Financing Social Security by Consumption Tax: A Political-Economy Perspective

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Abstract

In some developed nations, e.g. Japan, France, Denmark, and Germany, consumption tax is increasingly becoming a popular way to fund social security expenditures, while wage-based taxes have been traditionally used. The paper analyzes how these taxes are combined for social security financing in a political arena of overlapping generations, placing a focus on their effects on inter- and intra-generational redistribution, and examines how population aging affects the political equilibrium. Employing the concept of structure-induced equilibrium invented by Shepsle (1979), the paper shows that a society with slow population growth is likely to have multiple equilibria with social security financed respectively by only wage tax, only consumption tax, and both, while a society with rapid population growth has a unique equilibrium with social security financed by only wage tax.

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1 Introduction

This paper studies a political economy of financing social security by taxing wage income and consumption. Recently, in countries such as Japan, France, Germany, and Denmark, shifting the financial sources from traditional wage-based social security contributions to consumption taxes is increasingly becoming a popular idea. Although the popularity comes from several social and economic reasons, a key to understanding it from a political economy point of view is the notion of intergenerational fairness: the elderly should share the burdens of population aging with the young according to their ability to pay.[1] In fact, the notion has been already playing an important role in shaping tax policies on social security transfer in developed countries. As Adema and Ladaique (2005) report, most OECD countries employ indirect taxation on consumption out of benefit income, as well as direct taxation on pension transfers, to make a considerable amount of public social expenditure paid back to the governments.[2]

Table 1 shows how diversified the ways to finance social protection expenditure are across nations.[3] The shares of government contributions vary widely from 11.5% in Korea to 80.4% in Australia, and as the mirror image, the shares of social contributions differ widely between 80.5% in France and 11.6% in Australia. Reasonably, general revenues consist of various tax revenues such as consumption taxes, individual income taxes, corporate income taxes, capital income taxes, and so on, while a large part of social contributions are wage-based taxes. To be important, these components of general revenues include taxes that are theoretically considered to have the excise tax effect; part of their burdens are shifted to commodity prices and effectively

[1]The introduction of consumption tax financing has been called for not only from the ground of intergenerational fairness. In Japan, fairness within generations, especially among the young, is also emphasized because a substantial population of them are not paying the national pension contributions, in spite of being mandated. In European counties, more emphasis is placed on the international competitiveness of domestic firms. For example, in France, the idea of introducing a "social" value-added tax in exchange for lowering the labor costs of domestic firms has been very controversial at least since the last presidential election. Denmark has already introduced such a reform. Germany also raised its value-added tax by 3 % for the same purpose.

[2]According to their estimate of *net* social expenditure, which takes account of the impact of taxation, the international differences in the ratio of social expenditure to GDP are not very large in contrast to what we usually observe in terms of *gross* social expenditure. In terms of gross social expenditure Denmark and Sweden are the biggest social spenders, but in terms of net social expenditure, France, Germany, and Sweden are among the highest.

[3]Due to the limited data availability, the social protection programs underlying the figures in table 1 include not only old-age pension programs but also health care programs, unemployment programs, housing and social assistance programs, although transfers to the old undoubtedly constitute a dominating component.

paid by consumers.[4] To the extent of the excise tax effect prevailing, the share of general revenues may be a rough estimate of consumption tax financing.

Table 1 also shows the international diversity in the share-out ratio of social contributions between employers and protected persons. In Sweden, for example, employers pay more than four times as large as protected persons do, while the social contributions are split almost half and half in Japan and the Netherlands. In Denmark, in contrast, protected persons pay more than twice as large as employers. If we accept the conventional but somewhat questionable assumption that social contributions paid by employers are likely to be shifted to commodity prices while those paid by employees are not, then we may count the share of employers' contributions when estimating the extent of consumption tax financing.[5]

Table 1: The composition of social protection receipts, %

Nation	Social Contributions			Government contributions			Other receipts
	Employers' contributions	Contribution by protected persons	Total	Earmarked taxes	General revenues	Total	
Korea	32.8	47.7	80.5	0	11.5	11.5	8.0
Netherlands	33.8	33.7	67.4	0	18.4	18.4	14.1
France	45.9	20.7	66.6	19.8	10.0	29.8	3.6
Germany	37.0	27.4	64.4	0	33.9	33.9	1.7
Japan	31.5	29.0	60.5	0	24.5	24.5	15.0
Italy	42.7	14.7	57.3	0	40.9	40.9	1.8
Sweden	42.1	9.2	51.3	0	46.7	46.7	2.0
USA	28.7	22.3	51.0	0.6	35.7	36.3	12.7
Finland	39.0	10.9	49.9	0	44.3	44.3	5.7
UK	30.2	19.5	49.7	0.5	48.0	48.5	1.8
Denmark	9.7	20.7	30.3	0	63.0	63.0	6.7
New Zealand	8.8	3.9	12.7	1.3	75.9	77.2	10.0
Australia	8.7	2.9	11.6	5.0	80.4	85.4	3.0

Note: Other receipts include transfer from reserves. The data for European countries are based on Eurostat (2006), and those for the others are on International Labor Office (1997). The former provides the figures in 2001–2003 but the latter does those in 1995–97.

[4]See Atkinson and Stiglitz (1980, Ch.6) for the excise tax effect of corporate income tax.

[5]Theoretically, when the markets are perfectly competitive, the share-out ratios themselves should make no differences in the equilibrium prices and resource allocation. This theoretical argument, however, seems to be hardly accepted as convincing in practice.

Why is the composition of social protection receipts so diversified across countries? Why has consumption tax financing become so popular recently in some countries? Why are social contributions shared out between employees and employers so differently across nations? This paper will present a political economy perspective to answer these questions. In an overlapping generations model of workers and retirees with different income and wealth, we will analyze the public choice of the combination of wage and consumption tax to finance social security benefits.

We will focus the analysis on the differences between the two taxes in their intra- and intergenerational redistribution effects. They produce varieties in policy preferences among workers and retirees.

When used to finance social security benefits that are flat among retirees, consumption tax redistributes income not only across workers but also across retirees. The beneficiaries from consumption tax financing are a group of poor workers and poor retirees, while rich retirees as well as rich workers pay more than they receive from the social security program. Wage tax, on the other hand, redistributes income only across workers since retirees are just receivers of benefits. The beneficiaries from wage tax financing consist of poor workers and all retirees, and all retirees want to increase the wage tax rate to the level that maximizes the social security benefits. As far as the intragenerational redistribution effects are concerned, therefore, a majority including rich retirees support replacing consumption tax with wage tax, and hence wage tax financing is likely to be a unique outcome in the political choice of social security funding.

The differential effects of the two taxes on intergenerational transfer also play an important role in the political choice. By wage tax financing the revenues are transferred across generations to finance the benefits for retirees, and a higher wage tax will increase intergenerational transfer if we put the Laffer curve argument aside. By consumption tax financing, on the other hand, only the consumption taxes paid by workers are transferred to retirees. Thus, consumption tax redistributes smaller income across generations than wage tax in funding a given amount of social security benefits. This in turn implies that consumption tax financing leads workers to save more for their consumption after retirement. Moreover, a higher consumption tax will decrease the size of intergenerational transfer generated by a given wage tax, since it discourages labor supply and reduces wage tax revenues. In a dynamically-efficient economy, where the population growth rate is lower than the interest rate, intergenerational transfer is a more expensive way for workers to finance their consumption after retirement than private saving. A large number of workers may prefer reducing the cost of intergenerational transfer by financing social security with the combination of a higher consumption tax and a lower wage tax.

Generally, no Condorcet winner exists in the majority voting over multiple issues. Following Conde-Ruiz and Galasso (2005), we will employ the concept of structure-induced equilibrium invented by Shepsle (1979) to aggregate the policy preferences over the financing methods and describe the political equilibrium. Specifically, wage tax and consumption tax are put to the vote separately to choose a Condorcet winner with the other tax rate taken as given.

Owing to the differential impacts on intragenerational redistribution, a poorer worker turns out decisive in the majority voting over wage tax rates than in the voting over consumption tax rates. Since the poorer worker benefits the more from the social security program, a majority supports larger social security benefits per retiree in wage tax financing. We will show that in a dynamically-efficient economy with a small margin between the population growth and the interest rate as well as in a dynamically-inefficient economy, wage tax financing is a unique structure-induced equilibrium outcome. As we have traditionally observed in several countries, social security benefits are financed by only wage tax.

Population aging affects the policy preferences through several channels. Regarding the preferences over wage tax, even poor workers are induced to support a lower wage tax rate, since wage tax has smaller marginal revenues due to a decrease in the labor force. On the other hand, because of a reduction in the number of worker-voters, the political influence of retirees is strengthened to support a higher wage tax rate. Although these two changes counteract, we will show that if the density of the median worker-voters is sufficiently large, which seems quite natural in light of a standard wage distribution, the political influence of retirees is outweighed and aging shifts the aggregate policy preferences toward a lower wage tax rate. Regarding the preferences over consumption tax, workers are inclined to support a higher consumption tax rate, since intergenerational transfer costs higher for them due to slower population growth. Retirees are also induced to support a higher consumption tax rate to compensate the wage tax revenues lost due to a reduction in the labor force. Thus aging shifts the aggregate policy preferences toward a higher consumption tax rate.

Combining these effects, population aging may drastically change the political equilibrium. We will show that in a dynamically-efficient economy with a large margin between the population growth and the interest rate, there may occur multiple structure-induced equilibria; in the respective equilibria only wage tax, only consumption tax, and both are employed to finance social security. In particular, unless the margin is too large, financing by only wage tax is still an equilibrium outcome because the equilibrium wage tax rate is high enough to induce no majority to support funding social security by consumption tax. The high cost of intergenerational transfer sustains the co-existence of equilibria with consumption tax financing. With a sufficiently large margin, however, no majority supports wage tax financing and social security is funded by only consumption tax. The emergence of the three types of equilibrium explains the recently soaring popularity of consumption tax financing as well as the international diversity in the composition of social security funding as reported in Table 1.

At least to my knowledge no paper addresses the public choice of financing social security with a combination of different taxes. In the literature on political economy of income redistribution and social security, including Browning (1975), Meltzer and Richard (1981), Hu (1982), Boadway and Wildasin (1989), Cooley and Soares (1999), Tabellini (2000), Razin, Sadka, and Swagel (2002) among others, almost all the models employ only one wage tax to finance government spending for redistribution. As an exception, the model of Conde-Ruiz and Galasso (2005), on which ours is based, employs two different wage taxes, a social security tax for transfers to retirees and an income redistribution tax for transfers to poor workers, for the purpose of explaining the co-existence of the two social transfer programs.

The paper is organized as follows. Section 2 sets out the model. Section 3 examines the public choice of a wage tax rate through majority voting, taking a consumption tax rate as given. It is shown that population aging may lower the equilibrium wage tax rate in spite of retirees becoming politically more influential. Section 4 considers the public choice of a consumption tax rate through majority voting, taking a wage tax rate as given. It is shown that population aging increases the equilibrium consumption tax rate. Making use of the results of the two previous sections, section 5 analyzes the wage and consumption tax rates determined in the structure-induced equilibrium, and examines the consequences of population aging. Section 6 concludes the paper.

2 The model

2.1 Brief description of the economy

We will consider an economy with two overlapping generations, the young and the old, in which the population grows at the rate of $n > 0$. Every individual works and earns a wage income only when young and consumes only when old. Worker i has E_i units of leisure as her initial endowment, out of which she supplies N_i units of labor. She consumes C_i units of numeraire goods after retirement, financing it out of her savings, interest income, and social security benefit. Every retiree receives a flat social security benefit, the amount of which is denoted by B in real terms. Social security benefits are funded in a pay-as-you-go fashion by either wage tax revenues, consumption tax revenues, or both. A wage tax rate is denoted by τ_w and a net consumption tax rate is by $\hat{\tau}$, which we will convert into a gross consumption tax rate $\tau_c := \hat{\tau}/(1+\hat{\tau})$. There is no intra-family transfer. The interest rate and the wage rate are assumed to be constant; the former is denoted by r and the latter is normalized to unity.

2.2 Workers and retirees

Worker i faces a lifetime budget constraint,

$$(1+\hat{\tau})C_i = (1+r)(1-\tau_w)N_i + (1+\hat{\tau})B,$$

where $0 \leq N_i \leq E_i$ and the term $(1+\hat{\tau})B$ represents nominal social security benefits. Using a net consumption tax rate $\tau_c := \hat{\tau}/(1+\hat{\tau})$, we can rewrite the budget constraint into

$$C_i = (1+r)(1-\tau_c)(1-\tau_w)N_i + B.$$

Putting aside the difference in the timing of payment, consumption tax affects a worker's budget constraint in the same way as wage tax does.

Her leisure endowment, E_i, which we will regard as her ability, depends on her luck that follows a cumulative distribution function, $F(E_i)$, with support $[\underline{E}, \overline{E})$. The mean E and the median E_m are respectively defined by $E = \int_{\underline{E}}^{\overline{E}} x f(x) dx$ and $F(E_m) = 1/2$, where $f(x)$ denotes the density function. The ability distribution is skewed to the left like figure 1, yielding $E_m < E$.

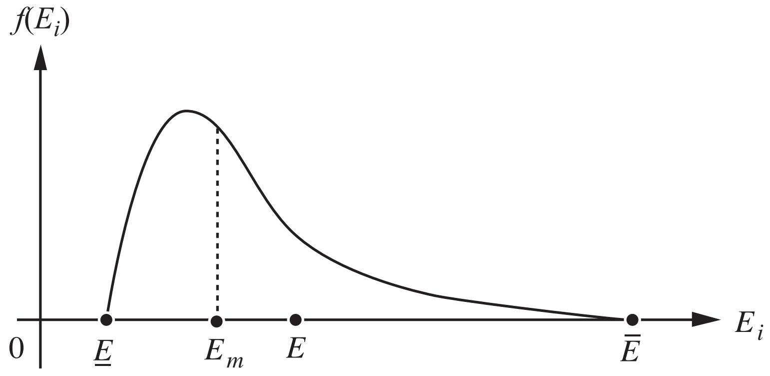

Figure 1: Distribution of ability

We will specify worker i's utility function as

$$u_i = \frac{C_i}{1+\delta} + \ln(E_i - N_i),$$

where δ is the rate of time preference, and set $\delta = r$ for simplicity. We will also assume that $\underline{E}$ is large enough to ensure $N_i > 0$ for every individual in equilibria. Her labor supply and the indirect utility are then obtained respectively as follows:

$$N_i = E_i - \frac{1}{(1-\tau_w)(1-\tau_c)}$$

and

$$U_i = (1-\tau_c)(1-\tau_w)E_i - \ln(1-\tau_w)(1-\tau_c) + \frac{B}{1+r}. \tag{1}$$

Her savings are written as $A_i = (1-\tau_w)N_i$. The distribution of before-tax wage income, which is equal to labor supply, is as skewed as the ability distribution.

Suppose that worker i was retired after saving A_i. Her utility in the retirement period comes only from her consumption, so that we will write it as

$$V_i = (1-\tau_c)(1+r)A_i + B. \tag{2}$$

2.3 Social security system

Social security expenditures in each period are financed by wage tax and consumption tax revenues collected in the same period. From the budget constraint, we can express the benefit per retiree in real terms as

$$B = \tau_c(1+r)A + \tau_w(1+n)N, \tag{3}$$

where

$$N = N(\tau_w, \tau_c) := E - \frac{1}{(1-\tau_w)(1-\tau_c)}$$

and

$$A = (1-\tau_w)N(\tau_w, \tau_c)$$

are labor supply and savings per retiree, respectively. The first term in (3) is the consumption tax revenues (except for taxes paid by consumption out of social security benefits) and the second is the wage tax revenues. If the tax rates are constant over time, we can write (3) into

$$B = B(\tau_w, \tau_c) := [(1+r)\tau_c(1-\tau_w) + (1+n)\tau_w]N(\tau_w, \tau_c). \tag{4}$$

3 Financing by wage tax

3.1 Policy preferences of retirees

Let us first consider the preferences of retirees over wage tax rates, taking a consumption tax rate as given. Plugging (3) into (2), we obtain the utility function of a retiree as

$$V_i(\tau_w, \tau_c) := (1+r)[(1-\tau_c)A_i + \tau_c A] + (1+n)\tau_w N(\tau_w, \tau_c). \tag{5}$$

Since A_i and A are predetermined, the most preferred wage tax rates of retirees are equal to τ_w^o, which maximizes the per-worker wage tax revenues, $\tau_w N(\tau_w, \tau_c)$. The first order condition,

$$\frac{\partial V_i}{\partial \tau_w} = (1+n)\left(N(\tau_w, \tau_c) + \tau_w \frac{\partial N}{\partial \tau_w}\right) = 0, \tag{6}$$

yields

$$\tau_w^o = \tau_w^o(\tau_c) := 1 - \frac{1}{\sqrt{(1-\tau_c)E}}. \tag{7}$$

We also find that the preferences are single peaked since

$$\frac{\partial^2 V_i}{\partial {\tau_w}^2} = -\frac{2}{(1-\tau_c)(1-\tau_w)^3} < 0.$$

Every retiree has the same preferences over wage tax rates because no redistribution occurs among retirees.

3.2 Policy preferences of workers

Next consider the policy preferences of workers. In contrast to the case of retirees, wage tax financing produces income redistribution across workers in favor of the poor.

Let us examine how a higher wage tax affects worker i's utility. Following the literature on political economy of social security, we will assume policy commitment; workers expect that the tax rate determined today will not be altered in the future.[6] From (1), then, the utility of worker i is written as

$$U_i(\tau_w, \tau_c) := (1-\tau_c)(1-\tau_w)E_i - \ln(1-\tau_w)(1-\tau_c) + \frac{B(\tau_w, \tau_c)}{1+r}. \tag{8}$$

Differentiating (8) and (4), we have

$$\frac{\partial U_i}{\partial \tau_w} = -(1-\tau_c)N_i + \frac{1}{1+r}\frac{\partial B}{\partial \tau_w} \tag{9}$$

and

$$\frac{\partial B}{\partial \tau_w} = (1+n)\left(N + \tau_w \frac{\partial N}{\partial \tau_w}\right) + (1+r)\tau_c\left(-N + (1-\tau_w)\frac{\partial N}{\partial \tau_w}\right). \tag{10}$$

A higher wage tax produces two effects on the amount of social security benefits that the current workers expect to receive after retirement. First, as represented by the first term in (10), it increases wage tax revenues collected from the next working generation to increase the amount of benefits. Second, as shown by the second term in (10), it reduces savings today and thus reduces the future consumption tax revenues to decrease the amount of benefits.

Combining (9) and (10), we obtain

$$\frac{\partial U_i}{\partial \tau_w} = (1-\tau_c)(E-E_i) + [1-(1-\tau_w)(1-\tau_c)]\frac{\partial N}{\partial \tau_w} - \frac{r-n}{1+r}\left(N + \tau_w \frac{\partial N}{\partial \tau_w}\right). \tag{11}$$

The first term here represents worker i's utility change through the intragenerational redistribution effect of a higher wage tax. It favors the workers with ability lower than the mean (and hence workers poorer than the average), though the advantage is partly offset with consumption tax. The second term, being always negative, represents a loss in utility due to aggravated distortions in labor supply.[7] The third term shows a loss caused by intergenerational redistribution. As known well, income transfer from the young to the old is dynamically inefficient when $r > n$.

Let τ_w^y be worker i's most-preferred wage tax rate. It satisfies the first order condition,

$$\frac{\partial U_i}{\partial \tau_w} = -(1-\tau_c)E_i + \left(\frac{1+n}{1+r} - \tau_c\right)E + \frac{1}{1-\tau_w}\left(1 - \frac{1+n}{(1+r)(1-\tau_c)(1-\tau_w)}\right) \leq 0. \tag{12}$$

With the strict inequality, $\tau_w^y = 0$. From (12) we will write it as a function of E_i, τ_c, and n:

$$\tau_w^y = \tau_w^y(E_i, \tau_c, n). \tag{13}$$

[6]In appendix C, following Conde-Ruiz and Galasso (2005), we relax this assumption and show that the same equilibrium outcomes are realized in the subgame-perfect equilibria of an infinitely repeated voting game.

[7]Because $1-(1-\tau_w)(1-\tau_c)$ is a difference between before-tax and after-tax wage rates, the term multiplying it with a change in the amount of labor supply yields a change in the deadweight loss in labor supply.

Whenever $\tau_w^y > 0$, the second order condition, $\partial^2 U_i/\partial\tau_w^2 < 0$, must be met. The condition is reduced into

$$1 - \frac{2(1+n)}{(1+r)(1-\tau_w)(1-\tau_c)} < 0, \tag{14}$$

which is guaranteed if $(1+n)/(1+r) > 1/2$.

Workers' most-preferred wage tax rates have the following properties. First, as we can see from (11), $\tau_w^y = 0$ for $E_i \geq E$ whenever $r \geq n$. Rich workers are better off without a social security system that makes dynamically inefficient transfer. Second, τ_w^y is decreasing in E_i. The richer is a worker, the lower is her most-preferred wage tax rate. This is because $U_i(\tau_w, \tau_c)$ satisfies the single crossing property:

$$\frac{\partial^2 U_i}{\partial E_i \partial\tau_w} = -(1-\tau_c) < 0. \tag{15}$$

This property also allows us to apply the median voter theorem to aggregate the voters' preferences over wage tax rates even though they are not single-peaked.[8]

3.3 Voting equilibrium

Suppose that a majority voting takes place to determine a wage tax rate for social security financing, with a consumption tax rate being taken as given. We will invoke the median voter theorem to find a Condorcet winner.

Comparing the policy preferences between retirees and workers, we have $\tau_w^y(E_i, \tau_c, n) < \tau_w^o(\tau_c)$ because τ_w^o is the tax rate that maximizes wage tax revenues. Since $n > 0$, this means that the decisive voter is among workers. If we denote her ability by E_w, her ability satisfies $1 + (1+n)F(E_w) = (2+n)/2$, or

$$F(E_w) = \frac{n}{2(1+n)}, \tag{16}$$

from which we have $E_w < E_m$. Since E_w increases with n, we will write the equilibrium wage tax rate, τ_w^*, as follows:

$$\tau_w^* = \tau_w^y(E_w, \tau_c, n) := \tau_w^*(\tau_c, n). \tag{17}$$

Figure 2 depicts the distribution of the most-preferred wage tax rates to explain the voting equilibrium diagrammatically. The rectangle areas associated with $\tau_w = 0$ and $\tau_w = \tau_w^o$ represent the population of workers and retirees who prefer these tax rates most, respectively. The hight of the graph at each tax rate other than 0 and τ_w^o corresponds to the density of workers who prefer it most. The hump-shaped schedule in the middle is drawn by reversing the right and left of the ability distribution and extending it upward by a factor of $1+n$. The equilibrium wage tax rate is determined at the level where the areas to the left and to the right under the graph coincide.

[8] See Gans and Smart (1996) for the details.

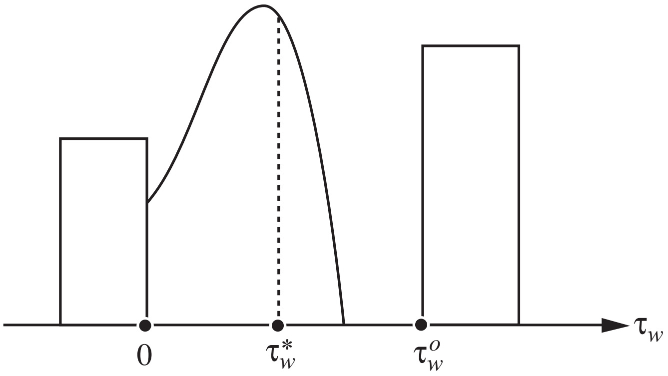

Figure 2: Voting equilibrium with wage taxes

3.4 Comparative statics

Consider how population aging, which we will identify as a decrease in n, and a consumption tax increase affect the equilibrium wage tax rate, assuming that $\tau_w^* > 0$.

Population aging affects the equilibrium wage tax rate in two opposing directions:

$$\frac{\partial \tau_w^*}{\partial n} = \frac{\partial \tau_w^y}{\partial n} + \frac{\partial \tau_w^y}{\partial E_i}\frac{\partial E_w}{\partial n}.$$

First, it reduces τ_w^* by lowering the most-preferred tax rate of the decisive worker-voter, because the cost of intergenerational transfer increases due to smaller labor force. This effect is represented by the first term, which is positive if $\tau_w^y > 0$, since differentiating (9) with respect to n and making use of (12) yields

$$\frac{\partial^2 U_i}{\partial n \partial \tau_w} = \frac{1}{1+n}\Big((1-\tau_c)N_i + \tau_c E\Big) > 0$$

when $\tau_w = \tau_w^y$. Second, it increases τ_w^* by strengthening the political influence of retirees to make a poorer worker-voter decisive in the voting. This effect is captured by the second term, which is negative if $\tau_w^y > 0$, since $\partial \tau_w^y / \partial E_i < 0$ and

$$\frac{\partial E_w}{\partial n} = \frac{1}{2(1+n)^2 f(E_w)} > 0.$$

Since these two effects counteract each other, it is generally ambiguous how population aging affects the equilibrium wage tax rate. Nonetheless, if the density of the decisive worker-voter, $f(E_w)$, is sufficiently large, as we can expect from the usual shape of earning distributions like figure 1, the first effect is likely to outweigh the second so that population aging decreases the equilibrium wage tax rate.[9]

[9]This observation is similar to the one presented by Razin, Sadka, and Swagel (2002). They used an overlapping generations model with human capital formation and showed that population aging may lead to the downsizing of the welfare state. They also tested this hypothesis with data

A consumption tax increase, on the other hand, unambiguously decreases the equilibrium wage tax rate. To see this, differentiate (9) with respect to τ_c, and we have

$$\frac{\partial^2 U_i}{\partial \tau_c \partial \tau_w} = -(E - E_i) - \frac{1+n}{(1+r)(1-\tau_w)^2(1-\tau_c)^2}. \tag{18}$$

The first term, which is negative for workers poorer than the average, shows that a consumption tax increase weakens the redistribution effect of wage tax financing. This is because a larger part of social security benefits are paid back to the government as consumption taxes. The second term represents the net of the following two effects; a consumption tax increase, on the one hand, aggravates labor market distortion by reducing labor supply, and on the other, decreases the costly intergenerational transfer by reducing wage tax revenues. Since the former outweighs the latter, the second term is always negative.

Overall, since $E_w < E$, both terms are negative for the decisive worker-voter and hence

$$\frac{\partial \tau_w^*}{\partial \tau_c} = \frac{\partial \tau_w^y}{\partial \tau_c} < 0$$

whenever $\tau_w^* > 0$. The following proposition summarizes the observations obtained in this section.

Proposition 1 *Suppose that $\tau_w^* > 0$. (i) It is generally ambiguous how population aging affects the equilibrium wage tax rate. Provided that the density of the decisive worker-voter is sufficiently large, then population aging lowers the equilibrium wage tax rate. (ii) A consumption tax increase unambiguously decreases the equilibrium wage tax rate.*

4 Financing by consumption tax

4.1 Policy preferences of retirees

In contrast to wage tax financing, consumption tax financing redistributes income not only among workers but also among retirees. In addition, it decreases intergenerational transfer by reducing wage tax revenues.

Let us start with the analysis of retirees' preferences over consumption tax rates. With a wage tax rate taken as given, (3) and (5) show that the most-preferred consumption tax rate of retiree j, denoted by τ_c^o, satisfies

$$\frac{\partial V_j}{\partial \tau_c} = -(1+r)A_j + \frac{\partial B}{\partial \tau_c} \leq 0, \tag{19}$$

and $\tau_c^o = 0$ with the strict inequality. For retirees, whose asset holdings are predetermined, the effect of a higher consumption tax on the size of benefits is written as

$$\frac{\partial B}{\partial \tau_c} = (1+r)A + (1+n)\tau_w \frac{\partial N}{\partial \tau_c}. \tag{20}$$

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for the US and 12 European countries over the period 1965–92 to obtain a positive empirical result. See also Disney (2007), Simonovits (2007), Galasso and Profeta (2007) among others for the controversies their paper initiated.

The first term shows an increase in consumption tax revenues per retiree, and the second does a reduction in wage tax revenues per retiree. Substituting (20) into (19) yields

$$\frac{1}{1+r}\frac{\partial V_j}{\partial \tau_c} = A - A_j - \frac{(1+n)\tau_w}{(1+r)(1-\tau_c)^2(1-\tau_w)} \leq 0. \tag{21}$$

Thus, $\tau_c^o = 0$ for every retiree wealthier than the average. Further, since $\partial^2 V_j/\partial \tau_c^2 < 0$, every retiree's policy preferences are single-peaked over consumption tax rates.

Suppose that the wage tax rate is fixed over time. If retiree j holds ability E_j when young, then $A - A_j = (1-\tau_w)(E - E_j)$, and hence from (21) her most-preferred consumption tax rate, τ_c^o, is determined as a function of E_j, τ_w, and n:

$$\tau_c^o = \tau_c^o(E_j, \tau_w, n). \tag{22}$$

Since the cross derivative of her utility function satisfies

$$\frac{\partial^2 V_j}{\partial E_j \partial \tau_c} = -(1+r)(1-\tau_w) < 0,$$

it follows that τ_c^o is decreasing in E_i; the richer is a retiree, the lower is her most-preferred consumption tax rate.

4.2 Policy preferences of workers

Consider next the preferences of workers over consumption tax rates, assuming policy commitment as in the case of wage tax financing. Differentiation of (8) shows that a higher consumption tax rate affects worker i's utility and the social security benefit as follows:

$$\frac{\partial U_i}{\partial \tau_c} = -(1-\tau_w)N_i + \frac{1}{1+r}\frac{\partial B}{\partial \tau_c} \tag{23}$$

and

$$\frac{\partial B}{\partial \tau_c} = (1+r)A + (1+r)(1-\tau_w)\tau_c\frac{\partial N}{\partial \tau_c} + (1+n)\tau_w\frac{\partial N}{\partial \tau_c}. \tag{24}$$

Comparing (24) with (20), the only difference is the second term in (24). This shows that workers' preferences over consumption tax rates take account of the change in future consumption, for it affects the benefits they receive after retirement.

Plugging (24) into (23) yields

$$\frac{\partial U_i}{\partial \tau_c} = (1-\tau_w)(E-E_i) + [1-(1-\tau_w)(1-\tau_c)]\frac{\partial N}{\partial \tau_c} - \frac{(r-n)\tau_w}{1+r}\frac{\partial N}{\partial \tau_c}. \tag{25}$$

Similar to (12), the first term represents the intragenerational redistribution, the second represents the distortion effect on labor supply, and the third represents the effect associated with intergenerational transfer. The second term is always negative. The third is positive when $r > n$, because a higher consumption tax decreases intergenerational transfer by reducing wage tax revenues.

Arranging the terms in (25), we find that worker i's most-preferred consumption tax rate, τ_c^y, satisfies

$$\frac{\partial U_i}{\partial \tau_c} = (1-\tau_w)(E-E_i) - \frac{(1+n)\tau_w}{(1+r)(1-\tau_c)^2(1-\tau_w)} - \frac{\tau_c}{(1-\tau_c)^2} \leq 0, \tag{26}$$

and $\tau_c^y = 0$ with the strict inequality. Since $\partial^2 U_i / \partial \tau_c^2 < 0$, every worker's preferences are single-peaked over consumption tax rates. We will write τ_c^y as a function of E_i, τ_w, and n:

$$\tau_c^y = \tau_c^y(E_i, \tau_w, n). \tag{27}$$

From (26), $\tau_c^y = 0$ for workers richer than the average. Moreover, since

$$\frac{\partial^2 U_i}{\partial E_i \partial \tau_c} = -(1-\tau_w) < 0,$$

τ_c^y is decreasing in E_i; similar to retirees, the richer is a worker, the lower is her most-preferred consumption tax rate.

4.3 Voting equilibrium

Suppose that a majority voting takes place to determine a consumption tax rate for social security financing, with a wage tax rate being taken as given. To obtain a voting equilibrium, we will first find a pair of a worker and a retiree whose preferences over consumption tax rates coincide.

Comparing between (21) and (26), we observe that the most-preferred consumption tax rate is the same for worker i and retiree j if and only if their ability levels are connected to satisfy

$$E_j = E^o(E_i, \tau_w, n) := E_i + \frac{\tau_c^y}{(1-\tau_w)(1-\tau_c^y)^2}. \tag{28}$$

It is thus possible to aggregate the policy preferences by combining a worker of ability E_i and a retiree of ability $E^o(E_i, \tau_w, n)$. To be important, the matching abilities satisfy

$$E^o(E_i, \tau_w, n) \geq E_i, \tag{29}$$

with the equality holding if and only if $\tau_c^y(E_i, \tau_w, n) = 0$. Since the most-preferred consumption tax rate is decreasing in ability for both workers and retirees, this inequality implies that with the same ability, retirees prefer a higher consumption tax rate than workers. In other words, for any given consumption tax rate, the share of population who want to raise it is larger in the old generation. This is because retirees do not take account of the impact of a higher consumption tax on future consumption tax revenues.

Figure 3 diagrammatically explains the voting equilibrium. The upper graph in figure 3 represents the distribution of the retirees' most-preferred consumption tax rates, and the lower graph does the workers'. Each rectangle shows the population who prefer a zero tax rate most for expositional convenience. As compared to the lower graph, the upper graph is extended to the right, since retirees prefer a higher consumption tax than workers of the same ability. The hight of the upper graph is shrunk by $1/(1+n)$, as compared to the lower, owing to the difference in

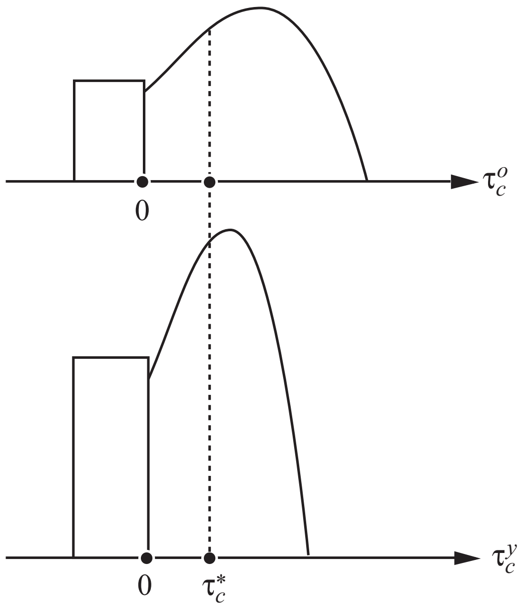

Figure 3: Voting equilibrium with consumption taxes

population. The equilibrium consumption tax rate, τ_c^*, separates the two graphs to equalize the sum of areas on the right hand side with the one on the left.

To formalize the above diagrammatic exposition, combine the workers and retirees of the same policy preferences, making use of (28). The ability of the median worker-voter, E_c^*, is then determined by

$$F(E^o(E_c^*, \tau_w, n)) + (1+n)F(E_c^*) = \frac{2+n}{2}. \tag{30}$$

The first term is the population of retirees who vote for increasing consumption taxes, and the second is that of workers who follow suit. E_c^* turns out to generally depend on n and τ_w. Hence we will write $E_c^* = E_c^*(\tau_w, n)$ and the equilibrium consumption tax rate as

$$\tau_c^* = \tau_c^y(E_c^*, \tau_w, n) := \tau_c^*(\tau_w, n). \tag{31}$$

Note that because of (29), $E_c^* < E_m$ whenever $\tau_c^* > 0$.

4.4 Comparative statics

We will next examine the effect of population aging and a wage tax increase on the equilibrium consumption tax rate, provided that $\tau_c^* > 0$.

Consider first the effect of population aging. As for retirees, since differentiation of (21) yields

$$\frac{\partial^2 V_j}{\partial n \partial \tau_c} = -\frac{\tau_w}{(1-\tau_c)^2(1-\tau_w)} \leq 0,$$

population aging induces every retiree to favor a higher consumption tax rate, irrespective of how wealthy she is. This is owing to a smaller labor force. Wage tax revenues decrease less in response to a consumption tax increase so that every retiree can receive more benefits from a consumption tax increase. As for workers, since differentiation of (26) yields

$$\frac{\partial^2 U_i}{\partial n \partial \tau_c} = -\frac{\tau_w}{(1+r)(1-\tau_c)^2(1-\tau_w)} \leq 0,$$

population aging leads workers also to prefer a higher consumption tax rate. Recall that population aging makes intergenerational transfer more costly for workers, as we have seen in the third term of (25). By reducing labor supply and thus wage tax revenues, a higher consumption tax decreases the amount of intergenerational transfer, which is beneficial to every worker. Moreover, population aging makes a poorer worker-voter decisive. Recall from the inequality in (29) that the share of people who favor a consumption tax increase is larger in the old generation. Since the political influence of retirees is made more powerful by population aging, the overall distribution of the most-preferred consumption tax rates changes in such a way to lower E_c^*. As a result, population aging unambiguously raises the equilibrium consumption tax rate.

Consider next the effect of a wage tax increase. As for retirees, since (21) yields

$$\frac{\partial^2 V_j}{\partial \tau_w \partial \tau_c} = -\frac{1+n}{(1-\tau_c)^2(1-\tau_w)^2} < 0, \tag{32}$$

a higher wage tax lowers their most-preferred consumption tax rates. If a higher wage tax rate is imposed, retirees can raise social security benefits in a smaller amount with a consumption tax increase, for it reduces wage tax revenues more. As for workers, a wage tax increase also lowers their most-preferred consumption tax rate, as we can see from (18). The reason is symmetric to why a consumption tax increase lowers their most-preferred wage tax rates. Overall, a wage tax increase lowers the equilibrium consumption tax rate, changing the policy preferences of all voters in the same direction. The following proposition summarizes what we observed in this section.

Proposition 2 *Suppose that $\tau_c^* > 0$. (i) Population aging increases the equilibrium consumption tax rate. (ii) A wage tax increase lowers the equilibrium consumption tax rate.*

Proof: See appendix A.

5 Political economy of social security financing

5.1 Structure-induced equilibria

We will now consider the public choice of the pair of the two tax rates. No Condorcet winner generally exists in voting over multiple issues without restricting either the voters' policy preferences or the structure of political decision making.[10] Taking the latter course, we will use the notion of structure-induced equilibrium, first introduced by Shepsle (1979). The focus of the analysis is on how population growth rates affect the outcome determined in the structure-induced equilibrium.

[10]See e.g. Persson and Tabellini (2000).

Assume that a wage tax rate and a consumption tax rate are within jurisdictions of separate committees in a legislature.[11] The members of each committee reflect the policy preferences of all voters in the society with no biases; the case that Shepsle (1979) called "the committee of the whole." Each committee puts the tax rate within its jurisdiction to the majority vote, taking the other as given. A pair of the two tax rates, (τ_c^e, τ_w^e), is a structure-induced equilibrium if and only if τ_w^e is a Condorcet winner given τ_c^e and *vice versa*, that is, $\tau_c^e = \tau_c^*(\tau_w^e, n)$ and $\tau_w^e = \tau_w^*(\tau_c^e, n)$.

Figures 4 and 5 depict the wage-tax reaction curve $W_1W_2\tau_c$, which plots the equilibrium relationship of the two taxes satisfying (17) with a given population growth rate. The equilibrium wage tax rate decreases in response to a higher consumption tax rate, and is equal to zero when the consumption tax rate exceeds the threshold rate at W_2. The reaction curve coincides with the schedule of the most-preferred wage tax rates of a worker with ability E_w.

Similarly, the consumption-tax reaction curve $C_1C_2\tau_w$ depicts the equilibrium relationship satisfying (31) with the same population growth rate. The equilibrium consumption tax rate decreases in response to a higher wage tax rate, and is equal to zero when the wage tax rate exceeds the threshold rate at C_2. In contrast to the wage-tax reaction curve, the consumption-tax reaction curve does not coincide with the schedule of a particular worker's most-preferred consumption tax rates, since the decisive worker-voter's ability depends on wage tax rates, as shown in (28).

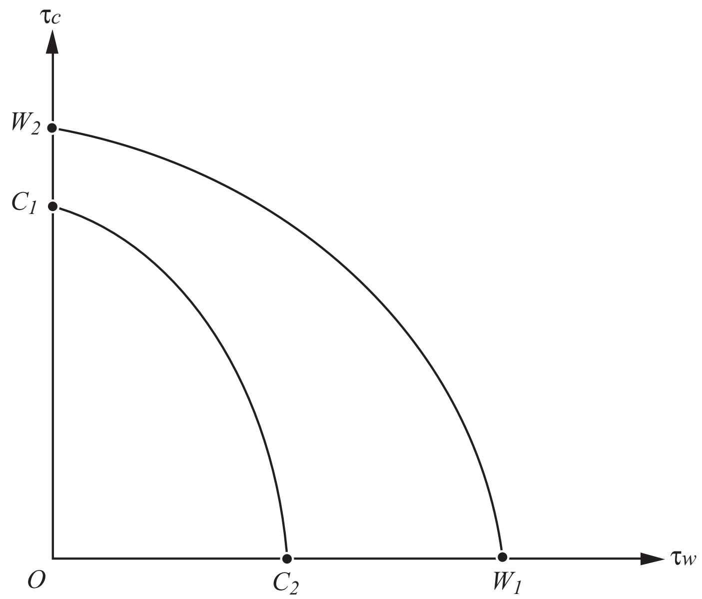

Figure 4: A unique structure-induced equilibrium

[11]In Japan, Committee on Health, Welfare, and Labor Labor in the lower house (as well as one in the upper house) has jurisdiction over wage-based social security contribution, while Committee on Financial Affairs does over consumption tax.

If $r > n$ with a small margin or if $n \geq r$, the wage-tax reaction curve is located over the consumption-tax one as in figure 4. The structure-induced equilibrium is then uniquely determined at W_1, where the social security benefits are financed by only wage tax. The equilibrium size of social security is driven so large that poor workers and poor retirees who support the introduction of consumption tax financing cannot constitute a majority. The economic advantage of consumption tax financing is limited due to the relatively low cost of intergenerational transfer. Conversely, the size of social security funded by only consumption tax cannot be an equilibrium outcome because poor workers and all retirees can form a majority to support imposing a wage tax for more benefits.

If the population growth rate decreases to validate $r > n$ with a large margin, the positions of the two schedules change. Provided that the effect of retirees' strengthened political influence is not very large, the wage-tax reaction curve shifts to the left and the consumption-tax reaction curve moves upward. The reaction curves are then likely to be positioned as illustrated in figure 5, where there occur three different types of structure-induced equilibrium. The social security system employs only wage tax at W_1, only consumption tax at C_1, and both at C_3.

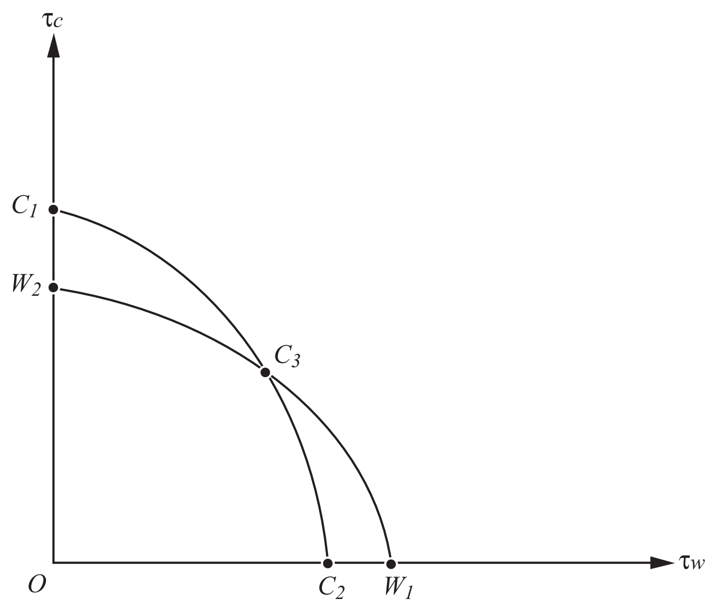

Figure 5: Multiple structure-induced equilibria

Comparison between figures 4 and 5 reveals that population aging may induce the society to shift the financing method from wage tax to consumption tax, totally or partially, as it chooses a different political equilibrium. Because slower population growth decreases the marginal revenue of wage tax and increases the cost of intergenerational transfer, an increasing number of poor workers and poor retirees support consumption tax financing, whereas an increasing number of middle-class and rich workers oppose raising the wage tax rate. In equilibria like C_1 and C_3 in figure 5, where the social security system employs consumption tax financing, all retirees support increasing benefits by raising the wage tax rate. However, too small a number of poor workers are aligned with the preferences of retirees to form a majority.

Nonetheless, as W_1 in figure 5 shows, financing by only wage tax is still one of the possible political equilibrium outcomes unless population growth slows down so much that the two reaction curves intersect only on the vertical axis. Once wage tax is levied as high as to satisfy the median worker-voter, no majority can support the introduction of consumption tax financing in spite of intergenerational transfer being rather costly to workers.

5.2 Formal presentation and simulation

To present these findings formally, we need some new notations. First, taking r and n as given, let τ_w^c be the wage tax rate at C_2 and τ_c^w the consumption tax rate at W_2 in the figures 4 and 5. More precisely, they are defined by $\tau_w^c := \min\{\tau_w | \tau_c^*(\tau_w, n) = 0\}$ and $\tau_c^w := \min\{\tau_c | \tau_w^*(\tau_c, n) = 0\}$. Second, let E_c be the ability level of the median worker-voter in voting on consumption tax rates when the wage tax rate is held fixed at zero, that is, $E_c := E_c^*(0, n)$, which satisfies $E_w < E_c < E_m$. Finally, define two threshold ability levels,

$$E_H := E_m - \frac{r-n}{1+r}\left(E - \frac{1}{1-\tau_w^c}\right)$$

and

$$E_L := E_c - \frac{r-n}{(1+r)(1-\tau_c^c)}\left(E - \frac{1}{1-\tau_c^c}\right).$$

Proposition 3 *(i) There exists a structure-induced equilibrium with $\tau_w^e > 0 = \tau_c^e$ if and only if $E_w \leq E_H$. (ii) There exists a structure-induced equilibrium with $\tau_w^e = 0 < \tau_c^e$ if and only if $E_w \geq E_L$. (iii) There exists a structure-induced equilibrium with $\tau_w^e > 0$ and $\tau_c^e > 0$, in conjunction with the equilibria of (i) and (ii), if and only if $E_L < E_w < E_H$.*

Proof: See appendix B.

As illustrated in figure 4, social security financed entirely by wage tax is a unique political equilibrium outcome in a society with $n \geq r$, for $E_H \geq E_m > E_w$ and $E_L \geq E_c > E_w$. Conversely, population aging must proceed to establish $n < r$ if consumption tax financing emerges as a political equilibrium outcome. The situation illustrated in figure 5 occurs if and only if $E_L < E_w < E_H$.[12] Social security funded by only consumption tax is a unique political outcome if r is so high relative to n that $E_L < E_H < E_w$ happens.

Finally, we will simulate the three patterns of equilibria to verify the above findings. Suppose that one period corresponds to 25 years and write the ability levels as $E_i = \underline{E} + z_i$. To make the wage distribution resemble one in Japan, we will let $\underline{E} = 1.2$ and z_i follow a log normal

[12]As shown in appendix B, the equilibrium with $\tau_w^e > 0$ and $\tau_c^e > 0$ is unique if it exists.

distribution with mean 0.5 and standard deviation 0.35.[13] This distribution produces $E = 2.95$ and $E_m = 2.85$.

Table 2 shows the simulation results, in which the annual interest rate is held fixed at 3% while the annual population growth rate is chosen out of three options, 1.5%, 2%, and 2.5%.

Table 2: Simulation results

Case 1:	$r = 1.03^{25} - 1, n = 1.02^{25} - 1$
	$(E_L, E_w, E_H) = (2.34, 2.42, 2.45)$
	$(\tau_w^e, \tau_c^e) = (0.137, 0), (0.077, 0.038), (0, 0.126)$
Case 2:	$r = 1.03^{25} - 1, n = 1.025^{25} - 1$
	$(E_L, E_w, E_H) = (2.56, 2.47, 2.64)$
	$(\tau_w^e, \tau_c^e) = (0.204, 0)$
Case 3:	$r = 1.03^{25} - 1, n = 1.015^{25} - 1$
	$(E_L, E_w, E_H) = (2.14, 2.36, 2.29)$
	$(\tau_w^e, \tau_c^e) = (0, 0.130)$

In case 1, where the annual population growth rate is supposed to be 2%, we obtain $E_L < E_w < E_H$ and three types of equilibrium occur. In the respective equilibria social security is financed by 13.7% wage tax, by 12.6% consumption tax, and by a combination of 7.7% wage tax and 3.8% consumption tax. In case 2, where the annual population growth rate is 2.5%, we obtain $E_w < E_L < E_H$, and in a unique equilibrium social security is financed by 20.4% wage tax. In case 3, where the annual population growth rate is 1.5%, we obtain $E_L < E_H < E_w$, and in a unique equilibrium social security is financed by 13.0% consumption tax.

6 Concluding remarks

The paper analyzed the political economy of financing social security by wage tax and consumption tax, using a model of majority voting and the concept of structure-induced equilibria within an overlapping-generations framework. The analytical focus was placed on the difference in the distributional impacts between the two taxes as well as on the cost of intergenerational transfer. It was shown that consumption tax financing, employed partially or entirely, emerges as a structure-induced equilibrium outcome when the society has a low population growth rate, while

[13]This ability distribution yields a before-tax wage distribution with quartile dispersion coefficient (3rd quartile - 1st quartile)/(2×median) = 0.23 and decile dispersion coefficient (9th decile - 1st decile)/(2× median) = 0.45. These are close to the numbers in Japanese wage distribution for males of age 40 to 44, which are, according to the Basic Survey on Wage Structure 2006, 0.23 and 0.46, respectively.

the traditional wage tax financing is an equilibrium outcome at the same time. The equilibrium multiplicity explains why the revenue sources for social security are diversified across nations as well as why the use of consumption tax for social security funding has recently become so popular in some countries with rapid population aging.

Our theoretical insight provides interesting testable hypotheses. First, social security is more likely to be funded by general tax revenues as population aging proceeds in a society. Second, if we admit the conventional argument that employer-pay social security contributions are shifted to the prices more than employee-pay contributions, a country with slower population growth tends to increase the share-out ratio of the former. The empirical analysis of these hypotheses is left for future research.

Our analysis also has an implication for the debate on the effect of aging on the size of the welfare state. Recently, Razin, Sadka, Swagel (2002) theoretically argued that population aging may downsize the welfare sate, taking account of a trade off between a political and an economic effect; aging makes stronger the political power of the old on the one hand, and it increases the cost of redistribution on the other. They also tested this hypothesis with data for the US and 12 European countries over the period 1965–92 and obtained positive empirical evidence. Their argument initiated theoretical and empirical discussion by several papers such as Disney (2007), Simonovits (2007), Galasso and Profeta (2007) among others. These papers, however, do not take account of the shift in the financing methods of the welfare state. Our analysis suggests that the shift from wage tax financing to consumption tax financing will make the impact of population aging more ambiguous on the size of the welfare state than these papers argue. In a dynamically-efficient economy, consumption tax can collect larger revenues than wage tax when they are imposed at the same rate, since the tax base is larger for the former. On the other hand, as argued in the text, wage tax financing makes a poorer worker decisive so that, other things being equal, the political factor leads to a higher wage tax rate. Since these two effects counteract, the shift to consumption tax financing adds another ambiguity in the overall effect of population aging on the size of the welfare state.

Several extensions of the model are also left to be studied in future research. First, the model assumed policy commitment that the tax rates determined in voting today also prevail in the future with no amendments. One of the ways to relax this assumption is to make the voting game dynamic and solve its subgame-perfect structure-induced equilibrium, the solution concept introduced by Conde-Ruiz and Galasso (2005). In appendix C we show that every structure-induced equilibrium described in the text is realized as such an equilibrium outcome in an infinitely repeated voting game. Second, we have assumed a small open economy with fixed rates of wage and interest. Extending the framework to a model of endogenous growth, we will be able to obtain richer insights about the relationship between the choice of the financial methods and economic growth.

Appendix A: Proof of proposition 2

Suppose $\tau_c^y(E_i, \tau_w, n) > 0$ and $\tau_w > 0$. Then, differentiating (26) yields

$$\frac{\partial \tau_c^y}{\partial E_i} = \frac{1-\tau_w}{U_{cc}} < 0,$$

$$\frac{\partial \tau_c^y}{\partial \tau_w} = \frac{1}{U_{cc}(1-\tau_c^y)^2(1-\tau_w)^2}\left[\frac{1+n}{1+r}(1+\tau_w) + \tau_c^y(1-\tau_w)\right] < 0,$$

and

$$\frac{\partial \tau_c^y}{\partial n} = \frac{\tau_w}{(1+r)U_{cc}(1-\tau_c^y)^2(1-\tau_w)} < 0,$$

where

$$U_{cc} := \frac{\partial^2 U_i}{\partial \tau_c^2} = -\frac{1}{(1-\tau_c^y)^3}\left[\frac{2(1+n)}{1+r}\frac{\tau_w}{1-\tau_w} + 1 + \tau_c\right] < 0.$$

To prove (i), differentiate (30) with respect to n, and we have

$$\left[f(E^o)\frac{\partial E^o}{\partial E_i} + (1+n)f(E_c^*)\right]\frac{\partial E_c^*}{\partial n} = \frac{1}{2} - F(E_c^*) - f(E^o)\frac{\partial E^o}{\partial n} > 0. \tag{A.1}$$

The sign follows since $F(E_c^*) < F(E_m) = 1/2$ and

$$\frac{\partial E^o}{\partial n} = \frac{\partial E_j}{\partial \tau_c^y}\frac{\partial \tau_c^y}{\partial n} < 0,$$

the latter of which comes from (28) since $\partial E_j/\partial \tau_c^y > 0$ and $\partial \tau_c^y/\partial n \leq 0$. From (28), on the other hand, we have

$$\frac{\partial E^o}{\partial E_i} = 1 + \frac{1+\tau_c^y}{(1-\tau_w)(1-\tau_c^y)^3}\frac{\partial \tau_c^y}{\partial E_i} = -\frac{2(1+n)\tau_w}{(1+r)U_{cc}(1-\tau_c^y)^3(1-\tau_w)} > 0.$$

Thus the bracketed term on the left hand side of (A.1) must be positive, and hence $\partial E_c^*/\partial n > 0$. From (31), then,

$$\frac{\partial \tau_c^*}{\partial n} = \frac{\partial \tau_c^y}{\partial E_i}\frac{\partial E_c^*}{\partial n} + \frac{\partial \tau_c^y}{\partial n} < 0$$

whenever $\tau_c^* > 0$.

The proof of (ii) is quite similar. Differentiating (30) with respect to τ_w yields

$$\left[f(E^o)\frac{\partial E^o}{\partial E_i} + (1+n)f(E_c^*)\right]\frac{\partial E_c^*}{\partial \tau_w} = -f(E^o)\frac{\partial E^o}{\partial \tau_w} > 0 \tag{A.2}$$

whenever $\tau_c^* > 0$, because from (28)

$$\begin{aligned}\frac{\partial E^o}{\partial \tau_w} &= \frac{\tau_c^y}{(1-\tau_c^y)^2(1-\tau_w)^2} + \frac{1}{1-\tau_w}\left[\frac{1}{(1-\tau_c^y)^2} + \frac{2\tau_c^y}{(1-\tau_c^y)^3}\right]\frac{\partial \tau_c^y}{\partial \tau_w} \\ &= \frac{(1+n)[1+\tau_w+\tau_c^y(1-\tau_w)]}{(1+r)(1-\tau_c^y)^5(1-\tau_w)^3 U_{cc}} < 0\end{aligned}$$

whenever $\tau_c^y > 0$. Since the bracketed term on the left hand side of (A.2) is positive, we obtain $\partial E_c^*/\partial \tau_w > 0$. From (31), then,

$$\frac{\partial \tau_c^*}{\partial \tau_w} = \frac{\partial \tau_c}{\partial E_i}\frac{\partial E_c^*}{\partial \tau_w} + \frac{\partial \tau_c}{\partial \tau_w} < 0$$

whenever $\tau_c^* > 0$. ||

Appendix B: Proof of proposition 3

Let τ_w^w and τ_w^c be the wage tax rates at W_1 and C_2 in figure 4, respectively. Formally, they are defined by $\tau_w^w := \tau_w^*(0, n)$ and $\tau_w^c := \min\{\tau_w | \tau_c^*(\tau_w, n) = 0\}$. If we let $T_w^w := 1/(1 - \tau_w^w)$ and $T_w^c := 1/(1 - \tau_w^c)$ to simplify the notations, then the equilibrium conditions, (12), (26), (28), and (30) yield

$$kE - E_w^* + T_w^w(1 - kT_w^w) = 0 \tag{A.3}$$

and

$$E - E_m + kT_w^c(1 - T_w^c) = 0, \tag{A.4}$$

where $k := (1+n)/(1+r)$. In (A.4) we make use of the fact that $E_c^* = E_m$ when $\tau_c^y = 0$, following from (28). An equilibrium with $\tau_w^e > 0 = \tau_c^e$ exists if and only if $T_w^w \geq T_w^c$. Because (A.4) is quadratic, subtracting (A.4) from (A.3) after substituting T_w^c for T_w^w in (A.3), we can rewrite the necessary and sufficient condition for $T_w^w \geq T_w^c$ into

$$E_w \leq E_H := E_m - (1 - k)(E - T_w^c). \tag{A.5}$$

Similarly, let τ_w^c and τ_c^c be the consumption tax rates at W_2 and C_1 in figure 4. Their formal definitions are $\tau_c^w := \min\{\tau_c | \tau_w^*(\tau_c, n) = 0\}$ and $\tau_c^c := \tau_c^*(0, n)$. Let us denote $T_c^w := 1/(1 - \tau_c^w)$ and $T_c^c := 1/(1 - \tau_c^c)$ for simplicity. Then the equilibrium conditions, (12), (26), (28), and (30) yield

$$E - E_w + (k - 1)ET_c^w + T_c^w(1 - kT_c^w) = 0 \tag{A.6}$$

and

$$E - E_c + T_c^c(1 - T_c^c) = 0. \tag{A.7}$$

The existence of an equilibrium with $\tau_c^e > 0 = \tau_w^e$ is guaranteed if and only if $T_c^c \geq T_c^w$. Subtracting (A.7) from (A.6) after substituting T_c^c for T_c^w in (A.6) reduces the condition into

$$E_w \geq E_L := E_c - (1 - k)T_c^c(E - T_c^c). \tag{A.8}$$

Rewriting the equilibrium conditions, (12), (26), and (30), we find that if an equilibrium with $\tau_w^e > 0$ and $\tau_c^e > 0$ exists, then the tax rates, τ_w and τ_c, and the ability of the median worker-voter in voting on consumption tax, E_c^*, are determined through the following system of equations:

$$E - E_w + E(k - 1)T_c + T_cT_w(1 - kT_cT_w) = 0 \tag{A.9}$$

$$E - E_c^* + kT_c^2T_w(1 - T_w) + T_cT_w(1 - T_c) = 0 \tag{A.10}$$

and

$$F(E^o) + (1 + n)F(E_c^*) = \frac{2 + n}{2}, \tag{A.11}$$

where $T_w := 1/(1-\tau_w)$ and $T_c := 1/(1-\tau_c)$. The definition of E^o is given by $E^o = E + kT_c^2 T_w(1-T_w)$, which we obtain from (26) and (28) in the case of $\tau_c^y > 0$. Then, subtracting (A.10) from (A.9) yields

$$(1-k)T_c(E - T_cT_w) = E_c^* - E_w.$$

Since $E - T_cT_w > 0$ owing to positive labor supply and $E_c^* > E_w$, it follows that $k < 1$ is necessary for the existence of an equilibrium with $\tau_w^e > 0$ and $\tau_c^e > 0$.

We will next show that as illustrated in figure 5 the consumption-tax reaction curve is steeper than the wage-tax reaction curve at their intersection.

Differentiating (A.9), we have the slope of the wage-tax reaction curve,

$$\frac{\partial T_c}{\partial T_w} = -\frac{T_c\lambda}{T_w\lambda + E(k-1)} < 0,$$

where $\lambda := 1 - 2kT_cT_w < 0$ owing to the second order condition, (14). Similarly, differentiating (A.10) and (A.11) and rearranging the terms, we obtain the slope of the consumption-tax reaction curve,

$$\frac{\partial T_c}{\partial T_w} = -\frac{T_c\lambda + \alpha}{T_w\lambda + \beta} < 0,$$

where

$$\alpha := T_c^2(k-1) + \frac{f(E^o)}{(1+n)f(E_c^*)}kT_c^2(1-2T_w) < 0$$

and

$$\beta := 2T_wT_c(k-1) + \frac{2f(E^o)}{(1+n)f(E_c^*)}kT_wT_c(1-T_w) < 0.$$

Simple calculation then demonstrates that the consumption-tax reaction curve is steeper, because

$$\frac{T_c\lambda + \alpha}{T_w\lambda + \beta} - \frac{T_c\lambda}{T_w\lambda + E(k-1)} = \frac{\Delta}{(T_w\lambda + \beta)(T_w\lambda + E(k-1))} > 0,$$

where

$$\begin{aligned}\Delta := {} & T_c(k-1)\{\lambda(E - T_wT_c) + ET_c(k-1)\} \\ & + \frac{kf(E^o)T_c^2}{(1+n)f(E_c^*)}\{-2\lambda T_w + E(k-1)(1-2T_w))\} > 0.\end{aligned}$$

Finally, given that the consumption-tax reaction curve is steeper, it follows that the equilibrium with $\tau_w^e > 0$ and $\tau_c^e > 0$ is unique and that it exists if and only if $T_w^w > T_w^c$ and $T_c^c > T_c^w$. This condition is reduced into $E_L < E_w < E_H$. ||

Appendix C: Subgame perfection of the equilibrium outcomes

Following Conde-Ruiz and Galasso (2005), we will show that every structure-induced equilibrium obtained in the text under the assumption of policy commitment is established as a subgame-perfect equilibrium outcome in an infinitely-repeated voting game without policy commitment.

Suppose that a voting game takes place in each period, where each worker and retiree announces a pair of wage and consumption tax rates. Let τ_{wt} and τ_{ct} be the voting outcomes in period $t \geq 1$, which are defined respectively as the medians of wage and consumption tax rates announced by voters. Let h_1 be the history at the start of the game and h_t be one at the start of period t. The latter is a combination of h_1 and the outcomes having been realized until period t. The set H collects all possible histories, and H^c contains only h_1 and h_t such that $\tau_{ws} = \tau_w^e$ and $\tau_{cs} = \tau_c^e$ for all $s \leq t-1$. We will denote by H_t the set of possible histories until period t.

Each voter's strategy in period t is a mapping from H_t to the set of the pairs of the two tax rates. Let $\sigma_i^o(h_t)$ be the strategy of a retiree with ability E_i when voting in period t. Following (5), her payoff function is defined as

$$V_{it} = (1+r)[A_i - \tau_{ct}(1-\tau_{wt-1})(E_i - E)] + (1+n)\tau_{wt}N(\tau_{wt}, \tau_{ct+1}),$$

where A and A_i are constant, satisfying $A_i < A$ if and only if $E_i < E$. Similarly, let $\sigma_i^y(h_t)$ be the strategy of a worker with ability E_i when voting in period t and, following (8), define her payoff function as

$$U_{it} = (1-\tau_{ct+1})(1-\tau_{wt})E_i - \ln(1-\tau_{wt})(1-\tau_{ct+1}) + \frac{B_{t+1}}{1+r},$$

where

$$B_{t+1} = (1+r)\tau_{ct+1}(1-\tau_{wt})N(\tau_{wt}, \tau_{ct+1}) + (1+n)\tau_{wt+1}N(\tau_{wt+1}, \tau_{ct+2}).$$

Now we will show that every structure-induced equilibrium (τ_w^e, τ_c^e) presented in proposition 3 is established as a stationary subgame-perfect equilibrium outcome of the infinitely-repeated voting game by the combination of strategies, $\sigma_i^o(h_t) = (\tau_{wi}^o(h_t), \tau_{ci}^o(h_t))$ and $\sigma_i^y(h_t) = (\tau_{wi}^y(h_t), \tau_{ci}^y(h_t))$ for $t \geq 1$, such that

$$\tau_{wi}^o(h_t) = \tau_w^o(\tau_c^e), \quad \tau_{ci}^o(h_t) = \tau_c^e \tag{A.12}$$

for $h_t \in H_t$ and $E_i \in [\underline{E}, \overline{E}]$;

$$\tau_{wi}^y(h_t) = \begin{cases} \tau_w^e & \text{if } h_t \in H^c \\ 0 & \text{otherwise,} \end{cases} \qquad \tau_{ci}^y(h_t) = \begin{cases} \tau_c^e & \text{if } h_t \in H^c \\ 0 & \text{otherwise} \end{cases} \tag{A.13}$$

for $E_i \in [\underline{E}, E_c^e]$; and

$$\tau_{wi}^y(h_t) = \begin{cases} \tau_w^y(E_i, \tau_c^e, n) & \text{if } h_t \in H^c \\ 0 & \text{otherwise,} \end{cases} \qquad \tau_{ci}^y(h_t) = \begin{cases} \tau_c^y(E_i, \tau_w^e, n) & \text{if } h_t \in H^c \\ 0 & \text{otherwise.} \end{cases} \tag{A.14}$$

for $E_i \in (E_c^e, \overline{E}]$, where E_c^e is the ability level that the equilibrium median worker-voter has in voting on consumption tax rates, implicitly defined by $\tau_c^y(E_c^e, \tau_w^e, n) = \tau_c^e$.

These strategies have the following properties. First, as (A.12) shows, concerning voting on wage tax rates, the equilibrium strategy of a retiree stipulates the same behavior as she chooses in the structure-induced equilibrium with policy commitment. As regards consumption tax rates, every retiree votes for τ_c^e whatever happens in the past. Second, as (A.13) shows, the votes of workers with $E_i \leq E_c^e$ cluster at the pair of tax rates realized in the structure-induced equilibrium with policy commitment, as far as it has been repeatedly realized in the past, and otherwise they all vote for abolishing the social security system. Third, as (A.14) shows, workers with $E_i > E_c^e$ vote as described in the text whenever the outcome (τ_w^e, τ_c^e) has been repeatedly realized, but otherwise they will vote for abolishing the social security system. Under these strategies, $(\tau_{wt}, \tau_{ct}) = (\tau_w^e, \tau_c^e)$ if $h_t \in H^c$, and otherwise $(\tau_{wt}, \tau_{ct}) = (0, 0)$.

Let us check if those strategies constitute a subgame-perfect equilibrium, assuming that even a single vote can affect the voting outcome.

To begin with, consider the strategy of retirees. If the consumption tax rate is τ_c^e, they all want to increase the wage tax rate above τ_w^e because $\tau_w^o(\tau_c^e) > \tau_w^e$. To do so they have to increase the votes for the tax rates higher than τ_w^e. However, their votes are already higher than the level and thus there is no room for them to change the voting outcome. With respect to the consumption tax rate, retirees with $E_i < E^o(E_c^e, \tau_w^e)$ want to increase it above τ_c^e. However, they cannot manipulate the voting outcome in their desired direction because they already vote for τ_c^e. The similar reasoning applies to the voting behavior of retirees with $E_i > E^o(E_c^e, \tau_w^e)$.

Turn to the strategy of workers. First, in the case of $h_t \in H^c$, the similar reasoning applies. There is no room for each worker to manipulate the voting outcome in period t in her desired direction because she already votes in that way. Next, suppose that workers with $E_i \leq E_w^e$ strategically voted for a wage tax rate below τ_w^e and successfully reduced τ_{wt} in period t. Then, the voting behavior stipulated in (A.13) will yield $\tau_{wt+1} = \tau_{ct+1} = 0$ in period $t+1$. This means that those workers receive no social security benefits. If so, their best outcome to happen in period t is $\tau_{wt} = 0$. However, even though it happens, $U_i(\tau_w^e, \tau_c^e) \geq U_i(0, 0)$ holds for the following reason. First, $U_i(\tau_w^e, \tau_c^e) \geq U_i(0, \tau_c^e)$ for $E_i \leq E_w^e$ because of the single-crossing property of the utility function. Second, $U_i(0, \tau_c^e) \geq U_i(0, 0)$ for $E_i < E_c^e$ because $\tau_c^e < \tau_c^y(E_i, 0, n)$ and $\partial^2 U_i / \partial \tau_c^2 < 0$. Accordingly they have no incentive to deviate from (A.13). Regarding workers with $E_i > E_w^e$, because the majority of votes cluster at τ_w^e, they cannot manipulate the voting outcome even if they change their votes on wage tax rates. The similar reasoning applies to the voting on consumption tax rates, and the above strategies turn out to form a subgame-perfect Nash equilibrium. ||

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