Longevity, Health Spending and the Variance of Lifetime Welfare
INCOMPLETE

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Abstract

This paper studies the normative problem of redistribution between agents who can influence their probability of survival through private health expenditures, but who differ on their attitude towards the degree of risk involved in the lotteries of life to be chosen. For that purpose, we develop a simple two-period model where agents's preferences over lotteries of life can be represented by a mean and variance utility function allowing, unlike the expected utility form, a sensitivity of decision-makers to what Allais (1953) calls the 'dispersion of psychological values'. In the present context of risky length of life, the mean and variance utility function has the virtue to allow some degree of risk-aversion with respect to the length of life. It is shown that if agents tend to ignore the impact of their health expenditures on the return of their savings, the decentralization of the first-best optimum requires not only intergroup lump sum transfers, but, also, group-specific taxes on private health expenditures. Under asymmetric information on individual sensitivities to the variance in welfare, we find that, in addition to existing taxes on health expenditures, a tax on annuity for the high-type individual is a way to make the problem incentive compatible.

Keywords: longevity, risk, lotteries of life, expected utility theory, health spending.

JEL codes: D81, H21, I12, I18, J18.

1 Introduction

Whereas human longevity depends on factors of various natures - genetic, environmental or sociocultural -, a large demographic literature emphasized the crucial influence of the lifestyles chosen by individuals on their own longevity.[1] Clearly, *how long* one lives is not independent from *how* one lives. To be precise, individual longevity depends significantly on the extent to which one is willing to 'make an effort' to improve or preserve his health.

As this is well-known among demographers, health-improving efforts can take various forms: the effort can be either *temporal* (e.g. physical activity, see Surault 1996, Kaplan et al. 1987 and Okamoto 2006), *physical* (e.g. abstinence of food, see Solomon and Manson, 1997), or *monetary* (e.g. health services, see Poikolainen and Eskola, 1986).

Despite numerous pieces of scientific work emphasizing the positive influence of health-improving efforts on longevity, the amount of efforts carried out by individuals of all ages still varies nowadays among populations. While some people do a lot of efforts, others do not, and those differentials in lifestyles tend to be reflected by longevity differentials.

What should a utilitarian government do in front of such a heterogeneity of lifestyles and longevities? The answer depends on the source of the heterogeneity, which can take various forms. Undoubtedly, whether longevity is exogenous or depends on individual efforts is likely to influence the optimal policy. If longevity depends on some exogenous health endowment, individuals should be compensated for their poor health conditions by means of transfers, as this is shown by Bommier *et al.* (2007 a,b). On the contrary, if individuals are (partly) responsible for their lower longevity, it is no longer optimal to redistribute from long-lived individuals to short-lived individuals. For instance, Leroux (2007) showed that under the expected utility hypothesis and additive lifetime welfare, the decentralization of the social optimum

[1] See Vallin *et al* (2002).

in a society of agents differing on their taste for efforts requires lump-sum transfers from agents with low taste of effort to agents with high taste of effort, as well as some - hardly implementable - taxes on efforts. Hence, under that framework, the optimal policy requires transfers from short-lived agents to long-lived agents.

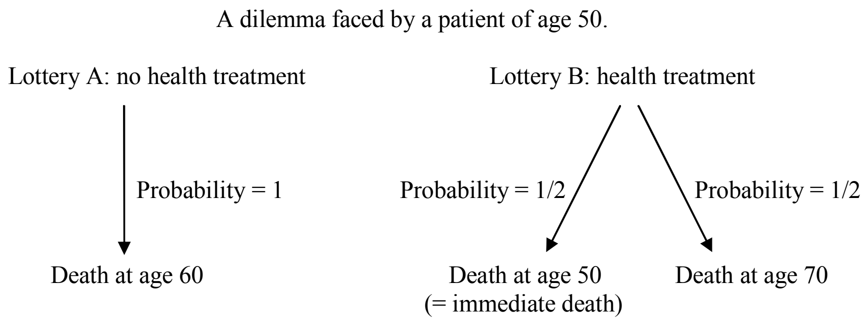
While Leroux's model enriches the study of optimal public intervention when heterogeneous populations can affect their longevities through health-improving efforts, that model, by focusing on one source of heterogeneity - the disutility of effort - tends to ignore other significant dimensions of heterogeneity. Among other things, Leroux's framework, by assuming expected utility and additive lifetime welfare, presupposes that all agents exhibit what Bommier (2005) calls net *risk-neutrality* with respect to the length of life (i.e. risk neutrality with respect to longevity under no pure time preferences). A major corollary of that assumption is to ignore a dimension of tastes that explains the heterogeneity of behaviors and longevities: the attitude of individuals towards risk.

Actually, when a person chooses a level of effort - whatever its form is -, that person does not choose a certain life, but, rather, only expresses a preference for a particular *lottery of life*, whose different scenarii involve different lengths of life (and, also, different levels of efforts). In other words, the chosen effort is not a guarantee of a long life, but, only, of a longer *expected* length of life. Moreover, the chosen level of effort is likely to affect not only the expected length of life, but, also, the *variance* of the length of life. To be precise, the chosen level of health-improving effort is likely to influence - either increase or decrease - the dispersion of the ages at death.

Thus, in the context of risk about the length of life, individual choices of health-improving efforts may reflect their attitudes towards risk about the length of life - rather than different disutilities from efforts -, so that the making of a particular *uniform* assumption on the attitude towards risk -

such as risk-neutrality with respect to the length of life - may simplify the problem of the optimal public intervention.

To see the crucial role played by risk about the length of life in individual decisions, let us consider the following example (see Figure 1). A person of age 50, who has some disease, can choose between two possible lotteries of life: either lottery A, 'no medical treatment', or lottery B, 'medical treatment'. Under no medical treatment, the patient is certain to live the next 10 years for sure, but not longer. On the contrary, under the medical treatment, the patient can die during the intervention with a probability 1/2, but can, if the intervention is a success, hope to live until the age of 70 years with a probability 1/2. What will the patient choose?



Choice between two lotteries of life

It is not straightforward to see what the patient will decide. Actually, each lottery exhibits the same expected length of life, equal to 60 years, but different degrees of risk about the length of life: whereas lottery A is risk-free, lottery B is risky regarding the length of life.[2]

[2]Note that, in general, the choice of an effort level influences not only the expected length of life and the variance of the age at death, but, also, the utility per period of life. That point is worth being stressed, as discussions on risk-aversion with respect to the

Under net risk-neutrality with respect to the length of life, a patient would be totally indifferent between lotteries A and B, and would toss a coin to decide whether he will undergo the medical treatment or not. However, such an indifference is highly unlikely, because the degree of risk about the length of life is a non-neutral information for decision-makers. Thus, it is likely that individuals differ largely regarding their attitude towards risk, and do not all exhibit risk-neutrality with respect to the length of life. Obviously, some patients, who are risk-averse with respect to the length of life, will choose no medical treatment (lottery A), while some others, who are risk-lover, will choose the medical treatment (lottery B).[3]

As this example illustrates, the observed heterogeneity in health-influencing efforts is likely to reflect the heterogeneity of preferences, and, in particular, the heterogeneity of individual attitudes towards risk. But the observed heterogeneity in individual's attitude towards risk raises the difficult question of the optimal public intervention in that context: what should a utilitarian government do in front of such a heterogeneity in the attitude towards risk?

The goal of this paper is to examine the optimal public intervention in an economy where individuals know the impact of their effort on probability of survival, but differ regarding their attitude towards risk with respect to the length of life. For that purpose, we consider a two-period model, and assume that there exist two groups of individuals with different attitudes towards risk, who have to choose a monetary effort - e.g. health expenditure - that determines the probability of survival to the second period. Naturally, by choosing their health expenditures, agents choose a lottery of life, with a particular expected length of life and a particular variance of the length of life.

length of life presuppose in general a constant temporal welfare under each lottery under comparison.

[3]Note that, if the medical treatment had the virtue not to raise, but to reduce the variance of the age at death, risk-averse individuals would *ceteris paribus* naturally opt for the medical treatment.

As this is well-known since Bommier's (2005) work, there exist two broad ways to depart from net risk-neutrality with respect to the length of life. One widely used way is to depart from additive lifetime welfare, as in Bommier's works; the alternative approach is to depart from the expected utility hypothesis. The former approach has the advantage to allow a reliance on the - highly convenient - expected utility theory, but suffers from the lack of intuition behind non-additive lifetime welfare. Given that this can hardly be defended without a reference to risk, it seems more natural to choose the other way, and to keep additive lifetime welfare but depart from the expected utility hypothesis.

This is the road we shall follow in this paper. Lifetime welfare is thus still assumed to be additive in temporal welfare (without pure time preferences), but the expected utility hypothesis is here replaced by an alternative, less restrictive assumption, which allows for risk-aversion with respect to the length of life. More precisely, it is assumed that agents's preferences over lotteries of life can be represented by a classical 'mean and variance' utility function of the kind defended by Allais (1953) in his seminal paper. Actually, Allais emphasized that, given that the dispersion of psychological values is 'the specific element of the psychology of risk' (Allais, 1953, p. 512), it follows that '[...] *even in a first approximation*, one should take into account the second order moment of the distribution of psychological values' (1953, p. 513).[4] Moreover, it was also quite clear in Allais's mind that '[...] one cannot regard as irrational a psychological attitude in front of risk that takes the dispersion of psychological values into account.'(see Allais, 1953, p. 520).[5] We shall thus follow Allais's intuitions and postulate a mean and variance utility function, which is a simple generalization of the EU hypothesis accounting for Allais's intuition. Naturally, other assumptions

[4] Original version: '[...] *même dans une première approximation*, on doit tenir compte du moment d'ordre deux de la distribution des valeurs psychologiques'.

[5] Original version: '[...] on ne saurait considérer comme irrationnelle une attitude psychologique devant le risque qui tient compte de la *dispersion* des valeurs psychologiques.'

were available (see Stigum and Wenstop, 1983), but the 'mean and variance' utility function has the advantage of simplicity.

The rest of the paper is organized as follows. Section 2 presents the model and derives the laissez-faire equilibrium, as well as the social optimum. After discussing its decentralization with and without moral hazard constraints (Section 3), the second-best problem is considered (Section 4). Conclusions are drawn in Section 5.

2 The model

2.1 Environment

Let us consider a stationary population of individuals who live a first period of life (whose length is normalized to one) with certainty, but survive to the second period only with a probability π. That probability depends positively on some monetary investment m:

$$\pi = \pi(m) \tag{1}$$

Equivalently, m can be seen as private health expenditures made by the individual in the first period of his life so as to increase his survival probability. We assume here that individuals have the same survival function $\pi(m)$.

However, agents are assumed to differ on their attitude towards risk. More precisely, the population is constituted of two groups of equal size where the first group exhibits a higher risk-aversion than the second. In order to introduce that difference, we shall assume that individual preferences over lotteries of life can be represented by a function having the classical 'mean and variance' form (see Allais, 1953):

$$U^i = \bar{u}^i - \gamma^i var(u^i) \tag{2}$$

where $\bar{u}^i$ is the expected lifetime welfare of an agent of type $i \in \{1, 2\}$, while $var(u^i)$ is the variance of lifetime welfare exhibited by a lottery. The parameter γ^i reflects the sensitivity to the variance of lifetime welfare exhibited by a lottery. Under complete insensitivity, γ^i equals 0 and we are back to traditional expected utility theory. On the contrary, if γ^i is positive, the individual prefer, *ceteris paribus*, lotteries with a lower variance of lifetime welfare across scenarios, while a negative γ^i reflects the tastes of risk-lover agents. Note that the above function, although more general than the usual expected utility function, could still be generalized by taking into account higher moments of the distribution of lifetime welfare across scenarios of lotteries of life.[6] Given that agents of type 1 are more risk-averse than agents of type 2, we have $\gamma^1 > \gamma^2$.

Under a zero utility from death and additive lifetime welfare (with no pure time preferences), the expected lifetime welfare $\bar{u}^i$ is:

$$\begin{aligned} \bar{u}^i &= \pi(m^i)\left[u(c^i) + u(d^i)\right] + (1 - \pi(m^i))\left[u(c^i)\right] \\ &= u(c^i) + \pi(m^i)u(d^i) \end{aligned} \tag{3}$$

where c^i and d^i denote, respectively, the first and second period consumption. As in Becker et al. (2005), we assume that the temporal welfare from consumption, $u(.)$ has the following form:

$$u(c) = f(c) + \alpha$$

with $f'(c) > 0$, $f"(c) < 0$. The constant α determines *"the level of annual consumption at which the individual would be indifferent between being alive or dead arising from the normalization of the utility of death to zero"* (Becker et al. pp 281). In our model, we assume that α is positive and large.[7]

In this two-scenarios world, the variance of lifetime welfare, $var(u^i)$ takes a quite simple form:

[6] For more general functions, see Machina (1983).

[7] This is equivalent to the assumption of a "priceless life context" found in Bommier (2006) which corresponds to a situation where $u(c(t)) = 1 + \lambda\omega(c(t))$ with $\lambda = 1/\alpha$ a small scalar and ω bounded.

$$\begin{aligned}
var(u^i) &= \left[\left(u(c^i)+u(d^i)\right)-\left(u(c^i)+\pi(m^i)u(d^i)\right)\right]^2 \\
&\quad +\left[u(c^i)-\left(u(c^i)+\pi(m^i)u(d^i)\right)\right]^2 \\
&= \left[(1-\pi(m^i))u(d^i)\right]^2+\left[\pi(m^i)u(d^i)\right]^2 \\
&= \left[u(d^i)\right]^2\left[(1-\pi(m^i))^2+\left(\pi(m^i)\right)^2\right] \qquad (4)
\end{aligned}$$

At this stage, let us note the ambiguous effect of the effort level on the variance of lifetime welfare. Actually,

$$\frac{\partial var(u^i)}{\partial m^i}=2\pi^{'}(m^i)\left[u(d^i)\right]^2\left[2\pi(m^i)-1\right] \begin{array}{c}>\\=\\<\end{array} 0 \Longleftrightarrow \pi(m^i) \begin{array}{c}>\\=\\<\end{array} \frac{1}{2} \qquad (5)$$

Hence, a higher effort level tends to raise the variance of lifetime welfare if $\pi(m^i)$ exceeds 1/2, whereas it tends to lower it if $\pi(m^i)$ is lower than 1/2. Further on, we shall postulate that $\pi(m^i) \leqslant 1/2$ so that $var(u^i)$ is *decreasing* in m^i.[8]

2.2 The laissez-faire

Agents of type $i \in \{1,2\}$ choose their first period and second period consumptions, as well as health expenditure, so as to maximize their objective function subject to their budget constraint:

$$\begin{array}{l}
\max\limits_{c,d,m} U^i(c^i,d^i,m^i) \\
\text{s.to} \left\{\begin{array}{c} c^i=w-s^i-m^i \\ d^i=\frac{s^i}{r}\end{array}\right.
\end{array}$$

where lifetime utility takes the following form

$$U^i(c^i,d^i,m^i)=u(c^i)+\pi(m^i)u(d^i)-\gamma^i\left[u(d^i)\right]^2\left[(1-\pi(m^i))^2+\left(\pi(m^i)\right)^2\right] \qquad (6)$$

[8]Given that life expectancy at birth is lower than 85 years, this assumption seems reasonable. Indeed, if agents in our model may live two periods of length 40 years beginning at the adult age of 25 years, it follows that life expectancy at age 25, equal to $1+\pi\left(m^i\right)$, must be smaller than 1.5, so that it is plausible to have $\pi\left(m^i\right) \leqslant 0.5$.

We assume that the individual's savings are entirely invested in private annuities and that r is the price of an annuity. The wealth endowment w is exogenous, supposed to be identical for all agents and equal to 1. Note also that there is no pure time preference, and that the interest rate is zero. The laissez faire solution is characterized by:

$$\begin{aligned} MRS^i_{c,d} &\equiv -\frac{U^i_d\,(c,d,m)}{U^i_c\,(c,d,m)} = -r \\ MRS^i_{c,m} &\equiv -\frac{U^1_m\,(c,d,m)}{U^1_c\,(c,d,m)} = -1 \end{aligned}$$

where $MRS^i_{x,y}$ stands for the marginal rate of substitution between x and y for individual with type γ^i. If the annuity market is actuarially fair and insurers can perfectly observe individuals monetary effort, the annuity price is set such that $r = \pi(m^i)$; in this case $MRS^i_{c,d} = -\pi(m^i)$. On the contrary, if the annuity is taxed, $r > \pi(m^i)$ and $MRS^i_{c,d} < -\pi(m^i)$. Assuming actuarially fair prices, the laissez-faire allocation for an individual of type i satisfies the following optimality conditions

$$u'(c^i) = u'(d^i)\left[1 - 2\gamma^i u(d^i)\left(2\pi(m^i) - 2 + \frac{1}{\pi(m^i)}\right)\right] \tag{7}$$

$$u'(c^i) = \pi'(m^i)u(d^i)\left[1 - 2\gamma^i u(d^i)\left(2\pi(m^i) - 1\right)\right] \tag{8}$$

Condition (7) characterizes the optimal saving decision. In the absence of any sensitivity to the variance of lifetime welfare ($\gamma^i = 0$), each agent would choose to smooth consumption perfectly over time (i.e. $c^i = d^i \forall i$), because of the conjunction of no pure time preference, an actuarially fair annuity price and a zero interest rate. However, under a positive γ^i, the term in brackets is necessarily smaller than 1, so that the sensitivity of agents to the variance of lifetime welfare makes them consume more in the first period (i.e. $c^i > d^i \forall i$). Actually, consuming during the first period is a simple way to insure oneself against undergoing a big loss of welfare if one dies at the end of the first period. Thus, consuming more during the first period is a straightforward way to protect oneself against a too large variation of lifetime welfare across scenarios of the lottery of life. Note also that the higher γ^i is, the steeper the intertemporal consumption profile will be *ceteris paribus*, because the more risk-averse the agent is, the more he will use that trick to avoid big welfare losses. This result is presented in the proposition below:

Proposition 1 *If the market of annuities is actuarially fair, $c^i > d^i$ for any individual with type $\gamma^i > 0$.*

Condition (8) characterizes the optimal health expenditure level. Under traditional expected utility theory, condition (8) would collapse to $\pi'(m^i)u(d^i) = u'\left(c^i\right)$, stating that the optimal health expenditure is such that the marginal welfare gain due to health expenditure (in terms of the second period of life) should equalize the marginal welfare cost of such an effort. However, under a positive γ^i, the marginal lifetime utility from health expenditure depends also on its impact on the variance of lifetime welfare (second term in brackets), which is always positive since we assume that $\pi(m^i)$ is lower than 1/2. Thus, under positive sensitivity to the variance in welfare, the level of health investment is always greater than under expected utility theory. Note that in the Laissez-Faire, the individual does not take into account the impact of health expenditures on the return of his savings $1/r = 1/\pi(m^i)$ so that the individual chooses a level of health expenditures which is too high compared to its optimal level.[9]

We now find the equilibrium levels of consumptions and of health expenditure of individuals with different sensitivity to the variance in welfare. Our results are summarized in the following proposition:

Proposition 2 *For any α sufficiently large, if the market for annuities is actuarially fair, the laissez allocation implies that for any two individuals with variance in welfare such that $\gamma^1 > \gamma^2$,*

[9] This result is emphasized in Becker and Philipson (1998).

(i) $m^1 > m^2$

(ii) $c^1 > c^2$

(iii) $d^1 < d^2$

Then, the individual with higher sensitivity to the variance in welfare invests more in health so as to increase his survival probability but also to decrease more his variance in lifetime welfare (since $\pi\left(m^i\right)$ is assumed to be lower than 1/2). He also consumes more in the first period and less in the second period than the less sensitive individual. Indeed, since the individual with type γ^1 prefers more early consumption as a way to partially compensate himself for the risk of dying young and thus having a low lifetime utility due to both low consumption levels and early death. Since both individuals have same initial endowments w, d^1 has to be smaller than d^2. In this case, the variance in welfare is lower for the individual with higher sensitivity since a lower level of future consumption decreases the variance in welfare (and health expenditures are higher for this individual).

3 First best

In this section, we assume that the social planner is utilitarian and that he perfectly observes individuals' types. The economy is assumed to be in a steady state equilibrium and the social planner can lend or borrow at a zero interest rate in order to balance the budget at any given period. The resource constraint of the economy is thus:

$$\sum_{i=1,2} n^i \left(c^i + \pi\left(m^i\right) d^i + m^i\right) \leq w \tag{9}$$

where n^i is the proportion of each group with different sensitivity γ^i. Thus, the social planner chooses consumption paths as well as health investments levels for each group of individuals in order to maximize

$$\sum_{i=1,2} n^i U^i \left(c^i, d^i, m^i\right)$$

subject to (9).

The first order conditions yield:

$$
\begin{aligned}
u'(c^i) &= \lambda &\quad (10)\\
u'(d^i)\left[1-2\gamma^i u(d^i)\left(2\pi(m^i)-2+\frac{1}{\pi(m^i)}\right)\right] &= \lambda &\quad (11)\\
\pi'(m^i)u(d^i)\left[1-2\gamma^i u(d^i)\left(2\pi(m^i)-1\right)-\lambda\frac{d^i}{u\left(d^i\right)}\right] &= \lambda &\quad (12)
\end{aligned}
$$

Combining (10) and (11), we obtain the optimal trade off between present and future consumption; this is identical to our laissez faire condition (7) when the price of the annuity is actuarially fair. Thus, first period consumption is still preferred to future consumption in the first best. On the contrary, (12) together with (10) differs from (8) by a term $-\lambda\pi'(m^i)d^i$. In the first best, the social planner takes into account the impact of health expenditure on the price of future consumption. Indeed, a higher level of effort m^i not only increases direct utility through higher survival but also increases the price of the annuity $\pi\left(m^i\right)$ which decreases consumption possibilities. Thus in the first best optimum, the social planner induces the individual to exert lower effort so as to reduce the negative impact of m^i over the individual's budget set. These results are summarized in the following proposition:

Proposition 3 *The first best allocation is such that, for any individual with sensitivity* $\gamma^i > 0$

(i) $m^{i,FB} < m^{i,LF}$,

(ii) $c^i > d^i$.

We now turn to the allocation of consumption and of health expenditures depending on individuals types. Obviously, first period consumption is equalized between individuals. However, considering (11) and (12), there is no reason for second period consumptions and health expenditures to be identical between individuals. Our results are summarized in the following proposition and solved in Appendix B.

Proposition 4 *Consider two groups of individuals with sensitivity to the variance in welfare such that $\gamma^1 > \gamma^2$. For any α sufficiently large, the first best yields*

(i) $c^1 = c^2$

(ii) $d^1 < d^2$

(iii) $m^1 > m^2$

In the first best optimum, first period consumption is equalized between individuals while future period consumption and health expenditures are differentiated between individuals. In such a case, the individual with higher sensitivity to the variance obtains lower level of future consumption but a higher level of health expenditures. As a result, whether the first best transfers resources from the high sensitivity individual toward the low sensitivity one is ambiguous.

We now study how to decentralize the first best optimum. Computing marginal rates of substitution in the first best

$$\begin{aligned} MRS^i_{c,d} &= -\pi\left(m^i\right) \\ MRS^i_{c,m} &= -\left[1 + \pi'\left(m^i\right) d^i\right] \end{aligned}$$

and comparing them with their laissez faire counterparts, one sees that the decentralization of the first best simply requires lump sum transfers between individuals and fair annuity prices. We also find that it is optimal to tax health expenditures for both individuals and the tax is higher for the individual with lower γ^i.

4 Second Best

In this section, we assume that the social planner cannot observe individuals' sensitivity to the variance of lifetime welfare. Using results of Proposition

2, if the social planner was offering first best bundles, one or the other individual would have interest in claiming he is of the other type so as to get higher consumption or benefit from higher health expenditures (depending on first best levels of c^i, d^i, m^i). However, we claim that for α large, future consumption only has marginal impacts on the level of lifetime utility so that individuals are more interested in increasing m than in increasing d. Under this assumption, only individual 2 has interest in lying on his type; in such a case, he would obtain a higher level of health expenditures at a price of lower future consumption (which only has secondary effects on his lifetime utility). Thus only the incentive constraint of individual with type γ^2 is relevant and the second best problem is written as

$$\sum_{i=1,2} n^i U^i \left(c^i, d^i, m^i\right)$$
$$s.to \left\{ \begin{array}{c} \sum_{i=1,2} n^i \left(c^i + \pi\left(m^i\right) d^i + m^i\right) \leq w \\ U^2\left(c^2, d^2, m^2\right) \geqslant U^2\left(c^1, d^1, m^1\right) \end{array} \right.$$

Full resolution of the above problem is provided in Appendix. In the second best, the optimal allocation is such that

$$\begin{aligned}
MRS^1_{c,d} &= -\pi\left(m^1\right)\left[\frac{1-\frac{\mu^2}{n^1}}{1-\frac{\mu^2}{n^1}\frac{MRS^2_{c,d}}{MRS^1_{c,d}}}\right] &(13)\\
MRS^2_{c,d} &= -\pi\left(m^2\right) &(14)\\
MRS^1_{c,m} &= -\left[1+\pi'\left(m^1\right)d^1\right]\left[\frac{1-\frac{\mu^2}{n^1}}{1-\frac{\mu^2}{n^1}\frac{MRS^2_{c,m}}{MRS^1_{c,m}}}\right] &(15)\\
MRS^2_{c,m} &= -\left[1+\pi'\left(m^2\right)d^2\right] &(16)
\end{aligned}$$

where the expression inside brackets is greater than 1 in expression (13) and lower than 1 in (16). We find the usual result of no distortion at the top for the low-type individual, the one who would lie on his type under asymmetric information. On the contrary, for the individual with type γ^1, the trade-off between two-period consumptions is distorted downward while the trade-off between consumption and health expenditures is distorted upward. Thus, early consumption as well as health expenditures are encouraged for this individual so as to make the problem incentive compatible. In such a case, the allocation of individual 1 is less attractive to type-2 individual as he would obtain too high a level of earlier consumption relatively to later consumption and too high a level of health expenditures.

We finally study how to decentralize this second best optimum. Comparing the above marginal rates of substitution with the laissez faire situation, we find that it is optimal to impose a tax on annuities for the individual with high sensitivity to the variance (as to induce him to consume more in early periods of his life). As, in the first best, both individuals face a Pigouvian tax on health expenditures so as to correct for the effect of health expenditures on the price of annuity and make the individual invest less in health. For the low sensitivity individual, the tax is the same as in the first best; yet, for individual 1, the tax might be lower so as to solve the incentive problem.

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Appendix

A Laissez Faire

The Laissez-faire problem can be rewritten as

$$\max_{s,m} U^i(c^i, d^i, m^i) = u(w - s^i - m^i) + \pi(m^i)u(\frac{s^i}{r}) - \gamma^i \left[u(\frac{s^i}{r})\right]^2 \left[(1-\pi(m^i))^2 + \left(\pi(m^i)\right)^2\right]$$

and first order conditions yield:

$$-u'(c^i)+\frac{\pi(m^i)}{r}u'(d^i)\left[1-2\gamma^i u(d^i)\left(2\pi(m^i)-2+\frac{1}{\pi(m^i)}\right)\right] = 0 \quad (17)$$

$$-u'(c^i)+\pi'(m^i)u(d^i)\left[1-2\gamma^i u(d^i)\left(2\pi(m^i)-1\right)\right] = 0 \quad (18)$$

Applying the implicit function theorem, one has

$$sign\left(\frac{dm^i}{d\gamma^i}\right)=sign\left(\frac{\partial^2 U^i(c^i,d^i,m^i)}{\partial mi\partial\gamma^i}+\frac{\partial^2 U^i(c^i,d^i,m^i)}{\partial mi\partial s^i}\frac{ds^i}{d\gamma^i}\right)$$

where, using (18), the right hand side is equal to

$$-2\pi'(m^i)u(d^i)^2\left(2\pi(m^i)-1\right)+\left[u''(c^i)-4\gamma^i\pi'(m^i)u'(d^i)u(d^i)\frac{\left(2\pi(m^i)-1\right)}{r}\right]\frac{ds^i}{d\gamma^i}$$

Under our assumption that α is large, first expression always dominates second part so that the sign of the above expression is positive. Thus $dm^i/d\gamma^i > 0$ and $m^1 > m^2$. We now determine how consumption levels c^i and d^i vary with individuals types in the laissez faire, by considering both the individual's budget constraint and equation (17):

$$c^i+\pi\left(m^i\right)d^i+m^i = w$$

$$u'(d^i)\left[1-2\gamma^i u(d^i)\left(2\pi(m^i)-2+\frac{1}{\pi(m^i)}\right)\right] = u'(c^i)$$

By contradiction, we prove that the only possible solution is to have $c^1 > c^2$ and $d^1 < d^2$ whenever $m^1 > m^2$.

B First Best

The first best level of health expenditures is defined by

$$\frac{\partial \pounds^i(c^i,d^i,m^i)}{\partial m^i}\equiv\pi'(m^i)u(d^i)\left[1-2\gamma^i u(d^i)\left(2\pi(m^i)-1\right)-\lambda\frac{d^i}{u\left(d^i\right)}\right]-\lambda=0$$

so that, using the implicit function theorem, one has

$$sign\left(\frac{dm^i}{d\gamma^i}\right)=sign\left(\frac{\partial^2 \pounds^i(c^i,d^i,m^i)}{\partial m^i\partial\gamma^i}+\frac{\partial^2 \pounds^i(c^i,d^i,m^i)}{\partial m^i\partial d^i}\frac{dd^i}{d\gamma^i}\right)$$

The expression on the right hand side is equal to

$$2\pi'(m^i)u\left(d^i\right)^2\left(1-2\pi(m^i)\right)+\left[\begin{array}{c}\pi'(m^i)u'(d^i)\left[1-2\gamma^i u(d^i)\left(2\pi(m^i)-1\right)-\lambda\frac{d^i}{u(d^i)}\right]\\ -\pi'(m^i)u(d^i)\left(2\gamma^i u'(d^i)\left(2\pi(m^i)-1\right)+\lambda\frac{u\left(d^i\right)-d^i u'\left(d^i\right)}{u(d^i)^2}\right)\end{array}\right]\frac{dd^i}{d\gamma^i}$$

and is positive if and only if

$$\begin{array}{cl} & 2u\left(d^i\right)\left(1-2\pi(m^i)\right)+\dfrac{u'(d^i)}{u(d^i)}\left[1-2\gamma^i u(d^i)\left(2\pi(m^i)-1\right)\right]\dfrac{dd^i}{d\gamma^i}\\ > & \left(2\gamma^i u'(d^i)\left(2\pi(m^i)-1\right)+\dfrac{\lambda}{u\left(d^i\right)}\right)\dfrac{dd^i}{d\gamma^i}\end{array}$$

which is always positive for any sign of $dd^i/d\gamma^i$ under the assumption that α is high enough. Using first order condition (11) with respect to d^i, we show by contradiction that the only possible solution is to have $d^1 < d^2$.

C Second best problem

The lagrangian of the second best problem is written as follows

$$\begin{aligned}\pounds &= \sum_{i=1,2} n^i U^i\left(c^i, d^i, m^i\right)+\lambda\left[w-\sum_{i=1,2} n^i\left(c^i+\pi\left(m^i\right)d^i\right)\right]\\ &\quad +\mu^2\left[U^2\left(c^2, d^2, m^2\right)-U^2\left(c^1, d^1, m^1\right)\right]\end{aligned}$$

First order conditions are

$$U_c^1\left(c^1, d^1, m^1\right)-\frac{\mu^2}{n^1}U_c^2\left(c^1, d^1, m^1\right) = \lambda \tag{19}$$

$$U_d^1\left(c^1, d^1, m^1\right)-\frac{\mu^2}{n^1}U_d^2\left(c^1, d^1, m^1\right) = \lambda\pi\left(m^1\right) \tag{20}$$

$$U_c^2\left(c^2, d^2, m^2\right)\left[1+\frac{\mu^2}{n^2}\right] = \lambda \tag{21}$$

$$U_d^2\left(c^2, d^2, m^2\right)\left[1+\frac{\mu^2}{n^2}\right] = \lambda\pi\left(m^2\right) \tag{22}$$

$$U_m^1\left(c^1, d^1, m^1\right)-\frac{\mu^2}{n^1}U_m^2\left(c^1, d^1, m^1\right) = \lambda\pi'\left(m^1\right)d^1 \tag{23}$$

$$U_m^2\left(c^2, d^2, m^2\right)\left[1+\frac{\mu^2}{n^2}\right] = \lambda\pi'\left(m^2\right)d^2 \tag{24}$$

Rearranging (19) and (20),

$$\begin{aligned} U_c^1\left(c^1,d^1,m^1\right)\left[1-\frac{\mu^2}{n^1}\frac{U_c^2\left(c^1,d^1,m^1\right)}{U_c^1\left(c^1,d^1,m^1\right)}\right] &= \lambda \\ U_d^1\left(c^1,d^1,m^1\right)\left[1-\frac{\mu^2}{n^1}\frac{U_d^2\left(c^1,d^1,m^1\right)}{U_d^1\left(c^1,d^1,m^1\right)}\right] &= \lambda\pi\left(m^1\right) \end{aligned}$$

and substituting one into the other, we obtain the marginal rate of substitution between present and future consumption for individual 1

$$\begin{aligned} MRS_{c,d}^1 &= -\pi\left(m^1\right)\frac{\left[1-\frac{\mu^2}{n^1}\right]}{\left[1-\frac{\mu^2}{n^1}\frac{U_d^2(c^1,d^1,m^1)}{U_d^1(c^1,d^1,m^1)}\right]} \\ &= -\pi\left(m^1\right)\left[\frac{1-\frac{\mu^2}{n^1}}{1-\frac{\mu^2}{n^1}\frac{\overline{MRS}_{c,d}^2}{MRS_{c,d}^1}}\right] \end{aligned}$$

where $U_c^2\left(c^1,d^1,m^1\right)/U_c^1\left(c^1,d^1,m^1\right)=1$ and $\overline{MRS}_{c,d}^2$ is the marginal rate of substitution of individual 2 mimicking individual 1. The expression inside brackets is always greater than 1 since, using (6), it is possible to show that $U_d^2(c,d,m)/U_d^1(c,d,m)>1$ for $\gamma^2<\gamma^1$.

We compute here the marginal rate of substitution between health expenditure and first period consumption. Rearranging (23) and (19),

$$\begin{aligned} U_c^1\left(c^1,d^1,m^1\right)\left[1-\frac{\mu^2}{n^1}\frac{U_c^2\left(c^1,d^1,m^1\right)}{U_c^1\left(c^1,d^1,m^1\right)}\right] &= \lambda \\ U_m^1\left(c^1,d^1,m^1\right)\left[1-\frac{\mu^2}{n^1}\frac{U_m^2\left(c^1,d^1,m^1\right)}{U_m^1\left(c^1,d^1,m^1\right)}\right] &= \lambda\pi'\left(m^1\right)d^1 \end{aligned}$$

one obtains

$$\begin{aligned} MRS_{c,m}^1 &= -\frac{U_m^1\left(c^1,d^1,m^1\right)}{U_c^1\left(c^1,d^1,m^1\right)}=-\pi'\left(m^1\right)d^1\frac{1-\frac{\mu^2}{n^1}}{\left[1-\frac{\mu^2}{n^1}\frac{U_m^2(c^1,d^1,m^1)}{U_m^1(c^1,d^1,m^1)}\right]} \\ &= -\pi'\left(m^1\right)d^1\left[\frac{1-\frac{\mu^2}{n^1}}{1-\frac{\mu^2}{n^1}\frac{\overline{MRS}_{c,m}^2}{MRS_{c,m}^1}}\right] \end{aligned}$$

Using (6), it possible to prove that $U_m^2(c,d,m)<U_m^1(c,d,m)$, so that the expression inside brackets is always lower than 1.

For individual 2, straightforward rearrangements of (21) with (22) yield (14) and of (24) with (21) yield (16).