Differential Mortality, Moral Hazard and Optimal Taxation[1]

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Abstract

This paper studies the normative problem of redistribution between individuals who can influence their longevity through a non monetary effort but who have different taste for effort. As benchmarks, we first present the laissez-faire and the first best. In the first best, the level of effort is always lower than in the laissez-faire since the social planner takes into account the impact of higher effort on survival and thus on the price of annuities. However, if effort is assumed to be private and non monetary (like doing sport), it is reasonable to think that the social planner has no control on it. Thus, we modify our framework and assume for the rest of the paper that effort is determined by the individual while the social planner only allocates consumptions. Under full information and moral hazard constraints, early consumption is preferred to future consumption and the high survival individual obtains higher future consumption. Under asymmetric information, the distortion is equal to the first best one for the low survival individual while the trade-off between two-period consumptions is distorted upward for the high survival individual as a way to solve the incentive problem. We finally show how to decentralize first best and second best allocations through a perfect annuity market and taxes on annuities.

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1 Introduction

Is our life expectancy predetermined? To what extent can we modify it? Several factors may determine human longevity. It surely depends on intrinsic characteristics (such as gender or heredity) or on environmental and socio-cultural factors. However, individuals certainly have some control over their life expectancy and be able to modify it either through monetary investments (for example, having an expensive surgery) or through non monetary ones. In this latter case, health improving effort can take either a temporal or a physical form (such as diets or physical activity). To illustrate this latter point, Kaplan et al. (1987) show that little or no physical activity is associated with higher mortality risks at all ages. In a more recent study, Okamoto (2006) also finds a significantly positive relationship between leisure time engaged in sports and the increase in life expectancy at 65 of Japanese men. More evidence on the relationship between physical activity and life expectancy can also be found in Ferucci et al. (1999), Franco et al. (2005, 2006).

Relating these questions to current Social Security debates, the determinants and the consequences of life extension are certainly a matter of public concern. For instance, many empirical studies (such as Coronado et al. 2000, Liebman 2001 and Bommier et al. 2006) find that life duration differentials reduce intra-generational redistribution since there is no link between the amount of per period benefits and expected length of life. They highlight the importance of considering aggregate redistribution rather than just marginal redistribution when life expectancy enters the problem. Indeed, individuals with lower income obtain higher replacement rates but due to positive correlation between life expectancy and income, part of this redistribution is neutralized.

From a theoretical point of view, the contributions of Bommier et al. (2007a, b) deal with this relation between life duration and Social Security benefits. They study the optimal pension design when individuals differ in their life expectancy which they assume to be exogenous. Their paper focus on two main points. A first technical problem of how to model life expectancy in optimal income taxation problems: they show that the "double additivity assumption" (i.e. a utilitarian social planner and additively separable individuals' preferences) implies that individuals exhibit temporal risk neutrality, which leads to very specific and questionable conclusions in terms of redistribution.[1] Their second point concerns optimal taxation results when relaxing the assumption of additive preferences. They find that at the first best optimum, long-lived individuals should receive lower consumption and work longer while short-lived individuals should be compensated for their unluckiness by getting higher consumption and retiring earlier.

On the contrary, in this paper, we propose a framework in which individuals' longevity is only the result of a private (and costly) effort (taking other possible determinants as fixed) and study the optimal redistribution problem. We assume a two-period model in which individuals live the second period with a probability which depends on the level of effort they make in the first period. Individuals may yet differ in their taste for efforts so that they end up having different survival probabilities. As opposed to Eeckhoudt and Pestieau (2006) and Becker and Philipson (1998), we also assume that individuals'effort is always non monetary so that the social planner cannot influence it directly. We then study the direction of transfers between individuals with different taste for effort and thus different survival functions. As opposed to Bommier et al. (2007a, b), lifetime utility is additively separable so that individuals exhibit temporal risk neutrality in our model. We decided to stick to this formulation in order to emphasize the role of private effort on the optimal allocation when it is determined by the individual prior

[1] On the notion of temporal risk aversion, see Bommier (2006b).

to the allocation of consumptions by the social planner.[2]

Under these assumptions, we first present the laissez-faire as a benchmark case and second we study the first best problem in which the social planner allocates consumptions and effort levels. We obtain that the optimal level of effort is smaller in the first best than in the laissez-faire. This is due to the fact that in the first best, the social planner takes into account that effort modifies the survival probability which in turn modifies the price of consumption.[3] Since effort is non monetary, the social planner cannot decentralize this first best optimum through a tax, for example. This is why in the following, we will resort to a constrained first best in which it is assumed that individuals decide ex-ante of their effort.

Thus, in this framework, the social planner can now only influence effort through the level of consumptions. We find that under full information, future consumption is always lower than present consumption as a way to induce individuals to exert lower effort and make it tend to its first best level. We also show that second period consumption is higher for the individual with higher taste for effort so that the optimal allocation transfers resources from low survival individuals toward high survival ones. Finally, we study the second best problem, when the social planner cannot observe the taste for effort and effort levels. Under asymmetry of information, the distortion is identical to the first best distortion for the low survival individual so that it is still optimal to encourage early consumption for this individual. On the contrary, for the high survival individual, the trade-off between present and future consumption is modified due to the introduction of the incentive

[2] As it is shown in Bommier (2006a-b) and in Bommier et al. (2007a-b), assuming both additively separable utility functions and a utilitarian social welfare function implies that individuals are risk neutral toward their length of life which leads to the equalisation of consumptions between individuals with different length of life. Relaxing the assumption of additivity accross time in the lifetime utility function would surely change our results.

[3] These findings are closely related to Becker and Philipson (1998) who studied the optimal trade-off between the quantity and the quality of life when health expenditures modify the length of life.

constraint. Indeed, under asymmetry of information and moral hazard constraints, two effects are going in opposite directions for this individual. On the one hand, the social planner wants to encourage early consumption relative to future consumption so as to induce him to exert lower effort (as under full information and moral hazard constraints) but on the other hand, he wants to discourage early consumption so as to make the problem incentive compatible (in this case, a low survival individual willing to mimic a high survival individual would obtain too high a level of future consumption). We find that the overall effect is such that early consumption should still be encouraged at the second best for the high survival individual. Yet, this effect would be smaller than in the constrained first best.

We also study how to decentralize these optima through a perfect annuity market. Decentralisation of the constrained first and second best optima requires to distort the price of annuities from its fair price for both individuals. However, the level of this tax may be different between individuals with different taste for effort.

The paper is constructed as follows. In Section 2, we present the model, derive the laissez-faire and first best problems. In Section 3, we present a modified framework with full information and moral hazard constraints in which the social planner has no direct control on individuals' effort while Section 4 gives the results under asymmetric information. Last section concludes.

2 The Model

2.1 Individuals' types and preferences

We consider a stationary population composed of two groups of individuals, indexed by $i = 1, 2$ who have different tastes for effort γ^i and represent a proportion n^i of the population. Individuals may live for two periods, each of them with lengths normalized to 1. The first time period is certain while individuals survive to the second period with a probability $\pi(e) \in [0,1]$ which depends on the effort level e they make; they die at the end of the first period with a probability $(1-\pi(e))$.[4] We assume that $\pi'(e) > 0$ and that $\pi''(e) < 0$.[5] The individual's effort is made in first period and it is assumed to be non monetary (it can be either a temporal or a physical effort) so that it does not enter in the individual's budget constraint. Regarding these previous assumptions, individuals with different tastes for efforts may eventually differ in their survival probability.

In this framework, individuals derive utility from consumption at each period and we denote c and d, consumptions at first and at second periods respectively. Setting the discount and interest rates equal to zero, the expected lifetime utility of an individual of type γ^i is simply given by:

$$U\left(c,d,e,\gamma^i\right) = \pi(e)\left[u(c) + u(d) - \gamma^i v(e)\right] + (1-\pi(e))\left[u(c) - \gamma^i v(e)\right]$$

In our framework, there is no bequest motive so that if the individual dies at the beginning of the second period, his current utility is zero. The above utility function simplifies to

$$U^i(c,d,e) = u(c) + \pi(e)\,u(d) - \gamma^i v(e) \tag{1}$$

where for ease of notation, we denote $U^i(c,d,e) \equiv U\left(c,d,e,\gamma^i\right)$. Per period utility of consumption, $u(c)$ and $u(d)$ are such that $u'(.) > 0$ and $u''(.) < 0$. In the above expression, γ^i can equivalently be seen as the intensity of effort disutility so that $\gamma^i v(e)$ represents total utility cost incurred by an individual of type i exerting an effort e. For the following, we assume that the form of the effort cost *per se* is such that $v'(e) > 0$ and $v''(e) = 0$.[6]

For the rest of the paper, we assume that $\gamma^1 > \gamma^2$ so that type-1 individuals are "bad"-type individuals (the ones with high disutility of effort or

[4] Note that for a given effort level, $\pi(e)$ takes the same form for both individuals.

[5] These assumptions on the shape of the survival function are standard (see Eeckhoudt and Pestieau, 2007). Later in Section 3, we make more precise assumptions on the form of the survival function.

[6] Note that assuming $v''(e) > 0$ leads to the same conclusions.

equivalently with low taste for effort) while type-2 individuals are "good"-type individuals.

2.2 The laissez-faire

We assume that individuals invest all their savings on a perfect annuity market. Deleting superscripts for this section only, an individual with disutility of effort γ determines optimal levels of consumption at first and second periods, as well as his optimal level of effort by solving the following problem:

$$\begin{aligned} \max_{c,d,e} U(c,d,e) &= u(c) + \pi(e)\,u(d) - \gamma v(e) \\ \text{s.to} \quad c + rd &\leq w \end{aligned}$$

where second line is the individual's budget constraint and r is the price of an annuity. We assume that the initial wealth endowment w is exogenous and identical for any individual. Rearranging first order conditions yield

$$MRS_{c,d} \equiv \frac{\pi(e)\,u'(d)}{u'(c)} = r \tag{2}$$

$$\pi'(e)\,u(d) = \gamma v'(e) \tag{3}$$

where $MRS_{c,d}$ stands for the marginal rate of consumption between present and future consumption in absolute value terms. Condition (2) gives the trade-off between present and future consumptions. If the market for annuities is actuarially fair and insurers can perfectly observe individuals' survival probability, the annuity price is set such that $r = \pi(e)$. In such a case, $MRS_{c,d} = -\pi(e)$ and $u'(c) = u'(d)$ so that individual's consumption is smoothed across periods. On the contrary, if the annuity is taxed, its price is such that $r > \pi(e)$ and $MRS_{c,d} > \pi(e)$; first period consumption is then higher than second period consumption. Condition (3) states that the individual's optimal level of effort is such that, at this level, the expected marginal utility of increased life expectancy must be equal to marginal disutility of effort.[7] This defines the optimal level of effort $e^*(\gamma, d)$ chosen by

[7] Under our assumptions, the second order condition $\pi''(e)\,u(d) - \gamma v''(e) < 0$ is always satisfied.

the individual as a function of both the level of second period consumption and of his taste for effort. Note that in the laissez-faire, the individual does not take into account that by choosing a specific level of effort, he changes the price of the annuity and thus his budget set. Indeed, if the annuity price is actuarially fair, $r = \pi(e)$ so that increasing e increases both the individual survival chance and the price of second period consumption. In the laissez-faire, the individual only takes into account first effect and the chosen level of effort is too high compared to the optimal one.[8] Our results on the laissez-faire allocation are proven in Appendix A and summarized in the following proposition:

Proposition 1 *When the annuity market is actuarially fair, the laissez-faire allocation is such that:*

(i) consumption is smoothed across periods: $c = d$,

(ii) $e^*\left(\gamma^1, d\right) < e^*\left(\gamma^2, d\right)$ *for any given* d,

(iii) $d^1 > d^2$.

where d^i is second period consumption of individual with type γ^i. As mentioned in point *(ii)*, the higher is the preference for leisure(or equivalently the lower is the taste for effort), the lower is the level of effort made by the individual in the laissez-faire solution. Thus, the probability of surviving into the second period is lower. We also find that consumption is higher for the high-disutility individual; indeed, this individual prefers higher consumption in first period since he has lower chances to enjoy any consumption in the following period. Finally, doing comparative statics on (3), we show in Appendix A, that the higher is the level of consumption in the second period, the more likely is the individual to exert high effort. This result seems

[8] This result was highlighted first by Becker and Philipson (1998) who studied the trade-off between the quantity and the quality of life and how individuals' attitude toward life extension affects mortality contingent claims.

reasonable: if second period consumption is high, the individual has more incentives to exert higher effort so as to increase his survival probability.

2.3 The first best problem

In this section, we assume that the social planner is utilitarian and that he perfectly observes individuals' types. The economy is assumed to be in a steady state equilibrium and the social planner can lend or borrow at a zero interest rate in order to balance the budget at any given period. The resource constraint of the economy is thus:

$$\sum_{i=1,2} n^i \left(c^i + \pi\left(e^i\right) d^i\right) \leq w \tag{4}$$

where $\left(c^i, d^i\right)$ is the consumption allocation of an individual of type $i = 1, 2$ and e^i is his effort level. Thus, the social planner chooses consumption paths as well as effort levels in order to maximize

$$\sum_{i=1,2} n^i U^i\left(c^i, d^i, e^i\right)$$

subject to (4). First order conditions of this problem yield:

$$MRS_{c,d}^{i,FB} = \pi\left(e^i\right) \tag{5}$$

$$\pi'\left(e^i\right) u\left(d^i\right)\left[1 - \frac{u'\left(d^i\right) d^i}{u\left(d^i\right)}\right] = \gamma^i v'\left(e^i\right) \tag{6}$$

for any $i = 1, 2$. Obviously, condition (5) states that consumption should be equalized across time and across agents. The second condition defines the optimal level of effort, $e^*\left(\gamma^i, d^i\right)$.[9] The expression $u'\left(d^i\right) d^i / u\left(d^i\right)$ is assumed to be lower than 1 which is standard in the literature and ensures that the value of life is large enough.[10] Indeed, the individual makes a strictly positive effort only when life is worth living, i.e. when this elasticity is lower than 1; otherwise, the optimal level of effort is zero. Comparing this first best

[9] The second order condition is always satisfied under our assumptions.

[10] See Murphy and Topel (2005) and Becker et al. (2005).

condition (6) with its laissez-faire counterpart (3), one clearly sees that the first best allocation now includes an additional term, $-u'\left(d^i\right)\pi'\left(e^i\right)d^i$. This corresponds to the effect of effort on the price of second period consumption; in the first best, the social planner takes into account that a higher effort decreases consumption possibilities in the second period through an effect in prices. Then, the first best level of effort is such that net marginal gain in utility (due to increased survival probability but to lower consumption levels) is equal to marginal disutility of effort. This leads us to the following proposition:

Proposition 2 *For any type of individual, the first best level of effort is lower than the laissez-faire one.*

This result is similar to Becker and Philipson (1998). Our second set of results is summarized hereafter:

Proposition 3 *The first best allocation is characterized by*
(i) $c^i = d^i = \bar{c}\ \forall i$,
(ii) $e^*\left(\gamma^1, \bar{c}\right) < e^*\left(\gamma^2, \bar{c}\right)$ *for any given* $\bar{c}$ *and* $\gamma^1 > \gamma^2$.

When individuals differ in their taste for effort, a utilitarian social planner provides individuals with the same consumption bundles. These results are standard: when individual preferences are additively separable and the social welfare function is utilitarian, the first best implies that consumption is equalized between individuals.[11] The social planner also requires lower effort from the individual with lower taste for effort. The survival chance for this individual is then smaller. Finally, the expected lifetime consumption of an individual of type γ^i is equal to $c^i + \pi\left(e^*\left(\gamma^i, d^i\right)\right)d^i$ so that the

[11] This is what Bommier et al. (2007) call "the double additivity assumption". As shown in Bommier (2006), under this assumption, individuals are risk neutral toward the length of life so that a utilitarian social planner corrects for intra-period inequality but does not take into account inter-period inequality (and thus, the fact that one individual might live longer than the other).

expected consumption of the high-taste individual is higher. The first best solution then transfers resources from low-taste (low-survival) individuals toward high-taste (high-survival) ones.

This first best allocation cannot be decentralized since effort is non monetary and thus, it cannot be taxed. This is why in the following section, we resort to a constrained first best in which the social planner lets individuals choose their effort level and can eventually influence it through the allocation of consumptions.

3 Full information with moral hazard

Since effort takes a non monetary form in our framework, it is reasonable to assume that the social planner has no control over it. Thus, we now assume that the social planner only decides of the allocation of consumptions, knowing that it may have consequences on individuals' choice of effort.

The timing of this problem is the following one. First, individuals choose their level of effort. For any $i = 1, 2$, the optimal level of effort $e^*\left(\gamma^i, d^i\right)$ is defined by (3); thus $e^*\left(\gamma^1, d\right) < e^*\left(\gamma^2, d\right)$ $\forall d$ and effort increases in the level of second period consumption.

Replacing for $e^*\left(\gamma^i, d^i\right)$ into (1), the individual's indirect utility function $V^i\left(c^i, d^i\right) \equiv U^i\left(c, d, e^*\left(\gamma^i, d^i\right)\right)$ becomes

$$V^i\left(c^i, d^i\right) = u\left(c^i\right) + \pi\left(e^*\left(\gamma^i, d^i\right)\right) u\left(d^i\right) - \gamma^i v\left(e^*\left(\gamma^i, d^i\right)\right)$$

Second, the social planner determines consumption paths of individuals with types γ^1 and γ^2. In this section, we assume that the social planner perfectly observes individuals' types. Our modified first best problem now takes the following form:

$$\begin{aligned} \max_{c^i, d^i} \quad & \sum_{i=1,2} n^i V^i\left(c^i, d^i\right) \\ \text{s.to} \quad & \sum_{i=1,2} n^i \left[c^i + \pi\left(e^*\left(\gamma^i, d^i\right)\right) d^i\right] \leq w \end{aligned}$$

The first order conditions with respect to c^i and d^i are, respectively:

$$u'\left(c^i\right) = \lambda \tag{7}$$

$$u'\left(d^i\right) = \lambda\left[1+\frac{\pi'\left(e^*\left(\gamma^i,d^i\right)\right)}{\pi\left(e^*\left(\gamma^i,d^i\right)\right)}d^i\frac{de^*\left(\gamma^i,d^i\right)}{dd^i}\right] \tag{8}$$

Substituting equation (7) into (8) and replacing for the expression of $de^*\left(\gamma^i,d^i\right)/dd^i$, one obtains the following marginal rate of substitution

$$MRS_{c,d}^{i,FBmh} = \pi\left(e^*\left(\gamma^i,d^i\right)\right)\left[1-\frac{\pi'\left(e^*\left(\gamma^i,d^i\right)\right)^2}{\pi\left(e^*\left(\gamma^i,d^i\right)\right)\pi''\left(e^*\left(\gamma^i,d^i\right)\right)}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right] \tag{9}$$

where $MRS_{c,d}^{i,FBmh}$ stands for the marginal rate of substitution between two-period consumptions in the constrained first best in absolute value. Comparing condition (9) with previous equation (5), we find that consumption is not smoothed across periods anymore and that present consumption is now higher than future consumption. The explanation is related to the optimal level of effort. As mentioned in Proposition 2, the first best level of effort should be lower than in the laissez-faire due to the effect of effort on the price of second period consumption. Thus, the social planner provides individuals with less second period consumption so as to induce them to exert lower effort and make it tend to its first best level. This limits the increase of individuals' survival probability and thus the negative impact of effort on the price of future consumption.[12] Our results are summarized in the following proposition:

Proposition 4 *Under full information and moral hazard constraints, $c^i > d^i$ for any individual with type γ^i.*

Using condition (7), we also obtain that first period consumption is equalized between individuals with different tastes for effort; as already mentioned, this directly follows from our assumptions on individual preferences and on the social welfare function and from the fact that first period

[12] This result is in line with Becker and Philipson (1998).

consumption has no impact on effort levels. On the contrary, second period consumption might be differentiated depending on our assumptions on survival probabilities and on per period utility. This stated formally in the following proposition.

Proposition 5 *Consider two groups of individuals with disutility of effort* γ^1 *and* γ^2 *such that* $\gamma^1 > \gamma^2$*. In the constrained first best,*

(i) *First period consumption is equalized between individuals,* $c^i = \bar{c}\ \forall i$*.*

(ii) *Second period consumption is such that*

- *if the survival probability has constant elasticity,* $d^1 = d^2$*,*
- *otherwise,* $d^1 \gtrless d^2$ *if and only if*

$$\frac{\pi'\left(e^*\left(\gamma^1, d^1\right)\right)}{\pi\left(e^*\left(\gamma^1, d^1\right)\right)} d^1 \frac{de^*\left(\gamma^1, d^1\right)}{dd^1} \lessgtr \frac{\pi'\left(e^*\left(\gamma^2, d^2\right)\right)}{\pi\left(e^*\left(\gamma^2, d^2\right)\right)} d^2 \frac{de^*\left(\gamma^2, d^2\right)}{dd^2}$$

Let us explicit point (ii) of the above proposition. Under constant elasticity of the survival probability, the individual' type disappears from (8) so that second period consumption is independant of individuals' tastes for effort. However, we believe that a case where the survival probability has decreasing elasticity is more reasonable.[13] As detailed in the Appendix C, we find that when specifying functional forms for the survival probability such that it has decreasing elasticity, second period consumption should be higher for the high-taste-for-effort individual ($d^1 < d^2$). In this case, the social planner rewards this individual by giving him higher level of second period consumption; equivalently, he gives more to the individual who is more likely to survive. The effort of the high-taste individual as well as his survival probability are then higher so that the optimal redistribution scheme is such that it transfers resources from the low-survival individual to the high-survival individual.[14]

[13] For instance, it is proven that in the case of physical activity, there exists an optimal level of effort above which additionnal effort may effectively decrease survival chances.

[14] These results are confirmed in the last section when doing simulations.

These results can be decentralized through lump sum transfers from the low survival toward the high survival individual. Moreover, as we mentioned in the laissez-faire, the individual's savings are invested on a perfect annuity market. Hence, comparing (9) with (2), one has $MRS_{c^i,d^i}^{i,FBmh} > \pi\left(e^*\left(\gamma^i, d^i\right)\right)$ so that a tax on annuities is a way to decentralize this first best. The level of the tax will be higher for the individual with low taste of effort.

4 Asymmetric information with moral hazard

In this section, we assume that the social planner can observe neither individuals' preference for leisure nor their levels of efforts.[15] Using results of Proposition 5, it is straightforward to see that if the social planner proposes first best bundles, the individual with low taste for effort γ^1 has interest in claiming to be high-taste for effort type and enjoy higher consumption d^2.[16] Then, to avoid mimicking behavior, we add an incentive constraint to our first best problem such that:

$$\begin{aligned} \max_{c^1,d^1,c^2,d^2} \quad & \sum_{i=1,2} n^i V^i\left(c^i, d^i\right) \\ & \text{s.to} \sum_{i=1,2} n^i \left[c^i + \pi\left(e^*\left(\gamma^i, d^i\right)\right) d^i\right] \leq w \\ & \text{s.to } V^1\left(c^1, d^1\right) \geq V^1\left(c^2, d^2\right) \end{aligned}$$

[15] Note that the social planner observes survival probabilities ex post. But he cannot obtain additional information on types from it: survival can always be due to luck and not because the individual lied on his type.

[16] Note that since effort and consumption are positively correlated, an increase in consumption not only increases direct utility derived from consumption but also survival and total effort disutility. However, it is easy to prove that $\partial V^i\left(c^i, d^i\right)/\partial d^i > 0$. Thus, type-1 individual has always interest in lying on his type.

First order conditions yield:

$$MRS^{1,SB}_{c,d} = \pi\left(e^*\left(\gamma^1, d^1\right)\right)\left(1+\frac{\pi'\left(e^*\left(\gamma^1, d^1\right)\right)}{\pi\left(e^*\left(\gamma^1, d^1\right)\right)} d^i \frac{de^*\left(\gamma^1, d^1\right)}{dd^1}\right)$$

$$MRS^{2,SB}_{c,d} = \pi\left(e^*\left(\gamma^2, d^2\right)\right)\left(1+\frac{\pi'\left(e^*\left(\gamma^2, d^2\right)\right)}{\pi\left(e^*\left(\gamma^2, d^2\right)\right)} d^i \frac{de^*\left(\gamma^2, d^2\right)}{dd^2}\right) \times \left[\frac{1-\frac{\mu}{n^2}}{1-\frac{\mu}{n^2}\frac{\overline{MRS}^1_{c,d}}{MRS^{2,SB}_{c,d}}}\right]$$

where μ is the Lagrange multiplier associated to the incentive constraint and $\overline{MRS}^1_{c,d}$ is the second best marginal rate of substitution of a type-1 individual mimicking a type-2 individual. The second best marginal rate of substitution between present and future consumptions is equivalent to (9) for individual 1. On the opposite, $MRS^{2,SB}_{c,d}$ is distorted downward since the expression inside brackets is lower than 1. Our results are summarized hereafter:

Proposition 6 *Assume two groups of individuals with disutility of effort* γ^1 *and* γ^2 *such that* $\gamma^1 > \gamma^2$*. The second best allocation implies*

(i) $MRS^{1,SB}_{c,d} = MRS^{1,FBmh}_{c,d}$

(ii) $MRS^{2,SB}_{c,d} < MRS^{2,FBmh}_{c,d}$

The above proposition states that second best distortion for the individual with low taste for effort is equivalent to the first best one. Thus, in the second best, the social planner simply imposes a distortion on this individual so as to induce him to exert lower effort, as in the full information case. This is *kind of* "no distortion at the top" result for type-1 individual who would like to lie on his type.

On the contray, the individual with high taste for effort now faces an additional distortion which is due to the introduction of the incentive constraint so that the marginal rate of substitution is now distorted downward compared to the full information case. Then, under assymetric information, the high taste for effort individual faces two types of distortions which go in opposite directions. For instance, in $MRS^{2,SB}_{c,d}$, the first expression inside parenthesis is greater than 1 and is related to the price of consumption; as in the first best, the social planner wants to induce lower levels of efforts (called afterward the first best effort effect) by encouraging present consumption relative to future consumption. On the other hand, the second expression inside brackets i lower than 1 and is due to the introduction of the incentive constraint. Under asymmetric information, it is desirable to encourage future consumption for this individual. This makes the low-taste-for-effort individual less interested in the allocation proposed to the high-taste individual as it would provide him with too high a level of future consumption relative to present consumption. It is a way to relax an otherwise binding self-selection constraint.

Thus, depending on whether the first best effort effect dominates the second best incentive effect, we obtain that early consumption should be encouraged reatively to future consumption at the second best; the difference between consumptions should be yet lower than in the symmetric information case for individual 2.

In the next subsection, we study numerically the direction of the overall distortion and find whether $MRS^{2,SB}_{c,d}$ is greater or lower than $MRS^{i,FB}_{c,d}$.

Finally, we study how to implement these results through a perfect annuity market. Comparing $MRS^{1,SB}_{c^1,d^1}$ and $MRS^{2,SB}_{c^2,d^2}$, with their laissez-faire counterparts, we find that the second best optimum can be decentralized through lump sum transfers and positive taxes on annuities. This tax is identical to the full information case for the low-taste-for-effort individual. On the contrary, the high-taste individual now faces a positive tax which is lower than under full information since the incentive effect partly neutralizes the first best effect. Thus, in the second best, the tax on annuities is lower for individual 2 than for individual 1 (since in the constrained first best, the tax for this individual was already higher). A positive tax on annuities is way to implement the second best optimum and to encourage early consumption for both groups of individuals.

5 Numerical examples

Consider the following specifications for the various components of our model. Types (γ^1, γ^2) are distributed on $[0,1]$ and we set $w = 10$.We consider three possible cases: a case where types are very close, a case where $\gamma^1 = 2\gamma^2$ and a case where types are very different. The utility function is isoelastic: $u(c) = c^{1-\varepsilon}/(1-\varepsilon)$ where ε is the coefficient of relative risk aversion. We assume that $\varepsilon = 0.83$. The survival probability takes the following form, $\pi(e) = e/(1+e)$ which ensures that the survival probability is always lower than one. In this case, the suvival probability has decreasing elasticity of substitution. In the following table, we report first- and second best results under the different assumptions on the γs:

types		c	d	e	$\pi(e)$	$\frac{u'(d)}{u'(c)}$
$(\gamma^1, \gamma^2) = (1, 0.9)$	individual 1 FB	6.1707	5.84015	1.8179	0.6451	
	SB	7.1752	6.798	1.8541	0.6496	1.046
	individual 2 FB	6.1707	5.8647	1.9714	0.6634	
	SB	5.1639	4.9283	1.9273	0.6583	1.04
$(\gamma^1, \gamma^2) = (1, 0.5)$	individual 1 FB	5.9995	5.6770	1.8111	0.6442	
	SB	6.2442	5.9101	1.8203	0.6454	1.047
	individual 2 FB	5.9995	5.7998	2.9827	0.7489	1.023
	SB	5.7552	5.5956	2.9700	0.7481	1.024
$(\gamma^1, \gamma^2) = (1, 0.1)$	individual 1 FB	5.7401	5.4298	1.8005	0.6429	
	SB	1.8129	1.6990	1.5368	0.6057	1.055
	individual 2 FB	5.7401	5.6665	7.8881	0.8874	1.011
	SB	9.6598	8.4135	8.1904	0.8912	1.121

This table confirms our first best results that under decreasing elasticity of the survival probability, future consumption is always higher for type 2 individuals. These results are yet more intersting when studying the allocation under assymmetric information. First we find that first period consumption is not smoothed anymore between individuals. Then under asymmetric information, individual 1 always gets higher levels of first period consumption as well as future consumption, except for very different tastes for efforts. However, the individual 2 always ends up making a higher level of effort and thus has higher survival probability. In the last column, we also present the level of the distortion under asymmetric information and find that it is always greater than 1 for individual 2 so that the overall distortion due to both the effort effect and to the incentive constraint results in encouraging first period consumption. STRANGE RESULTS FOR GAMMA ELOIGNES

6 Conclusion

This paper studies the problem of redistribution between individuals who can influence their survival probabilities by exerting some non monetary effort. We assume that the effort level is taking the form of a physical activity which is typically a variable that the social planner cannot control. In such a case, depending on his taste for effort, the individual chooses his level of effort while the social planner decides of the allocation of consumptions. We first highlight the trade-off between the quantity and the quality of life by expliciting the relations between survival probabilities, effort and the price of annuities. On the one hand, higher effort increases expected length of life but it also decreases consumption possibilities through an increase in the price of future consumption. We showed that in the laissez-faire, the effort level is higher than in the first best, because in the former, the individual does not integrate the consequence of higher effort over his budget constraint. Thus, under full information and moral hazard constraints, we find that early consumption is encouraged relative to future consumption as a way to make individuals exert less effort. Under asymmetry of information, the second best distortion in the trade-off between present and future consumption is identical to the first best one for the individual with low taste for effort. However, for the high-taste individual, the distortion is lower than under full information but we find that even if the incentive effect partly neutralizes the first best effort effect, present consumption should still be encouraged for this individual.

We also studied how to decentralize these optima through a perfect annuity market. We find that imposing a tax on annuities is a way to implement the first best for both individuals. However, its level will be different between individuals with different taste for effort and different survival chances. Under asymmetry of information, a tax on annuities is still optimal for both groups of individuals but its level will be lower than in the constrained first best for the high survival individual.

There are several directions in which we could extend the model. For instance, we have neglected the fact that life expectancy depends on intrinsic characteristics (such as gender) and that efforts and genetics may be correlated. Adding this additional characteristic, how would our model be modified? Moreover, we assume additively separable preferences which implies risk neutrality towards the length of life. This is a strong assumption and relaxing it may modify substantially our results. Answering these questions are on our research agenda.

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Appendix

A Laissez Faire

Using (3) and the fact that $\pi''(e) < 0$ and $v''(e) > 0$, we obtain that for a given level of consumption d, the only possible solution is $e^*\left(\gamma^1, d\right) < e^*\left(\gamma^2, d\right)$. This proves point (ii) of Proposition 1. From full differentiation of (3), we find that effort increases with second period consumption:

$$\frac{de^*(\gamma, d)}{dd} = -\frac{\pi'(e)\, u'(d)}{\pi''(e)\, u(d)} > 0 \tag{10}$$

If the price of the annuity is actuarially fair, the individual's budget constraint is $d^i\left(1 + \pi\left(e^*\left(\gamma^i, d^i\right)\right)\right) = w$ for any individual i; using our assumptions on $\pi(e)$ and (10), the only possible solution is $d^1 > d^2$. This proves point (iii).

B First best

Proof of Proposition 2: Comparing (3) with (6), one has that for any γ,

$$\left.\frac{\gamma^i v'\left(e^i\right)}{\pi'\left(e^i\right)}\right|_{FB} < \left.\frac{\gamma^i v'\left(e^i\right)}{\pi'\left(e^i\right)}\right|_{LF}$$

We get Proposition 2.

Proof of Proposition 3: Using (5), we obtain point (i) of Proposition 3. Point (ii) is obtained from full differentiation of (6):

$$\frac{de^i}{d\gamma^i} = \frac{\gamma^i v'\left(e^i\right)}{\pi''\left(e^i\right)\left[u\left(d^i\right) - u'\left(d^i\right) d^i\right]} < 0$$

C Full information with moral hazard

Proof of Proposition 5: First order conditions with respect to c^i and d^i of the modified first best problem are:

$$u'\left(c^i\right) - \lambda = 0 \tag{11}$$

$$\begin{aligned} & \left[\begin{array}{c} \pi'\left(e^*\left(\gamma^i,d^i\right)\right)u\left(d^i\right) \\ -\gamma^i v'\left(e^*\left(\gamma^i,d^i\right)\right) - \lambda\pi'\left(e^*\left(\gamma^i,d^i\right)\right)d^i \end{array}\right] \frac{de^*\left(\gamma^i,d^i\right)}{dd^i} \\ + \quad & \pi\left(e^*\left(\gamma^i,d^i\right)\right)\left[u'\left(d^i\right) - \lambda\right] = 0 \end{aligned} \tag{12}$$

where λ is the Lagrange multiplier associated to the resource constraint. Replacing for (3) into (12) and rearranging terms, one obtains

$$\pi\left(e^*\left(\gamma^i,d^i\right)\right)u'\left(d^i\right) - \pi\left(e^*\left(\gamma^i,d^i\right)\right)\lambda\left[1 + \frac{\pi'\left(e^*\left(\gamma^i,d^i\right)\right)}{\pi\left(e^*\left(\gamma^i,d^i\right)\right)}d^i\frac{de^*\left(\gamma^i,d^i\right)}{dd^i}\right] = 0 \tag{13}$$

Using both (11)and (13), we obtain (9). Replacing for (10) in the above expression, we obtain

$$u'\left(d^i\right) - \lambda\left[1 - \frac{\pi'\left(e^*\left(\gamma^i,d^i\right)\right)^2}{\pi\left(e^*\left(\gamma^i,d^i\right)\right)\pi''\left(e^*\left(\gamma^i,d^i\right)\right)}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right] = 0$$

Let first assume that $\pi(e)$ *has constant elasticity* denoted ε with $\varepsilon < 1$ and $e \in [0,1]$. Thus,

$$u'\left(d^i\right) - \lambda\left[1 - \frac{\varepsilon}{\varepsilon - 1}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right] = 0$$

so that first order condition on d^i is independent of individual's types and $d^1 = d^2$ for $\gamma^1 > \gamma^2$.

Let now assume that $\pi(e)$ *takes a log form with* $e \in [1, \exp]$. In this case, $\pi'\left(e^*\left(\gamma^i,d^i\right)\right)^2 / \left(\pi\left(e^*\left(\gamma^i,d^i\right)\right)\pi''\left(e^*\left(\gamma^i,d^i\right)\right)\right) = -1/\log e^*\left(\gamma^i,d^i\right)$ and (13) is equal to

$$\begin{aligned} u'\left(d^i\right) - \lambda\left[1 + \frac{1}{\log e^*\left(\gamma^i,d^i\right)}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right] &= 0 \\ u'\left(d^i\right) - \lambda\left[1 + \frac{1}{\pi\left(e^*\left(\gamma^i,d^i\right)\right)}\frac{u'\left(d^i\right)d^i}{u\left(d^i\right)}\right] &= 0 \end{aligned} \tag{14}$$

If $u\left(d^i\right)$ *has constant elasticity of substitution* β, (14) simplifies to

$$u'\left(d^i\right) - \lambda\left[1 + \frac{\beta}{\pi\left(e^*\left(\gamma^i, d^i\right)\right)}\right] = 0$$

Fully differentiating the above expression,

$$\frac{dd^i}{d\gamma^i} = -\frac{\frac{\lambda\beta}{\pi(e^*(\gamma^i,d^i))^2}\pi'\left(e^*\left(\gamma^i, d^i\right)\right)\frac{de^*\left(\gamma^i,d^i\right)}{d\gamma^i}}{u''\left(d^i\right) + \frac{\lambda\beta}{\pi(e^*(\gamma^i,d^i))^2}\pi'\left(e^*\left(\gamma^i, d^i\right)\right)\frac{de^*(\gamma^i,d^i)}{dd^i}}$$

where the denominator is negative by second order condition and $de^*\left(\gamma^i, d^i\right)/d\gamma^i < 0$. Thus, the above expression is negative and $d^1 < d^2$ for $\gamma^1 > \gamma^2$.

On the contrary, *if* $u\left(d^i\right)$ *takes a log form*, (14) is now equal to

$$u'\left(d^i\right) - \lambda\left[1 + \frac{1}{2\pi\left(e^*\left(\gamma^i, d^i\right)\right)u\left(d^i\right)}\right] = 0$$

Again fully differentiating the above expression,

$$\frac{dd^i}{d\gamma^i} = \frac{-\lambda\frac{\pi'\left(e^*\left(\gamma^i,d^i\right)\right)u\left(d^i\right)\frac{de^*\left(\gamma^i,d^i\right)}{d\gamma^i}}{\left[\pi\left(e^*\left(\gamma^i,d^i\right)\right)u\left(d^i\right)\right]^2}}{u''\left(d^i\right) + \lambda\frac{\pi'(e^*(\gamma^i,d^i))u(d^i)\frac{de^*(\gamma^i,d^i)}{dd^i} + \pi(e^*(\gamma^i,d^i))u'(d^i)}{\left[\pi(e^*(\gamma^i,d^i))u(d^i)\right]^2}}$$

where the denominator is negative by second order condition. This expression is negative and $d^1 < d^2$ for $\gamma^1 > \gamma^2$. This proves Proposition 5.

D Asymmetric information with moral hazard

D.1 Marginal rate of substitution

First order conditions of the second best problem are:

$$V_c^1\left(c^1, d^1\right)\left(1 + \frac{\mu}{n^1}\right) = \lambda \tag{15}$$

$$V_c^2\left(c^2, d^2\right)\left(1 - \frac{\mu}{n^2}\frac{V_c^1\left(c^2, d^2\right)}{V_c^2\left(c^2, d^2\right)}\right) = \lambda \tag{16}$$

$$V_d^1\left(c^1, d^1\right)\left(1 + \frac{\mu}{n^1}\right) = \lambda\left[\pi\left(e^*\left(\gamma^1, d^1\right)\right) + \pi'\left(e^*\left(\gamma^1, d^1\right)\right)\frac{de^*\left(\gamma^1, d^1\right)}{dd^1}d^1\right]$$

$$V_d^2\left(c^2, d^2\right)\left(1 - \frac{\mu}{n^2}\frac{V_d^1\left(c^2, d^2\right)}{V_d^2\left(c^2, d^2\right)}\right) = \lambda\left[\pi\left(e^*\left(\gamma^2, d^2\right)\right) + \pi'\left(e^*\left(\gamma^2, d^2\right)\right)\frac{de^*\left(\gamma^2, d^2\right)}{dd^2}d^2\right]$$

so that we are able to write marginal rates of substitution of individuals with types γ^1 and γ^2 in absolute value terms:

$$\begin{aligned} MRS^{1,SB}_{c,d} \equiv \frac{V^1_d\left(c^1,d^1\right)}{V^1_c\left(c^1,d^1\right)} &= \pi\left(e^*\left(\gamma^1,d^1\right)\right)\left(1+\frac{\pi'\left(e^*\left(\gamma^1,d^1\right)\right)}{\pi\left(e^*\left(\gamma^1,d^1\right)\right)}d^1\frac{de^*\left(\gamma^1,d^1\right)}{dd^1}1\right) \\ MRS^{2,SB}_{c,d} \equiv \frac{V^2_d\left(c^2,d^2\right)}{V^2_c\left(c^2,d^2\right)} &= \pi\left(e^*\left(\gamma^2,d^2\right)\right)\left(1+\frac{\pi'\left(e^*\left(\gamma^2,d^2\right)\right)}{\pi\left(e^*\left(\gamma^2,d^2\right)\right)}d^2\frac{de^*\left(\gamma^2,d^2\right)}{dd^2}\right) \\ &\times\left[\frac{1-\left(\mu/n^2\right)\left(V^1_c\left(c^2,d^2\right)/V^2_c\left(c^2,d^2\right)\right)}{1-\left(\mu/n^2\right)\left(V^1_d\left(c^2,d^2\right)/V^2_d\left(c^2,d^2\right)\right)}\right] \end{aligned}$$

where $V^1_c\left(c^2,d^2\right)/V^2_c\left(c^2,d^2\right)=1$ when replacing by functional forms. It is also possible to rewrite the right hand side so that

$$\begin{aligned} MRS^{2,SB}_{c,d} &= \pi\left(e^*\left(\gamma^2,d^2\right)\right)\left(1+\frac{\pi'\left(e^*\left(\gamma^2,d^2\right)\right)}{\pi\left(e^*\left(\gamma^2,d^2\right)\right)}d^2\frac{de^*\left(\gamma^2,d^2\right)}{dd^2}\right) \\ &\times\left[\frac{1-\left(\mu/n^2\right)}{1-\left(\mu/n^2\right)\left(\frac{-V^1_d(c^2,d^2)/V^1_c(c^2,d^2)}{V^2_d(c^2,d^2)/V^2_d(c^2,d^2)}\right)}\right] \\ &= \pi\left(e^*\left(\gamma^2,d^2\right)\right)\left(1+\frac{\pi'\left(e^*\left(\gamma^2,d^2\right)\right)}{\pi\left(e^*\left(\gamma^2,d^2\right)\right)}d^2\frac{de^*\left(\gamma^2,d^2\right)}{dd^2}\right)\times\left[\frac{1-\left(\mu/n^2\right)}{1-\left(\mu/n^2\right)\left(\frac{\overline{MRS}^1_{c,d}}{MRS^{2,SB}_{c,d}}\right)}\right] \end{aligned}$$

where $\overline{MRS}^1_{c,d}=-V^1_d\left(c^2,d^2\right)/V^1_c\left(c^2,d^2\right)$. We finally show that the expression inside brackets is lower than one if and only if $V^1_d\left(c^2,d^2\right)<V^2_d\left(c^2,d^2\right)$, i.e.

$$\begin{aligned} &\left[\pi'\left(e^*\left(\gamma^1,d^2\right)\right)u\left(d^2\right)-\gamma^1v'\left(e^*\left(\gamma^1,d^2\right)\right)\right]\frac{de^*\left(\gamma^1,d^2\right)}{dd^2} \\ &\quad+\pi\left(e^*\left(\gamma^1,d^2\right)\right)u'\left(d^2\right) \\ <\; &\left[\pi'\left(e^*\left(\gamma^2,d^2\right)\right)u\left(d^2\right)-\gamma^2v'\left(e^*\left(\gamma^2,d^2\right)\right)\right]\frac{de^*\left(\gamma^2,d^2\right)}{dd^2} \\ &\quad+\pi\left(e^*\left(\gamma^2,d^2\right)\right)u'\left(d^2\right) \end{aligned}$$

Using (3), the above expression simplifies to $\pi\left(e^*\left(\gamma^1,d^2\right)\right)<\pi\left(e^*\left(\gamma^2,d^2\right)\right)$ which is always verified under our assumptions.

D.2 Distortion of a type 2 individual

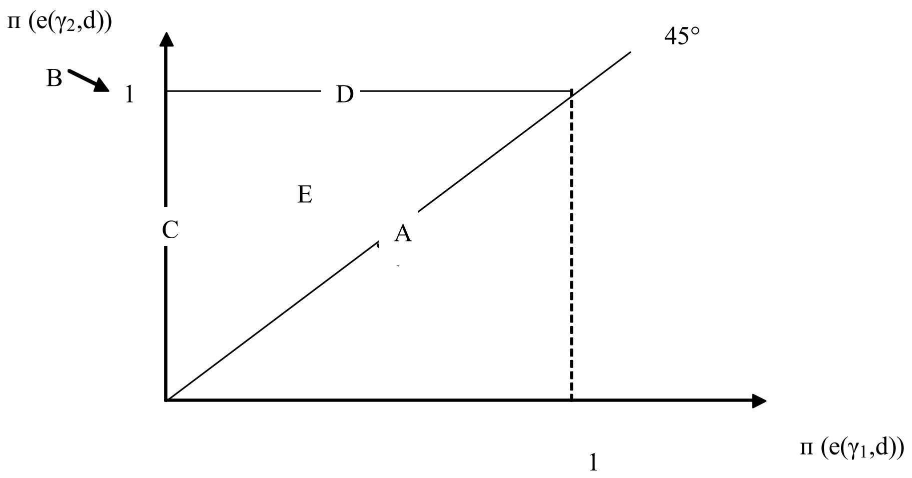
In this section, we determine whether the level of the distortion of a type 2 individual, equal to

$$\Lambda = \left(1 + \frac{\pi'\left(e^*\left(\gamma^2, d^2\right)\right)}{\pi\left(e^*\left(\gamma^2, d^2\right)\right)}\frac{de^*\left(\gamma^2, d^2\right)}{dd^2}d^2\right) \times \left[\frac{1-\frac{\mu}{n^2}}{1-\frac{\mu}{n^2}\frac{MRS^1_{c,d}}{MRS^{2,SB}_{c,d}}}\right]$$

is greater or lower than 1. In the case where $\pi(e)$ takes a log form, we rewrite the above expression as

$$\Lambda = \left(1 + \frac{u'\left(d^2\right)d^2/u\left(d^2\right)}{2\pi\left(e^*\left(\gamma^2, d^2\right)\right)}\right) \times \left[\frac{1-\frac{\mu}{n^2}}{1-\frac{\mu}{n^2}\frac{\pi\left(e^*\left(\gamma^1, d^2\right)\right)}{\pi\left(e^*\left(\gamma^2, d^2\right)\right)}}\right]$$

where Λ is monotone and continuous in $\pi\left(e^*\left(\gamma^1, d^2\right)\right)$ and $\pi\left(e^*\left(\gamma^2, d^2\right)\right)$. Assuming successively different values for $\pi\left(e^*\left(\gamma^1, d^2\right)\right)$ and $\pi\left(e^*\left(\gamma^2, d^2\right)\right)$, we prove that $D > 1$ always. Looking at the following figure,



the set of feasible survival probabilities is represented by the area E and includes segments A,C, D, since $\pi\left(e^*\left(\gamma^1, d^2\right)\right) \leq \pi\left(e^*\left(\gamma^2, d^2\right)\right)$. We study the sign of distortion D in each of these cases.

- Let first begin by segment A. In this case, $\pi\left(e^*\left(\gamma^1, d^2\right)\right) = \pi\left(e^*\left(\gamma^2, d^2\right)\right)$ (this is the case where γ^1 and γ^2 are very close) so that

$$\Lambda = \left(1 + \frac{u'\left(d^2\right) d^2 / u\left(d^2\right)}{2\pi\left(e^*\left(\gamma^2, d^2\right)\right)}\right) > 1$$

- Let then study segment C which also includes point $B = (0,1)$. In this case, $\pi\left(e^*\left(\gamma^1, d^2\right)\right) \to 0$ so that

$$\Lambda \to \left(1 + \frac{u'\left(d^2\right) d^2 / u\left(d^2\right)}{2\pi\left(e^*\left(\gamma^2, d^2\right)\right)}\right) \times \left[1 - \frac{\mu}{n^2}\right]$$

A priori, one cannot find whether Λ is greater or smaller than 1. Using the incentive constraint and the fact that $\pi\left(e^*\left(\gamma^1, d^2\right)\right) \to 0$, one necessarily have that $u\left(c^1\right) \geq u\left(c^2\right)$. Using (15) and (16), this is also the case that

$$u'\left(c^1\right) = \frac{\lambda}{1 + \frac{\mu}{n^1}} \leq u'\left(c^2\right) = \frac{\lambda}{1 - \frac{\mu}{n^2}}$$

Assume that the incentive constraint is not binding so that $c^1 > c^2$ and $\mu = 0$. In this case, the above condition implies that $c^1 = c^2$ which is a contradiction. Thus, the only possible solution is that the incentive constraint is binding so that $c^1 = c^2$ and $\mu \geq 0$. Again, due to the above condition, the only possible solution is $\mu = 0$. Thus the distortion is such that

$$\Lambda \to \left(1 + \frac{u'\left(d^2\right) d^2 / u\left(d^2\right)}{2\pi\left(e^*\left(\gamma^2, d^2\right)\right)}\right) > 1$$

- Let now study the distortion on segment D. In this case, $\pi\left(e^*\left(\gamma^2, d^2\right)\right) = 1$ so that

$$\Lambda = \left(1 + \frac{u'\left(d^2\right) d^2 / u\left(d^2\right)}{2}\right) \times \left[\frac{1 - \frac{\mu}{n^2}}{1 - \frac{\mu}{n^2}\pi\left(e^*\left(\gamma^1, d^2\right)\right)}\right]$$

Starting from point B, which also belongs to segment D, it is straightforward to see that increasing $\pi\left(e^*\left(\gamma^1, d^2\right)\right)$ increases the level of the distortion. In the extreme case where $\pi\left(e^*\left(\gamma^2,d^2\right)\right)=\pi\left(e^*\left(\gamma^1,d^2\right)\right)=1$, the distortion is also greater than one (this point belongs to segment A). Thus, starting from a point where the distortion is greater than 1, the distortion increases monotonically and continuously until point $(1,1)$ where the distortion is also greater than 1. Thus, along segment D, $\Lambda>1$.

- We finally study the level of the distortion in area E. In this case, both survival probabilities are lower than 1 so that

$$\Lambda=\left(1+\frac{u'\left(d^2\right)d^2/u\left(d^2\right)}{2\pi\left(e^*\left(\gamma^2,d^2\right)\right)}\right)\times\left[\frac{1-\frac{\mu}{n^2}}{1-\frac{\mu}{n^2}\frac{\pi(e^*(\gamma^1,d^2))}{\pi(e^*(\gamma^2,d^2))}}\right]$$

  Starting from any point on segment D, we find that decreasing $\pi\left(e^*\left(\gamma^2,d^2\right)\right)$ always increases Λ monotonically and continuously until segment A where the distortion is simply equal to $\left(1+u'\left(d^2\right)d^2/2u\left(d^2\right)\pi\left(e^*\left(\gamma^2,d^2\right)\right)\right)$. Thus, starting from a point where the distortion is greater than 1, the level of the distortion increases until any point of segment A where the distortion is greater than 1. Thus any point in area E implies that $\Lambda>1$. (Similar conclusion is obtained starting from segment A and increasing $\pi\left(e^*\left(\gamma^2,d^2\right)\right)$ until segment D)

Thus, we proved that for any possible levels of $\pi\left(e^*\left(\gamma^1,d^2\right)\right)$ and $\pi\left(e^*\left(\gamma^2,d^2\right)\right)$, the second best distortion of a type 2 individual is always greater than 1.