The Private Provision of Public Inputs

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Abstract

We extend the private provision game to public inputs like knowledge, information and innovation, which are non-rival in its effects and are not final consumption goods but intermediate factors. The literature on public inputs is reviewed and the individual payoff maximizing input choice is compared to the socially optimal level. We present the private provision game for linear and for best-shot contribution technology, since the latter approach may be better suited to model public inputs. The results from the model are then contrasted with the traditional view of intellectual property protection as an incentive mechanism for innovation effort.

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1 Introduction

The private provision of public goods has been extensively analyzed in the literature after the seminal contributions of Bergstrom et al. (1986) and Cornes and Sandler (1984a). Its basic results concerning underprovision of the public good, invariance of the result with respect to income redistribution, etc. are well known by now. In this literature, the public good is a public *consumption* good with non-rival consumption. In reality many public goods are not consumption goods, but *input* factors in the production of consumption goods. The public infrastructure, the rule of law, information and knowledge: all these inputs enter firm's production functions, increase their productivity, and do so in a non-rival way. These public input factors also constitute a big portion of public budgets.

Kaizuka (1965) analyzed the efficient condition describing the efficient allocation in the presence of such a public intermediate good. The condition resembles the classical Samuelson condition describing the efficient allocation rule for a pure public consumption good. Kaizuka's assumptions were discussed in several papers, Feehan (1989,1998) summarizes this literature. While several public inputs are customarily provided by government or public institutions (e. g., public infrastructure, the rule of law, basic research in labs and universities), in many situations it is are private firms who provides this public input privately (specially in the case of knowledge, innovation, information, software). Knowledge can be shared in a non-rival way, and may actually grow through sharing, if more agents contribute to its diffusion and extension, as for instance in scientific research where we all publish and share our papers. The aim of this paper is to extend the analysis of private provision of a public good to the case of an input good, an analysis which is lacking in the literature.

Consider as an example the problem of investing in research and innovation. From an economic point of view, the problem lies on the incentives of a rational agent to invest private effort on an enterprise producing public benefits.[1] Because of the non-rivalry of the public input, the social benefits of innovation are always higher than the private benefits of the innovating firm. To set an incentive to innovate, the innovator is granted a temporary monopoly on the use of her innovation, a patent or a copyright on the intellectual property. There is a large literature on the optimal design of intellectual property protection to optimize the trade-off between the incentives to innovate and the welfare loss due to monopoly power.[2] Newer contributions extend the basic innovation models to include cumulative innovation, licensing and many other aspects associated with intellectual property rights (for a survey, see Kamien (1992)).

In this paper, we will not focus on this "positive" strand of the patent literature. Instead, we will pursue a more "normative" approach. The focus is on the input property of the public good. Thus, we concentrate on the production side of the economy and for the sake of simplicity disregard the consumer side of the economy with its concern with market structure, which has been the focus of most literature until now. Innovation and knowledge are modeled as public inputs which are non-rival in production and which are provided privately by firms, increasing the knowledge stock of the economy for all firms. Formally our paper is also related to the private provision literature as Bergstrom et al. (1986) and Cornes and Sandler (1984a). In general, private provision of a public consumption good leads to underprovision (locally always and in most cases also globally). While in the private provision of a public consumption good the contributions of the other players increase the budget in terms of the public good, in the case of a public input good the contributions of the other agents increase the production possibility sets of all agents. We argue that this effect has been neglected in the literature and suggest that for a public input, the deviation of the private provision equilibrium from the socially efficient Samuelson level may be in proportion

[1]For an extended treatment of this incentive problem, see Suzanne Scotchmer's book *Innovation and Incentives*.

[2]To name just a few seminal contributions, see Nordhaus (1972), Gilbert and Shapiro (1990), Klemperer (1990), and Denicolo (1996).

greater than for public consumption goods. This will also depend on the technology assumed for the public good. In most models in the literature, the contributions of all players are added up linearly, $\sum_{i=1}^{n} g_i = G$, and Nash contributing behaviour is assumed. There are exceptions like the non-Nash model analyzed by Cornes and Sandler (1984b) and the weakest-link and best-shot technology introduced by Hirshleifer (1983). We will argue that these non-standard contribution technologies may a play a greater role when the public good to be provided privately is a public input factor.

We proceed as follows. In the next section we summarize the literature of public inputs and state the Kaizuka condition characterizing the efficient level of the public input. Section 3 determines the optimal social level of the public input and it is showed that a market equilibrium does not lead to a social optimum. Section 4 and 5 present the private provision extension with firms contributing to the public input factor, which is the main new contribution of the present paper. We compare the resulting Nash equilibrium level with private provision to the socially efficient Samuelson-Kaizuka level for different public good technologies in. In Section 6 we relate our results to the "positive" literature on intellectual property. Section 7 concludes.

2 A model with a public input

In this section we present a summary of the literature on public inputs and present the Kaizuka conditions and the underlying assumptions following Feehan (1998). Consider an economy with two primary factors labour L and capital K. Assume in a standard way that both factors are used with constant returns to scale technology to produce a public input G:

$$G = F_G(L_G, K_G), \tag{1}$$

where F_G is a linear homogenous production function and L_G and K_G denote the input levels of L and K, respectively, devoted to the production of G. There are n final private goods $x_i, i = 1, \ldots, n$ which are produced with the primary factors L and K and the public input G (which can be considered an intermediate good). Each good x_i is produced competitively with the same technology by many firms in each industry i, denote with N_i the number of firms in industry i. The output of good is then given by:

$$x_i = N_i F_i(L_i/N_i, K_i/N_i, G/N_i^{\alpha}), \tag{2}$$

where L_i and K_i denote the input factors of L and K, respectively, devoted to the production of x_i, and α is a congestion parameter. If $\alpha = 0$, the public good is available to all firms without congestion and for any number of firms. If $\alpha = 1$, the public input is completely congested for firms in industry i, but not across different industries. In his original contribution, Kaizuka (1965) assumed $\alpha = 1$ and linear homogeneity in all arguments of the production function F_i:

$$x_i = F_i(L_i, K_i, G). \tag{3}$$

With given factor endowments, the first-order conditions determining the efficient production from a social point of view require that the input factors L_G and K_G should be allocated such that

$$\frac{\frac{\partial F_i}{\partial L_i}}{\frac{\partial F_i}{\partial K_i}} = \frac{\frac{\partial F_G}{\partial L_G}}{\frac{\partial F_G}{\partial K_G}}, \quad i = 1, \ldots, n, \tag{4}$$

$$\sum_{i=1}^{n} \frac{\frac{\partial F_i}{\partial G}}{\frac{\partial F_i}{\partial L_i}} = \frac{1}{\frac{\partial F_G}{\partial L_G}}, \tag{5}$$

$$\sum_{i=1}^{n} \frac{\frac{\partial F_i}{\partial G}}{\frac{\partial F_i}{\partial K_i}} = \frac{1}{\frac{\partial F_G}{\partial K_G}}. \tag{6}$$

The first n conditions require that the marginal rate of substitution between input factors are equalized across the production of all goods. Conditions (5) and (6) resemble the Samuelson condition for efficient provision level of a public output good. Each primary input L and K should be allocated in a such a way that the marginal product of the input is equal whether it is used directly in the production of the private goods x_i or it is used indirectly through the additional product of the public input G. While a competitive equilibrium will ensure that conditions (4) are fulfilled, Kaizuka (1965) pointed out that conditions (5) and (5) will, in general, fail to obtain in a competitive equilibrium, since the single firm does not take into consideration the positive input externality generated by an increased level of G.

However, the above formulation is problematic. First, it is strange that the public input gets congested within an industry, but not across industries. Second, constant returns to all input factors to (3) lead to "firm augmenting" public input, where the public input G augments the rents of a single firm. This means that the smaller and more numerous the firms, the higher the output, potentially leading to an unbounded production possibility set if there is no limit to firm divisibility. Boadway (1973) claimed that the Kaizuka condition only applies in certain, specific circumstances. A consensus emerged in the literature (see Hillman (1978), McMillan (1979), and Feehan (1998)) that the most plausible assumption is to set $\alpha = 0$, where there is no congestion in the use of public good by all firms in all industries (this also corresponds to our interpretation of G as knowledge and innovation) and to assume constant returns to scale only in the primary factors L_i and K_i. The input factor G is then called "factor augmenting", because it increases the marginal output of the primary factors. The production function of the final goods x_i is

$$x_i = F_i(L_i, K_i, G), \tag{7}$$

which looks like (3), but is not identical since F_i is only linear homogenous in L_i and K_i. This implies increasing returns to scale in all arguments, which may result in a non-convex production set. McMillan (1979) suggests the following specification

$$x_i = h_i(G)F_i(L_i, K_i), \tag{8}$$

where $h_i(G)$ can be interpreted as technological change. **?** show that separability in the production function between the primary factors and the public input is a sufficient condition to guarantee a convex production possibility set. In the following we will always assume formulation (8) with constant returns to scale in L_i and K_i.[3]

3 The social optimum

Assume an economy with 2 factors L and K. The only public input G is produced under constant returns to scale:

$$G = F_G(L_G, K_G). \tag{9}$$

There exist two private final goods x_1 and x_2 which are produced under constant returns to scale in the primary factors:

$$x_i = h_i(G)F_i(L_i, K_i), i = 1, 2. \tag{10}$$

We assume that factor augmenting effect of the public input increases in G with diminishing returns: $h'(G) > 0, h'' < 0$. The production functions have the usual positive, diminishing returns: $\frac{\partial F_i}{\partial L_i} > 0$ and $\frac{\partial^2 F_i}{\partial L_i^2} > 0$, $\frac{\partial F_i}{\partial K_i} > 0$ and $\frac{\partial^2 F_i}{\partial K_i^2} > 0$, for $i = 1, 2$ Within each industry the output is produced under competitive conditions by identical firms producing only one output good. The social optimum is found at the solution of the Lagrangian

$$\begin{aligned} \mathcal{L} &= \sum_{i=1,2} \lambda_i h_i(G)F_i(L_i, K_i) + \lambda_G(F_G(L_G, K_G) - G) \qquad (11) \\ &\quad + \mu_L(L_1 + L_2 + L_G - \bar{L}) + \mu_K(K_1 + K_2 + K_G - \bar{K}), \end{aligned}$$

[3]The public good G acts as a positive externality in the spirit of the "atmosphere" good in Meade (1952).

where $\bar{L}$ and $\bar{K}$ represent the primary factor endowment of L and K. We obtain the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial L_i} = \lambda_i h_i(G)\frac{\partial F_i}{\partial L_i} + \mu_L = 0 \qquad i = 1,2 \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial K_i} = \lambda_i h_i(G)\frac{\partial F_i}{\partial K_i} + \mu_K = 0 \qquad i = 1,2 \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial L_G} = \lambda_G \frac{\partial F_i}{\partial L_G} + \mu_L = 0 \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial K_G} = \lambda_G \frac{\partial F_i}{\partial K_G} + \mu_K = 0 \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial G} = \sum_i \lambda_i \frac{\partial h_i}{\partial G} F_i(L_i, K_i) - \lambda_G = 0 \tag{16}$$

Rearranging the first six conditions (12) to (15), we arrive at the efficient factor input condition

$$\frac{\frac{\partial F_1}{\partial L_1}}{\frac{\partial F_1}{\partial K_1}} = \frac{\frac{\partial F_2}{\partial L_2}}{\frac{\partial F_2}{\partial K_2}} = \frac{\frac{\partial F_G}{\partial L_G}}{\frac{\partial F_G}{\partial K_G}}. \tag{17}$$

requiring that the marginal rate of technical substitution in production must be equal across the production of all goods. This condition is attainable under a competitive economy. Using conditions (12) to (15) to eliminate the Lagrangian multipliers $\lambda_i, i = 1,2$ and λ_G from equation (16), we obtain the conditions that describe the efficient level of the public input G:

$$\sum_{1,2} \frac{\frac{\partial h_i}{\partial G} F_i(L_i, K_i)}{h_i(G)\frac{\partial F_i}{\partial L_i}} = \frac{1}{\frac{\partial F_G}{\partial L_G}} \tag{18}$$

$$\sum_{1,2} \frac{\frac{\partial h_i}{\partial G} F_i(L_i, K_i)}{h_i(G)\frac{\partial F_i}{\partial K_i}} = \frac{1}{\frac{\partial F_G}{\partial K_G}} \tag{19}$$

As will be shown in brief, this second set of conditions will not the outcome of a competitive economy. They resemble the Samuelson rule for a pure public consumption good and are the Kaizuka conditions (5) and (6) for our assumed production functions. The sum of all marginal products of the public good (e. g., the marginal benefit of providing an additional unit of input G) equals the marginal cost of this in units of foregone consumption goods.

The factors L and K as the only primary factor of production should be allocated between the production of the private goods x_1 and x_2 and the public intermediate good G in such a way that the marginal contribution of the factors is equal whether directly in the production of the private good or indirectly due to the increase in the public input level. Notice that the presence of a public input (intermediate) good G leads to three distinct effects, which we describe for an increase of one unit of G:

1. To increase the output of G, primary input factors labour and capital need to be shifted away from the production of the private goods. This effect reduces the output of the final consumption goods.

2. The increase in the public input G increases the productivity of labour and capital, indirectly increasing the output of final goods.

3. The output proportion between the final goods x_1 and x_2 may change, depending on the productivity effects $h_1(G)$ and $h_2(G)$. Suppose good 1 benefits more from technological advances, $h_1(G) > h_2(G)$, then the efficient production after an increase provision of G will shift output from x_2 to x_1.

Perfectly competitive firms do not choose their labor input according to the Kaizuka conditions (18) and (19). Consider the market behaviour of the representative firm of industry i, which maximizes her profit

$$\Pi_i(L_i, K_i, L_G, K_G) = p_i h_i(F_G(L_G, K_G))F_i(L_i, K_i) - w(L_i + L_G) - r(K_i + K_G), \tag{20}$$

where p_i is the price of x_i, and w and r are the factor prices of labour and capital, respectively. The first order conditions resulting from maximizing

(20) are given by

$$\begin{aligned}
\frac{\partial \Pi_i}{\partial L_i} &= p_i h_i(G)\frac{\partial F_i}{\partial L_i} - w = 0, && (21)\\
\frac{\partial \Pi_i}{\partial K_i} &= p_i h_i(G)\frac{\partial F_i}{\partial K_i} - r = 0, && (22)\\
\frac{\partial \Pi_i}{\partial L_G} &= p_i \frac{\partial h_i}{\partial G}\frac{\partial F_G}{\partial L_G} F_i(L_i, K_i) - w = 0, && (23)\\
\frac{\partial \Pi_i}{\partial K_G} &= p_i \frac{\partial h_i}{\partial G}\frac{\partial F_G}{\partial K_G} F_i(L_i, K_i) - r = 0. && (24)
\end{aligned}$$

From the first two conditions we see that the ratio of marginal products will be equal across the production of the final goods x_i. In contrast, the choice of the profit maximizing input level of G is characterized by

$$h_i(G)\frac{\partial F_i}{\partial L_i} = \frac{\partial h_i}{\partial G}\frac{\partial F_G}{\partial L_G} F_i(L_i, K_i) \quad \Longleftrightarrow \quad \frac{\frac{\partial h_i}{\partial G} F_i(L_i, K_i)}{h_i(G)\frac{\partial F_i}{\partial L_i}} = \frac{1}{\frac{\partial F_G}{\partial L_G}} \qquad (25)$$

An analogous result applies to the input level of K_i. Clearly, an individual firm does not take into consideration that its output of G has a positive effect on the other firms. Thus, comparing condition (25) with (18), the sum is missing, which contains the factor augmenting effect on the production of the other good(s).[4] This resembles the standard result regarding the Samuelson rule according to which a single individual, when consuming a public good, takes into consideration only his private profit and not the social positive externality benefits accruing to other individuals.

[4] Actually, the firm may want to harm competitors or prevent them from having access to new technologies. This incentive would strengthen our argument. But since it is not the aim of our analysis, we disregard those incentives in our settings.

4 The private provision of a public input with linear contribution technology

Suppose now that there are the representative firms of each industry interact. Each firm may contribute labour and capital to the public input good. Denote with L_G^1 and L_G^2 the contributions of firms 1 and 2 to labour input L_G^N and with K_G^1 and K_G^2 the contributions of firms 1 and 2 to capital input K_G^N:

$$L_G^N = L_G^1 + L_G^2 = L_G^1 + L_G^{-1} \tag{26}$$

$$K_G^N = K_G^1 + K_G^2 = K_G^1 + K_G^{-1} \tag{27}$$

We use the superscript N for the private provision level because we will soon assume that the firms contribute jointly to the public input good in a Nash behaviour way. The superscript -1 denotes, as usual, the contributions to the public input by the other agent.

The firm now maximizes its profit

$$\begin{aligned} \Pi_i(L_i, K_i, L_G^i, \bar{L}_G^{-i}, K_G, \bar{K}_G^{-i}) &= p_i h_i(F_G(L_G^N, K_G^N))F_i(L_i, K_i) \\ &\quad -w(L_i + L_G^i) - r(K_i + K_G^i), \end{aligned} \tag{28}$$

taking into account that the other firm contributes L_G^{-i} and K_G^{-i}. Firm i takes this contributions as given (Nash behaviour) and calculates her best response to the given values $\bar{L}_G^{-i}$ and $\bar{K}_G^{-i}$. The first order conditions resulting from maximizing (28) are the following:

$$\begin{aligned} \frac{\partial \Pi_i}{\partial L_i} &= p_i h_i(G^N)\frac{\partial F_i}{\partial L_i} - w = 0, \\ \frac{\partial \Pi_i}{\partial K_i} &= p_i h_i(G^N)\frac{\partial F_i}{\partial K_i} - r = 0, \\ \frac{\partial \Pi_i}{\partial L_G} &= p_i \frac{\partial h_i(G^N)}{\partial G}\frac{\partial F_G(L_G^N, K_G^N)}{\partial L_G}F_i(L_i, K_i) - w = 0, \\ \frac{\partial \Pi_i}{\partial K_G} &= p_i \frac{\partial h_i(G^N)}{\partial G}\frac{\partial F_G(L_G^N, K_G^N)}{\partial K_G}F_i(L_i, K_i) - r = 0. \end{aligned}$$

These equations look similar to the conditions corresponding to the case of a single individual firm, but the arguments of the functions may, in principle, differ. The conditions describing the profit maximizing choice of primary inputs are

$$\begin{aligned}
\frac{\partial h_i(F_G(L_G^N, K_G^N))}{\partial G} F_i(L_i, K_i) \frac{\partial F_G(L_G^N, K_G^N)}{\partial L_G^1} - h_i(F_G(L_G^N, K_G^N)) \frac{\partial F_i}{\partial L_i} &= 0 \\
\frac{\partial h_i(F_G(L_G^N, K_G^N))}{\partial G} F_i(L_i, K_i) \frac{\partial F_G(L_G^N, K_G^N)}{\partial K_G^1} - h_i(F_G(L_G^N, K_G^N)) \frac{\partial F_i}{\partial K_i} &= 0
\end{aligned}$$

These equations define implicitly the best-response functions of firm i for given contributions L_G^{-i} and K_G^{-i}: $\frac{dL_G^i}{L_G^{-i}}$ and $\frac{dK_G^i}{K_G^{-i}}$. Denote the above first order conditions as FOC_L^i and FOC_K^i. In the following, we will focus on the 2-industry and 2-firm case and analyze the best response functions of firm 1 to given contributions of firm 2. The reaction functions are:

$$\frac{dL_G^1}{L_G^2} = -\frac{\frac{\partial FOC_L^1}{\partial L_G^2}}{\frac{\partial FOC_L^1}{\partial L_G^1}} \tag{29}$$

$$\frac{dK_G^1}{K_G^2} = -\frac{\frac{\partial FOC_K^1}{\partial K_G^2}}{\frac{\partial FOC_K^1}{\partial K_G^1}} \tag{30}$$

Since the input factors enter the technological function in an additive way, L_G^1 and L_G^2 are perfect substitutes for each other (analogous applies to K_G^i). So the derivatives with respect to L_G^1 and L_G^2 are equal up to the effect of L_G^1 on the output $F^1(L_1, K_1)$:

$$\frac{\partial FOC_L^1}{\partial L_G^2} = \frac{\partial FOC_L^1}{\partial L_G^1} - \left(\frac{\partial h_i(G)}{\partial G} \cdot \frac{\partial F_G}{\partial L_G^1} \cdot \frac{\partial F_1}{\partial L_1} \cdot \frac{\partial L_1}{\partial L_G^1} - h_1(G) \cdot \frac{\partial^2 F_1}{\partial (L_1)^2} \cdot \frac{\partial L_1}{\partial L_G^1} \right) \tag{31}$$

Since $\frac{\partial L_1}{\partial L_G^1} = -1$, the term in brackets is negative and thus the second term is being added to the first. This means

$$\frac{\partial FOC_L^1}{\partial L_G^2} < \frac{\partial FOC_L^1}{\partial L_G^1} \iff \frac{dL_G^1}{L_G^2} = -\frac{\frac{\partial FOC_L^1}{\partial L_G^2}}{\frac{\partial FOC_L^1}{\partial L_G^1}} \in (-1, 0)$$

An analogous result obtains for the reaction function corresponding to the choice of joint capital input K_G^1. Besides, all the calculations also apply for the reaction functions for L_G^2 and K_G^2. To sum up, we confirm the usual result that the slope of the reaction functions is negative and that its slope lies (in absolute value) between 0 and 1. As can be seen from expression (31), the exact value of the slope of the reaction function depends on the technology functions $h_i(\cdot)$ and on the production functions $F^i(L_i, K_i)$. While in private provision games one usually has to assume additionally normality to arrive at this result, the assumptions about the technology are sufficient here for this result to obtain.

Regarding the provision level of the public input, the result above is, in principle, only valid at the local level. It is well known that this result cannot be generalized to a global statement (Buchanan and Kafoglis, 1963). Buchholz and Peters (2001) analyze the case where an "overprovision" anomaly is possible when providing a public consumption good privately. They show that the anomaly can be avoided if the goods are normal and all agents are contributors. In our setting, the role of normality is satisfied by our production technology. To guarantee underprovision of G, it thus suffices that the benefit of the new technology is not too unbalanced and that both firms want to contribute and be at an inner solution.

5 The private provision of a public input with best-shot contribution technology

It may be argued that the linear contribution technology is not the right assumption in the case of a public input like knowledge, innovation, information of software. Rather, the firms invest factors in producing the public input, say, a new technology, and then the best technology is adapted. This would correspond to Hirshleifer's (1983):

$$L_G^N = \max\{L_G^1, L_G^2\} \tag{32}$$

$$K_G^N = \max\{K_G^1, K_G^2\} \tag{33}$$

Firm 1 now maximizes its profit

$$\Pi_1(L_1, K_1, L_G^1, \bar{L}_G^2, K_G^1, \bar{K}_G^2) \tag{34}$$
$$= p_1 h_1(F_G(\max\{L_G^1, \bar{L}_G^2\}, \max\{K_G^1, \bar{K}_G^2\})) \cdot F_1(L_1, K_1) - w(L_1 + L_G^1) - r(K_1 + K_G^1),$$

taking into account that firm 2 contributes L_G^2 and K_G^2. For firm 2 the profit is given by

$$\Pi_2(L_2, K_2, \bar{L}_G^1, L_G^2, \bar{K}_G^1, K_G^2) \tag{35}$$
$$= p_2 h_2(F_G(\max\{L_G^1, \bar{L}_G^2\}, \max\{\bar{K}_G^1, K_G^2\})) \cdot F_2(L_2, K_2) - w(L_2 + L_G^2) - r(K_2 + K_G^2),$$

taking into account that firm 2 contributes L_G^1 and K_G^1. We do not need to calculate the first order conditions here, and in general it is not possibly because the profit functions are not differentiable on the whole domain. Since only the "best shot", i. e., the best technology, is relevant, only the firm which benefits most from the public input will invest in the public input. But the provision level will still be determined by the individual profit of that firm and not by the socially efficient level:

$$h_i(G)\frac{\partial F_i}{\partial L_i} = \frac{\partial h_i}{\partial G}\frac{\partial F_G}{\partial L_G}F_i(L_i, K_i), i = 1, 2. \tag{36}$$

The firm only considers her private benefit and cost. The firm with the largest productivity function $h_i(G)$ and the largest output productivity will be the only contributor.

In this case, and in the spirit of Hirshleifer (1983), there is always underprovision of the public input G. The more firms there are in the economy, the greater the gap between the Pareto optimal outcome and the private provision outcome.

6 Relation to patent and contest literature

Nordhaus (1972) was among the first to state the trade-off between the incentives to innovate and the welfare loss due to monopoly power. The innovator is granted a patent to set an incentive to innovate in the first place. Patent races are contests where firms invest in R&D to find newer products or improve on existing technology. An alternative incentive mechanism is to award prizes (see Shavell and van Ypersele, 2001). In this literature, the socially efficient innovation level (and R&D effort level) in general depend on the social surplus on the demand side of the economy.

In contrast, the present paper presents a new approach to modeling innovation and knowledge. If we consider knowledge a public input good, the welfare analysis has to be done from a different and new perspective. If the provision of this public good is provided privately by individual firms, this means that, in almost all case we will obtain and supoptimal level of investment in innovation. This not only restricts the production possibility set, but it also skews the economy's output towards goods which benefit less from technological innovation.

7 Discussion

In this section we want to discuss several assumptions in our model that may meet some criticism.

The specification of the production functions. Remember that we have assumed that the production functions are

$$x_i = h_i(G)F_i(L_i, K_i), \quad i = 1, 2 \tag{37}$$

with constant returns to scale in L_i, K_i, and $h_i(G)$ representing technological change. It may be argued that this assumption is restrictive or that this formulation is even driving our results. While this formulation gives our model some analytical tractability, it is not as restrictive as it may seem. From a theoretical point of view, this formulation where the public input effect is factor-augmenting, is the only one with sound theoretical foundation, as stressed by McMillan (1979). The more general looking formulations lead to a firm-augmenting public input, a non-convex production possibility set and, plausibly, to non-existence of equilibrium.

The competitive behaviour of firms. We have assumed in our model that each industry produces a single good in a competitive setting and have modeled the behaviour of a representative firm. Due to constant returns to scale, this is equivalent to assuming many different firms competitively producing the single final good. More restrictive is to assume that the firms only produce a single output good. The reason to do this is to be able to focus on a public input good that accrues to all firms and to all industries. If we allowed the firms to produce both (all) output goods, we would artificially internalize the externality. This would not be much more realistic and is not the focus or our model.

8 Conclusion

Two are the main contributions from the present paper. First, from a technical point of view, it extends the private provision framework to the case of public inputs like knowledge, information and innovation, which was a gap in the literature until now. The standard and well-known results from the private provision of a public consumption good carry over, in general, but there are some new different characteristics. Second, this approach allows to analyze innovation and knowledge from a new methodological perspective, not as a final consumption good but as a public input good or intermediate good. The results from the model are then contrasted with the traditional view of intellectual property protection as an incentive mechanism for innovation effort.

We find that if knowledge and innovation are considered public inputs, the private provision is associated with a suboptimal provision level, with the gap being specially large if the provision technology is a best-shot one.

Our approach suggests that the standard patent models may underestimate the welfare loss associated with intellectual property protection and may provided a further rationale for government support of basic research and innovation.

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