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Introduction

The two last decades have seen decentralization of fiscal authority coming to the forefront in both well-established developed countries and in developing and transitional economies. Benefits of decentralization have extensively been dealt with in the literature on fiscal federalism, most of which relate to (i) improvement in the level and quality of local services and revenue sources, (ii) better matching of local services to the preferences of local constituencies, and (iii) greater accountability. However, fiscal decentralization is also raising several problems as shown by both practitioners and scholars (see Oates, 1999 and Treisman, 2007 for a survey on the pros and cons of decentralization). Firstly, decentralization is likely to enhance fiscal competition as sub-national governments are granted with a substantial autonomy. Indeed, local jurisdictions may set strategically their tax rates (and in some extent their public expenditures) in order to attract mobile tax bases such as footloose location firms or high-skilled workers. A common wisdom in the recent literature on fiscal federalism (see Wilson, 1999 for a survey) is that fiscal competition results in tax rates being too low to provide efficiently local public services (note however that a challenging view is provided by Brennan and Buchanan among others (1977, 1980) who underline that mobility of tax payers is desirable in order to tame the Leviathan). Secondly, decentralization may create disparities among regions in the ability to provide public goods and public services so that equalization can be seen as a necessary counter-part to decentralization. In most countries, equalization transfers can take various forms and account for a large part of vertical transfers (Boadway, 2004). Finally, higher levels of government may not be able to commit themselves not to bail-out lower-level governments when the later are in financial trouble leading to the well-known problem of soft budget constraint (Rodden, Eskeland and Litvak, 2003 ; Vigneault, 2005). In this case, sub-national governments 'spending and borrowing decisions are likely to be influenced by the expectation of receiving additional resources from the upper-layer of government. The surprising point is that these three issues which are in some extend inherent to any federation have been for long analysed separately whereas they interact with each other. For instance, sub-national governments may be incited to cut their tax rates (and hence their tax revenue) if they expect to receive additional grants from the central government. Consequently, analysing the problem of soft budget constraint issue supposes to take into account both strategic fiscal interactions between lower levels of governments and equalization issues.

The purpose of this paper is precisely to analyse the impact of two widely-used equalization transfer schemes on the degree of "softness" of regional budget constraints when regions compete among each other to attract mobile capital and finance two kinds of public expenditures – current expenditures and investment expenditures (see above). Following Rodden, Eskeland and Litvack (2003), soft budget constraint can be defined as "the situation when an entity (say, a province) can manipulate its access to funds in undesirable ways". Hence, the inability of the rescuer to generate expectations of no bail out entails a soft budget constraint. There is a long line of empirical papers and case studies which have dealt with issues arising from soft budget constraints in federations (see among others Jones, Sanguinetti and Tomassi (1999), Von Hagen (1991), Poterba (1995), Borge and Rattso (1999), Garcia-Mila, Goodspeed and McGuire (2001), Von Hagen and Dahlberg (2002), Rodden, Eskeland and Litvack (2003)). What emerges from all theses studies is that the soft budget constraint is in some extent present in all countries but "its severity and the proper mechanisms to handle it depend on each country's institutions" (Rodden and alii (2003), p.4). Furthermore, it is shown that the upper-level of government is more likely to be unable to resist bail out demands when there is a large vertical tax gap and when sub-national governments fail to provide public goods which are of highly importance from its own point of view or in case where the upper-layer of government aims at equalizing the provision of public goods across regions. A good example of such a situation is given by the German fiscal federalism as Rodden (2000) points out that "the constitution and the history of intergovernmental transfers have led politicians and their constituents to believe that

expenditures will not be allowed to fall below the national average, regardless of a state's fiscal performance or debt levels".

There is a growing theoretical literature trying to connect public finance with soft budget constraint literature. Papers by Wildasin (1997), Qian and Roland (1998), Goodspeed (2002), Köthenbürger (2004), Akai and Sato (2005), Breuillé, Madiès and Taugourdeau (2006), Akai and Silva (2007) are representative of such an approach (see also the survey by Kornai, Maskin and Roland, 2003). A key-point of almost all these papers is that the soft budget constraint is formulated in the context of a sequential game where the first move is made by a regional governments which generally borrow; the federal government has the second move and at that point the costs to the central government of no providing additional funds may exceed these of providing one. There are only few papers using such a theoretical setting which consider the soft budget constraint issue as the result of the federal government's aim at equalizing public good provision across regions among which Goodspeed (2002); Köthenbürger (2004) and Breuillé and alii (2006). Goodspeed (2002) shows that transfers from higher layers of government to lower layers generally involve a "common pool" effect as a part of the bailout must be paid for through increased taxes and then shared by all the regions. Goodspeed endogenously derives soft budget constraint bailout behaviour on the part of the central government but ignores tax interactions among governments as transfers are financed through an immobile and exogenous tax base. Köthenbürger (2004) makes a further step as he explicitly introduces capital mobility among regional governments and shows that ex-post federal transfer policy results in internalizing the impact of interregional tax competition on regional tax policy but at the expense of a new source of inefficiency that turns to be welfare deteriorating relative to tax competition. Breuillé and alii (2006) analyse the impact of both horizontal and vertical tax competition on the degree of softness of the regional budget constraint and show that "horizontal tax competition" has no impact on the ex post federal government transfer to regions but hardens the regional budget constraint when the regional debt is not too heavy.

Our paper is in line with the papers quoted above as we explicitly assume that the federal government has a redistributive role and allocates transfers to regions so as to equalize public good provision across regions. Furthermore, the basic two-tier structure of the model is broadly the same as Köthenbürger (2004) and the soft budget constraint problem is modelled as a sequential game between an overarching federal government and regions which compose the lower level of government. We assume that regions behave as Nash payers as they compete with each other in order to attract mobile capital using a source-based tax rate on capital and providing firms with a productivity-enhancing public good (the later is not present in Köthenbürger (2003)'s paper). The federal government provides lump-sum grants which are financed by contributions made by regions of the federation. The sequence of actions makes that regions may be considered as Stackelberg leaders vis-à-vis the federal government. However, our paper departs from Breuillé and alii (2006) and enriches previous literature in two main directions. Firstly, we compare two widely-used equalization transfers (see Boadway, 2004, p. 213): Gross scheme which involves transfers from the central government to regions financed from central tax revenues and net equalization schemes which consists of self financing regions-to-regions transfers. This is in line with Bahl and Linn (1992, Chapter 13) who, drawing into case studies, have developed a taxonomy of grants that shows that intergovernmental transfers have two dimensions: the size of the divisible pool devoted by the upper-layer of government to equalization, and the distribution of this pool among eligible sub-national government units. Notice that a reason why the central government may not have the ability to modify the size of the equalization pool is that there is an external enforcer such as for instance the European Stability and Growth Pact which imposes a constraint on the public budget constraint. We argue that such a distinction is important as it is likely to have an impact on both the ability of the federal government to commit itself not to bail out regional governments and on the opportunistic behaviour of regional government to get extra-funds from the upper-layer of government. Secondly, we assume that regional governments provide both a public good which enters the citizens'sutility and a productivity-enhancing public input which benefits firms (see

Zodrow and Mieszkowsky (1986); Noiset, 1995; Matsumoto, 1998; Upmann-Bayindir, 1999 among the few papers dealing with public inputs). The public good (which can alternatively be seen as current expenditures) enters the utility function of the representative citizen and corresponds broadly to consumption items such as recreational facilities or social services. The public input (which may be considered as capital expenditures) must include a real element of publicness such as transportation expenditures as pointed out by Keen and Marchand (1997). Hence, one can expect public input to have an impact on the emergence of a soft budget constraint and in turn on the optimal composition of regional public spending. In this respect, this paper contributes to the scarce literature dealing with the optimal pattern of public expenditure in a tax competition setting (see Keen and Marchand, 1997).

Three main results emerge from our paper. Firstly, public input provision always reduces the opportunity cost of the federal government's ex post transfers and hence reduces the ability of the federal government to commit itself not to bail out regions in comparison with a situation where only public goods are provided to citizens. Secondly, a net equalization scheme can be considered as commitment device with respect to a gross equalization scheme as the federal government is always incited to reduce its bail out to regions which cut their tax rate and increase their public expenditure in order to attract capital. In other words, gross equalization schemes lead to soften the regional budget constraint (all things being equal). Finally, public good provision is efficiently provided and public good is over-provided under a gross equalization scheme. Under a net equalization scheme public good provision turns to be under- provided while the public input is surprisingly efficiently provided.

The paper is organised as follows. The set-up of the model is presented in section 2. Both the first-best solution and the hard budget constraint benchmark are presented in section 3. The federal government's incentive to bail out is presented in part 4 making a distinction between gross and net equalization schemes. Regional government behaviour and regional optimal fiscal policy is presented in section 5. Section 6 concludes.

The framework

The federation is composed of n identical regions. Each region i (for $i = 1,\ldots,n$) finances both current public expenditures (e.g. public goods) in quantity G_i and capital public expenditures (e.g. public inputs) in quantity I_i.

The representative household

Let $U(c_i, G_i)$ be the utility of the representative household of the region i derives from the provision of public goods, G_i, and from the consumption of a private good, denoted by c_i:

$$U(c_i, G_i) = c_i + v(G_i), \qquad \#$$

where the utility function v is increasing in its argument, twice differentiable and concave. This representative household is initially endowed with $\widetilde{K}$ units of capital, i.e. $\frac{1}{n}$ of the capital available in the federation, that are remunerated at the before-tax interest rate r_i on which a regional tax τ_i is levied according to the source principle. A unique firm is located in each region i and owned by the representative household, who receives the profit, Π_i. The private consumption thus amounts to the profit of the firm, plus the remuneration of the initial capital endowment at the net return $\rho = r_i - \tau_i$, minus a lump-sum tax Γ levied by the federal government. The representative household's budget constraint is given by:

$$c_i = \Pi_i + \rho\widetilde{K} - \Gamma. \qquad \#$$

The capital market

The firm located in region i produces an output $F(K_i, I_i)$ from the capital, K_i, borrowed on the market at the interest rate r_i and from the capital public expenditures, I_i. The production function is monotonously increasing in both production factors ($\frac{\partial F}{\partial K_i} \equiv F_{K_i} > 0$ and $\frac{\partial F}{\partial I_i} \equiv F_{I_i} > 0$) and has decreasing marginal products. Based on a review of empirical evidences (see Sturn and al (1996)), we assume that capital public expenditures increase the marginal productivity of capital ($\frac{\partial^2 F}{\partial K_i \partial I_i} \equiv F_{K_i I_i} > 0$), due to a complementary between the two production factors. The profit of the firm located in region i, which totally accrues to the representative household is given by

$$\Pi_i = F(K_i, I_i) - r_i K_i$$

Firm's profit maximizing behavior implies the familiar condition of remuneration at the marginal productivity of capital, $\frac{\partial F}{\partial K_i} = r_i \ \forall i$. The resulting demand for capital $K_i(r_i, I_i)$ and profit $\Pi_i(r_i, I_i)$ are decreasing functions of the interest rate r_i, *i.e.* $\frac{\partial K_i(.)}{\partial r_i} \equiv K_{r_i} = \frac{1}{F_{K_i K_i}} < 0$ and $\frac{\partial \Pi_i(.)}{\partial r_i} \equiv \Pi_{r_i} = -K_i < 0$, and increasing functions of the capital public expenditures, *i.e.* $\frac{\partial K_i(.)}{\partial I_i} \equiv K_{I_i} = -\frac{F_{K_i I_i}}{F_{K_i K_i}} > 0$ and $\frac{\partial \Pi_i(.)}{\partial I_i} \equiv \Pi_{I_i} = F_{I_i} > 0$. The capital market clearing condition in the federation,

$$\sum_{i=1}^{n} K_i(r_i, I_i) = n\widetilde{K},$$

characterizes the capital market equilibrium, i.e. it defines in a symmetric setting the net return n $\rho(\tau_1,\ldots,\tau_n, I_1,\ldots,I_n)$ as a decreasing function of the regional tax rate:

$$\frac{\partial \rho}{\partial \tau_i} = -\frac{\frac{\partial K_i}{\partial r_i}}{\sum_{i=1}^{n} \frac{\partial K_i}{\partial r_i}} = -\frac{1}{n} \qquad \in]-1, 0[,$$

and as an increasing function of the capital public expenditures:

$$\frac{\partial \rho}{\partial I_i} = -\frac{\frac{\partial K_i}{\partial I_i}}{\sum_{i=1}^{n} \frac{\partial K_i}{\partial r_i}} = -\frac{\frac{\partial K_i}{\partial I_i}}{n\frac{\partial K_i}{\partial r_i}} > 0 ,$$

which implies that the interest rate moves as follows:

$$\frac{\partial r_i}{\partial \tau_i} = 1 + \frac{\partial \rho}{\partial \tau_i} = \frac{n-1}{n} \in]0,1[\qquad \text{and} \qquad \frac{\partial r_i}{\partial \tau_{-i}} = \frac{\partial \rho}{\partial \tau_{-i}} = -\frac{1}{n} \in]-1,0[.$$

In line with empirical findings, we postulate that the elasticity of the regional tax base with respect to the regional tax rate, denoted by $\varepsilon_i \equiv \frac{\partial K_i}{\partial \tau_i}\frac{\tau_i}{K_i}$ $\forall i$ belongs to the interval $]-1,0[$.

A rise in the capital public expenditures thus produces two opposite effects on both the demand for capital and the profit: i) a direct positive effect ($K_{I_i} > 0$ and $\Pi_{I_i} > 0$) and ii) an indirect negative effect through the net return on capital ($K_{r_i}\frac{\partial \rho}{\partial I_i} < 0$ and $\Pi_{r_i}\frac{\partial \rho}{\partial I_i} < 0$). Without further assumptions, the sign of the cumulative effect is undetermined.

The federal and regional governments

Both federal and regional governments are benevolent. Each regional government acts so as to maximize the utility of the representative household located in its region. Region i chooses an amount of capital and current public expenditures and finances it by both a federal transfer, T_i, and the revenue from regional taxation τ_i on capital K_i. The budget constraint of the region i is thus:

$$I_i + G_i = T_i + \tau_i K_i(r_i, I_i). \qquad \#$$

As for the federal government, it allocates the transfers to regions so as to maximize the aggregated utility $\sum_{i=1}^{n} U(c_i, G_i)$ of households. These transfers are financed by the lump-sum tax Γ, levied on each household:

$$\sum_{i=1}^{n} T_i = n\Gamma. \qquad \#$$

In what follows, we alternately consider two broad forms of redistribution among regions, according to the distinction proposed by Boadway (2004). Firstly, a "gross equalization scheme" where transfers to regions are financed from federal tax revenues, i.e. Γ endogenous chosen by the federal government. Secondly, a kind of "net equalization scheme" where region-to-region transfers are self-financed beyond a given amount $n\overline{\Gamma}$; the federal government devotes an exogenous amount $n\overline{\Gamma}$ of federal tax revenues, which can be equal to zero, and every additional transfer to one region must be compensated by contributions made by the other regions. In a way, a gross equalization scheme means that the size and the cutting of the cake "transfers to regions" are both determined by the federal government whereas a net equalization scheme means that only the rules for dividing the cake are determined by the federal government, given the fixed size of the cake.

The timing of the SBC game

A favorable environment to the emergence of a soft budget constraint problem is a Stackelberg game. Regional governments are assumed to act as Stackelberg leaders vis-à-vis the federal government. Specifically, regional governments first select simultaneously their taxes and

capital public expenditures, taking into account the reaction of the federal government, to maximize the welfare of the representative household located in its territory. In doing so, they play as Nash competitors vis-à-vis each other, but as Stackelberg leaders vis-à-vis the federal government.

Then, given the regional budgetary decisions, the federal government allocates transfers to regions, and chooses federal tax revenues if we consider a gross equalization scheme, to maximize the welfare of all households within the federation. The federal government is thus a Stackelberg follower vis-à-vis the regional governments.

Finally, transfers are paid, taxes are collected, current public expenditures are provided as residuals of the budgetary decisions and households consume. We determine the subgame-perfect equilibrium by solving the governments' choice problems backwards, i.e. from the federal to the regional governments.

The benchmark cases

Before we proceed to the analysis of the SBC game, we first present the outcome of the first-best solution and the outcome of the HBC game, which both will serve as a benchmark for comparison purposes.

The first-best solution

In the benchmark case of centralization, a benevolent social planner takes all the budgetary decisions so as to maximize the aggregated welfare $\sum_i U(c_i, G_i)$ subject to the budget constraints (ref: CBagent), (ref: CBregion1) and (ref: CBCG). These choices perfectly internalize the budgetary externalities linked to fiscal competition among regions. At the first-best optimum, regardless the form of the equalization scheme, the transfers to regions $\{T_i\}_i$ equalize the marginal utilities from the current public expenditures; i.e. $\frac{\partial v}{\partial G_i} = \frac{\partial v}{\partial G_k} = v' \ \forall i,k$. Like Zodrow and Mieszkowski (1986), both current and capital public expenditures are optimally provided; formally, $F_{I_i} = v'(G_i) = 1 \ \forall i$.

The HBC benchmark case

In this benchmark case of regional hard budget constraints, the federal government and the regional governments select simultaneously their budgetary choices. Due to the absence of decentralized leadership, the regional governments are unable to manipulate their choices in order to attract a bailout from the higher level. The transfers to regions $\{T_i\}_i$, chosen by the federal government to maximize the aggregated welfare $\sum_i U(c_i, G_i)$, equalize the marginal utilities from the current public expenditures; i.e. $\frac{\partial v}{\partial G_i} = \frac{\partial v}{\partial G_k} \ \forall i,k$. Each region i simultaneously chooses I_i and τ_i in order to maximize the regional welfare $U(c_i, G_i)$; i.e. $v'(G_i) = \frac{1}{(1+\varepsilon_i)} > 1$ and $F_{I_i} = v'[1 - \tau_i K_{I_i} - \tau_i K_{r_i} \frac{\partial \rho}{\partial I_i}] = v'[1 - \tau_i K_{I_i}(1 - \frac{1}{n})]$. An interior solution implies that $1 - \tau_i K_{I_i} - \tau_i K_{r_i} \frac{\partial \rho}{\partial I_i} \geq 0$, which we assume subsequently. This assumption means that a decrease in the capital public expenditures increases the current public expenditures *ceteris paribus*, *i.e.* for T_i given. As it is well-know in the literature, the capital tax competition results in an underprovision of the current public expenditures. Similarly, the capital public expenditures are sub-optimally provided due to the fiscal externalities.

The federal government's problem

We now introduce decentralized leadership and analyze the impact of a gross versus net equalization scheme on the incentives to bail out a region.

With a gross equalization scheme

The federal government chooses both the amount of aggregated transfers, or indifferently the value of the lump-sum tax Γ, and the dividing of transfers among regions, to maximize the aggregated utility of households in the federation. It solves:

$$\underset{\mathbf{T},\Gamma}{Max} \quad \sum_{i=1}^{n} [c_i + v(G_i)] \qquad \#$$

$$s.t.$$

$$c_i = \Pi_i(r_i, I_i) + \rho\widetilde{K} - \Gamma, \qquad \#$$

$$I_i + G_i = T_i + \tau_i K_i(r_i, I_i), \qquad \#$$

$$\sum_{i=1}^{n} T_i = n\Gamma. \qquad \#$$

At the symmetric equilibrium, the federal transfers policy satisfies the following first-order conditions:

$$\frac{\partial v}{\partial G_i} = \frac{\partial v}{\partial G_k} \quad \forall i,k \Rightarrow \; G_i = G_k \; \forall i,k, \qquad \#$$

$$\frac{\partial v}{\partial G_i} = 1 \quad \forall i. \qquad \#$$

The federal government allocates transfers to regions so as to i) equalize the marginal utilities of current public expenditures in the federation, which boils down to identical current public expenditures in every region since households have the same preferences, and ii) ensure an optimal amount of current public expenditures in each region. To derive the best-reply of the federal government, $\{\widehat{T}_i(\tau_1, \dots \tau_n, I_1, \dots, I_n)\}_{i=1,\dots,n}$ and $\widehat{\Gamma}(\tau_1, \dots \tau_n, I_1, \dots, I_n)$, to a change in a region i's tax rate τ_i and capital public expenditures I_i, we differentiate the condition (ref: equal 1) w.r.t. T_i, $\mathbf{T}_{-i}$ and τ_i, I_i and we use the federal budget constraint $\sum_{i=1}^{n} T_i = n\Gamma$:

$$\frac{\partial \widehat{T}_i}{\partial \tau_i} = -\frac{\partial \tau_i K_i}{\partial \tau_i} = -(1 + \varepsilon_i)K_i < 0, \qquad \frac{\partial \widehat{T}_{-i}}{\partial \tau_i} = -\frac{\partial \tau_{-i} K_{-i}}{\partial \tau_i} < 0, \qquad \#$$

$$\frac{\partial \widehat{T}_i}{\partial I_i} = 1 - \tau_i K_{I_i} - \tau_i K_{r_i} \frac{\partial \rho}{\partial I_i}, \qquad \frac{\partial \widehat{T}_{-i}}{\partial I_i} = -\tau_{-i} K_{r_{-i}} \frac{\partial \rho}{\partial I_i} > 0, \qquad \#$$

$$\frac{\partial \widehat{\Gamma}}{\partial \tau_i} = -\frac{1}{n}\left(\frac{\partial \tau_i K_i}{\partial \tau_i} + (n-1)\frac{\partial \tau_{-i} K_{-i}}{\partial \tau_i} \right) < 0, \qquad \#$$

$$\frac{\partial \widehat{\Gamma}}{\partial I_i} = \frac{1}{n}\left(1 - \tau_i K_{I_i} - \tau_i K_{r_i} \frac{\partial \rho}{\partial I_i} - (n-1)\tau_{-i} K_{r_{-i}} \frac{\partial \rho}{\partial I_i} \right). \qquad \#$$

Achieving an optimal amount of current public expenditures in each region, $v' = 1 \; \forall i$, implies that any reduction in a region i's tax rate must be compensated by a raise in the region i's transfer and a raise in the other regions' transfers. Indeed, a reduction in the region i's tax rate τ_i lowers tax revenues and thus current public expenditures in all regions, *ceteris paribus,* which no longer satisfies the condition (ref: equal 1), in the absence of federal *ex post* intervention. As the

capital locates where the net return of capital is the highest, a reduction in τ_i generates a capital inflow in the region i, at the expense of the other regions whose tax bases are reduced. But the resulting increase in the region i's tax base, $-\varepsilon_i K_i > 0$, does not compensate the loss of tax revenues, $-K_i$, given that $(1 + \varepsilon_i) > 0$. The federal government is obliged to react ex post to these tax revenues losses by bailing out all the regions including region i. Through a reduction in τ_i, the region i is thus able to extract a bailout for itself, and also for the other regions. These bailouts are financed by a raise in the lump-sum tax Γ.

Achieving an optimal amount of current public expenditures in each region also implies that the budgetary impact of any increase in a region i's capital public expenditures must be compensated by the federal government. The negative externality generated by an increase in I_i on the tax base of each region - which goes through the net return of capital ρ - always induces a bailout from the top to entirely compensate the tax revenues losses. However, the sign of the variation of the transfer granted to region i is not clear-cut in the absence of further assumptions. As shown before, a rise in the capital public expenditures produces two opposite effects on the tax base K_i, i.e. a direct positive effect ($K_{I_i} > 0$) and an indirect negative effect through the net return on capital ($K_{r_i}\frac{\partial \rho}{\partial I_i} < 0$), which leads to an indeterminate sign of the cumulative effect. The impact of public capital expenditures on the regional fiscal discipline can be summarized by the following proposition:

Proposition *With a gross equalization scheme, when a region i finances capital public expenditures in addition to current public expenditures,*

i) it increases the amount of the bailout from the federal government to region i under the assumption $1 - \tau_i K_{I_i} - \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} > 0$,

ii) it always increases the amount of the bailout to the other regions, i.e. $\frac{d\widehat{T}_{-i}}{dI_i} > 0$.

When the region i does not finance any capital public expenditures, the whole bailout from the federal government is given by $d\widehat{T}_i = \frac{\partial \widehat{T}_i}{\partial \tau_i} d\tau_i$ whereas when the region i does finance some capital public expenditures, the whole bailout is given by $d\widehat{T}_i = \frac{\partial \widehat{T}_i}{\partial \tau_i} d\tau_i + \frac{\partial \widehat{T}_i}{\partial I_i} dI_i$. We show that for $1 - \tau_i K_{I_i} - \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} > 0$, the capital public expenditures always worsen the commitment problem of the federal government. Note that $\frac{\partial \widehat{T}_i}{\partial \tau_i} = \frac{\partial \widehat{T}_{-i}}{\partial \tau_i} = \frac{\partial \widehat{T}_i}{\partial I_i} = \frac{\partial \widehat{T}_{-i}}{\partial I_i} = \frac{\partial \widehat{\Gamma}}{\partial \tau_i} = \frac{\partial \widehat{\Gamma}}{\partial I_i} = 0$ ensures a hard budget constraint at the regional level because the federal government wouldn't deviate *ex post* from the budgetary decisions taken *ex ante*.

With a net equalization scheme

We now assume that the federal government is no longer able to manipulate the size of the cake to be distributed among regions, i.e. Γ is exogenously given, but it still determines the cutting. In that setting, any bailout to one region must be financed by a reduction in transfers granted to the other regions. The problem of the federal government is the same as (ref: fg) except that Γ is not a policy instrument anymore:

$$\underset{\mathbf{T}}{Max} \quad \sum_{i=1}^{n} [c_i + v(G_i)] \qquad \#$$

$$s.t.$$

$$c_i = \Pi_i(r_i, I_i) + \rho\widetilde{K} - \overline{\Gamma}, \qquad \#$$

$$I_i + G_i = T_i + \tau_i K_i(r_i, I_i), \qquad \#$$

$$\sum_{i=1}^{n} T_i = n\overline{\Gamma}. \qquad \#$$

At the symmetric equilibrium, the federal government still allocates transfers to regions so as to equalize the marginal utilities of current public expenditures in the federation:

$$\frac{\partial v}{\partial G_i} = \frac{\partial v}{\partial G_k} \quad \forall i,k \Rightarrow \; G_i = G_k \; \forall i,k, \qquad \#$$

but nothing guarantees an optimal amount of current public expenditures. To derive the best-reply of the federal government, $\{\widetilde{T}_i(\tau_1, \ldots \tau_n, I_1, \ldots, I_n)\}_{i=1,\ldots,n}$, to a change in a region i's tax rate τ_i and capital public expenditures I_i, we differentiate the condition (ref: equal 2) w.r.t. T_i, $\mathbf{T}_{-i}$ and τ_i, I_i, $\boldsymbol{\tau}_{-i}$, $\mathbf{I}_{-i}$:

$$v''\Big[\frac{\partial \tau_i K_i}{\partial \tau_i} d\tau_i + \sum_{-i} \tau_i \Big(\frac{\partial K_i}{\partial \tau_{-i}} d\tau_{-i} + K_{r_i} \frac{\partial \rho}{\partial I_{-i}} dI_{-i}\Big) + \Big(\tau_i K_{I_i} + \tau_i K_{r_i} \frac{\partial \rho}{\partial I_i}\Big) dI_i + dT_i - dI_i\Big] =$$

$$v''\Big[\frac{\partial \tau_j K_j}{\partial \tau_j} d\tau_j + \sum_{-j} \tau_j \Big(\frac{\partial K_j}{\partial \tau_{-j}} d\tau_{-j} + K_{r_j} \frac{\partial \rho}{\partial I_{-j}} dI_{-j}\Big) + \Big(\tau_j K_{I_j} + \tau_j K_{r_j} \frac{\partial \rho}{\partial I_j}\Big) dI_j + dT_j - dI_j\Big]$$

Summing across j, for $j \neq i$, and using the federal budget constraint $\sum_{i=1}^{n} T_i = n\overline{\Gamma}$ yields to:

$$\frac{\partial \widetilde{T}_i}{\partial \tau_i} = -\frac{(n-1)}{n} \frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{1}{n} \sum_{-i} \tau_{-i} \frac{\partial K_{-i}}{\partial \tau_i} < 0, \qquad \frac{\partial \widetilde{T}_{-i}}{\partial \tau_i} = \frac{1}{n} \frac{\partial \tau_i K_i}{\partial \tau_i} - \frac{1}{n} \tau_{-i} \frac{\partial K_{-i}}{\partial \tau_i} > 0 \qquad \#$$

$$\frac{\partial \widetilde{T}_i}{\partial I_i} = \frac{(n-1)}{n} \Big(1 - \tau_i K_{I_i} - \tau_i K_{r_i} \frac{\partial \rho}{\partial I_i}\Big) + \frac{1}{n} \sum_{-i} \tau_{-i} K_{r_{-i}} \frac{\partial \rho}{\partial I_i} \qquad \#$$

$$\frac{\partial \widetilde{T}_{-i}}{\partial I_i} = -\frac{1}{n} \Big(1 - \tau_i K_{I_i} - \tau_i K_{r_i} \frac{\partial \rho}{\partial I_i}\Big) - \frac{1}{n} \tau_{-i} K_{r_{-i}} \frac{\partial \rho}{\partial I_i} \qquad \#$$

Like Köthenbürger (2004), a decrease in τ_i exerts two opposite effects on the federal transfer to region i: on the one hand, any increase in region i's tax revenues is captured by the federal government to be redistributed equally among all the regions; on the other hand, any capital outflow from region i, which increases other regions' tax revenues, is partially compensated by contributions made by the other regions, which ensures the equalization of marginal utilities *ex post*. The global influence of a raise in region i's tax effort, combining these two effects, is a reduction in region i's federal transfer which benefits to the other regions. In a standard way, the externalities linked to capital mobility across regions are perfectly internalized by the transfers scheme designed by the federal government. But unlike Köthenbürger (2004), the federal transfers also react to the regional capital public expenditures. The federal government smooths the impact of a change in I_i across regions so as to equalize marginal utilities *ex post*.

When the region i does not finance any capital public expenditures, the whole bailout from the federal government is given by $d\widetilde{T}_i = \frac{\partial \widetilde{T}_i}{\partial \tau_i} d\tau_i$ whereas when the region i does finance some capital public expenditures, the whole bailout is given by $d\widetilde{T}_i = \frac{\partial \widetilde{T}_i}{\partial \tau_i} d\tau_i + \frac{\partial \widetilde{T}_i}{\partial I_i} dI_i$. Without further

assumptions, we cannot sign $\frac{\partial \widetilde{T}_i}{\partial I_i}$ and $\frac{\partial \widetilde{T}_{-i}}{\partial I_i}$.

The comparison between the best-reply with a gross equalization scheme (ref: brg) and the best-reply with a net equalization scheme (ref: brn) yields the following proposition:

Proposition *The net equalization scheme, compared with the gross equalization scheme, always acts as a commitment device*

i) to reduce the bailout following a reduction in the regional tax rate,

ii)to reduce the bailout following an increase in the regional capital public expenditures under the assumption $1 - \tau_i K_{I_i} - \tau_i K_{r_i} \frac{\partial \rho}{\partial I_i} > 0$.

As a result, the net equalization scheme improves the ability of the federal government to commit not to bailout. Whatever the amount of tax revenues $n\overline{\Gamma}$ devoted to equalization, i.e. the federal revenues with a net equalization scheme $n\overline{\Gamma}$ can be much more important than the federal revenues with a gross equalization scheme $n\Gamma$, what does matter is the way the bailout is financed and the incentives at the margin. The self-financed region-to-region bailouts with a net equalization scheme reduces the deviation from the optimum *ex post*.

To determine the impact on fiscal discipline at the regional level, we will analyze how a gross versus net equalization scheme affects the opportunistic behaviour of the regional governments.

The regional government's problem

With a gross equalization scheme

Each regional government acts as a Stackelberg leader vis-à-vis the federal government. It maximizes the utility of the representative household located in its territory taking into account the best-reply of the federal government, $\{\widehat{T}_i(\tau_1,\dots\tau_n,I_1,\dots,I_n)\}_{i=1,\dots,n}$ and $\widehat{\Gamma}(\tau_1,\dots\tau_n,I_1,\dots,I_n)$:

$$\begin{aligned} &\underset{I_i,\tau_i}{Max} \quad c_i + v(G_i) \\ &s.t. \\ c_i &= \Pi_i(r_i,I_i) + \rho\widetilde{K} - \widehat{\Gamma}, \\ I_i + G_i &= \widehat{T}_i + \tau_i K_i(r_i,I_i), \\ \sum_{i=1}^{n} \widehat{T}_i &= n\widehat{\Gamma}, \end{aligned}$$

which yields to the following first-order conditions:

$$\Pi_{I_i} + \Pi_{r_i}\frac{\partial \rho}{\partial I_i} + \frac{\partial \rho}{\partial I_i}\widetilde{K} - \frac{\partial \widehat{\Gamma}}{\partial I_i} + v'\left[-1 + \tau_i K_{I_i} + \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} + \frac{\partial \widehat{T}_i}{\partial I_i}\right] = 0,$$

$$\Pi_{r_i}\frac{\partial r_i}{\partial \tau_i} + \frac{\partial \rho}{\partial \tau_i}\widetilde{K} - \frac{\partial \widehat{\Gamma}}{\partial \tau_i} + v'\left[\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial \widehat{T}_i}{\partial \tau_i}\right] = 0.$$

At the symmetric equilibrium, using $\Pi_{r_i} = -K_i$ and the capital market clearing condition $\sum_{i=1}^{n} K_i(r_i, I_i) = n\widetilde{K}$, replacing $\frac{\partial \widehat{T}_i}{\partial I_i}$ and $\frac{\partial \widehat{T}_i}{\partial \tau_i}$ by their value, the conditions boil down to:

$$\Pi_{I_i} = \frac{\partial \widehat{\Gamma}}{\partial I_i} \Leftrightarrow F_{I_i} = \frac{1}{n} < 1, \quad \#$$

$$\Pi_{r_i} = \frac{\partial \widehat{\Gamma}}{\partial \tau_i} \Leftrightarrow v' = 1. \quad \#$$

The terms in square brackets are null as the federal government entirely compensates, via its transfers policy, the impact of a variation in the regional taxation or in the capital public expenditures, to achieve an optimal amount of current public expenditures. The amount of capital public expenditures at the equilibrium equalizes the marginal profit (Π_{I_i}) to the marginal increase in the federal lump-sum tax ($\frac{\partial \widehat{\Gamma}}{\partial I_i}$). As for the level of the regional tax rate at the equilibrium, it equalizes the marginal cost in terms of profit reduction (Π_{r_i}) to the marginal decrease in the federal taxation ($\frac{\partial \widehat{\Gamma}}{\partial \tau_i}$). Since regions are perfectly symmetric, these conditions simplify into two meaningful ones; i.e. $F_{I_i} = \frac{1}{n}$ and $v'(G_i) = 1$. The condition $F_{I_i} = \frac{1}{n} < 1$ implies that capital public expenditures are overprovided relative to the first-best optimum. By contrast, current public expenditures are optimally provided, since the federal government bails out the regions to compensate the loss of tax revenue and to ensure $v'(G_i) = 1$. These results can be summarized by the following proposition:

Proposition *The SBC equilibrium with a gross equalization scheme is characterized by an optimal amount of current public expenditures, i.e.* $v'(G_i) = 1$*, and an upward distortion of the amount of capital public expenditures w.r.t. the optimum, i.e.* $F_{I_i} = \frac{1}{n} < 1$.

With a net equalization scheme

The regional government in i maximizes the utility of the representative household located in its territory taking into account $\{\widetilde{T}_i(\tau_1, \ldots \tau_n, I_1, \ldots, I_n)\}_{i=1,\ldots,n}$, the best-reply of the federal government:

$$\begin{aligned} \underset{I_i, \tau_i}{Max} \quad & c_i + v(G_i) \\ s.t. & \\ c_i &= \Pi_i(r_i, I_i) + \rho\widetilde{K} - \overline{\Gamma}, \\ I_i + G_i &= \widetilde{T}_i + \tau_i K_i(r_i, I_i), \\ \sum_{i=1}^{n} \widetilde{T}_i &= n\overline{\Gamma}. \end{aligned}$$

which yields the following first-order conditions:

$$\Pi_{I_i} + \Pi_{r_i}\frac{\partial \rho}{\partial I_i} + \frac{\partial \rho}{\partial I_i}\widetilde{K} + v'[-1 + \tau_i K_{I_i} + \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} + \frac{\partial \widetilde{T}_i}{\partial I_i}] = 0,$$

$$\Pi_{r_i}\frac{\partial r_i}{\partial \tau_i} + \frac{\partial \rho}{\partial \tau_i}\widetilde{K} + v'\left[\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial \widetilde{T}_i}{\partial \tau_i}\right] = 0.$$

At the symmetric equilibrium, using $\Pi_{r_i} = -K_i$ and the capital market clearing condition

$\sum_{i=1}^{n} K_i(r_i, I_i) = n\widetilde{K}$, replacing $\frac{\partial \widetilde{T}_i}{\partial I_i}$ and $\frac{\partial \widetilde{T}_i}{\partial \tau_i}$ by their value, the conditions boil down to:

$$F_{I_i} = 1, \quad \#$$

$$v' = n > 1. \quad \#$$

The condition $F_{I_i} = 1$ implies that capital public expenditures are optimally provided. By contrast, current public expenditures are underprovided with respect to the first-best equilibrium. These results can be summarized by the following proposition:

Proposition *The SBC equilibrium with a net equalization scheme is characterized by a downward distortion of the amount of current public expenditures w.r.t. the optimum, i.e.* $v'(G_i) = n > 1$*, and an optimal provision of the amount of capital public expenditures, i.e.* $F_{I_i} = 1$.

The comparison between (ref: rg1) and (ref: rn1) emphasizes that the regional government provides more capital public expenditures when the federal government implements a gross equalization scheme than when it implements a net equalization scheme. Moreover, the comparison between (ref: rg2) and (ref: rn2) emphasizes that the regional government sets a lower tax rate, and provides more current public expenditures, when the federal government implements a gross equalization scheme than when it implements a net equalization scheme. The following proposition summarizes the findings:

Proposition *The gross equalization scheme, compared with the net equalization scheme, softens the regional budget constraint. The capital public expenditures are distorted upwards and the tax rate is distorted downwards.*

Too much is spent in capital public expenditures and too little taxes are levied by the region. This strategic behaviour is driven by the fact that it aims at extracting more bailout from the top. The net equalization scheme must be favored over the gross equalization scheme if the federal government wants to instill more fiscal discipline at the regional level.

Conclusion

The soft budget constraint problem arises when sub-national governments can behave strategically as they expect to receive additional resources from an upper-layer of government. In our paper such expectations come from our assumption that the federal government has a redistributive role and allocates transfers to regions so as to equalize public good provision across regions. The originality of our paper is to show that the choice of the equalization scheme must be done carefully as we have seen that it may entail - especially in the case of gross equalization scheme - perverse incentives for sub-national governments and hence on the degree of softness of sub-national budget constraints. In addition public spending at the lower-level of government may result distorted. This paper could be extended in at least two directions. First of all, the soft budget constraint issue should be explicitly modelled in a dynamic setting as expectations of bailout are likely to depend on the bailouts received in the past (see on this point Dahlberg and Petterson,-Lidbom (2003)). On the other hand, a way of improving our understanding of soft budget constraint consists in introducing uncertainty as in the real word regional government do not know precisely at which point in time and how the federal government will rescue them, which in turn lead all things being equal to harden the regional budget constraint.

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