On the Budget-Constrained IRS: Equilibrium and Welfare

Meng-Yu Liang

Institute of Economics, Academia Sinica, Nankang, Taipei 115, Taiwan

C.C. Yang*

Institute of Economics, Academia Sinica, Nankang, Taipei 115, Taiwan

Department of Public Finance, National Chengchi University, Wenshan, Taipei 116, Taiwan

March 2008

Abstract

This paper extends Graetz, Reinganum and Wilde's (1986) seminal work on tax compliance to the real-world scenario where the IRS (Internal Revenue Service) faces a budget constraint imposed upon her by the Congress. The paper consists of two parts. First, we characterize the equilibria resulting from the interaction between taxpayers and the budget-constrained IRS. Second, we examine the welfare implication of varying the size of the budget allocated to the IRS. It is shown that, to mitigate or eliminate the so-called "congestion effect," the IRS should be sufficiently budgeted and, in particular, we provide a case for the policy prescription that the size of the budget allocated to

*Corresponding author, C.C. Yang. Mailing address: Institute of Economics, Academia Sinica, Nankang, Taipei 115, Taiwan. Email: ccyang@econ.sinica.edu.tw

the IRS should be expanded as long as an additional dollar allocated could return more than an additional dollar of revenue.

1 Introduction

"Unlike other government agencies, there is a positive return on money invested in the IRS. ... In its FY2007 budget recommendation, the Board calls for a modest increase in enforcement that would result in a real return on investment, ranging from three to six dollars on every dollar spent, resulting in $730 million revenue increase by FY2009 on a $242 million investment."

IRS Oversight Board (2006, pp. 12-13)[1]

On the basis of a 3-1 to 6-1 return for an additional dollar invested, does it make sense for the Board to recommend an expanded IRS budget on enforcement? This paper provides a case for the positive answer.

We consider a model of tax compliance, which extends the seminal work of Graetz, Reinganum and Wilde (1986, hereafter GRW) to the real-world scenario where the IRS faces a budget constraint imposed upon her by the Congress. The paper consists of two parts. The first part is positive. We characterize the equilibria resulting from the interaction between taxpayers and the budget-constrained IRS, and study the impact of imposing budget constraints on the IRS. The second part is normative. We examine the welfare implication of varying the size of the budget allocated to the IRS and, in particular, we ask: how much should we fund the IRS?

Unlike the classical work of Allingham and Sandmo (1972) and Yitzhaki (1974) on tax evasion, which treats the IRS actions as exogenous, the GRW model views the IRS as a

[1] "The IRS Oversight Board was created by the IRS Restructuring and Reform Act of 1998 (RRA 98), which was enacted to improve the IRS so that it may better serve the public and meet the needs of taxpayers." (see the web-site of the Board)

strategic player that interacts with taxpayers. The GRW model also differs from the principal-agent tax evasion model first introduced by Reinganum and Wilde (1985). As pointed out by GRW, the principal-agent model suffers from the time inconsistency problem since it requires that the IRS announce and commit to an audit policy, even though the precommitted audit policy will typically prove suboptimal once taxpayers submit their reported income. GRW emphasize that their interactive model follows the natural temporal sequence of decisions: first, taxpayers report their income, and only then does the IRS decide whether to perform tax audits.[2]

Graetz, Reinganum and Wilde (1984, hereafter GRW1984) also extend the GRW model to account for the effect of imposing budget constraints on the IRS. However, their way of deriving equilibria is complicated becuase they take into account *direct* interactions between taxpayers. We greatly simplify the analysis by utilizing the property of a large game so that only *indirect* interactions between taxpayers need be considered (see Section 3.3 for more elaboration). Note also that GRW1984 analyze the case where taxpayers are risk neutral, while we allow for risk-adverse taxpayers. Most importantly, we further address the welfare issue across equilibria whereas they do not.

Slemrod and Yitzhaki (1987, hereafter SY) investigate the same normative question as our paper. The main differences in modeling include: (i) while they study tax auditing with commitment, we study tax auditing without commitment; and (ii) while they subsume the IRS and the Congress under the rubric of a single player called "government," we treat the IRS and the Congress as two different players. Perhaps more interestingly, the policy prescription derived from our model starkly contrasts that derived from their model. SY prescribe that the size of the budget allocated to the IRS *should not* be expanded to the level where an additional dollar allocated would return just an additional dollar of revenue,

[2]For recent surveys of the tax evasion literature, see Andreoni et al (1998), Slemrod and Yitzhaki (2002), Cowell (2004) and Sandmo (2005).

no matter how high the benefit that an addtional dollar of revenue may bring about to a society.[3] By contrast, we prescribe that, as long as the benefit that an additonal dollar of revenue brings about is high enough, the size of the budget allocated to the IRS *should* be expanded until an additional dollar allocated would return just an additional dollar of revenue. We will make more comparisons between our result and SY's when we address the welfare issue.

The remainder of this article is organized as follows. Section 2 describes the model. Section 3 provides the full characterization of equilibrium outcomes. Section 4 addresses the welfare issue and Section 5 concludes.

2 Model

Our model is essentially the same as the GRW model. For ease of exposition, however, we transform the GRW model into an equivalent, but simpler, model. GRW assume that taxpayers earn either high or low income. High-income taxpayers need to pay a high tax, while low-income taxpayers need to pay a low tax. We normalize the low income of the GRW model to zero so that a taxpayer either earns an income or does not. Only the taxpayers who earn the income need to pay tax in our model. GRW defend their simple setup by arguing that "the model might also be viewed as addressing issues of noncompliance across a relatively small range of income–for example, within a given audit class" (GRW, p. 17).

Suppose that there is a unit mass of continuum taxpayers who may earn an income $y > 0$. This income need not be the total income earned. It may simply represent a particular type of income, say, income from vehicle sales or tip income. Those taxpayers who have income y are obliged to pay a positive tax T $(< y)$, while those who do not have income y are obliged to

[3] This conclusion is also drawn by Usher (1986), Kaplow (1990), Mayshar (1991), and Sanchez and Sobel (1993).

pay nothing. The IRS knows that there is a $q \in (0,1)$ portion of taxpayers who have y, and a $1-q$ portion of taxpayers who do not have y. However, the IRS cannot identify a priori which taxpayer has y and which taxpayer does not have y. As emphasized by GRW, one of the distinct features of modern systems of income taxation is their self-reporting nature: the tax law requires taxpayers to file tax returns and report their own income to the IRS. The taxpayers who do not have y will always report nothing to the IRS truthfully. However, some taxpayers who have y may cheat and also report nothing. A cheater is subject to a fine $F > 0$ if he is discovered cheating by the IRS. This fine is imposed in addition to the tax T due with $T+F \leq y$ (the limited liability constraint). The taxpayers who have y are assumed to possess a common von Neumann-Morgenstern utility function over income, namely, $u : \mathbb{R}_+ \to \mathbb{R}_+$ with $u' > 0$ and $u'' < 0$. They maximize the expected utility by choosing to report y honestly or to masquerade as taxpayers who do not have y and report nothing.

After receiving a taxpayer's report, the IRS can decide whether to perform an investigative tax audit. Auditing is costly and the IRS has to bear a cost of $c > 0$ to verify each taxpayer. We assume as in GRW that the truth will be discovered once a tax audit is performed and that it always pays off for the IRS to audit an evader (i.e. $T + F > c$). Under a given budget constraint I, the IRS's objective is to maximize the tax revenue collected (including taxes and fines), net of audit costs through auditing. Because auditing is costly, it is clear that the "profit-maximizing" IRS will only audit those taxpayers who report nothing. We assume that none of the taxpayers bear any additional cost during the auditing process. We also assume that the IRS cannot use the taxes or fines collected to finance her own auditing expenses.[4] The IRS takes the tax T, the fine F and the budget I as given in her auditing

[4] Wertz (1979, p. 144) describes the rule: "An agency [the IRS] may not spend more on enforcement activities in a budget period than its legislature has appropriated for them. Deficiencies collected throughout the period are transmitted to the general government; they may not be used by the agency to expand its activities."

since these variables are predetermined by the Congress. Our focus is on the impact of I.[5]

The timing of this game is as follows. Given the realization of y, c, T, F and I, those taxpayers who do not have y always report nothing, and those taxpayers who have y simultaneously and independently choose whether to report y or not. After observing the taxpayers' reports, the IRS randomly chooses to audit a fraction of taxpayers who do not report y. We solve the equilibrium of this game under the condition that the IRS's strategies are restricted to depend on the distribution of the taxpayers' reports only.[6]

3 Equilibrium

3.1 Characterization

We use the notation α to denote the portion of cheaters among all taxpayers,[7] and the notation $\beta(\alpha)$ to denote the IRS's best audit response to α. It is clear that $\alpha \in [0, q]$ and $\beta(\alpha) \in [0, 1]$. Note that the IRS can observe α in our model. This is because the IRS knows by assumption that there is a $q \in (0, 1)$ portion of taxpayers who have y, but only a $q - \alpha$ portion of taxpayers report having y to the IRS.

For any $\alpha \in [0, q]$, let $R(\alpha)$ denote the IRS's (gross) expected revenue from a single audit. Thus,

$$R(\alpha) = \frac{\alpha}{\alpha + (1 - q)} (T + F)$$

where $\alpha + (1 - q)$ is the portion of taxpayers who do not report y. $R(\alpha)$ represents the marginal

[5] Both the tax T and the fine F are also under the control of the Congress. However, their determination is not on the yearly basis as the budget I.

[6] This restriction implies that a unilateral deviation by a single taxpayer cannot influence the course of our game. It is a natural regularity requirement when there are many players; see, for example, Gul et al (1986).

[7] The notation α in the GRW model denotes the portion of actual cheaters among those who are *potential cheaters*, while it denotes the portion of cheaters among *all taxpayers* in our model. That is, our α equals GRW's $q\alpha$.

revenue of tax collection to the IRS when there is the α amount of cheaters. Observe that $R(0) = 0$, $R(q) = q(T + F)$, and $\frac{\partial R}{\partial \alpha} > 0$.

To characterize the equilibrium, we need to define several other notations. First, define $\bar{\beta}$ as the probability of audit such that a taxpayer who has income y is merely indifferent between reporting y and not reporting y. That is,

$$u(y - T) = \bar{\beta} u(y - T - F) + \left(1 - \bar{\beta}\right) u(y).$$

Secondly, define $\bar{\alpha}$ as the amount of α such that the IRS is merely indifferent between auditing and not auditing. That is,

$$R(\bar{\alpha}) = c. \tag{1}$$

Finally, define $\hat{\alpha}$ as the amount of α such that the IRS uses $\bar{\beta}$ as the audit probability and just exhausts all her budget. That is,

$$\bar{\beta}(\hat{\alpha} + 1 - q)c = I.$$

For convenience, we shall call "the taxpayer who has y" simply "the taxpayer" from now on. We use (α^*, β^*) to denote the outcome of an equilibrium and let Γ be the set of all equilibrium outcomes. The following proposition characterizes the set of equilibrium outcomes of the game.[8]

Proposition 1 *(i) If $R(q) \in [0, c)$, then $\Gamma = \{(q, 0)\}$.*

(ii) If $R(q) = c$, then $\Gamma = \{(q, \beta) : \beta \in [0, \min\{\frac{I}{c}, \bar{\beta}\}]\}$.

(iii) If $R(q) > c$, then

a) $\Gamma = \{(q, \frac{I}{c})\}$ for $I \in [0,\ \bar{\beta}(\bar{\alpha} + 1 - q)c)$;

b) $\Gamma = \{(\bar{\alpha}, \bar{\beta}), \left(\hat{\alpha}, \bar{\beta}\right), \left(q, \frac{I}{c}\right)\}$ for $I \in [\bar{\beta}(\bar{\alpha} + 1 - q)c, \bar{\beta}c]$;

c) $\Gamma = \{(\bar{\alpha}, \bar{\beta})\}$ for $I \in (\bar{\beta}c,\ \infty)$.

[8] The proofs of our propositions are all relegated to the Appendix.

There are the so-called “hard-to-tax” taxpayers, who could be defined as those whose “tax amounts are quite low compared with the administration costs that would have to be incurred by the tax administration to assess the proper amount of tax.” (Thuronyi, 2004, p. 102). This definition corresponds to the case where $R(q) \in [0, c]$ in our model. “Hard-to-tax” is not the same as “impossible-to-tax” after all. It is simply not profitable for the IRS to audit these taxpayers.[9] As a result of lacking the motivation to audit, a very high level of evasion results in equilibrium ($\alpha^* = q$ in Proposition 1 (i) and (ii)).

Wertz (1979) observes that the IRS is often expected by the Congress to “show a profit” on her enforcement activities. This could aggravate the “hard-to-tax” problem since spending audit efforts on “hard-to-tax” taxpayers is simply not cost-effective. Fixing the “hard-to-tax” is a thorny task, and alternative strategies such as exempting these taxpayers or simply ignoring them have been proposed. We refer those who are interested in the issue to Alm et al. (2004). For the rest of this paper, our analysis will be confined to Proposition 1 (iii) where $R(q) > c$ holds.

3.2 Graphic illustration

The intuition underlying Proposition 1 (iii) is best understood in terms of the best-response graphs of the taxpayers and of the IRS. Figures 1a, 1b and 1c illustrate a), b) and c) of Proposition 1 (iii), respectively. In each figure, the dotted curve represents the taxpayers' best response while the solid curve represents the IRS's. Note that taxpayers adopt pure rather than mixed strategies in our model. In drawing the dotted curve, one can imagine that there exists a representative taxpayer who will choose evasion with a probability of $\frac{\alpha}{q}$ and compliance with a probability of $\frac{q-\alpha}{q}$. The pure-strategy outcomes are then realized

[9] Former IRS Commissioner Lawrence B. Gibbs was reported to have stated that the IRS will not collect “small amounts owed by such a huge number of taxpayers that collection efforts would not be cost-effective” (Los Angeles Times, 1987).

through the representative taxpayer's infinite trials so that, due to the law of large numbers, the α amount of taxpayers will evade while the $q-\alpha$ amount of taxpayers will comply.

[Insert Figure 1 about here]

Note that the shape of the taxpayers' best-response curve remains the same as that in GRW. Note also that the shape of the IRS's best-response curve remains the same as that in GRW if $\alpha < \bar{\alpha}$. However, the shape of the IRS's best-response curve changes if $\alpha \geq \bar{\alpha}$. Two salient features in the change stand out. First, the result $\beta(\alpha) \in [0,1]$ would be true at $\alpha = \bar{\alpha}$ if there were no budget constraint. This is because $\bar{\alpha}$ is by definition the amount of α such that the IRS is indifferent between auditing ($\beta = 1$) and not auditing ($\beta = 0$). However, when the budget constraint is imposed, it may no longer be feasible for the IRS to support any $\beta \in [0,1]$ as she would wish. Instead, we have $\beta(\alpha) \in [0, \min\{\frac{I}{c(\alpha+1-q)}, 1\}]$ at $\alpha = \bar{\alpha}$. In terms of the graph, the height of the IRS's best response curve at $\alpha = \bar{\alpha}$ may fall short of 1 as shown in Figures 1a and 1b. Secondly, when $\alpha > \bar{\alpha}$, $R(\alpha) > c$ will hold so that an incremental dollar of audit input could return more than an incremental dollar of revenue. As a result, the "profit-maximizing" IRS would audit for sure if there were no constraint on her budget. That is, $\beta(\alpha) = 1$ would hold for all $\alpha > \bar{\alpha}$. This may no longer be true when the budget constraint is imposed. Specifically, a binding budget constraint will bring down the feasible probability of audit that the IRS can support and, moreover, the larger the amount of evaders (i.e. a higher α), the lower the probability of audit that these evaders will face (i.e. a lower β). This "congestion" effect is captured in Figure 1 by the downward-sloping part of the IRS's best response curve as $\alpha > \bar{\alpha}$.

The intersection of the dotted and the solid curve in Figure 1 pins down the equilibrium of the game. There are three intersections in Figure 1b, while there is a single intersection in both Figures 1a and 1c. The former intersections represent the three possible equilibria characterized in b) of Proposition 1 (iii), while the latter intersection represents the unique equilibrium characterized in a) and c) of Proposition 1 (iii), respectively.

3.3 Discussion

Habitual compliers

The original GRW model incorporates taxpayers who are inherently honest, in the sense that they report thier incomes truthfully regardless of the incentive to cheat. GRW call these taxpayers "habitual compliers." Nothing essentially changes by introducing these habitual compliers to the model. In view of this, our analysis is confined to the "strategic" taxpayers. One may view the revenue collected from these strategic tapxayers as the extra revenue in addition to that from the habitual compliers.

Nash versus subgame-perfect equilibrium

Taxpayers report their income to the IRS before auditing and, therefore, they are first movers in the auditing game. However, unlike the leader in a Stackelberg game, knowing the IRS's response $\beta(\alpha)$ does not help the taxpayers much because no taxpayer can alter α by his single deviation and hence no taxpayer can affect β by his income report (remember our regularity assumption that the IRS's strategies only depend on the distribution of the taxpayers' reports and that all taxpayers simultaneously and independently report their incomes). As a result of this "impotent" feature, the taxpayers de facto take the IRS's audit probability β as given. This explains why the subgame perfect equilibrium coincides with the Nash equilibrium in our model. This coincidence is consistent with Kalai's (2004) observation that the equilibria of simultaneous-move games are robust to a large variety of sequential modifications when there are many players.

Comparison with GRW1984

The defining feature that distinguishes our model (with budget constraints) from the GRW model (without budget constraints) is the presence of the congestion effect: holding the IRS's budget constant, the higher the incidence of evasion, the lower the audit probability that an evader will face. This congestion effect is exhibited in Figure 1 through the modification of the IRS's best response in the GRW model. As to the taxpayers' best response, it remains the same as that in the GRW model.

GRW1984 adopt a different approach: the congestion effect is incorporated into their model through the modification of the taxpayers' rather than the IRS's best response in the GRW model. They first derive the so-called "taxpayer equilibrium" (all the taxpayers make mutually best responses to each other) as an intermediate step to characterize the full equilibria of the game. As might be imagined, their way of deriving equilibria is more complicated and may not be easy to follow intuitively. In particular, they characterize their equilibria by the so-called "probability of audit given exposure" rather than the "probability of audit" in the GRW model. We avoid the step of deriving "taxpayer equilibrium" altogether by utilizing the property of a large game. The likelihood of audit facing one taxpayer also depends on the reporting strategy of other taxpayers in our model. However, unlike GRW1984, there is no *direct* interaction between the reporting strategies of the (continuum) taxpayers. We characterize the equilibria by the "probability of audit" as in the GRW model. Similar to GRW, our equilibria are characterized simply by considering the interaction between the IRS and a *representative* taxpayer (compare our Proposition 1 or Figure 1 with GRW1984's Proposiiton 4 or Figure 2).

Budget surplus

The profit-maxmizing objective function of the IRS follows GRW and is standard in the tax evasion literature. GRW consider other possible IRS objective functions, but conclude that the profit-maximizing objective "adequately captures both the general and the specific deterrence objectives often attributed to IRS enforcement policy." (GRW, p. 29)

Sticking to the assumption that the IRS maximizes her "profit," a budget surplus becomes possible as long as the IRS is not budget constrained in equilibrium. For example, when the equilibrium outcome $(\alpha^*, \beta^*) = (\bar{\alpha}, \bar{\beta})$ occurs, it is possible that the audit cost expended by the IRS will be less than the budget size appropriated by the Congress (i.e. $\bar{\beta}(\bar{\alpha}+1-q)c < I$). This possiblity raises a subtle question which seemed to go unnoticed in the past literature, namely, how the profit-maximizing IRS would "deal with" the budget surplus. It should be enphasized that the situation of a budget surplus will always result in GRW's budget-unconstrained model. This is because the budget size in the GRW model is always feasible for the IRS to support $\beta = 1$ for all $\alpha \geq \bar{\alpha}$, but $\beta^* = \bar{\beta} < 1$ in equilibrium. We will not address the "budget surplus" question directly in this paper. Instead, we keep the basic framework of the GRW model and devise a simple scheme to achieve two ends: (i) preserving the IRS objective of profit maximization, and (ii) forcing the IRS to conserve the use of the allocated budget and return the unused money back to the Congress.

Melumad and Mookherjee (1989) argue that it is difficult for a government to commit to the *allocation* of aggregate audit costs or aggregate revenues collected, but it is reasonable to assume that the government can make commitments based on these *aggregate* variables since they are publicly available as part of the process of budgetary appropriations and reviews of tax-collection agencies. In line with this argument, our scheme consists of two aggregate variables: the total tax revenue collected ($G \equiv (q - \alpha)T + \beta(\alpha + 1 - q)R(\alpha) = (q - \alpha)T + \alpha\beta(T + F)$) and the budget surplus generated ($B \equiv I - \beta\,(\alpha + 1 - q)\,c$). The Congress uses the sum $G + B$ to evaluate the IRS's performance (the larger the sum, the higher the score), or even offers a fixed fraction of the sum $G + B$ to the IRS as her bonus.

If the IRS were to maximize G alone, she would exhaust all the budget with $B = 0$ even though an additional dollar of audit input could not return an additonal dollar of tax revenue (i.e. $R(\alpha) < c$). On the other hand, if the IRS were to be motivated to maximize B alone, she would simply do nothing and generate the budget surplus $B = I$, even though an additional dollar of audit input could return more than an additional dollar of tax revenue (i.e. $R(\alpha) > c$). Since the IRS is motivated to maximize the sum of G and B rather than either of them alone, she needs to trade off the loss of B against the gain in G when carrying out a tax audit. If $R(\alpha) > c$, the loss of B through expended audit cost will be more than compensated by the gain in G through collected tax revenue and, as a result, it will be worthwhile for the

IRS to carry out the tax audit. On the other hand, if $R(\alpha) < c$, the loss of B will not be compensated by the gain in G and, therefore, it will be not worthwhile for the IRS to carry out the tax audit. This trade-off between G and B at the margin will drive the IRS to equate $R(\alpha)$ (the marginal revenue of tax collection) with c (the marginal cost of tax collection) as far as possible and, at the same time, conserve the use of the allocated budget as much as possible. In other words, the proposed scheme has achieved the two ends stated within our framework.[10]

4 Welfare

In this section we turn our attention to the welfare issue across different equilibria as the budget size I varies. Our purpose is to make an attempt to answer the normative question in our context: how much should we fund the IRS?

4.1 Cost of evasion

Following Cowell (1990), we define the "cost of evasion" as the monetary amount that a taxpayer would just be prepared to pay in order to be guaranteed that he will get away with the tax evasion. It is an amount C such that

$$u(y-C) = \beta u(y-T-F) + (1-\beta) u(y). \tag{2}$$

The amount C can be decomposed into two components: the tax (including the fine) that a taxpayer expects to pay (r) and the risk premium that the taxpayer would be ready to pay in order to eliminate the exposure to audit risk (θ). That is, $C = r + \theta$, where $r = \beta(T+F)$. Note that $\theta > 0$ as long as $u'' < 0$. Yitzhaki (1987) calls the risk premium θ the "excess

[10] It must be admitted that many practical complications may arise if our scheme is put into effect in the real world. Nevertheless, the scheme may still serve as a useful start for taking into consideration these complications.

burden of tax evasion" since it represents a deadweight loss beyond what would be imposed on the taxpayer if the expected tax revenue r were somehow collected by a lump-sum tax.

By the implicit function theorem, we have C as a function of β. Taking the total derivative of equation (2) with respect to β gives

$$u'(y-C)(-C_\beta) = u(y-T-F) - u(y).$$

Hence,

$$C_\beta = \frac{u(y) - u(y-T-F)}{u'(y-C(\beta))} > 0 \tag{3}$$

and

$$C_{\beta\beta} = \frac{[u(y) - u(y-T-F)]u''(y-C(\beta))C_\beta}{[u'(y-C)]^2} < 0. \tag{4}$$

We then have

Lemma 1 *$C(\beta)$ is a strictly increasing and concave function with C(0)=0.*

If $\beta = 0$, it is clear from (2) that $C = 0$ must hold.

Note that if $\alpha^* = \bar{\alpha}$ or $\alpha^* = \hat{\alpha}$, we can replace T (the tax paid by compliers) with $C(\bar{\beta})$ (the cost imposed on evaders). The reason is that when $\alpha^* = \bar{\alpha}$ or $\alpha^* = \hat{\alpha}$, some taxpayers will evade while others will not. All these taxpayers must be indifferent between evasion and compliance and, therefore, we have $C(\bar{\beta}) = T$. Since $T = C(\bar{\beta}) = \bar{\beta}(T+F) + \theta$ with $\theta > 0$, this immediately leads to

Lemma 2 $C(\bar{\beta}) = T$ *impies that* $T > \bar{\beta}(T+F)$

4.2 Social welfare

Let I^* denote the amount of audit cost expended by the IRS in equilibrium. Under an equilibrium outcome (α^*, β^*), the total full cost imposed upon the q taxpayers (the private sector) equals $qC(\beta^*)$, while the corresponding total net tax revenue collected by the IRS equals $G(\alpha^*, \beta^*) - I^*$. We assume that the social welfare of the economy can be represented by the function $W = v(G - I) - qC$ with $v' > 0$ and $v'' < 0$. This welfare function is standard, in the sense that while tax enforcement causes a reduction in the economy's welfare (i.e. $\frac{\partial W}{\partial qC} < 0$), the resulting net revenue collected can be used to provide, say, public goods to enhance the economy's welfare (i.e. $\frac{\partial W}{\partial (G-I)} > 0$). In particular, this welfare function is basically the same as the benevolent government's objective considered by Mayshar (1991).

SY (1987) emphasize that the revenue collected from taxpayers merely represents a transfer from the private to the public sector and, therefore, it should not be counted as a cost to society. They employ the excess burden of tax evasion imposed on evaders plus the audit cost expended by the IRS as the total social cost to society. Assume that the government is constrained to raise a given amount of revenue, the social optimization problem in SY can be interpreted as minimizing the total social cost of evasion (see SY, p. 187).

SY subsume the IRS and the Congress under the rubric of a single player called "government." Through choosing both the probability of audit and the tax rate, the society is constrained to raise a given amount of revenue in their model. By contrast, the IRS and the Congress are two different players in our model. Because the tax structure is predetermined by the Congress and so is beyond the IRS's control, the revenue collected by the IRS will be as a rule variable and not fixed. Note that $C = r + \theta$ (cost of evasion equals the tax that a taxpayer expects to pay plus the excess burden of tax evasion) and hence $qC = G + EB$, where EB denotes the sum of θ over all evaders. Thus, minimizing $qC - [G - I]$ is equivalent to minimizing $EB + I$ (i.e. the excess burden of tax evasion imposed on evaders plus the audit cost expended by the IRS). In other words, minimizing $qC - [G - I]$ in our model can be interpreted as minimizing the total social cost of evasion as in SY (1987).

However, there is a key difference: while $G = G_0$ (a fixed G) in SY, $G = G(\alpha^*, \beta^*)$ (a variable G) in our model. A possible defect with the objetive of minimizing $qC - [G - I]$ as G is variable is that a low value of $qC - [G - I]$ may be associated with a small amount of net revenue collection. Indeed, without taking into account G, the best option for minimizing $qC-[G-I]$ is simply to let $I=0$ so that $qC-[G-I]=0$ (or $I\to 0$ so that $qC-[G-I]\to 0$ if $G-I>0$ is required).

4.3 Optimal size of IRS budget

By our assumption that the IRS is not allowed to use the taxes or fines collected to finance her own audit cost, we have $I^*\leq I$. Let $\bar{I}\equiv\bar{\beta}\left(\bar{\alpha}+1-q\right)c$, which is the minimal size of the budget that is capable of supporting $(\bar{\alpha},\bar{\beta})$ (see Proposition 1 (iii)). Table 1 summarizes the full cost imposed qC and the net revenue collected $G-I$ as the equilibrium outcome (α^*,β^*) varies with I. Note that both qC and $G-I$ remain the same for all $I\geq\bar{I}$ if $(\alpha^*,\beta^*)=(\bar{\alpha},\bar{\beta})$. This result is due to our scheme of forcing the IRS to conserve the use of the allocated budget and return the unused money back to the Congress (see Section 3.3).

Table 1.

IRS's budget I	(α^*,β^*)	Full cost imposed qC	Net revenue collected $G-I$
$I<\bar{I}$	$(q,\frac{I}{c})$	$qC\left(\frac{I}{c}\right)$	$\frac{I}{c}q(T+F)-I$
$I\in[\bar{I},\bar{\beta}c]$	$(q,\frac{I}{c})$	$qC\left(\frac{I}{c}\right)$	$\frac{I}{c}q(T+F)-I$
	$\left(\hat{\alpha},\bar{\beta}\right)$	$qC\left(\bar{\beta}\right)$	$\left(q-\hat{\alpha}\right)T+\hat{\alpha}\bar{\beta}(T+F)-I$
	$(\bar{\alpha},\bar{\beta})$	$qC\left(\bar{\beta}\right)$	$\left(q-\bar{\alpha}\right)T+\bar{\alpha}\bar{\beta}(T+F)-\bar{I}$
$I>\bar{\beta}c$	$(\bar{\alpha},\bar{\beta})$	$qC\left(\bar{\beta}\right)$	$\left(q-\bar{\alpha}\right)T+\bar{\alpha}\bar{\beta}(T+F)-\bar{I}$

Using Lemma 2, we prove a useful result.

Lemma 3 *Let $\hat{\alpha}\neq\bar{\alpha}$, then all equilibrium outcomes $\left\{\left(\hat{\alpha},\bar{\beta}\right)\right\}$ are strictly dominated by the equilibrium outcome $(\bar{\alpha},\bar{\beta})$ in terms of the social welfare W.*

We know that $\hat{\alpha}>\bar{\alpha}$ if $\hat{\alpha}\neq\bar{\alpha}$ (Figure 1b) and that $T>\bar{\beta}(T+F)$ (Lemma 2). Invoking these two results, it is straightforward to see from Table 1 that $G\left(\hat{\alpha},\bar{\beta}\right)<G\left(\bar{\alpha},\bar{\beta}\right)$ when

$I \in [\bar{I}, \bar{\beta}c]$ and $\hat{\alpha} \neq \bar{\alpha}$. This then leads to $G\left(\hat{\alpha}, \bar{\beta}\right) - I < G\left(\bar{\alpha}, \bar{\beta}\right) - \bar{I}$ since $\bar{I} \leq I$. From Table 1, we also see that $qC = qC(\bar{\beta})$ for both $\left(\bar{\alpha}, \bar{\beta}\right)$ and $(\hat{\alpha}, \bar{\beta})$. Putting these results together yields Lemma 3.

Lemma 3 allows us to narrow the welfare comparison simply between $(\bar{\alpha}, \bar{\beta})$ and $\{\left(q, \frac{I}{c}\right)\}$. That is, we want to know whether $W(\bar{\alpha}, \bar{\beta})$ is higher or lower than $W\left(q, \frac{I^{\#}}{c}\right)$, where

$$W(\bar{\alpha}, \bar{\beta}) \equiv v(G\left(\bar{\alpha}, \bar{\beta}\right) - \bar{I}) - qC(\bar{\beta})$$

$$W(q, \frac{I^{\#}}{c}) \equiv \max_{I \in [0, \bar{\beta}c]} [v(G\left(q, \frac{I}{c}\right) - I) - qC(\frac{I}{c})]$$

Figure 2-1 plots the resulting $G(\alpha^{*}, \beta^{*}) - I^{*}$ against the corresponding $qC(\beta^{*})$ as I varies. Note that $\frac{\partial(G(q, \frac{I}{c}) - I)/\partial I}{\partial(qC(\frac{I}{c}))/\partial I} = \frac{\frac{1}{c}q(T+F)-1}{qC'(\frac{I}{c})\frac{1}{c}}$, which is positive (since $C' > 0$ and $q(T+F) > c$) and increasing in I (since Lemma 1). Note also that $qC = qC(\bar{\beta})$ for $\left(\bar{\alpha}, \bar{\beta}\right)$, $(q, \bar{\beta})$ and $\{\left(\hat{\alpha}, \bar{\beta}\right)\}$, but $G\left(\bar{\alpha}, \bar{\beta}\right) - \bar{I} > G(\hat{\alpha}, \bar{\beta}) - I > G\left(q, \bar{\beta}\right) - I$ if $\bar{\alpha} \neq \hat{\alpha} \neq q$. These results explain the shape of the loci shown in Figure 2-1. Since $qC(\beta^{*}) > G(\alpha^{*}, \beta^{*}) - I^{*}$ at any I, the curve is located below the 45 degree line in the figure.

[Insert Figure 2 about here]

From the slope of the loci in Figure 2-1, one can derive the curve representing the term $\frac{\partial(qC)}{\partial(G-I)}$, which is the marginal cost of public funds (i.e. the full cost to the private sector of raising an additonal dollar of net tax revenue, denoted by MCPF). This MCPF curve is shown in Figure 2-2. It is interesting to observe that there is a range of $G - I$ in which MCPF is equal to zero. This is due to that while $qC = qC(\bar{\beta})$ for both $\left(\bar{\alpha}, \bar{\beta}\right)$ and $(q, \bar{\beta})$, $G\left(\bar{\alpha}, \bar{\beta}\right) - \bar{I} > G\left(q, \bar{\beta}\right) - I$.

[Insert Figure 3 about here]

The term v' represents the marginal benefit of public projects if funded by net tax revenue. Given the MCPF curve, whether $W(\bar{\alpha}, \bar{\beta})$ is higher or lower than $W\left(q, \frac{I^{\#}}{c}\right)$ critically depends on the position of the curve representing the term v'. There are three possibilities:

(i) Low v'

This possibility is shown in Figure 3-1. Figure 3-1a shows that there exists an interior $I^{\#}$ such that $W\left(q, \frac{I^{\#}}{c}\right) > W(\bar{\alpha}, \bar{\beta})$.[11] Note that $\left(\frac{d(G-I)}{d(qC)}\right)_{W=\text{constant}} = \frac{1}{v'(G-I)}$. Thus, this case tends to be associated with the situation where $G - I$ brings about a small v' or v' declines sharply as $G-I$ increases. Figure 3-1b shows the MCPF curve and the (low) v' curve ($v'' < 0$ by assumption).[12]

(ii) Medium v'

This possibility is shown in Figure 3-2. Figure 3-2a shows that there exists an interior $I^{\#}$ such that $W\left(q, \frac{I^{\#}}{c}\right) < W(\bar{\alpha}, \bar{\beta})$. Although $I^{\#}$ is the best choice in terms of W within $\{\left(q, \frac{I}{c}\right)\}$, the social welfare resulting from $(q, \frac{I^{\#}}{c})$ is lower than that from $(\bar{\alpha}, \bar{\beta})$. It is clear from Figure 3-2a that discrete jumps in $G(\alpha^*, \beta^*) - I^*$ as I varies is the key for the result. Figure 3-2b shows the MCPF curve and the (medium) v' curve.

(iii) High v'

This case is associated with a corner solution with $I^{\#} = \bar{\beta}c$. When $I^{\#} = \bar{\beta}c$, the equilibrium outcome $(q, \frac{I^{\#}}{c})$ coincides with the equilibrium outcome $\left(\hat{\alpha}, \bar{\beta}\right)$ such that $(q, \frac{I^{\#}}{c}) = \left(\hat{\alpha}, \bar{\beta}\right)$. This can be seen directly from the definition of $\hat{\alpha}$. From Lemma 3, we then have $W(\bar{\alpha}, \bar{\beta}) > W\left(q, \frac{I^{\#}}{c}\right)$. Figure 3-3a reflects this result. Note again that $\left(\frac{d(G-I)}{d(qC)}\right)_{W=\text{constant}} = \frac{1}{v'(G-I)}$. Thus, this case tends to be associated with the situation where $G - I$ brings about a large v' or v' declines slowly as $G - I$ increases. Again, discrete jumps in $G(\alpha^*, \beta^*) - I^*$ as I varies is clearly the key for the result. Figure 3-3b shows the MCPF curve and the (high)

[11] If v' is low enough, it is obvious that $I^{\#} = 0$ will be possible.

[12] If $v'' = 0$, then $I^{\#} = \bar{\beta}c$; if $v'' > 0$, then either $I^{\#} = 0$ or $I^{\#} = \bar{\beta}c$. Ruling out the unrealistic or unteresting case of $I^{\#} = 0$, we would have $I^{\#} = \bar{\beta}c$, which would in turn lead to $W\left(q, \frac{I^{\#}}{c}\right) < W(\bar{\alpha}, \bar{\beta})$; see possibility (iii).

v' curve.

To sum up, we state

Proposition 2 *Suppose that v' is high enough. Then, of all possible equilibrium outcomes, $(\bar{\alpha}, \bar{\beta})$ yields the highest social welfare.*

The discontinuous or discrete jumps in $G(\alpha^*, \beta^*) - I^*$ as I varies is the key to uphold Proposition 2. The reason for these discontinuous or discrete jumps is obviously attributable to the existence of multiple equilibria, which are in turn attributable to the congestion effect resulting from the IRS's constrained budget. The statement "v' is high enough" in Proposition 2 is admittedly imprecise or arbitrary to some extent. However, as will be seen shortly, this is a crucial difference between our result and the previous ones.

[**Insert Figure 4 about here**]

Suppose that $(\bar{\alpha}, \bar{\beta})$ is not the equilibrium outcome at the status quo. If we pour more resources into the IRS, the IRS's best-response curve will be shifted upward as shown in Figure 4. If we keep on pouring, it is clear that $(\bar{\alpha}, \bar{\beta})$, similar to that shown in Figure 1c, will eventually result as the unique equilibrium outcome. In contrast to outcomes $(\hat{\alpha}, \bar{\beta})$ and $(q, \frac{I}{c})$ where the marginal revenue is greater than the marginal cost of tax collection (i.e. $R(\hat{\alpha}) > c$ and $R(q) > c$), the IRS equates the marginal revenue to the marginal cost of tax collection under the outcome $(\bar{\alpha}, \bar{\beta})$ (i.e. $R(\bar{\alpha}) = c$). Since outcome $(\bar{\alpha}, \bar{\beta})$ yields the highest welfare across all possible equilibrium outcomes provided that v' is high enough, we obtain

Corollary 1 *Suppose that v' is high enough. Then the size of the budget allocated to the IRS should be expanded as long as an additional dollar allocated could return more than an additional dollar of revenue.*

A tax farmer, who is interested only in profit maximization, will expand the size of her audit resources if an additional dollar of audit input could return more than an additional dollar of revenue. Corollary 1 requires that the Congress support the "IRS as tax farmer" *provided that v' is high enough.*[13] This policy prescription contrasts with SY's (1987) finding that the marginal revenue exceeds the marginal cost of tax collection at the optimum *regardless of whether v' is high or low* and, consequently, the "IRS as tax farmer" would lead to a socially excessive amount of resources devoted to tax collection. Put differently, the Congress should always provide a smaller budget than the IRS would wish according to SY, whereas the Congress should provide the budget that the IRS would wish as long as v' is high enough according to our model.

To ensure that $(\bar{\alpha}, \bar{\beta})$ is the unique equilibrium outcome, $I > \bar{\beta}c$ must hold (see Proposition 1 or Table 1). Since the size of the population who report no income equals $(\alpha^* + 1 - q)$ in equilibrium and since the IRS incurs a cost c for each person it verifies, the meaning of the inequality $I > \bar{\beta}c$ is clear: even if $\alpha^* = q$ (i.e. all taxpayers evade) so that $\alpha^* + 1 - q = 1$, the size of the budget allocated should still enable the IRS to support an audit probability higher than $\bar{\beta}$ (i.e. $\frac{I}{c(\alpha^*+1-q)} = \frac{I}{c} > \bar{\beta}$ when $\alpha^* = q$). The intuition behind this result is simple. The taxpayers will comply if they expect $\beta > \bar{\beta}$. When $I > \bar{\beta}c$, it is feasible for the IRS to support an audit probability higher than $\bar{\beta}$ at all possible realized α's. This feasibility completely eliminates the taxpayers' self-fulfilling expectations that a widespread and rampant evasion may "congest" the IRS's tax administration to such an extent that it

[13] Should the IRS be simply privatized? Answering this question would take us beyond the scope of the present paper. Some would argue that collection costs tend to be lower for private than public agents (Toma and Toma, 1992). However, one might worry about whether taxpayers' private information should be possessed by private agents. Through H.R. 4520, American Jobs Creation Act of 2004, the Congress gives the IRS the authority to use private collection agencies to collect IRS debt and pay them a bounty of up to 25 percent of the money they collect. This statute is strongly opposed by National Treasury Employees Union. One reason raised for the opposition is: "the IRS does not have the technology in place to ensure that taxpayer information is kept secure and confidential when it is handed over to the private collection agencies." (Kelley, 2005)

becomes impossible for the IRS to maintain $\beta > \bar{\beta}$ at some high α's. In other wrods, the congestion effect resulting from the IRS's constrained budget is completely eliminated so that $(\alpha^*, \beta^*) = (\bar{\alpha}, \bar{\beta})$ must result. By contrast, the taxpayers' self-fulfilling expectations could support the realization of $\alpha^* = \hat{\alpha}$ or $\alpha^* = q$ as long as $I \leq \bar{\beta}c$ (see Table 1). As a consequence, $(\alpha^*, \beta^*) = (\bar{\alpha}, \bar{\beta})$ may no longer result.[14]

Our finding is a bit ironic in the following sense. While the IRS's budget constraint matters since it creates the congestion effect, yet the policy prescribed by Corollary 1, if implemented, would elimite the congestion effect, but at the same time take us back to the original GRW equilibrium without budget constraints imposed.

4.4 Intuition

As noted before, Slemrod and Yitzhaki (1987) and others, including Usher (1986), Kaplow (1990), Mayshar (1991) and Sanchez and Sobel (1993), all conclude that the size of the budget allocated to the IRS should fall short of equating the marginal revenue with the marginal cost of tax collection from the viewpoint of the IRS, whereas we conclude that it should equate the marginal revenue with the marginal cost of tax collection provided that the benefit that an additonal dollar of revenue brings about to a society is high enough. Is there any intuition behind the difference? In this subsection we provide one.

Mayshar (1991) views the maximal revenue collected as a function of the IRS's enforcement budget and other variables such as the tax base and tax structure. He calls this function a "tax technology." Like the standard production function of the firm, the tax technology is a "black box" and its details are left unspecified. Because of the "black box" nature, Mayshar's

[14] At $\alpha^* = q$, the maximal probability of audit that the IRS can support equals $\frac{I}{c}$. If $\frac{I}{c}$ is greater than $\bar{\beta}$, the equilibrium $(\bar{\alpha}, \bar{\beta})$ can be ensured. If $\frac{I}{c}$ is not greater than $\bar{\beta}$, the equilibrium $(\bar{\alpha}, \bar{\beta})$ cannot be ensured. We focus on lifting $\frac{I}{c}$ above $\bar{\beta}$ by increasing I. There is another side of the same coin: lifting $\frac{I}{c}$ above $\bar{\beta}$ by reducing c. We comment on this alternative possibility at the end of the paper.

tax technology can be interpreted to accommodate a variety of models, including Slemrod and and Yitzhaki's commitment model and our non-commitment model. Specifically, in terms of our notation, we can simply write $G = G(I)$, where $G(.)$ represents the tax technology. The net revenue or "profit" is then represented by $G(I) - I$.

What does $G(I) - I$ look like? Mayshar (1991) argues that it takes the shape of a Laffer curve. This shape seems to be typical for a profit function. Figure 5-1 basically duplicates Figure 1 in Mayshar (1991), in which a set of indifference curves of the economy's social welfare function is imposed on the "Laffer curve." It is easy to see from the figure that the optimal level of I is always lower than the level selected by the "profit-maximizing" IRS. SY (p. 187) offer an intuition behind the result: "This result follows immediately from the fact that increasing p [equivalent to increasing I in our model] decreases the representative individual's welfare, *ceteris paribus*." In other words, as long as the slopes of the indifference curve associated with the social welfare function is positive, the result follows. This contrasts with our finding that not only the sign but also the value of the slopes matters in the determination of the optimal level of I (i.e. the statement "if v' is high enough" in Corollary 1).

[Insert Figure 5 about here]

To see more clearly the difference, let us transform the graphs in Figure 5-1 into the graphs similar to those in Figure 3.[15] Suppose that the social welfare function is represented by $W = v(G - I) - k(I)$ with $k'(.) > 0$. The term $k(I)$ is similar to the term $qC(\beta)$ in our model, and $k'(.) > 0$ because tax enforcement by itself causes a reduction in an economy's welfare. Figure 5-2 plots the transformed graphs. Two key features survive after the transformation: (i) the slopes of the indifference curves associated with W remains positive (i.e. $\left(\frac{d(G-I)}{dk}\right)_{W=\text{constant}} = \frac{1}{v'(G-I)}$, and (ii) $G(I) - I$ against $k(I)$ still takes the shape of the

[15]One may also transform the graphs in Figure 3 into the graphs similar to those in Figure 5-1. The problem with this alternative route is that it is difficult to plot the indifference curves for $W = v(G-I) - qC$, since both $G - I$ and qC may jump as I varies.

Laffer curve even though it need not be concave any longer. Again, it is easy to see from Figure 5-2 that the optimal level of I is always lower than the level selected by the "profit-maximizing" IRS. Note in particular that whether v' is high or low does not matter for this result. By contrast, Figure 3 clearly shows that whether v' is high or low matters for our result. In summry, the policy prescription that the size of the budget allocated to the IRS should be expanded until an additional dollar allocated would return just an additional dollar of revenue will never be optimal according to Figure 5-2. By contrast, this policy prescription will be optimal according to Figure 3 as long as the benefit that an additonal dollar of revenue brings about is high enough.

5 Concluding remarks

We conclude our paper with three remarks. First, the model presented may well represent a particular audit class only, where the audit class is sorted on the basis of some observable taxpayer characteristics such as zip code, reported income level/source, occupation or age. GRW and Erard and Feinstein (1994), among others, interpret their audit rules within, not across, audit classes. The same kind of interpretation is equally applicable to our model.

Second, Kau and Rubin (1981, p. 262) hypothesize that "there have been changes in production technologies which have directly led to an increase in the proportion of income which is subject to taxation." These changes are attributable to factors such as fewer self-employed individuals, improved record keeping due to increased incorporation, and the substitution of market production for home production. All of these changes presumably lower the IRS's cost of tax audits. North (1985, p. 392) puts forth a similar hypothesis: "The supply of government was made possible by new technology which, coupled with the consequences of growing market specialization, lowered the costs of government monitoring of income and wealth and increased the efficiency of government taxation." Kau and Rubin (1981) find empirical support for their hypothesis, and Ferris and West (1996) provide additional empirical support. In terms of our model, a lower cost of tax audit has three main effects: (i) it turns some taxpayers from being "hard-to-tax" into "not-so-hard-to-tax" (i.e. from $R(q) \leq c$ to $R(q) > c$ in Proposition 1), (ii) it lowers the threshold evasion that makes the IRS indifferent between auditing and not auditing (i.e. a lower $\bar{\alpha}$ defined in equation (1)), and (iii) it raises the probability of audit that the IRS can support under a budget constraint (i.e. a higher $\frac{I}{c(\alpha+1-q)}$). These effects are obviously important and should not be ignored. Nevertheless, as far as the yearly budget appropriation is concerned, it does not seem unreasonable to view the audit cost c as a parameter, which is beyond the control of both the IRS and the Congress.[16]

Third, we provide a case for the policy prescription that the size of the budget allocated to the IRS should be expanded as long as an additional dollar allocated could return more than an additional dollar of tax revenue (Corollary 1). Of course, like findings in other theoretical models, this result is built upon several assumptions which abstract a parsimonious model from the complicated real world. An assumption of the GRW model, on which our model is based, is that individual incomes take one of only two values (either high or low). This assumption may be restrictive in that it reduces the taxpayer problem to a simple comply/do not comply decision. Other assumptions such as that true income will be discovered once a tax audit is performed, and that taxpayers suffer no additional cost during the auditing process may be problematic as well. It is arguable that the tax code itself is imperfect and that tax auditors are not uniform in interpreting the tax code. As a result, the so-called "true income" may never be known. "Mention the IRS, most people think of the dreaded tax audit." This vivid description of the IRS's tax audit by Slemrod and Bakija (2004, p. 180) suggests that the auditing process itself may be highly costly to taxpayers.

[16] The IRS is modernizing its forty-year-old information system through the Business Systems Modernization program. The implementation of this program is expected to reduce the IRS's audit cost in the future; see IRS Oversight Board (2006).

Note also that filing tax returns per se is assumed costless for individuals in our model. This seems inconsistent with the substantial efforts exerted by the IRS to provide the so-called "taxpayer service." Indeed, according to Professor Slemrod's (2005) testimony to the President's Advisory Panel on Federal Tax Reform, complying with the tax code per se costs individual taxpayers approximately $85 billion a year. Despite these and other possible limitations of our model, we believe we have brought a fresh perspective to the important issue of how much to fund the IRS. Kaplow (1996, p. 144) wrote:

> "In the academic literature, it is well understood (although not always remembered or emphasized) that the proper cost-benefit analysis does not simply compare the enforcement cost to the revenue raised."

This claim may need to be qualified based on the thrust of this paper.

6 Appendix

Proposition 1.

Proof. (i) If $R(q) \in [0, c)$, the IRS's incremental expected revenue from a tax audit will always be less than her audit cost spent, regardless of what α is. Hence, the IRS never audits, that is $\beta^* = 0$. Since the IRS never audits, the taxpayer has no incentive to report y and, as a result, $\alpha^* = q$.

(ii) Suppose that $R(q) = c$. If $\alpha^* < q$, the IRS has no incentive to audit since $R(\alpha) < c$ for all $\alpha < q$. This implies that $\beta(\alpha^*) = 0$. However, with $\beta(\alpha^*) = 0 < \bar{\beta}$, every taxpayer would strictly prefer cheating, that is, $\alpha^* = q$, which yields a contradiction. This leaves us only the case of $\alpha^* = q$. Given $R(q) = c$, the IRS is indifferent between auditing and not auditing, that is, $\beta(q) \in [0, \min\{\frac{I}{c}, 1\}]$. However, for all taxpayers to choose cheating, we require that $\beta(q) \leq \bar{\beta}$. Hence, we obtain $\beta^* \in [0, \min\{\frac{I}{c}, \bar{\beta}\}]$.

(iii) Suppose that $R(q) > c$. Since $R(0) = 0$ and $\frac{\partial R}{\partial \alpha} > 0$, we have a unique $\alpha \in (0, q)$ such that $R(\alpha) = c$. This unique α is the $\bar{\alpha}$ defined in (1). Note that the sign of $R(\alpha) - c$ is the same as the sign of $\alpha - \bar{\alpha}$. The IRS's best audit response to α with the budget constraint is thus given by

$$\beta(\alpha) = \begin{cases} \min\{\frac{I}{c(\alpha+1-q)}, 1\} & \text{if} \quad \alpha > \bar{\alpha} \\ \in [0, \min\{\frac{I}{c(\bar{\alpha}+1-q)}, 1\}] & \text{if} \quad \alpha = \bar{\alpha} \\ 0 & \text{if} \quad \alpha < \bar{\alpha} \end{cases}$$

where $\alpha > \bar{\alpha}$ implies $R(\alpha) > c$ so that the IRS will either exhaust all her budget with $\beta(\alpha) = \frac{I}{c(\alpha+1-q)}$ or reach $\beta(\alpha) = 1$; $\alpha < \bar{\alpha}$ implies $R(\alpha) < c$ so that it is not profitable for the IRS to carry out any tax audit with $\beta(\alpha) = 0$; and $\alpha = \bar{\alpha}$ implies $R(\alpha) = c$ so that the IRS is indifferent between auditing and not auditing.

A taxpayer's best response will depend on his expectation concerning β. If he expects $\beta > \bar{\beta}$, he will report y. If $\beta < \bar{\beta}$, he will report nothing. If $\beta = \bar{\beta}$, he is indifferent.

Suppose $\alpha^* < \bar{\alpha}$, then $\beta(\alpha^*) = 0$, which implies that every taxpayer strictly prefers cheating, that is, $\alpha^* = q > \bar{\alpha}$, a contradiction.

Suppose $\alpha^* = \bar{\alpha}$, then $\beta(\alpha^* = \bar{\alpha}) \in [0, \min\{\frac{I}{c(\bar{\alpha}+1-q)}, 1\}]$. Since $\bar{\alpha} \in (0, q)$, it is required that a taxpayer be indifferent between reporting y and reporting nothing. Because $\bar{\beta}$ is the audit probability that makes the taxpayer indifferent between reporting and not reporting, the only equilibrium in this case is $\beta(\alpha^* = \bar{\alpha}) = \bar{\beta}$. Note that $\beta(\alpha^* = \bar{\alpha}) \in [0, \min\{\frac{I}{c(\bar{\alpha}+1-q)}, 1\}]$. Therefore, $\frac{I}{c(\bar{\alpha}+1-q)} \geq \bar{\beta}$ or, equivalently, $I \geq \bar{\beta}\, c(\bar{\alpha} + 1 - q)$.

Suppose $\alpha^* \in (\bar{\alpha}, q)$, then $\beta(\alpha^*) = \min\{\frac{I}{c(\alpha^*+1-q)}, 1\}$. To support $\alpha^* \in (\bar{\alpha}, q)$, which implies that a taxpayer is indifferent between reporting y and not reporting, we need $\beta(\alpha^*) = \frac{I}{c(\alpha^*+1-q)} = \bar{\beta}$, that is, $\alpha^* = \hat{\alpha}$ and $\beta^* = \bar{\beta}$. Since $\hat{\alpha} \in (\bar{\alpha}, q)$, we have $I = \bar{\beta} c(\hat{\alpha} + 1 - q) \in (\bar{\beta}(\bar{\alpha} + 1 - q) c, \bar{\beta} c)$.

Suppose $\alpha^* = q$, then $\beta(\alpha^*) = \min\{\frac{I}{c(q+1-q)}, 1\} = \min\{\frac{I}{c}, 1\}$. To support $\alpha^* = q$, which implies that a taxpayer prefers cheating, it is required that $\beta(\alpha^* = q) = \frac{I}{c} \leq \bar{\beta}$. Hence, we obtain $I \leq c\bar{\beta}$.

To sum up, $(\alpha^*, \beta^*) = (\bar{\alpha}, \bar{\beta})$ could result if $I \geq \bar{\beta}c(\bar{\alpha} + 1 - q)$; $(\alpha^*, \beta^*) = (\hat{\alpha}, \bar{\beta})$ could ∇result if $\bar{\beta}(\bar{\alpha} + 1 - q) c < I < \bar{\beta}c$; and $(\alpha^*, \beta^*) = (q, \frac{I}{c})$ could result if $I \leq \bar{\beta}c$. ■

References

[1]

[2] Allingham, M.G, Sandmo, A., 1972. Income tax evasion: a theoretical analysis. Journal of Public Economics 1, 323-338.

[3] Alm, J., Martinez-Vazquez, J., Wallace, S. (Eds.), 2004. Taxing the Hard-to-Tax: Lessons from Theory and Practice. Elsevier Science, Amsterdam.

[4] Andreoni, J., Erard, B., Feinstein, J. 1998. Tax compliance. Journal of Economic Literature 36, 818-860.

[5] Bird, R.M., Casanegra de Jantscher, M., 1992. The reform of tax administration. In: R.M. Bird, M. Casanegra de Jantscher (Eds.), Improving Tax Administration in Developing Countries. IMF, Washington, DC.

[6] Cowell, F.A., 1990. Tax sheltering and the cost of evasion. Oxford Economic Papers 42, 21-243.

[7] Cowell, F.A., 2004. Sticks and carrots in enforcement. In: H.J. Aaron, J. Slemrod (Eds.), The Crisis in Tax Administration. Brookings Institution Press, Washington, DC.

[8] Cowell, F.A., Gordon, J.P.F., 1995. Auditing with ghosts. In: G. Fiorentini, S. Peltzman (Eds.), The Economics of Organized Crime. Cambridge University Press, Cambridge.

[9] Erard, B., Feinstein, J.S., 1994. Honesty and evasion in the tax compliance game. RAND Journal of Economics 25, 1-19.

[10] Erard, B., Ho, C.-C., 2001. Searching for ghosts: who are the nonfilers and how much tax do they owe? Journal of Public Economics 8, 25-50.

[11] Erard, B., Ho, C.-C., Mapping the US tax compliance continuum. In: J. Alm, J. Martinez-Vazquez, S. Wallace (Eds.), Taxing the Hard-to-Tax: Lessons from Theory and Practice. Elsevier Science, Amsterdam.

[12] Ferris, J.S., West, E.G., 1996. Testing theories of real government size – US experience, 1959-89. Southern Economic Journal 62, 537-553.

[13] Graetz, M.J., Reinganum, J.F., Wilde, L.L., 1984. A model of tax compliance with budget-constrained auditors. Social Science Working Paper no. 506, California Institute of Technology.

[14] Graetz, M.J., Reinganum, J.F., Wilde, L.L., 1986. The tax compliance game: toward an interactive theory of tax enforcement. Journal of Law, Economics, and Organization 2, 1-32.

[15] Gul, F., Sonnenschein, H., Wilson, R., 1986. Foundations of dynamic monopoly and the Coase conjecture. Journal of Economic Theory 39, 155-190.

[16] IRS Oversight Board, 2006. FY2007 IRS Budget Recommendations, Special Report. IRS Oversight Board, Washington, DC.

[17] Kalai, E., 2004. Large robust games. Econometrica 72, 1631-1665.

[18] Kaplow, L., 1990. Optimal taxation with costly enforcement and evasion. Journal of Public Economics 43, 221-236.

[19] Kaplow, L., 1996. How tax complexity and enforcement affect the equity and efficiency of the income tax. National Tax Journal 49, 135-150.

[20] Kau, J.B., Rubin, P.H., 1981. The size of the government. Public Choice 37, 261-274.

[21] Kelley, C.M., 2005. Statement of Collen M. Kelley, National Treasury Employees Union. Hearing Archives, Committee on Ways and Means.

[22] Los Angeles Times, 1987. Last-minute tax rush to delay refunds. April 14.

[23] Macho-Stadler, I., Perez-Castrillo, D.J., 1997. Optimal auditing with heterogeneous income sources. International Economic Review 38, 951-968.

[24] Mayshar, J., 1991. Taxation with costly administration. Scandinavian Journal of Economics 93, 75-88.

[25] Melumad, N.D., Mookherjee, D., 1989. Delegation as commitment: the case of income tax audits. RAND Journal of Economics 20, 139-163.

[26] North, D.C., 1985. The growth of government in the United States: an economic historian's perspective. Journal of Public Economics 28, 383-399.

[27] Reinganum, J.F., Wilde, L.L., 1985. Income tax compliance in a principal agent framework. Journal of Public Economics 26, 1-18.

[28] Reinganum, J.F., Wilde, L.L., 1986. Equilibrium verification and reporting policies in a model of tax compliance. International Economic Review 27, 739-760.

[29] Sanchez, I., Sobel, J., 1993. Hierarchical design and enforcement of income tax policies. Journal of Public Economics 50, 345-369.

[30] Sandmo, A., 2005. The theory of tax evasion: a retrospective view. National Tax Journal 58, 643-663.

[31] Slemrod, J., 2005. Statement of Professor Joel Slemrod, University of Michigan Ross School of Business, before the President's Advisory Panel on Federal Tax Reform. March 3.

[32] Slemrod, J., Bakija, J. 2004. Taxing Ourselves: A Citizen's Guide to the Debate over Taxes, Third Edition. MIT Press, Cambridge.

[33] Slemrod, J., Yitzhaki, S., 1987. The optimal size of a tax collection agency. Scandinavian Journal of Economics 89, 183-192.

[34] Slemrod, J., Yitzhaki, S., 2002. Tax avoidance, evasion, and administration. In: A.J. Auerbach, M, Feldstein (Eds.), Handbook of Public Economics, Vol. 3. North-Holland, Amsterdam.

[35] Thuronyi, V. 2004. Presumptive taxation of the hard-to-tax. In: J. Alm, J. Martinez-Vazquez, S. Wallace (Eds.), Taxing the Hard-to-Tax: Lessons from Theory and Practice. Elsevier Science, Amsterdam.

[36] Toma, E.-F., Toma, M., 1992. Tax collection with agency costs: private contracting or government bureaucrats? Economica 59, 107-120.

[37] Ueng, K.L.G., Yang, C.C., 2000. Taxation with little administration. Journal of Public Economics 75, 145-156.

[38] Wertz, K.L., 1979. Allocation by and output of a tax-administering agency. National Tax Journal 32, 143-56.

[39] Yitzhaki, S., 1974. A note on 'income tax evasion: a theoretical analysis'. Journal of Public Economics 3, 201-202.

[40] Yitzhaki, S., 1987. On the excess burden of tax evasion. Public Finance Quarterly 15, 123-137.

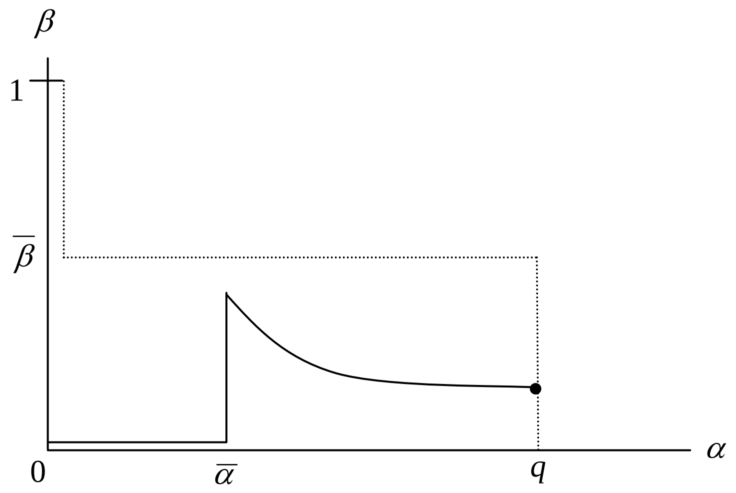

Figure 1a. $\alpha^* = q$

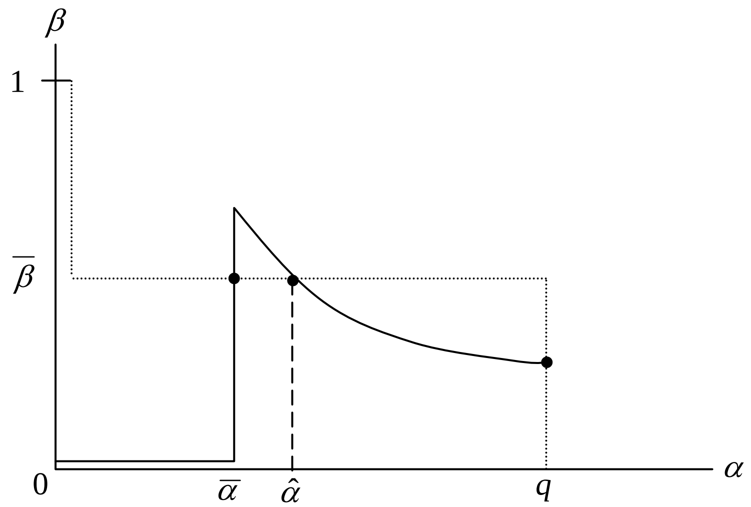

Figure 1b. $\alpha^* = \overline{\alpha}, \hat{\alpha},$ or q

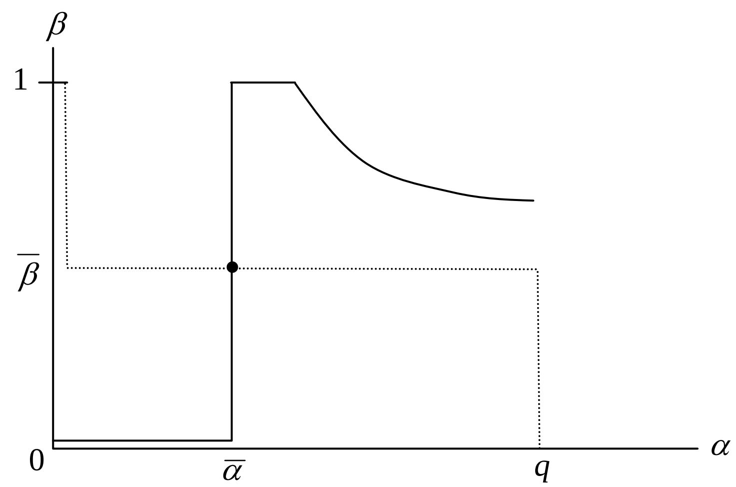

Figure 1c. $\alpha^* = \overline{\alpha}$

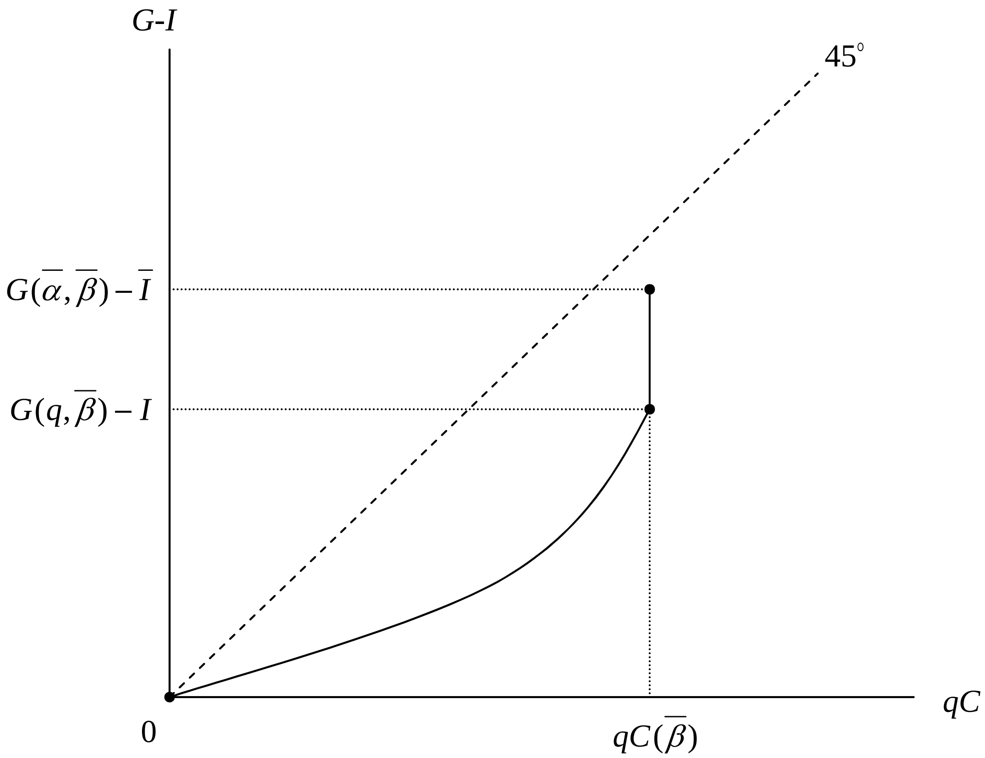

Figure 2-1. Loci of $G(\alpha^*,\beta^*)-I^*$ against $qC(\beta^*)$

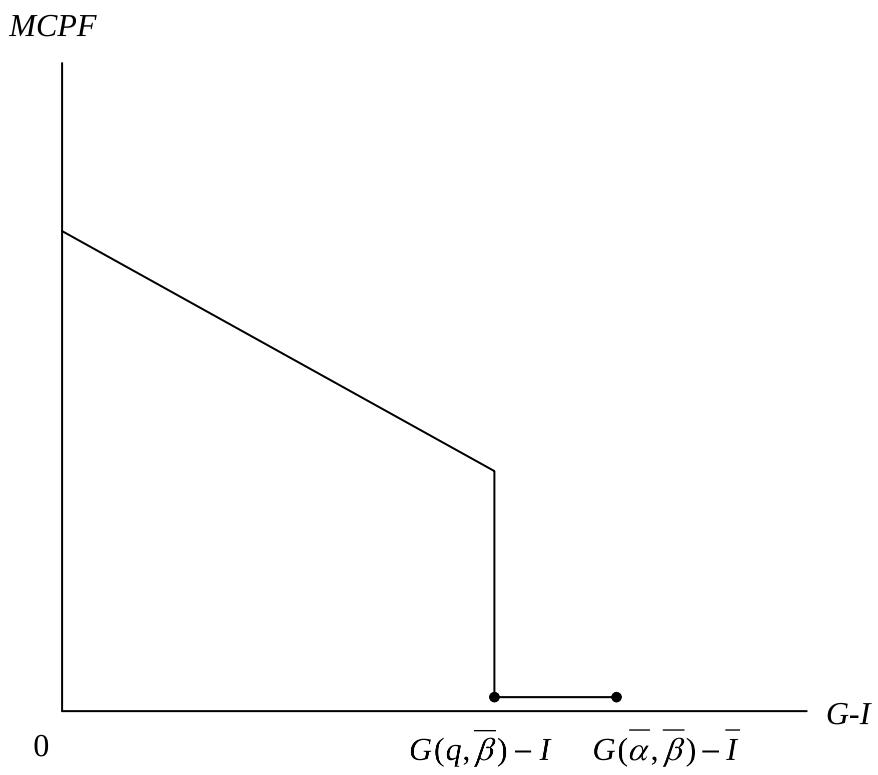

Figure 2-2. MCPF

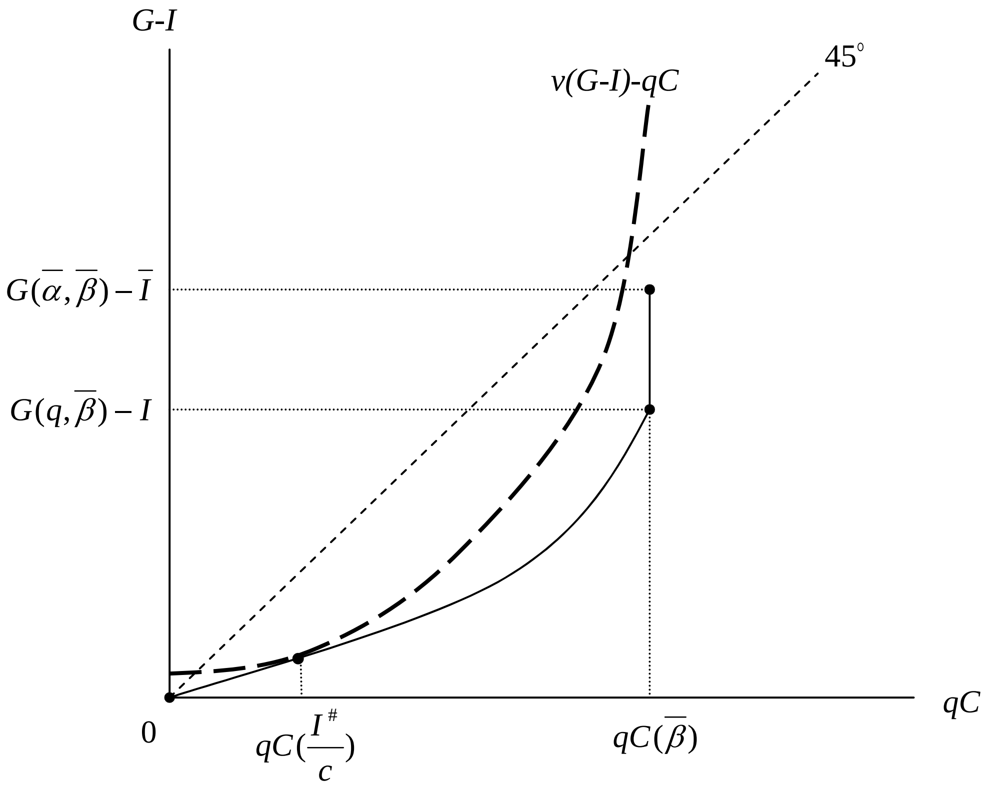

Figure 3-1a. An interior $I^{\#}$ with $W(q,\frac{I^{\#}}{c}) > W(\overline{\alpha},\overline{\beta})$

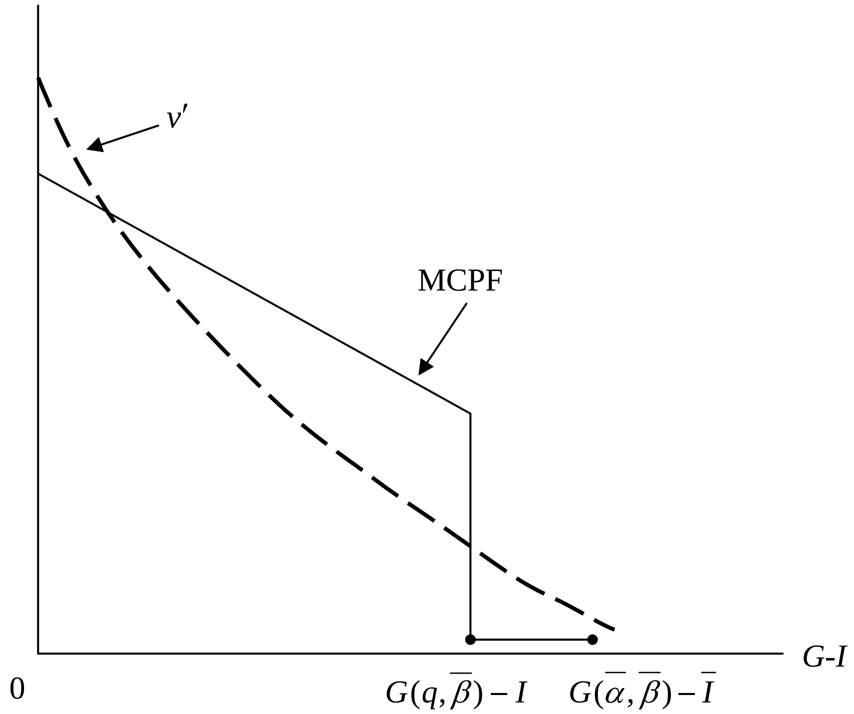

Figure 3-1b. MCPF vs. low v'

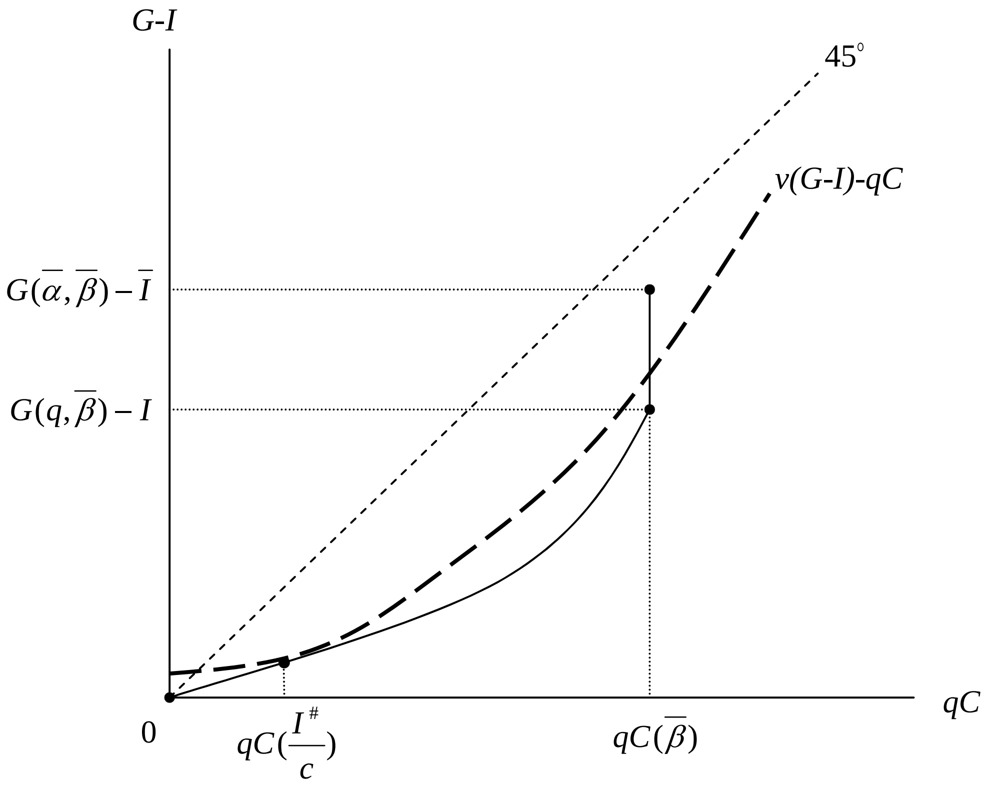

Figure 3-2a. An interior $I^{\#}$ with $W(q,\frac{I^{\#}}{c}) < W(\overline{\alpha},\overline{\beta})$

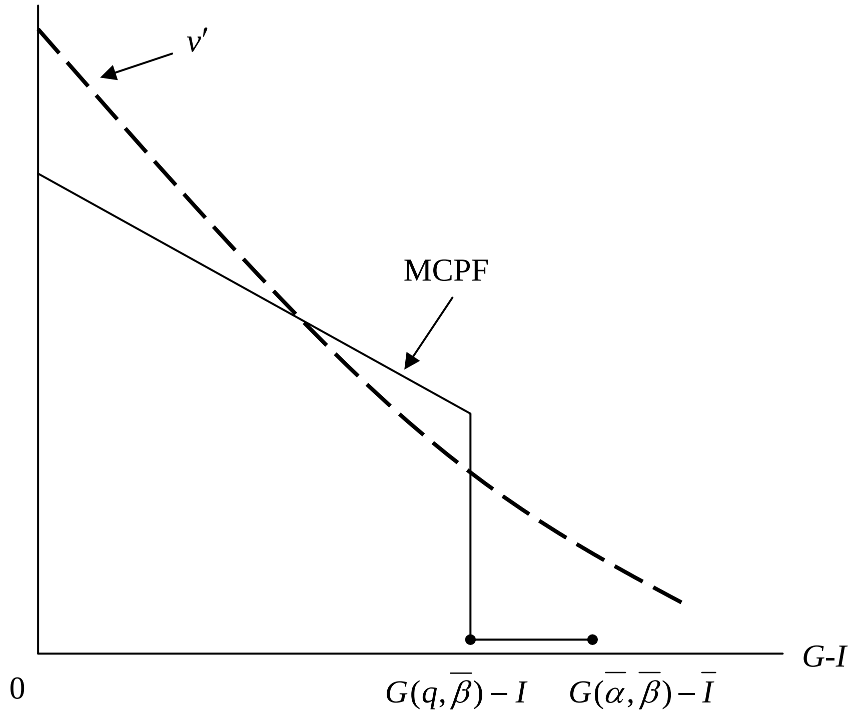

Figure 3-2b. MCPF vs. medium v'

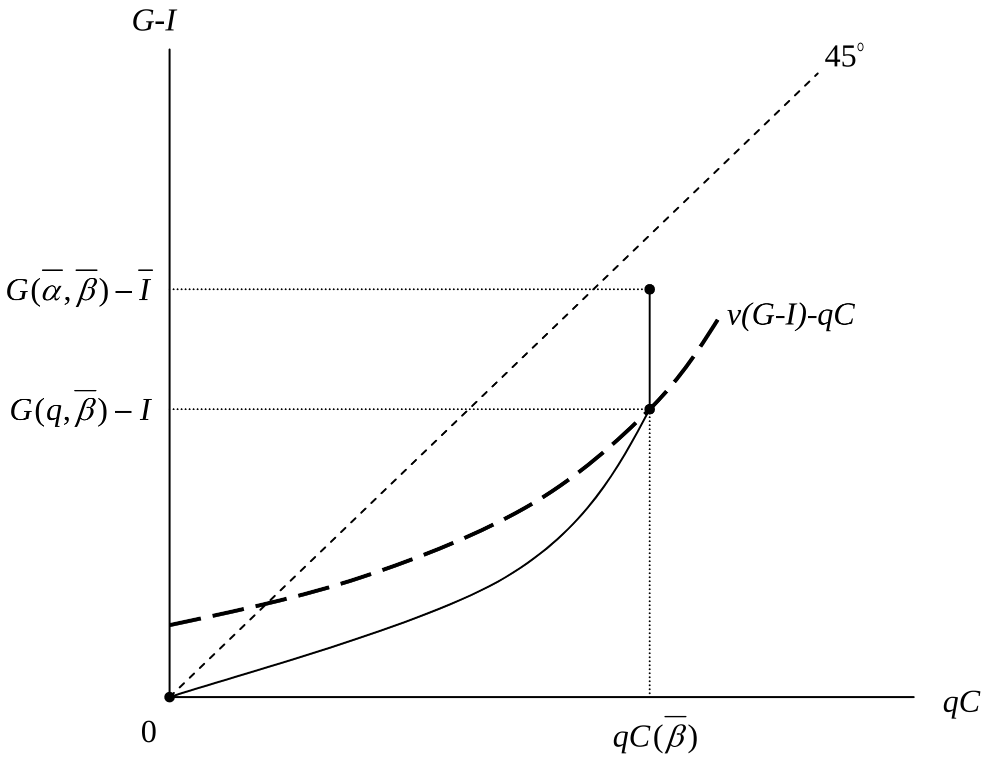

Figure 3-3a. A corner $I^{\#}=\bar{\beta}c$ with $W(q,\bar{\beta})<W(\bar{\alpha},\bar{\beta})$

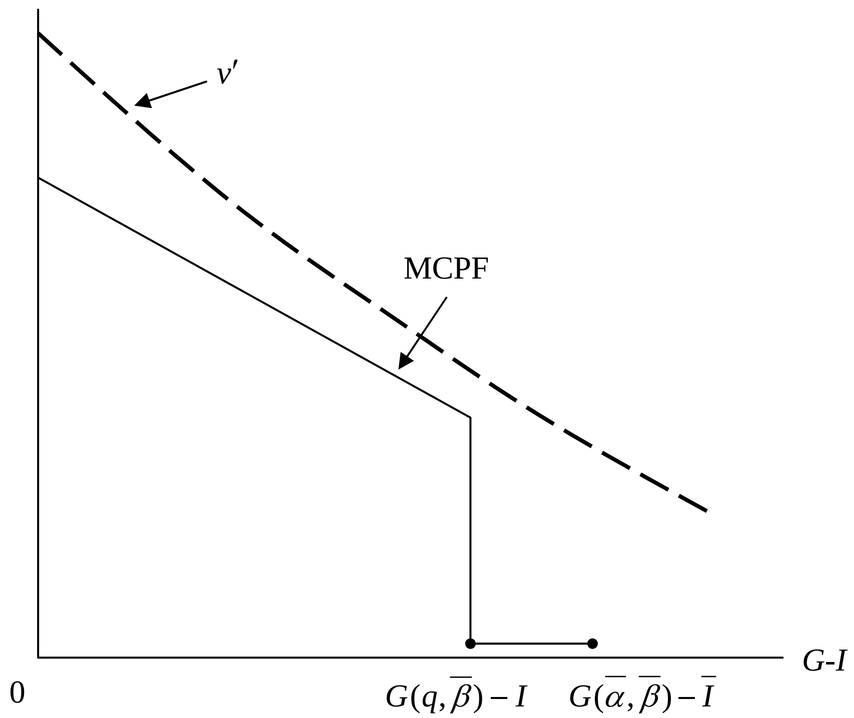

Figure 3-3b. MCPF vs. high v'

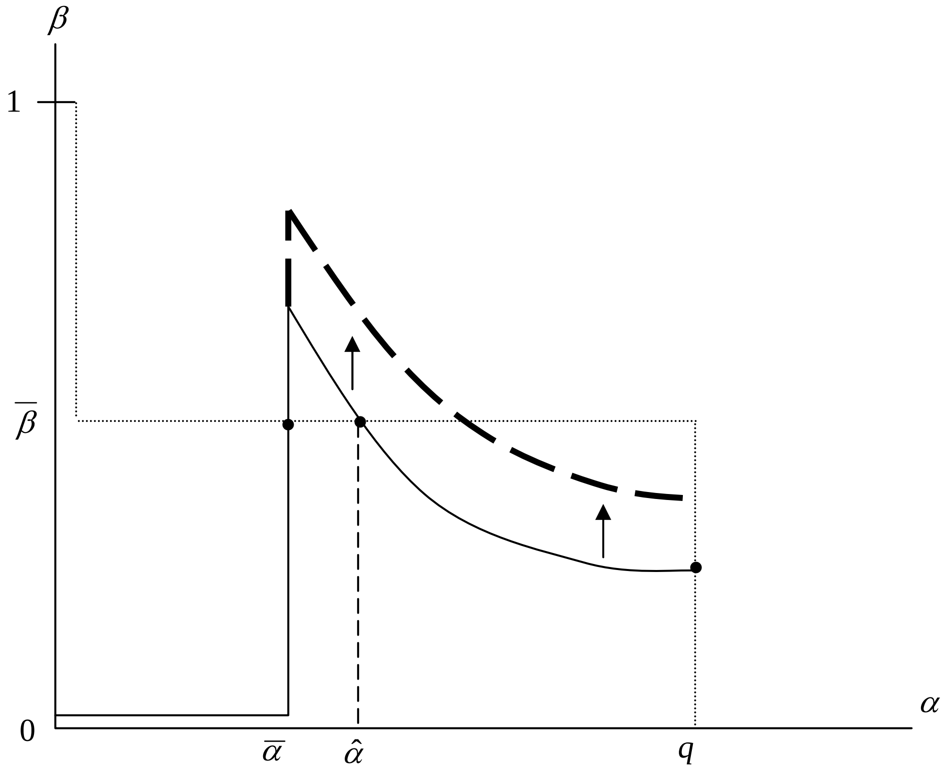

Figure 4. Shift of $\beta(\alpha)$ due to an increase in I

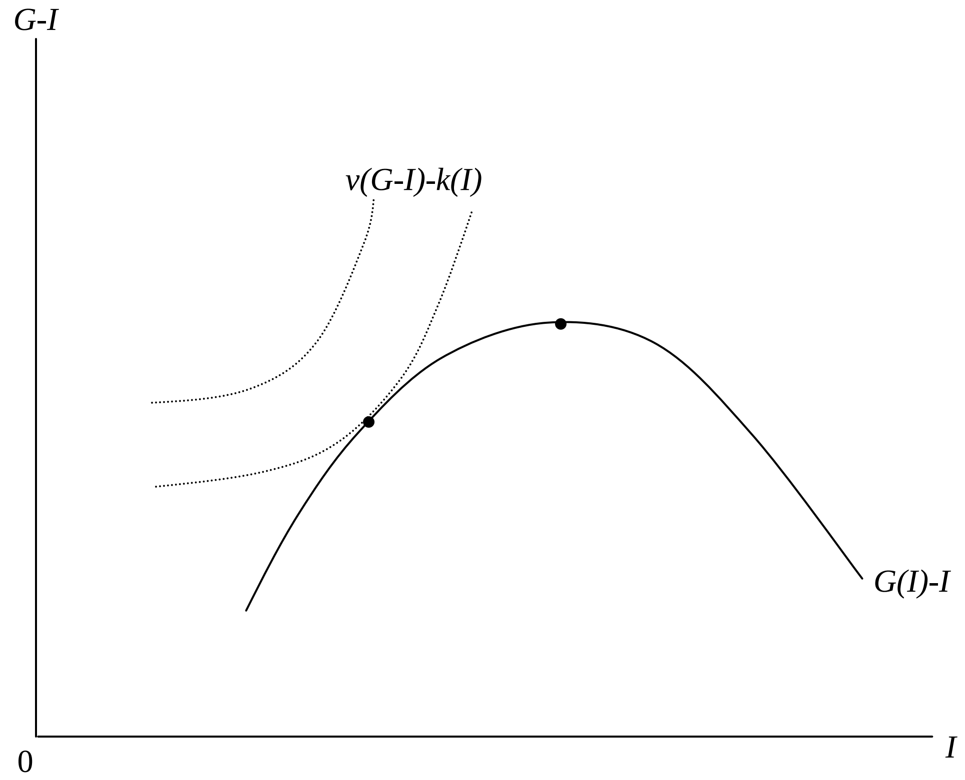

Figure 5-1. A typical Laffer curve

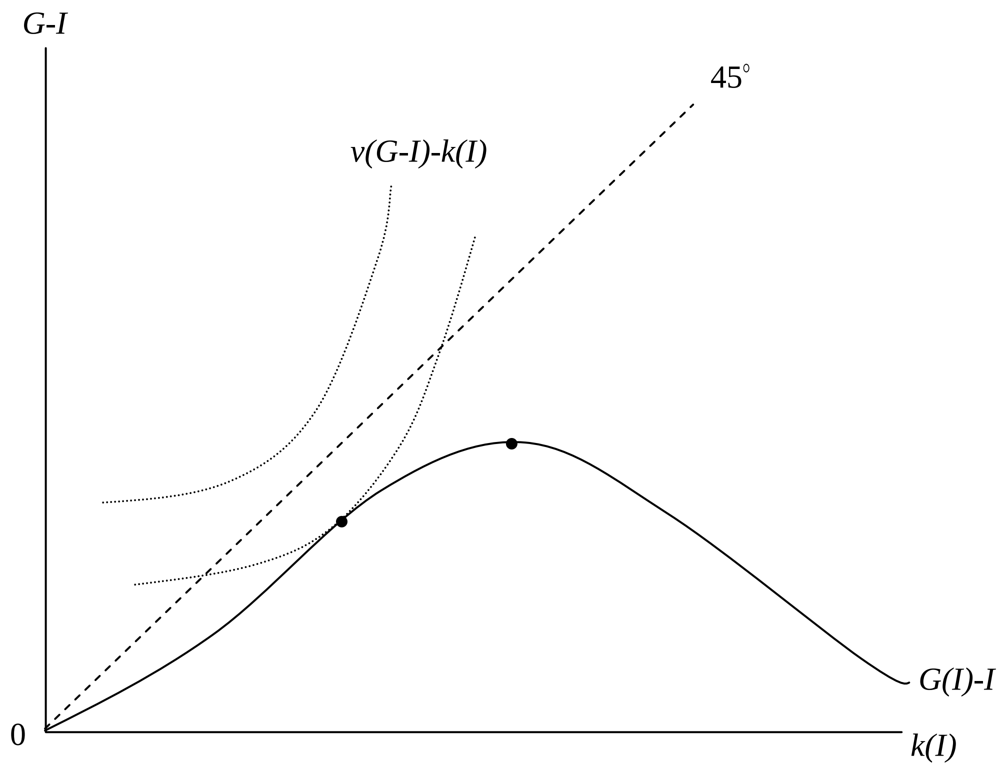

Figure 5-2. A typical Laffer curve again