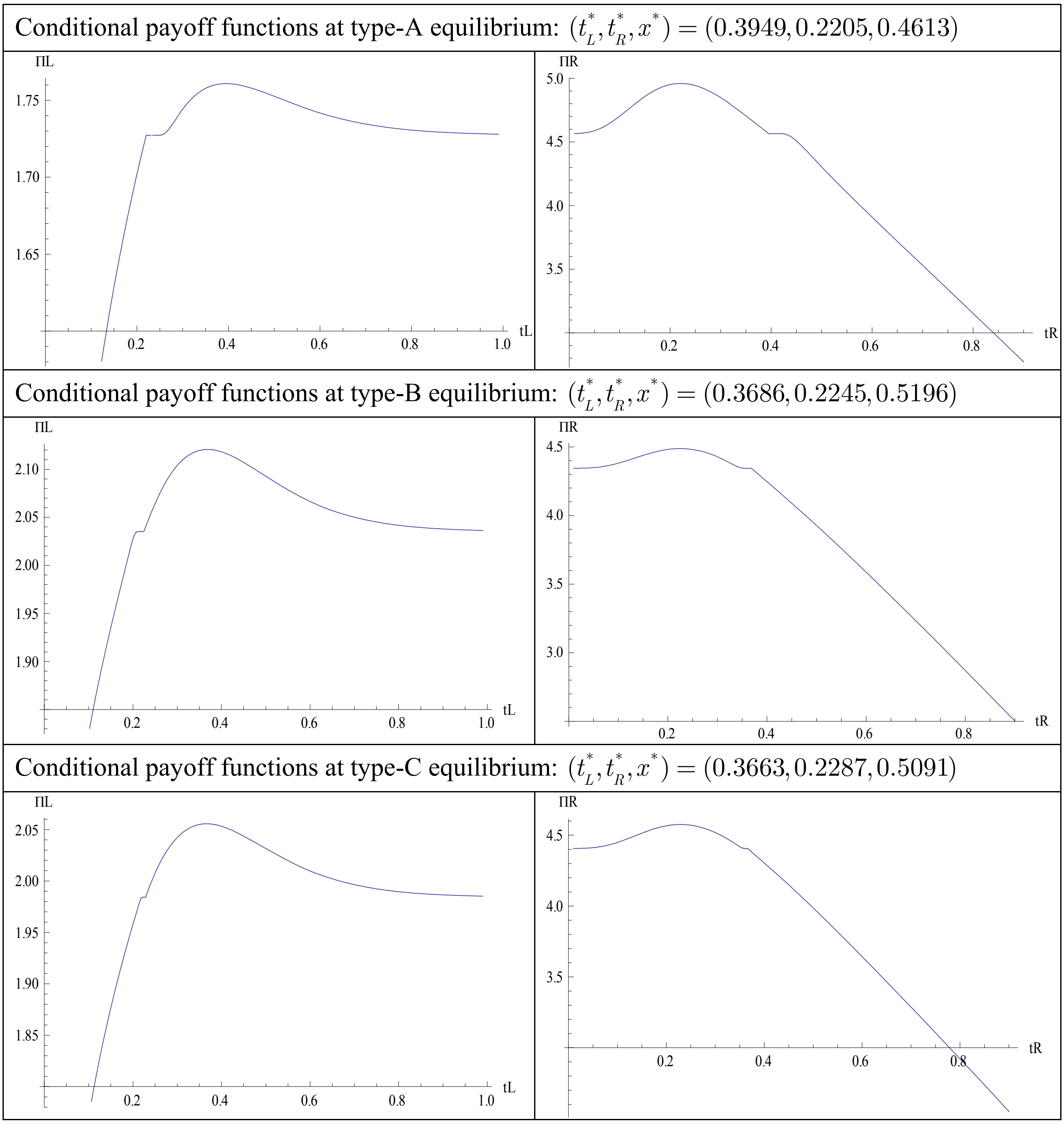
Parameter values are: m=1.40804; s=0.8860; a=0; b=0.05.

Figure 3: Equilibrium fraction of voters who prefer L to R



Parameter values are: m=1.40804; s=0.8860; a=0; b=0.05; k=1.5; α=1.8.

Figure 4: Conditional payoff functions when $\theta = 0.6$



Other parameter values are: m=1.40804; s=0.8860; a=0; b=0.05; k=1.5; α=1.8.

Figure 5.A: Type-A equilibrium

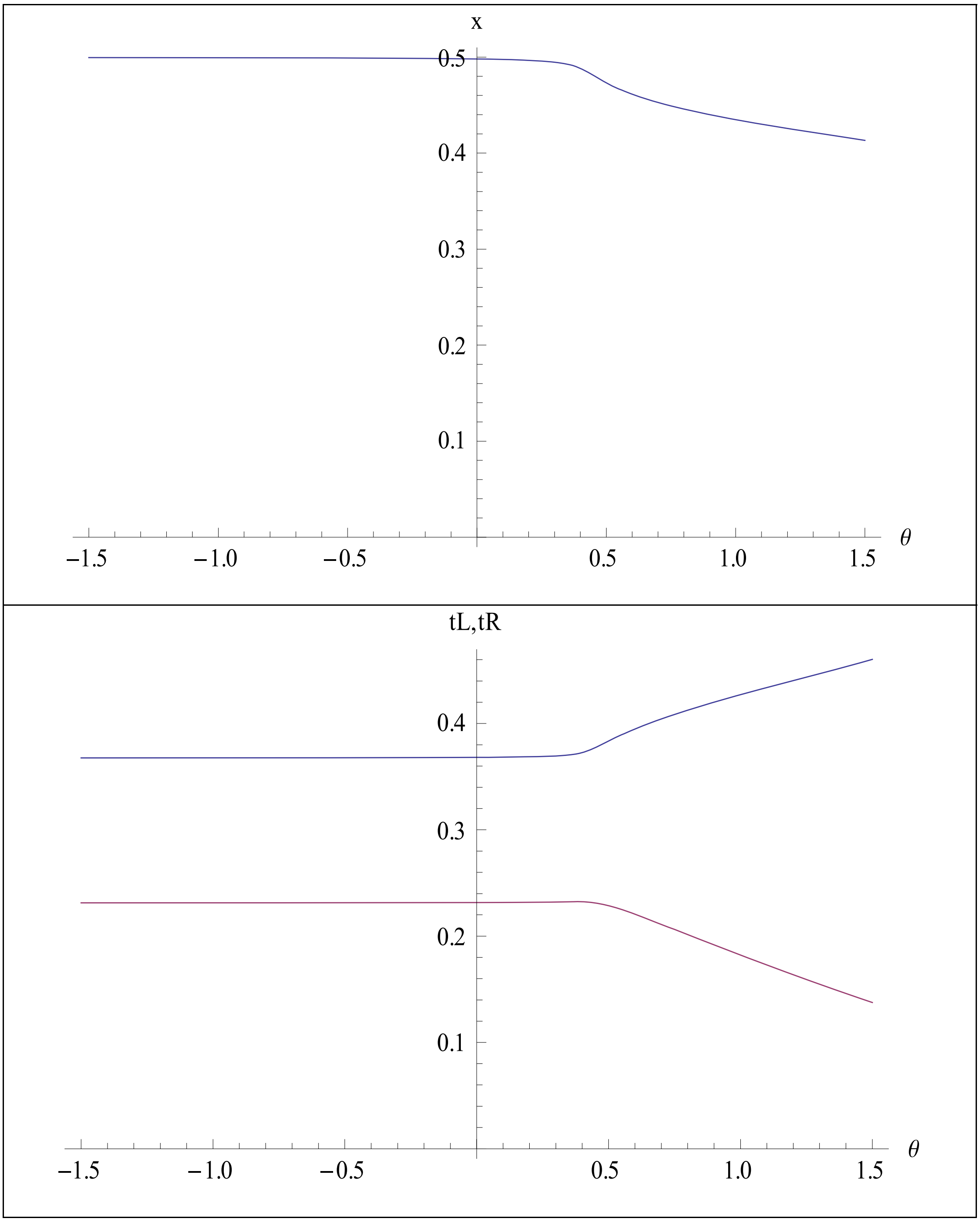


Figure 5.B: Type-B equilibrium

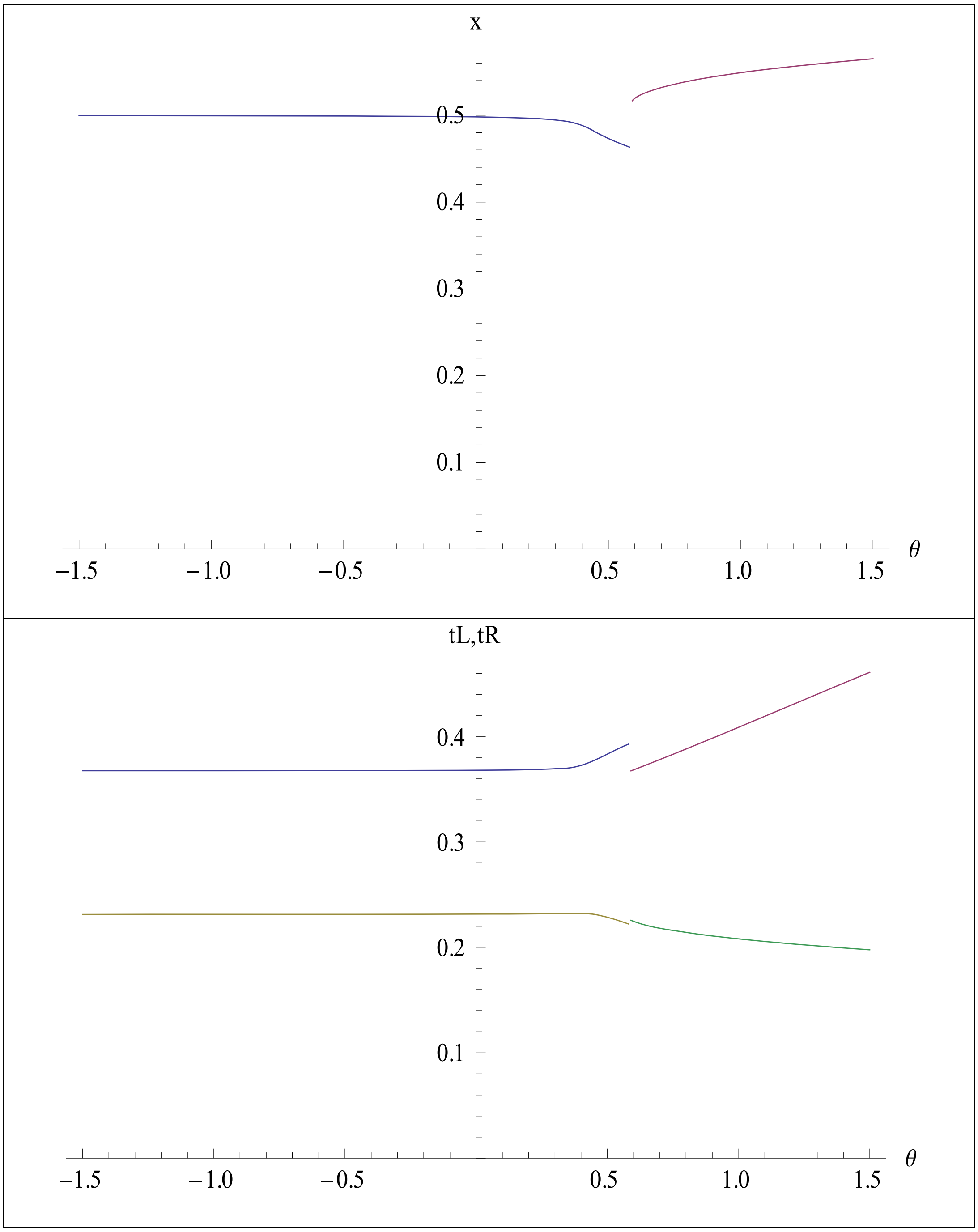


Figure 5.C: Type-C equilibrium