

Revolving Doors: a Channel of Influence Peddling*

Seung Han Yoo[†]

March 1, 2008

Abstract

A significant number of high-level government officials are employed as executives, consultants, lobbyists or members of advisory boards after retirement. The majority of these officials previously worked in departments involved in government regulations or procurement. This situation leads to problems of influence peddling. This paper attempts to develop a framework for a theoretical analysis of these issues and to examine implications of related economic policy. The main results of this paper are the following: (1) unlike the arguments in the canonical statistical discrimination models, in a one-stage game, discrimination is not always a coordination problem, and (2) in an infinitely repeated game, government officials and an employer can collude so that each official's expected payoff strictly increases, and post-government employment restrictions may not be an effective policy.

Keywords and Phrases: corruption, revolving doors, statistical discrimination, repeated games

JEL Classification Numbers: D73, D82, J71, C73

*I would like to thank Mukul Majumdar for valuable guidance and encouragement. I am also grateful to Talia Bar, Kaushik Basu and Ani Guerdjikova for helpful comments and suggestions. Thanks are due to seminar participants at The Royal Economic Society Third PhD Presentation Meeting at UCL, Cornell, Hanyang University, KAIST, KIPF, KISDI and KDI for helpful comments. Of course, all remaining errors are mine.

[†]Department of Economics, Cornell University, Ithaca, NY 14853, USA
(e-mail: sy239@cornell.edu).

1 Introduction

Influence peddling appears as a common theme in the literature on corruption defined roughly as the use of public office for private gains. It typically involves elected public officials or senior administrators and private firms or citizens. This paper deals with the interaction of public administrators and private firms: the focus is the possibility of improving future employment prospects and generating enhanced remuneration of public servants after mandatory or early retirement. Anecdotal evidence and laws on “revolving doors” or “cooling off periods” suggest that a move from the public to the private sector has not been viewed just as an example of “mobility” or the typical transfer of labor from one employer to another.¹ At risk is the performance of public servants’ duties to interpret and enforce the rule of the state or laws to protect the legitimate rights of citizens.

A significant number of *high-level* government officials are employed as executives, consultants, lobbyists or members of advisory boards after retirement.² The majority of these officials previously worked in departments involved in government regulations or procurement. They are offered salaries much higher than those the government pays and are working for companies that they had *once* regulated or attempted to procure services and products from for government projects.

One reason they are so highly compensated is the expertise gained from their experience in government. Another reason is that firms wish to capitalize on their

¹In the U.S., a 1962 act (18 U.S.C. 207(a)) provided for a one-year cooling off period (Gely and Zardkoohi (2001)). Most countries have similar post-government employment restrictions. According to a survey by Brezis and Weiss (1997), Canada uses a period of 1.5 or 2 years, the U.K. 2 years, France 5 years, Japan 2 years and Israel 1 year.

²Almost 51% of 142 ex-commissioners took related private-sector jobs (Eckert (1981)). Adams (1982) shows that 1,455 former military and 186 civilian employees of the Department of Defense were hired by eight major defense companies during the period 1970-1979, and 31 former employees of NASA were hired by these companies during the period 1974-1979. According to the *New York Times* (June 18, 2006), among the highest-level executives of the Department of Homeland Security in its beginning years, over two-thirds have moved through the revolving door.

connection and accessibility. Experience in government is not the only attribute valued in executives, consultants, lobbyists or members of advisory boards. Other attributes such as technical expertise and know-how gained from working as managers in relevant industries are also considered important. However, if incumbent government officials favor companies that employ former government officials when applying regulations or making procurement decisions, private firms will hire increasing number of government officials or establish business contracts with consulting or lobbying firms with former officials.

This paper shows that government officials can secure greater employment opportunities and higher salaries than others by employing the above tactic. In addition, they make distorted or unjust decisions on regulations or procurements by favoring companies that have connections with former officials.

In this game, we assume that two workers, one a former regulator and one not a former regulator, compete to be employed by a firm being regulated by an incumbent regulator.³ In an infinitely repeated game, each worker is identified by the firm based not on his or her name but on his or her former position. Thus, incumbent government officials care about their *group reputation* because if the firm believes that employing retired regulators is more profitable than employing workers from other positions, the future expected earnings of such officials after retirement will be greater.⁴

First, we examine a one-stage game. The model is closely related with statistical discrimination theory that was first introduced by Arrow (1973) and Phelps (1972) and further extended by Coate and Loury (1993). In the literature on statistical discrimination, the term “discrimination” is used to denote discrimination *against* a certain group whereas in this paper, discrimination occurs *in favor of* a

³Under many circumstances, this is more realistic. Moreover, most conflicts between different groups occur in this type of environment.

⁴Hence, our framework is distinct from the studies on social norms that use evolutionary game theory and from the group reputation effect studied by Tirole (1996).

certain group. In a canonical statistical discrimination model, the essential element of discrimination is employers' preconceptions about workers' qualifications. The existence of multiple equilibria, for example, an equilibrium with a preconception that a group is qualified and an equilibrium with a preconception that a group is unqualified, is derived from the relationship between an employer and a group of workers (without considering the other group). Then, discrimination arises from *self-fulfilling* equilibria; the equilibrium between the employer and one group is caught in the former, and the other equilibrium in the latter. Since both can get caught in the equilibrium where the employer believes that a group is qualified, discrimination is always an allocation that is not Pareto optimal. In contrast, we show that the set of equilibrium strategies has a property demonstrating *conflicts* between two different groups. As a result, under a certain condition, discrimination is an allocation that is Pareto optimal.⁵

Second, in an infinitely repeated game, government officials and an employer can collude so that each official's expected payoff strictly increases. In addition, post-government employment restrictions may not be an effective policy. For the collusion, the regulators manipulate decisions on regulations for their own benefit. This has two important social welfare implications; (1) the firm can further exploit loopholes, which deteriorates social welfare and (2) such regulations make non-regulators less qualified in a society. Two groups, regulators and non-regulators, are competing to be employed, so the less qualified non-regulators the higher the payoff of regulators.

This paper is related to two bodies of literature: papers on statistical discrimination and papers on regulatory capture. We use the same information structure as in Coate and Loury (1993). However, there is no such competition between different groups in the canonical statistical discrimination models (see Arrow (1973), Phelps (1972) and Coate and Loury (1993) among others). Moreover, these models do not

⁵Moro and Norman (2004) also show that with a general equilibrium model in which there are a continuum of workers and two firms, discrimination may not be a coordination problem.

consider a repeated game.

The papers on regulatory capture (see Che (1995), Salant (1995), Brezis and Weiss (1997) and Martimort (1999) among others) mostly deal with decisions on the rate of regulation.⁶ This paper includes both officials' endogenous human capital investment decisions and their decisions on the rate of regulation. In addition, the majority of their models are based on the principal-agent model whereas this paper is closer to signaling games.

This paper is organized as follows. In section 2, we consider a one-stage game where we show that if the value of a project is sufficiently large, there exist multiple non-trivial equilibria, and under a certain condition, discrimination is not a coordination problem. In section 3, we consider an infinitely repeated game and show that given any best equilibrium for officials in the one-stage game, there exists an equilibrium in the repeated game such that the expected payoff of each official strictly increases and post-government employment restrictions may not be an effective policy. Some concluding remarks are provided in section 4, and proofs are collected in an appendix.

2 A One-stage Game

A one-stage game consists of two periods. In each period, there are three players: a worker in a regulatory position in government; a worker in a non-regulatory position in government or in private sector; and a firm being regulated by the worker in the regulatory position (regulator).⁷ Each worker retires after one period, but the firm is the same. Given a period $t = 1, 2$, denote by R_t the worker in the regulatory position in period t , N_t the worker in the non-regulatory position or in private sector in period t , and F the firm.

⁶A notable exception is Che (1995). He provides both approaches in two separate models.

⁷In the model, we focus on a case in which a firm is regulated by the government. However, with a slight change in interpretation, this can also be applied to a case where a firm is making a procurement contract with the government.

In every period, the players make the following decisions. Each worker decides whether or not to make a human capital investment in being qualified, which is denoted by $q_i = 1$ or 0 for each worker i . Only each individual worker knows the cost of the investment c_i , and the other players know that c_i is drawn independently and identically from an absolutely continuous probability distribution function F over $[\underline{c}, \bar{c}]$ where $0 < \underline{c} < \bar{c}$. Hence, every *type* of each player $i \in \{R_t, N_t\}$ for $t = 1, 2$ corresponds to c_i . The firm can make a gain $V > 0$ by exploiting *loopholes* if it employs a qualified worker in each period. If the firm recruits an unqualified one, no attempt to exploit the loopholes is possible. The regulator also makes a decision on the rate of aggressiveness $a \in [0, 1]$ in regulating the firm if the firm exploits loopholes. The rate of regulation a is interpreted as the probability that the firm is caught by the regulator.

After the workers retire, they apply to the firm for a single position. Their qualifications are not directly observable by the firm, so the firm requires them to take a test such as an interview. Each retired worker's test result θ_i is given by a twice continuously differentiable probability distribution function $G(\theta_i|q_i)$ over $[\underline{\theta}, \bar{\theta}]$ where $\underline{\theta} < \bar{\theta}$ for each $q_i = 0, 1$. Let its density $g(\theta_i|q_i) > 0$ for all θ and define $\phi : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}_{++}$ by $\phi(\theta_i) \equiv g(\theta_i|0)/g(\theta_i|1)$. We assume that⁸

$$\phi(\theta_i) \text{ is strictly decreasing.} \tag{1}$$

Hence, a higher θ_i is more likely for $q_i = 1$. The firm obtains V *only* if it hires a qualified retired worker and an incumbent in the regulatory position is not aggressive. The firm gets 0 if it hires an unqualified worker.

Since our main interest is the difference in the expected wages between the two groups, not the wage decision process between the firm and each worker, we assume a simple wage decision process; given the test results, the firm chooses one of the workers and suggests $\lambda \in (0, 1)$ of the total expected gain as his or her wage.⁹

⁸Note that this implies that $G(\theta_i|0) > G(\theta_i|1)$ for all θ_i .

⁹Of course, we can find a bargaining process between the firm and a worker chosen by the firm to generate this rule.

Period 2 is the last period, so the human capital investment is not relevant for the workers in the second period. In the first period, since there is no retired worker, the firm's employment decision is irrelevant in period 1 as is the rate of regulation by the first period regulator. Hence, in the one-stage game, we focus on the human capital investment decisions of the first period's workers, the rate of regulation by the second period's regulator, and the employment by the firm after period 1. This sequence can be seen in the following time line:

Step 1. Nature determines c_i for each $i \in \{R_1, N_1\}$.

Step 2. Each worker decides whether to make an investment in being qualified.

Step 3. After they retire, workers take the test for possible recruitments.

Step 4. Given the test results, the firm decides whom to employ.

Step 5. If the firm employs a qualified worker, it exploits loopholes. After R_2 observes the employee of the firm, R_2 decides how aggressive he is in regulating the firm.

Now, we list the players' actions:

$q_i = 1$ or 0 according to whether each worker i does or does not make the investment,

$h = 1$ or 0 according to whether the firm hires R_1 or N_1 ,

$a \in [0, 1]$ is the rate of regulation.

Let $\Omega \equiv [\underline{\theta}, \bar{\theta}]^2$ and a vector $\theta \equiv (\theta_{R_1}, \theta_{N_1})$. The strategy of each worker in period 1 is a mapping $Q_i : [\underline{c}, \bar{c}] \rightarrow \{0, 1\}$, the strategy of the firm is a mapping $H : \Omega \rightarrow \{R_1, N_1\}$, and the strategy of the regulator in period 2 is a mapping $A : \{R_1, N_1\} \rightarrow [0, 1]$. The payoff to each of the first period workers, $i \in \{R_1, N_1\}$, is given by

$$u_i \equiv \delta\omega - c_i q_i,$$

where $\delta \in (0, 1)$ is the common discount factor, and ω is the wage. The payoff to the firm is

$$u_F \equiv \begin{cases} Vq_j - \omega & \text{if } F \text{ is not caught} \\ -\omega & \text{if } F \text{ is caught,} \end{cases}$$

where Vq_j is the benefit from hiring retired worker j . Let the payoff to R_2 be

$$u_{R_2} \equiv d(a),$$

where $d : [0, 1] \rightarrow \mathbb{R}_+$ is a continuous and strictly concave function and is assumed to satisfy $d(0) = d(1)$ and there exists $a' \in (0, 1)$ such that $d(a') > d(0)$. d captures the trade-off between expected penalties for not being aggressive in regulating the firm and personal costs from being aggressive. Hence, a unit increase in the rate of regulation has both marginal benefit and cost. Every player is risk-neutral and has *perfect recall*.

Let $\varphi(c_i|\theta_i)$ be the firm's posterior assessment of retired worker i 's type after observing θ_i . This assessment is a conditional probability density function updated by Bayes's rule given the value θ_i . For each $i \in \{R_1, N_1\}$, define a set

$$\mathcal{A}(q_i, Q_i) \equiv \{c_i \in [\underline{c}, \bar{c}] \mid Q_i(c_i) = q_i\}.$$

$\mathcal{A}(q_i, Q_i)$ is a set of the investment costs for which i makes an investment when $q_i = 1$ and does not make an investment when $q_i = 0$. Note that

$$\begin{aligned} & \int_{[\underline{c}, \bar{c}]} \sum_{q_i=0}^1 g(\theta_i|q_i) \mathbf{1}_{\mathcal{A}(q_i, Q_i)} f(c_i) dc_i \\ &= \int_{[\underline{c}, \bar{c}]} (g(\theta_i|q_i = 0) (1 - \mathbf{1}_{\mathcal{A}(q_i=1, Q_i)}) + g(\theta_i|q_i = 1) \mathbf{1}_{\mathcal{A}(q_i=1, Q_i)}) f(c_i) dc_i \\ &= \int_{[\underline{c}, \bar{c}]} (g(\theta_i|q_i = 0) + [g(\theta_i|q_i = 1) - g(\theta_i|q_i = 0)] \mathbf{1}_{\mathcal{A}(q_i=1, Q_i)}) f(c_i) dc_i. \end{aligned}$$

Since for all c_i ,

$$g(\theta_i|q_i = 0) + [g(\theta_i|q_i = 1) - g(\theta_i|q_i = 0)] \mathbf{1}_{\mathcal{A}(q_i=1, Q_i)} > 0,$$

φ can be written by

$$\begin{aligned} \varphi(c_i|\theta_i) &= \frac{\sum_{q_i=0}^1 g(\theta_i|q_i) \mathbf{1}_{\mathcal{A}(q_i, Q_i)} f(c_i)}{\int_{[\underline{c}, \bar{c}]} \sum_{q_i=0}^1 g(\theta_i|q_i) \mathbf{1}_{\mathcal{A}(q_i, Q_i)} f(c_i) dc_i} \\ &= \frac{\sum_{q_i=0}^1 g(\theta_i|q_i) \mathbf{1}_{\mathcal{A}(q_i, Q_i)} f(c_i)}{\sum_{q_i=0}^1 g(\theta_i|q_i) \int_{\mathcal{A}(q_i, Q_i)} f(c_i) dc_i}. \end{aligned}$$

Given θ_i and the strategy Q_i , the probability that worker i is qualified, μ , is

$$\mu(\theta_i, Q_i) \equiv \int_{[\underline{c}, \bar{c}]} Q_i(c_i) \varphi(c_i | \theta_i) dc_i = \frac{g(\theta_i | q_i = 1) \int_{\mathcal{A}(q_i=1, Q_i)} f(c_i) dc_i}{\sum_{q_i=0}^1 g(\theta_i | q_i) \int_{\mathcal{A}(q_i, Q_i)} f(c_i) dc_i}.$$

Perfect Bayesian equilibrium is adopted as the solution concept for the one-stage game. Denote $a^* = \arg \max_{a \in [0,1]} d(a)$. Then, by sequential rationality, the equilibrium strategy A^* can be derived as $A^*(R_1) = A^*(N_1) = a^*$. Thus, $Q_{R_1}^*$, $Q_{N_1}^*$ and H^* with the belief φ is a perfect Bayesian equilibrium if for every $c_{R_1} \in [\underline{c}, \bar{c}]$ of R_1 ,

$$Q_{R_1}^*(c_{R_1}) = \arg \max_{q_{R_1} \in \{0,1\}} \lambda \delta (1 - a^*) V \cdot E[\mu(\theta_{R_1}, Q_{R_1}^*) H^*(\theta) | q_{R_1}] - c_{R_1} q_{R_1},$$

for every $c_{N_1} \in [\underline{c}, \bar{c}]$ of N_1 ,

$$Q_{N_1}^*(c_{N_1}) = \arg \max_{q_{N_1} \in \{0,1\}} \lambda \delta (1 - a^*) V \cdot E[\mu(\theta_{N_1}, Q_{N_1}^*) (1 - H^*(\theta)) | q_{N_1}] - c_{N_1} q_{N_1},$$

and for each $\theta \in \Omega$,

$$\begin{aligned} H^*(\theta) = \arg \max_{h \in \{0,1\}} & (1 - \lambda) (1 - a^*) V \cdot \mu(\theta_{R_1}, Q_{R_1}^*) h \\ & + (1 - \lambda) (1 - a^*) V \cdot \mu(\theta_{N_1}, Q_{N_1}^*) (1 - h). \end{aligned}$$

Given the payoff to each $i \in \{R_1, N_1\}$, the following lemma can be shown.

Lemma 1 *In an equilibrium, the strategy Q_i^* must satisfy that for each $i \in \{R_1, N_1\}$, there exists a cutoff point $k_i \in [\underline{c}, \bar{c}]$ such that*

$$Q_i^* = \begin{cases} 1 & \text{if } c_i \leq k_i \\ 0 & \text{otherwise.} \end{cases}$$

Then, for each $i \in \{R_1, N_1\}$, we can denote

$$\int_{\mathcal{A}(q_i=1, Q_i)} f(c_i) dc_i = F(k_i) \quad \text{and} \quad \int_{\mathcal{A}(q_i=0, Q_i)} f(c_i) dc_i = 1 - F(k_i).$$

It follows that a function $\mu : [\underline{\theta}, \bar{\theta}] \times [\underline{c}, \bar{c}] \rightarrow [0, 1]$ can be rewritten by

$$\mu(\theta_i, k_i) \equiv \begin{cases} 1 / (1 + \phi(\theta_i) \pi(k_i)) & \text{if } k_i \in (\underline{c}, \bar{c}] \\ 0 & \text{if } k_i = \underline{c} \end{cases}, \quad (2)$$

where $\pi : (\underline{c}, \bar{c}] \rightarrow \mathbb{R}_+$ is defined by

$$\pi(k_i) \equiv \frac{1 - F(k_i)}{F(k_i)}.$$

Given each $\theta \in \Omega$, the firm chooses the one whose signal indicates a higher probability of being qualified than the other; that is, it picks a worker i if $\mu(\theta_i, k_i) > \mu(\theta_j, k_j)$.

Define a function $P : \{0, 1\} \times [\underline{c}, \bar{c}]^2 \rightarrow [0, 1]$ by

$$\begin{aligned} & P(k_i, k_j) \\ \equiv & F(k_j) \int_{\Omega} \zeta(\theta, k_i, k_j) dG(\theta_i | q_i) \times G(\theta_j | 1) \\ & + (1 - F(k_j)) \int_{\Omega} \zeta(\theta, k_i, k_j) dG(\theta_i | q_i) \times G(\theta_j | 0), \end{aligned} \tag{3}$$

where $\zeta : \Omega \times [\underline{c}, \bar{c}]^2 \rightarrow [0, 1]$ is

$$\zeta(\theta, k_i, k_j) \equiv \begin{cases} \mu(\theta_i, k_i) & \text{if } \mu(\theta_i, k_i) > \mu(\theta_j, k_j) \\ \mu(\theta_i, k_i) / 2 & \text{if } \mu(\theta_i, k_i) = \mu(\theta_j, k_j) \\ 0 & \text{if } \mu(\theta_i, k_i) < \mu(\theta_j, k_j). \end{cases}$$

ζ is the *ex post* return from qualifications contingent on the signal θ_i and given the firm's belief about the cutoff points of retired workers, (k_i, k_j) , which is *ex ante* a random variable. If worker i 's qualifications are better than those of worker j , i will be rewarded based on his qualifications; if worker i 's qualifications are worse than worker j 's, i will get nothing; and if their qualifications happen to be the same, the firm will choose one of them with an equal probability. Denote $W \equiv \lambda\delta(1 - a^*)V$. Then, $W \cdot P(1, k_i, k_j)$ is the expected return if worker i makes a human capital investment, and $W \cdot P(0, k_i, k_j)$ is the expected return if not. Hence, given c_i , worker i 's payoff is $W \cdot P(1, k_i, k_j) - c_i$ if i makes an investment and $W \cdot P(0, k_i, k_j)$ if not. Denote the increase in P from the investment by ΔP .

$$\Delta P(k_i, k_j) \equiv P(1, k_i, k_j) - P(0, k_i, k_j).$$

$W\Delta P$ is the *marginal* expected return on a human capital investment of worker i given the firm's belief about (k_i, k_j) . As a result, worker i makes an investment if

$$c_i \leq W\Delta P(k_i, k_j).$$

As a function of k_i and k_j , ΔP has the following properties.

Lemma 2 (i) $\Delta P(\underline{c}, k_j) = 0$ for all $k_j \in [\underline{c}, \bar{c}]$.

(ii) $\Delta P(\bar{c}, k_j) = 0$ for all $k_j \in [\underline{c}, \bar{c}]$.

(iii) $\Delta P(k_i, k_j)$ is a continuous function of k_i on $[\underline{c}, \bar{c}]$ given a fixed $k_j \in [\underline{c}, \bar{c}]$.

(iv) $\Delta P(k, k)$ is a continuous function of k on $[\underline{c}, \bar{c}]$.

Define a function $\Delta B : [\underline{c}, \bar{c}]^2 \rightarrow [\underline{c}, \bar{c}]$ by

$$\Delta B(k_i, k_j, W) = \begin{cases} \underline{c} & \text{if } W\Delta P(k_i, k_j) \leq \underline{c} \\ W\Delta P(k_i, k_j) & \text{if } W\Delta P(k_i, k_j) \in [\underline{c}, \bar{c}] \\ \bar{c} & \text{if } W\Delta P(k_i, k_j) \geq \bar{c}. \end{cases} \quad (4)$$

ΔB is the cutoff point for worker i given the firm's belief about the cutoff points of retired workers, (k_i, k_j) . Thus, a perfect Bayesian equilibrium is the same as a combination $(k_{R_1}^*, k_{N_1}^*)$ such that for all $i \in \{R_1, N_1\}$,

$$\Delta B(k_i^*, k_j^*, W) = k_i^*.$$

Before proceeding further, we introduce types of equilibria. If $(k_i^*, k_j^*) = (\underline{c}, \underline{c})$, we call it a trivial equilibrium. A symmetric equilibrium is an equilibrium with the property $k_i^* = k_j^*$, and an asymmetric equilibrium is an equilibrium with $k_i^* \neq k_j^*$. It follows from Lemma 2 that there always exists a trivial equilibrium. We establish a sufficient condition for which multiple asymmetric equilibria exist.

Lemma 3 *If there exists $k \in (\underline{c}, \bar{c})$ such that $W\Delta P(k, \underline{c}) > k$, there exist two sets of asymmetric equilibria, (i) (k', \underline{c}) , (\underline{c}, k') with $k' \in (\underline{c}, k)$ and (ii) (k'', \underline{c}) , (\underline{c}, k'') with $k'' \in (k, \bar{c})$.*

If there exists $k \in (\underline{c}, \bar{c})$ such that $W\Delta P(k, \underline{c}) > k$, we have an asymmetric equilibrium in which no type of worker i is qualified and types of worker j with a measure between 0 and 1 are qualified. Since their prior conditions are symmetric, this implies that a *reversed* one is also an asymmetric equilibrium. Now, we establish a sufficient condition for which a non-trivial symmetric equilibrium exists.

Lemma 4 *If there exists $k \in (\underline{c}, \bar{c})$ such that $W\Delta P(k, k) > k$, there exists a non-trivial symmetric equilibrium, (k', k') with $k' \in (\underline{c}, k)$.*

$\phi'(\theta_i) < 0$ in (1) implies that $G(\theta_i|1) < G(\theta_i|0)$ for all θ_i . Using Theorem 1 in Hadar and Russell (1971), a result on first order stochastic dominance, the following Lemma can be obtained.

Lemma 5 *For each $k \in (\underline{c}, \bar{c})$, (i) $\Delta P(k, \underline{c}) > 0$ and (ii) $\Delta P(k, k) > 0$.*

Now, we are ready to show the existence of multiple asymmetric equilibria and a non-trivial symmetric equilibrium.

Proposition 1 *If V is sufficiently large, there exist multiple asymmetric equilibria and a non-trivial symmetric equilibrium.*

We can have self-fulfilling equilibria like those in Arrow (1973) and Coate and Loury (1993), but their natures are different. In this model, not only do an employer's expectations about each group's qualifications matter, but so do one group's expectations about the other group's qualifications.

In addition, an employer's belief about a potential employee's productivity in terms of the probability that V is realized from hiring the employee consists of two parts: (i) expectation about the *qualifications* of a retired worker from a certain group, and (ii) expectation about the *reaction* of an incumbent regulator to a retired official who is currently working for the employer. If an incumbent regulator has a discriminatory taste for retired regulators, then it can be easily seen that even if the firm does not have any taste for that particular group, its belief about a potential employee's productivity will be influenced by the incumbent regulator's attitude towards who is working for the firm. However, without such a taste on the part of the current regulator, in a one-stage game, (ii) does not have any role in the analysis. In the next section, we examine a repeated game, where (ii) becomes relevant.

Our interest is whether in this model, discrimination in a one-stage game can also be considered a coordination problem. We first show that if k'_i and k_i are two

equilibrium cutoffs for i such that $k'_i > k_i$, given a fixed k_j , each type of worker i is better off. In addition, if k'_j and k_j are two equilibrium cutoffs for j such that $k'_j > k_j$, given a fixed k_i , each type of worker i is worse off.

Proposition 2 (i) *If (k'_i, k_j) and (k_i, k_j) are two equilibria with $k'_i > k_i$, each type of worker i is better off.*

(ii) *If (k_i, k'_j) and (k_i, k_j) are two equilibria with $k'_j > k_j$, each type of worker i is worse off.*

This implies that as evaluated at the interim stage, if given an asymmetric equilibrium (k, k') with $k' > k$, there exists a symmetric equilibrium (k'', k'') such that $k'' > k'$, discrimination *may* be treated as a coordination failure. Otherwise, one of them must be worse off; if $k'' \leq k$, by Proposition 2, the worker with k is worse off, and if $k < k'' \leq k'$, the worker with k' is worse off.

The main result of this section establishes that the one-stage game involving a human capital investment decision of agents from different groups is not always a coordination problem.

Proposition 3 *If $(1 - [F(k)G(\theta_i|1) + (1 - F(k))G(\theta_i|0)])\mu(\theta_i, k)$ is an increasing function of θ_i at k , the symmetric equilibrium (k, k) implies a set of asymmetric equilibria (k', \underline{c}) and (\underline{c}, k') where $k' \in [k, \bar{c})$.*

If (k, k) is the maximum symmetric equilibrium, resolving a coordination failure cannot be a solution to discrimination since there is no way to make one worker better off without making the other worse off. In the canonical statistical discrimination models, discrimination is always an allocation that is not Pareto optimal whereas in this model, discrimination can be an allocation that is Pareto optimal. Thus, it is reasonable to consider the possibility that a group of workers becomes dominant not because employers have certain prior beliefs about workers' qualifications but because they are playing certain strategies in order to become dominant and to maintain their dominant status.

3 A Repeated Game

Consider an infinitely repeated game in which there is a sequence of workers from each group $(R_t, N_t)_{t=1,2,\dots}$, and in each period $t \in \mathbb{N}$, R_t , N_t and a single firm F play the one-period game in the one-stage game described in the previous section. $(R_t)_{t=1,2,\dots}$ are *ex ante* identical, and $(N_t)_{t=1,2,\dots}$ are also *ex ante* identical. The subscript t of any variable or function in this section indicates that it is a variable or function of period t . For the infinitely repeated game, it is assumed that for each $i \in \{R_t, N_t\}$, c_{it} are drawn independently and identically from a differentiable probability distribution F , and θ_τ , a_τ and h_τ are publicly observable to all players in every period $t > \tau$. Let $\delta \in (0, 1)$ be the common discount factor. Finally, in what follows, we shall simply focus on a case where $V_t = V$ for all t and V is sufficiently large so that there is at least a non-trivial equilibrium in the one-stage game.

Note that difficulties in analysis arise since the cutoff point k_i cannot be used as an equilibrium strategy in a repeated game. If a worker does not qualify, that may be because the worker's cost of investment is greater than a cutoff point or because the worker defects from an equilibrium cutoff in a repeated game. Hence, the firm does not know whether a defection has taken place or not. However, the strategies using θ_t , a_t and h_t are implementable.

The payoff of R_t given $i \in \{R_{t-1}, N_{t-1}\}$ can be written as

$$\begin{aligned} u(k_{R_t}, A_t, k_{N_t}, A_{t+1}) &\equiv F(k_{R_t}) \lambda \delta (1 - A_{t+1}(R_t)) V \cdot P(1, k_{R_t}, k_{N_t}) \\ &\quad + (1 - F(k_{R_t})) \lambda \delta (1 - A_{t+1}(R_t)) V \cdot P(0, k_{R_t}, k_{N_t}) \\ &\quad - \int_{\underline{c}}^{k_{R_t}} c_R dF(c_R) + d(A_t(i)). \end{aligned} \quad (5)$$

The first two terms are the expected benefit from the post-government employment opportunity in the next period, the third term is the cost of a human capital investment in the present period and the last is the payoff from the rate of regulation. Denote by \mathcal{K} a set of the one-stage equilibrium decisions of two workers, each of whom comes from a different group. Let

$$(k_R^*, k_N^*) \equiv \arg \max_{(k_R, k_N) \in \mathcal{K}} u(k_R, A^*, k_N, A^*),$$

where $A^*(i) = a^*$ for all i . Then, (k_R^*, k_N^*) maximizes the *ex ante* one-stage equilibrium payoff of R . Let us call any set $\hat{\mathcal{B}} \subseteq \Omega$ and a function $\hat{\mu}$ a *collusive hiring rule* if given some (k_R, k_N) , the set of the test results for which the firm chooses a worker i is fixed as $\hat{\mathcal{B}}$ and the probability that a worker i is qualified is $\hat{\mu}_i(\theta_i)$. Given \mathcal{F} , a set of continuously differentiable functions from $[\underline{\theta}, \bar{\theta}]$ to $[0, 1]$, define a function $\hat{P} : \{0, 1\} \times \mathcal{F} \times \{\mathcal{B} \subseteq \Omega\} \times [\underline{c}, \bar{c}] \rightarrow [0, 1]$ by

$$\begin{aligned} \hat{P}(q_i, \hat{\mu}_i, \hat{\mathcal{B}}, k_j) &\equiv F(k_j) \int_{\hat{\mathcal{B}}} \hat{\mu}_i(\theta_i) dG(\theta_i|q_i) \times dG(\theta_j|1) \\ &\quad + (1 - F(k_j)) \int_{\hat{\mathcal{B}}} \hat{\mu}_i(\theta_i) dG(\theta_i|q_i) \times dG(\theta_j|0). \end{aligned} \quad (6)$$

Then, in analogy with (4) and (5), we can have functions $\Delta \hat{B}$ and \hat{u} such that $\Delta \hat{B} : \mathcal{F} \times \{\mathcal{B} \subseteq \Omega\} \times [\underline{c}, \bar{c}] \rightarrow [\underline{c}, \bar{c}]$ is given by

$$\begin{aligned} &\Delta \hat{B}(\hat{\mu}_i, \hat{\mathcal{B}}, k_j, \delta(1 - A(i))V) \\ &= \begin{cases} \underline{c} & \text{if } \lambda\delta(1 - A(i))V\Delta \hat{P}(\hat{\mu}_i, \hat{\mathcal{B}}, k_j) \leq \underline{c} \\ \lambda\delta(1 - A(i))V\Delta \hat{P}(\hat{\mu}_i, \hat{\mathcal{B}}, k_j) & \text{if } \lambda\delta(1 - A(i))V\Delta \hat{P}(\hat{\mu}_i, \hat{\mathcal{B}}, k_j) \in [\underline{c}, \bar{c}] \\ \bar{c} & \text{if } \lambda\delta(1 - A(i))V\Delta \hat{P}(\hat{\mu}_i, \hat{\mathcal{B}}, k_j) \geq \bar{c}, \end{cases} \end{aligned}$$

where

$$\Delta \hat{P}(\hat{\mu}_i, \hat{\mathcal{B}}, k_j) \equiv \hat{P}(1, \hat{\mu}_i, \hat{\mathcal{B}}, k_j) - \hat{P}(0, \hat{\mu}_i, \hat{\mathcal{B}}, k_j),$$

and $\hat{u} : \mathcal{F} \times \{\mathcal{B} \subseteq \Omega\} \times [\underline{c}, \bar{c}]^2 \times [0, 1] \rightarrow \mathbb{R}_+$ is

$$\begin{aligned} \hat{u}(\hat{\mu}_R, \hat{\mathcal{B}}, k_R, k_N, A) &\equiv F(k_R) \lambda\delta(1 - A(R))V \cdot \hat{P}(1, \hat{\mu}_R, \hat{\mathcal{B}}, k_N) \\ &\quad + (1 - F(k_R)) \lambda\delta(1 - A(R))V \cdot \hat{P}(0, \hat{\mu}_R, \hat{\mathcal{B}}, k_N) \\ &\quad - \int_{\underline{c}}^{k_R} c_R dF(c_R) + d(A(i)). \end{aligned}$$

We show that there exist a collusive hiring rule, A' , and $(k'_R, k'_N) \in [\underline{c}, \bar{c}]^2$, such that player R 's payoff from (k'_R, k'_N) is greater than $u(k_R^*, A^*, k_N^*, A^*)$.

Proposition 4 *There exist a collusive hiring rule $(\hat{\mathcal{B}}, \hat{\mu}_R, \hat{\mu}_N)$, A' and $(k'_R, k'_N) \in [\underline{c}, \bar{c}]^2$ such that*

$$\hat{u}(\hat{\mu}_R, \hat{\mathcal{B}}, k'_R, k'_N, A') > u(k_R^*, A^*, k_N^*, A^*).$$

The main step of the proof of Proposition 4 is to let $A'(k_R) < a^* < A'(k_N)$ such that $d(A'(k_R)) = d(A'(k_N))$ given the collusive hiring rule $\hat{B} = B(k_R^*, k_N^*)$, $\hat{\mu}_R(\theta_R) = \mu(\theta_R, k_R^*)$ and $\hat{\mu}_N(\theta_N) = \mu(\theta_N, k_N^*)$. As a result, we can make (k'_R, k'_N) satisfy $k'_R > k_R^*$ and $k'_N \leq k_N^*$.

Once we show that there exists (k'_R, k'_N) satisfying Proposition 4, the existence of a Bayes-Nash equilibrium that supports it in the repeated game can be shown given the following *grim* strategy. Under collusion between $(R_t)_{t=1,2,\dots}$ and F , in each period t , an R_t makes an investment if the cost of the investment is at most as large as k'_R and chooses $A'(R)$ and $A'(N)$, which are from Proposition 4. A N_t makes an investment if the cost of the investment is at most as large as k'_N . F employs the collusive hiring rule. Define a defection of R_t as enforcing $a > A'(R)$, and a defection of F as adopting a hiring rule for R_t such as $\mathcal{B} \subset \hat{\mathcal{B}}$, $\mu_R \neq \hat{\mu}_R$ or $\mu_N \neq \hat{\mu}_N$. If and when $(R_t)_{t=1,2,\dots}$ and F learn that a defection has taken place, then no type of R_t makes a human capital investment, every R_t will play a^* and F will play a hiring rule for R_t such as $\mathcal{B} = \phi$ for infinitely many periods. If there exists A' satisfying Proposition 4 such that the firm's payoff on the equilibrium path in the repeated game is greater than its payoff in the one-stage game with a^* , there exists a Bayes-Nash equilibrium of the infinitely repeated game given that δ is sufficiently close to 1.

Proposition 5 *If δ is sufficiently close to 1, there exists a Bayes-Nash equilibrium of the infinitely repeated game.*

Hence, on the equilibrium path, the regulators are regulating less aggressively if the firm employs a former regulator and more aggressively if not. The following corollary establishes that the post-government employment restriction that bans retired regulators from working in the regulated industry for $n \geq 2$ periods may not be an effective policy.

Corollary 1 *If given $n \geq 2$, $\delta \in (0, 1)$ and $A'(R) \in (0, 1)$, there exists $A''(R)$ such*

that

$$\delta^n (1 - A''(R)) \geq \delta (1 - A'(R)),$$

the post-government employment restriction does not decrease the regulator's cutoff.

The longer the restricted periods, the less aggressive the regulators will be in regulating the firm that employs a former regulator. This result implies that what matters may be not the restrictions on post-government employment opportunities but how much a government can keep regulators from manipulating the rate of regulation for their own benefit. In this model, this boils down to how to design the function d and its domain, which decide the benefit and cost of choosing a and the related “policy space.”

4 Concluding Remarks

We show that it is possible that discrimination is not an outcome of self-fulfilling equilibria but an outcome of collusion in a repeated interaction between regulators and an employer. That is, the reason that regulators are more qualified is that the higher expected payoff of each of them is guaranteed through cooperation in a repeated interaction between them and a firm that is being regulated.

The correct diagnosis of the cause of discrimination is necessary for correct policy analysis. Hence, it is of interest for subsequent work to assess the welfare implications of post-government employment restrictions.

Appendix: Proofs

Proof of Lemma 1. Suppose not. There exist $c'_i, c''_i \in [\underline{c}, \bar{c}]$ such that

$$(c'_i - c''_i) (Q_i^*(c'_i) - Q_i^*(c''_i)) < 0.$$

Without loss of generality, let $c'_i < c''_i$ and $Q_i^*(c'_i) > Q_i^*(c''_i)$. Then, for $i = R_1$, the following relations are satisfied.

$$\begin{aligned}\lambda\delta V(1-a^*)E[\mu(\theta_{R_1}, Q_{R_1}^*)H^*(\theta)|1] - c'_i &\geq \lambda\delta V(1-a^*)E[\mu(\theta_{R_1}, Q_{R_1}^*)H^*(\theta)|0], \\ \lambda\delta V(1-a^*)E[\mu(\theta_{R_1}, Q_{R_1}^*)H^*(\theta)|1] - c''_i &\leq \lambda\delta V(1-a^*)E[\mu(\theta_{R_1}, Q_{R_1}^*)H^*(\theta)|0].\end{aligned}$$

It follows that $c''_{R_1} \geq c'_{R_1}$, which contradicts $c'_{R_1} < c''_{R_1}$. Similarly, for $i = N_1$, we have a contradiction. ■

Proof of Lemma 2. Denote

$$\mathcal{B}(k_i, k_j) \equiv \{\theta \in \Omega \mid \mu(\theta_i, k_i) > \mu(\theta_j, k_j)\}. \quad (7)$$

$B(k_i, k_j)$ is the set of the test results for which the firm chooses a worker i .

(i) By (2), $\mu(\theta_i, \underline{c}) = 0$ for all θ_i . It follows from (3) that for each q_i , $P(q_i, \underline{c}, k_j) = 0$ for all $k_j \in [\underline{c}, \bar{c}]$.

(ii) By (2), $\mu(\theta_i, \bar{c}) = 1$ for all θ_i . It follows from (7) that $B(\bar{c}, k_j) = \Omega$ for all $k_j \in [\underline{c}, \bar{c}]$. (3) implies that for each q_i , $P(q_i, \bar{c}, k_j) = 1$ for all $k_j \in [\underline{c}, \bar{c}]$. In addition, for each q_i , $P(q_i, \bar{c}, \bar{c}) = \lambda$. Hence, $\Delta P(\bar{c}, k_j) = 0$ for all $k_j \in [\underline{c}, \bar{c}]$.

(iii) Case 1: $k_j = \underline{c}$. From (2), $\mu(\theta_j, \underline{c}) = 0$. By (7), $B(k_i, \underline{c}) = \Omega$ for all $k_i \in (\underline{c}, \bar{c}]$. It follows from (3) that for each q_i ,

$$P(q_i, k_i, \underline{c}) = \int_{[\underline{\theta}, \bar{\theta}]} \mu(\theta_i, k_i) dG(\theta_i|q_i). \quad (8)$$

Fix θ_i . Since μ is a continuous function of k_i , given $\varepsilon > 0$, there exists $\delta > 0$ such that if $|k'_i - k_i| < \delta$, then $|\mu(\theta_i, k'_i) - \mu(\theta_i, k_i)| < \varepsilon$. This in turn implies that

$$\begin{aligned}&\left| \int_{[\underline{\theta}, \bar{\theta}]} \mu(\theta_i, k'_i) dG(\theta_i|q_i) - \int_{[\underline{\theta}, \bar{\theta}]} \mu(\theta_i, k_i) dG(\theta_i|q_i) \right| \\ &\leq \int_{[\underline{\theta}, \bar{\theta}]} |\mu(\theta_i, k'_i) - \mu(\theta_i, k_i)| dG(\theta_i|q_i) < \varepsilon.\end{aligned}$$

Then, $P(q_i, k_i, \underline{c})$ is a continuous function of k_i on $(\underline{c}, \bar{c}]$. In addition, the continuity of μ implies that

$$\lim_{k_i \rightarrow \underline{c}} P(q_i, k_i, \underline{c}) = P(q_i, \underline{c}, \underline{c}) = 0.$$

Case 2: $k_j \in (\underline{c}, \bar{c})$. Since ϕ is strictly decreasing, for $k_i \in (\underline{c}, \bar{c})$, (7) can be written by

$$\mathcal{B}(k_i, k_j) = \left\{ \theta \in \Omega \mid \phi^{-1} \left(\phi(\theta_i) \frac{\pi(k_i)}{\pi(k_j)} \right) > \theta_j \right\}.$$

It follows from (3) that for each q_i ,

$$\begin{aligned} P(q_i, k_i, k_j) &= F(k_j) \int_{\mathcal{B}(k_i, k_j)} \mu(\theta_i, k_i) dG(\theta_i | q_i) \times G(\theta_j | 1) \\ &\quad + (1 - F(k_j)) \int_{\mathcal{B}(k_i, k_j)} \mu(\theta_i, k_i) dG(\theta_i | q_i) \times G(\theta_j | 0). \end{aligned} \quad (9)$$

Then,

$$\begin{aligned} &\int_{\mathcal{B}(k_i, k_j)} \mu(\theta_i, k_i) dG(\theta_i | q_i) \times G(\theta_j | q_j) \\ &= \int_{[\underline{\theta}, \bar{\theta}]} \mu(\theta_i, k_i) G \left(\phi^{-1} \left(\phi(\theta_i) \frac{\pi(k_i)}{\pi(k_j)} \right) \middle| q_j \right) dG(\theta_i | q_i). \end{aligned} \quad (10)$$

Using the same argument as above, it can be shown that $P(q_i, k_i, k_j)$ is a continuous function of k_i on (\underline{c}, \bar{c}) . In addition, the continuity of μ , π , G and $B(\bar{c}, k_j) = \Omega$ for $k_j \in (\underline{c}, \bar{c})$ implies that

$$\begin{aligned} \lim_{k_i \rightarrow \underline{c}} P(q_i, k_i, k_j) &= P(q_i, \underline{c}, k_j) = 0, \\ \lim_{k_i \rightarrow \bar{c}} P(q_i, k_i, k_j) &= P(q_i, \bar{c}, k_j) = 1. \end{aligned}$$

This shows that $\Delta P(k_i, k_j)$ is a continuous function of k_i on $[\underline{c}, \bar{c}]$ for all $k_j \in [\underline{c}, \bar{c}]$.

(iv) Let $k_i = k_j = k$. By (7), $B(k, k) = \{\theta \in \Omega \mid \theta_i > \theta_j\}$ for all $k \in (\underline{c}, \bar{c})$. It follows from (3) that for each q_i ,

$$\begin{aligned} P(q_i, k, k) &= F(k) \int_{\theta_i > \theta_j} \mu(\theta_i, k) dG(\theta_i | q_i) \times G(\theta_j | 1) \\ &\quad + (1 - F(k)) \int_{\theta_i > \theta_j} \mu(\theta_i, k) dG(\theta_i | q_i) \times G(\theta_j | 0). \end{aligned} \quad (11)$$

Since F is continuous, by the same argument as (iii), it can be shown that $\Delta P(k, k)$ is a continuous function of k on $[\underline{c}, \bar{c}]$. ■

Proof of Lemma 3.

(i) $\Delta P(\underline{c}, \underline{c}) = 0$ and $\underline{c} > 0$, so $W\Delta P(\underline{c}, \underline{c}) < \underline{c}$. Since by Lemma 2 (iii), $\Delta P(k_i, \underline{c})$ is a continuous function of k_i on $[\underline{c}, \bar{c}]$, the Intermediate Value Theorem entails that there

exists $k' \in (\underline{c}, k)$ such that $W\Delta P(k', \underline{c}) = k'$. This implies that $W\Delta P(k', \underline{c}) \in (\underline{c}, \bar{c})$, so $\Delta B(k', \underline{c}, W) = k'$. On the other hand, from Lemma 2 (i), $\Delta P(\underline{c}, k') = 0$, we have $\Delta B(\underline{c}, k', W) = \underline{c}$. This shows that there exist asymmetric equilibria, (k', \underline{c}) , (\underline{c}, k') with $k' \in (\underline{c}, k)$.

(ii) By Lemma 2 (ii), $\Delta P(\bar{c}, \underline{c}) = 0$ and $\underline{c} > 0$, so $W\Delta P(\bar{c}, \underline{c}) < \underline{c}$. Using the same argument as (i) above, it can be shown that there exist asymmetric equilibria, (k'', \underline{c}) , (\underline{c}, k'') with $k'' \in (k, \bar{c})$. ■

Proof of Lemma 4. $\Delta P(\underline{c}, \underline{c}) = 0$ and $\underline{c} > 0$, so $W\Delta P(\underline{c}, \underline{c}) < \underline{c}$. Since by Lemma 2 (iv), $\Delta P(k, k)$ is a continuous function of k on $[\underline{c}, \bar{c})$, the Intermediate Value Theorem entails that there exists $k' \in (\underline{c}, k)$ such that $W\Delta P(k', k') = k'$. This implies that $W\Delta P(k', k') \in (\underline{c}, \bar{c})$, so $\Delta B(k', k', W) = k'$. This shows that there exist a non-trivial symmetric equilibrium, (k', k') with $k' \in (\underline{c}, k)$. ■

Proof of Lemma 5.

(i) Since $\phi(\theta_i)$ is strictly decreasing and twice differentiable, for any $k_i \in (\underline{c}, \bar{c})$, $\partial\mu(\theta_i, k_i)/\partial\theta_i$ exists, continuous and

$$\frac{\partial\mu(\theta_i, k_i)}{\partial\theta_i} = -\frac{\phi'(\theta_i)\pi(k_i)}{(1 + \phi(\theta_i)\pi(k_i))^2} > 0 \text{ on } [\underline{\theta}, \bar{\theta}].$$

Clearly, μ is bounded. It follows from (8) that

$$\Delta P(k, \underline{c}) = \int_{[\underline{\theta}, \bar{\theta}]} \mu(\theta_i, k_i) dG(\theta_i|1) - \int_{[\underline{\theta}, \bar{\theta}]} \mu(\theta_i, k_i) dG(\theta_i|0). \quad (12)$$

$\phi'(\theta_i) < 0$ implies that $G(\theta_i|1) < G(\theta_i|0)$ for all θ_i . By Theorem 1 in Hadar and Russell (1971), $\Delta P(k, \underline{c}) > 0$.

(ii) It follows from (11) that

$$\begin{aligned} \Delta P(k, k) &= \int_{[\underline{\theta}, \bar{\theta}]} [F(k)G(\theta_i|1) + (1 - F(k))G(\theta_i|0)] \mu(\theta_i, k) dG(\theta_i|1) \\ &\quad - \int_{[\underline{\theta}, \bar{\theta}]} [F(k)G(\theta_i|1) + (1 - F(k))G(\theta_i|0)] \mu(\theta_i, k) dG(\theta_i|0). \end{aligned} \quad (13)$$

$[F(k)G(\theta_i|1) + (1 - F(k))G(\theta_i|0)] \mu(\theta_i, k)$ is a strictly increasing function of θ_i , and its first derivative exists and is continuous. In addition, it is bounded. Since $G(\theta_i|1) < G(\theta_i|0)$ for all θ_i , using Theorem 1 in Hadar and Russell (1971), $\Delta P(k, k) > 0$. ■

Proof of Proposition 1. Since by Lemma 5, for each $k \in (\underline{c}, \bar{c})$, (i) $\Delta P(k, \underline{c}) > 0$ and (ii) $\Delta P(k, k) > 0$. Let V be large enough that there exists $k' \in (\underline{c}, \bar{c})$ such that

$$\lambda \delta (1 - a^*) V > \frac{k'}{\Delta P(k', \underline{c})}.$$

It follows from Lemma 3 that there exist multiple asymmetric equilibria. Similarly, let V be large enough that there exists $k' \in (\underline{c}, \bar{c})$ such that

$$\lambda \delta (1 - a^*) V > \frac{k'}{\Delta P(k', k')}.$$

It follows from Lemma 4 that there exists a non-trivial symmetric equilibrium. ■

Proof of Proposition 2. First, note that by Lemma 2 (ii), (\bar{c}, k_j) or (k_i, \bar{c}) cannot be an equilibrium.

(i) If $k_j = \underline{c}$, since μ is a strictly increasing of k_i on $[\underline{c}, \bar{c}]$, it is satisfied. If $k_j \in (\underline{c}, \bar{c})$, it follows from (10) that for $k_i \in (\underline{c}, \bar{c})$,

$$\partial G \left(\phi^{-1} \left(\phi(\theta_i) \frac{\pi(k_i)}{\pi(k_j)} \right) \middle| q_j \right) / \partial k_i = g \left(\phi^{-1} \left(\phi(\theta_i) \frac{\pi(k_i)}{\pi(k_j)} \right) \middle| q_j \right) \frac{\phi(\theta_i) \pi'(k_i)}{\phi' \pi(k_j)} > 0.$$

From Lemma 2 (iii), $P(q_i, k_i, k_j)$ is a continuous function of k_i on $[\underline{c}, \bar{c}]$ for all $k_j \in [\underline{c}, \bar{c}]$. Since μ is a strictly increasing function of k_i on $[\underline{c}, \bar{c}]$, from (9), for each q_i , P is a strictly increasing function of k_i on $[\underline{c}, \bar{c}]$. Hence, for $c_i \leq k_i$, $P(1, k'_i, k_j) - c_i > P(1, k_i, k_j) - c_i$; for $c_i \in (k_i, k'_i]$, $P(1, k'_i, k_j) - c_i > P(0, k'_i, k_j) > P(0, k_i, k_j)$; and for $c_i > k'_i$, $P(0, k'_i, k_j) > P(0, k_i, k_j)$.

(ii) Note that for $k_i, k_j \in (\underline{c}, \bar{c})$,

$$\partial G \left(\phi^{-1} \left(\phi(\theta_i) \frac{\pi(k_i)}{\pi(k_j)} \right) \middle| q_j \right) / \partial k_j = g \left(\phi^{-1} \left(\phi(\theta_i) \frac{\pi(k_i)}{\pi(k_j)} \right) \middle| q_j \right) \frac{\phi(\theta_i) - \pi(k_i) \pi'(k_j)}{\phi' \pi(k_j)^2} < 0.$$

Moreover,

$$\begin{aligned} & \int_{\mathcal{B}(k_i, k_j)} \mu(\theta_i, k_i) dG(\theta_i | q_i) \times G(\theta_j | 1) - \int_{\mathcal{B}(k_i, k_j)} \mu(\theta_i, k_i) dG(\theta_i | q_i) \times G(\theta_j | 0) \\ &= \int_{[\underline{\theta}, \bar{\theta}]} \mu(\theta_i, k_i) G \left(\phi^{-1} \left(\phi(\theta_i) \frac{\pi(k_i)}{\pi(k_j)} \right) \middle| 1 \right) dG(\theta_i | q_i) \\ & \quad - \int_{[\underline{\theta}, \bar{\theta}]} \mu(\theta_i, k_i) G \left(\phi^{-1} \left(\phi(\theta_i) \frac{\pi(k_i)}{\pi(k_j)} \right) \middle| 0 \right) dG(\theta_i | q_i) \\ &< 0. \end{aligned}$$

It follows from (9) that for each q_i , P is a strictly decreasing function of k_j on $[\underline{c}, \bar{c}]$. Hence, for $c_i \leq k_i$, $P(1, k_i, k'_j) - c_i < P(1, k_i, k_j) - c_i$ and for $c_i > k_i$, $P(0, k_i, k'_j) < P(0, k_i, k_j)$. ■

Proof of Proposition 3. From (12) and (13),

$$\begin{aligned} & \Delta P(k, \underline{c}) - \Delta P(k, k) \\ &= \int_{[\underline{\theta}, \bar{\theta}]} (1 - [F(k)G(\theta_i|1) + (1 - F(k))G(\theta_i|0)]) \mu(\theta_i, k) dG(\theta_i|1) \\ & \quad - \int_{[\underline{\theta}, \bar{\theta}]} (1 - [F(k)G(\theta_i|1) + (1 - F(k))G(\theta_i|0)]) \mu(\theta_i, k) dG(\theta_i|0). \end{aligned}$$

If $(1 - [F(k)G(\theta_i|1) + (1 - F(k))G(\theta_i|0)]) \mu(\theta_i, k)$ is an increasing function of θ_i at k , since $G(\theta_i|1)$ first order stochastically dominates $G(\theta_i|0)$, we have

$$\Delta P(k, \underline{c}) - \Delta P(k, k) \geq 0.$$

If (k, k) is a symmetric equilibrium, it is satisfied that

$$W\Delta P(k, k) = k.$$

Hence,

$$W\Delta P(k, \underline{c}) \geq k.$$

If $W\Delta P(k, \underline{c}) = k$, we are done. If $W\Delta P(k, \underline{c}) > k$, by Lemma 3, there exist asymmetric equilibria, (k', \underline{c}) , (\underline{c}, k') with $k' \in (k, \bar{c})$. ■

Proof of Proposition 4. Let $\hat{B} = B(k_R^*, k_N^*)$, $\hat{\mu}_R(\theta_R) = \mu(\theta_R, k_R^*)$ and $\hat{\mu}_N(\theta_N) = \mu(\theta_N, k_N^*)$. Then, given $i = R$,

$$\frac{\partial \hat{u}(\hat{\mu}_R, \hat{B}, k_R^*, k_N^*, A^*)}{\partial A(R)} = - \left[\begin{array}{c} F(k_R) \lambda \delta V \cdot \hat{P}(1, \hat{\mu}_R, \hat{B}, k_N) \\ + (1 - F(k_R)) \lambda \delta V \cdot \hat{P}(0, \hat{\mu}_R, \hat{B}, k_N) \end{array} \right] + d'(a^*) < 0$$

and

$$\frac{\partial \hat{u}(\hat{\mu}_R, \hat{B}, k_R^*, k_N^*, A^*)}{\partial k_R} = f(k_R^*) \left(\lambda \delta (1 - a^*) V \Delta \hat{P}(\hat{\mu}_R, \hat{B}, k_N^*) - k_R^* \right) = 0.$$

Hence, given any $A(R) < a^*$,

$$\frac{\partial \hat{u}(\hat{\mu}_R, \hat{\mathcal{B}}, k_R, k_N^*, A)}{\partial k_R} > 0 \text{ for } k_R \in [k_R^*, \lambda \delta (1 - A(R)) V \Delta \hat{P}(\hat{\mu}_R, \hat{\mathcal{B}}, k_N^*)].$$

It follows from the proof of Proposition 2 (ii) that \hat{u} is a strictly decreasing function of k_N on (\underline{c}, \bar{c}) .

For $(\hat{\mathcal{B}}, \hat{\mu}_R, \hat{\mu}_N)$, the following relations are satisfied.

$$\begin{aligned} \lambda \delta (1 - a^*) V \Delta \hat{P}(\hat{\mu}_R, \hat{\mathcal{B}}, k_N^*) &= k_R^*, \\ \lambda \delta (1 - a^*) V \Delta \hat{P}(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, k_R^*) &= k_N^*. \end{aligned} \tag{14}$$

$\Delta P(\hat{\mu}_i, \mathcal{B}, \bar{k}_j)$ can be rewritten by

$$\Delta P(\hat{\mu}_i, \hat{\mathcal{B}}, \bar{k}_j) = F(\bar{k}_j) \Delta \Phi^1(\hat{\mu}_i, \hat{\mathcal{B}}) + (1 - F(\bar{k}_j)) \Delta \Phi^0(\hat{\mu}_i, \hat{\mathcal{B}}),$$

where

$$\begin{aligned} \Delta \Phi^1(\hat{\mu}_i, \hat{\mathcal{B}}) &\equiv \int_{\hat{\mathcal{B}}} \hat{\mu}_i(\theta_i) dG(\theta_i|1) \times G(\theta_j|1) \\ &\quad - \int_{\hat{\mathcal{B}}} \hat{\mu}_i(\theta_i) dG(\theta_i|0) \times G(\theta_j|1); \\ \Delta \Phi^0(\hat{\mu}_i, \hat{\mathcal{B}}) &\equiv \int_{\hat{\mathcal{B}}} \hat{\mu}_i(\theta_i) dG(\theta_i|1) \times G(\theta_j|0) \\ &\quad - \int_{\hat{\mathcal{B}}} \hat{\mu}_i(\theta_i) dG(\theta_i|0) \times G(\theta_j|0). \end{aligned}$$

We do not know whether $\Delta P(\hat{\mu}_R, \hat{\mathcal{B}}, k_N)$ is strictly increasing in k_N , decreasing or neither, and it is the same with $\Delta P(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, k_R)$.

Part 1. $k_N^* = 0$. By simply choosing $A'(R) < a^*$, we can have $k'_R > k_R^*$, which in turn implies that $\hat{u}(\hat{\mu}_R, \hat{\mathcal{B}}, k'_R, k_N^*, A') > u(k_R^*, A^*, k_N^*, A^*)$. Since $k_N^* = 0$, no type of N is qualified and thus none is hired by the firm. Hence, $A(N)$ is not relevant.

Part 2. $k_N^* \neq 0$. We divide this into three cases.

Case 1: $\Delta \Phi^1(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}) = \Delta \Phi^0(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}})$. Since $\Delta P(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, k_R)$ does not depend on k_R , let $A'(R) < a^*$ and $A'(N)$ satisfy $A'(N) > a^*$ and $d(A'(N)) = d(A'(R))$. Then, we have $k'_R \in (k_R^*, \bar{c}]$ and $k'_N \in [\underline{c}, k_N^*)$. It follows that $\hat{u}(\hat{\mu}_R, \hat{\mathcal{B}}, k'_R, k_N^*, A') > u(k_R^*, A^*, k_N^*, A^*)$.

Case 2: $\Delta\Phi^1(\hat{\mu}_N, \Omega/\hat{\mathcal{B}}) < \Delta\Phi^0(\hat{\mu}_N, \Omega/\hat{\mathcal{B}})$. $\Delta P(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, k_R)$ is a strictly decreasing function of k_R . It follows from choosing $A'(R) < a^*$ and $A'(N)$ satisfy $A'(N) > a^*$ and $d(A'(N)) = d(A'(R))$ that

$$\begin{aligned}\lambda\delta(1 - A'(R))V\Delta\hat{P}(\hat{\mu}_R, \hat{\mathcal{B}}, k_N^*) &> k_R^*, \\ \lambda\delta(1 - a^*)V\Delta\hat{P}(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, k_R^*) &< k_N^*.\end{aligned}$$

Hence,

$$\lambda\delta(1 - A'(R))V\Delta\hat{P}(\hat{\mu}_R, \hat{\mathcal{B}}, \lambda\delta(1 - a^*)V\Delta\hat{P}(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, k_R^*)) > k_R^*.$$

Note that $\Delta\hat{B}(\hat{\mu}_R, \hat{\mathcal{B}}, k_N, \delta(1 - A(R))V) \leq \bar{c}$ for all k_N . By the Intermediate Value Theorem, there exists $k'_R \in (k_R^*, \bar{c}]$ such that

$$\lambda\delta(1 - A'(R))V\Delta P(\hat{\mu}_R, \hat{\mathcal{B}}, \lambda\delta(1 - A'(N))V\Delta P(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, k'_R)) = k'_R.$$

This is equivalent to saying that there exist $k'_R \in (k_R^*, \bar{c}]$ and $k'_N \in [\underline{c}, k_N^*)$ such that

$$\begin{aligned}\lambda\delta(1 - A'(R))V\Delta P(\hat{\mu}_R, \hat{\mathcal{B}}, k'_N) &= k'_R, \\ \lambda\delta(1 - A'(N))V\Delta P(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, k'_R) &= k'_N.\end{aligned}$$

Hence, $\hat{u}(\hat{\mu}_R, \hat{\mathcal{B}}, k'_R, k'_N, A') > u(k_R^*, A^*, k_N^*, A^*)$.

Case 3: $\Delta\Phi^1(\hat{\mu}_N, \Omega/\hat{\mathcal{B}}) > \Delta\Phi^0(\hat{\mu}_N, \Omega/\hat{\mathcal{B}})$. $\Delta P(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, k_R)$ is a strictly increasing function of k_R . Given each $k_R \in (k_R^*, \lambda\delta V\Delta P(\hat{\mu}_R, \hat{\mathcal{B}}, k_N^*)]$, there exists $A''(N) \in (a^*, \bar{c})$ such that

$$\lambda\delta(1 - A''(N))V\Delta P(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, k_R) = k_N^*.$$

Let $A'(N)$ satisfy $\lambda\delta(1 - A'(N))V\Delta P(\hat{\mu}_N, \Omega \setminus \hat{\mathcal{B}}, \lambda\delta V\Delta P(\hat{\mu}_R, \hat{\mathcal{B}}, k_N^*)) = k_N^*$ and $A'(R) < a^*$ and $d(A'(R)) = d(A'(N))$. Then, k_N is fixed as k_N^* , and $k'_R \in (k_R^*, \bar{c}]$ such that

$$\lambda\delta(1 - A'(R))V\Delta P(\hat{\mu}_R, \hat{\mathcal{B}}, k_N^*) = k'_R.$$

Thus, $\hat{u}(\hat{\mu}_R, \hat{\mathcal{B}}, k'_R, k_N^*, A') > u(k_R^*, A^*, k_N^*, A^*)$. Therefore, this establishes the result.

■

Proof of Proposition 5. It is sufficient to show that there exists A' such that the firm's payoff on the equilibrium path in the repeated game is greater than its payoff in the one-stage game equilibrium (\underline{c}, k_N'') . Given the collusive hiring rule, choose $A'(N)$ such that $A'(N)$ satisfies Proposition 4 and

$$E \left[\{ (1 - A'(N)) V \cdot \mu(\theta_N, k_N'') - \lambda (1 - A'(N)) V \cdot \hat{\mu}(\theta_N) \} \mathbf{1}_{\Omega/\mathcal{B}(k_R^*, k_N^*)} \right] \geq 0.$$

Then, we have

$$\begin{aligned} & E \left[\{ (1 - A'(R)) V \cdot \mu(\theta_R, k_R') - \lambda (1 - A'(R)) V \cdot \hat{\mu}(\theta_R) \} \mathbf{1}_{\hat{\mathcal{B}}} \right] \\ & + E \left[\{ (1 - A'(N)) V \cdot \mu(\theta_N, k_N'') - \lambda (1 - A'(N)) V \cdot \hat{\mu}(\theta_N) \} \mathbf{1}_{\Omega/\hat{\mathcal{B}}} \right] \\ \geq & E \left[\{ (1 - A'(R)) V \cdot \mu(\theta_R, k_R') - \lambda (1 - A'(R)) V \cdot \hat{\mu}(\theta_R) \} \mathbf{1}_{\hat{\mathcal{B}}} \right] \\ = & E \left[\{ (1 - A'(R)) V \cdot \mu(\theta_R, k_R') - \lambda (1 - A'(R)) V \cdot \mu(\theta_R, k_R^*) \} \mathbf{1}_{\mathcal{B}(k_R^*, k_N^*)} \right] \\ > & E \left[\{ (1 - A^*(R)) V \cdot \mu(\theta_R, k_R^*) - \lambda (1 - A^*(R)) V \cdot \mu(\theta_R, k_R^*) \} \mathbf{1}_{\mathcal{B}(k_R^*, k_N^*)} \right]. \end{aligned}$$

The last strict inequality follows since μ is a strictly increasing function of k_R and $A'(R) < A^*(R)$. Now, we claim that $k_R'' \leq k_R^*$ for any one-stage game equilibrium (k_R'', \underline{c}) . Suppose $k_R'' > k_R^*$. Then, by Proposition 2, $u(k_R'', A^*, \underline{c}, A^*) > u(k_R^*, A^*, \underline{c}, A^*) > u(k_R^*, A^*, k_N^*, A^*)$, a contradiction with the definition of (k_R^*, k_N^*) . It follows that

$$\begin{aligned} & E \left[\{ (1 - A^*(R)) V \cdot \mu(\theta_R, k_R^*) - \lambda (1 - A^*(R)) V \cdot \mu(\theta_R, k_R^*) \} \mathbf{1}_{\mathcal{B}(k_R^*, k_N^*)} \right] \\ \geq & E \left[\{ (1 - A^*(R)) V \cdot \mu(\theta_R, k_R^*) - \lambda (1 - A^*(R)) V \cdot \mu(\theta_R, k_R^*) \} \mathbf{1}_{\mathcal{B}(k_R^*, k_N^*)} \right] \\ & - \frac{1}{\delta} \left(\frac{1 - \lambda}{\lambda} \right) \int_{\underline{c}}^{k_R^*} c_R dF(c_R) + \frac{1}{\delta} \left(\frac{1 - \lambda}{\lambda} \right) \int_{\underline{c}}^{k_R''} c_R dF(c_R) \\ \geq & E \left[(1 - A^*(R)) V \cdot \mu(\theta_R, k_R'') - \lambda (1 - A^*(R)) V \cdot \mu(\theta_R, k_R'') \right]. \end{aligned}$$

The last inequality follows from the definition of (k_R^*, k_N^*) .

$$\begin{aligned} & E \left[\{ (1 - \lambda) (1 - A^*(R)) V \cdot \mu(\theta_R, k_R^*) \} \mathbf{1}_{\mathcal{B}(k_R^*, k_N^*)} \right] - \frac{1}{\delta} \left(\frac{1 - \lambda}{\lambda} \right) \int_{\underline{c}}^{k_R^*} c_R dF(c_R) \\ \geq & E \left[(1 - \lambda) (1 - A^*(R)) V \cdot \mu(\theta_R, k_R'') \right] - \frac{1}{\delta} \left(\frac{1 - \lambda}{\lambda} \right) \int_{\underline{c}}^{k_R''} c_R dF(c_R) \end{aligned}$$

Thus,

$$\begin{aligned} & E \left[\{ \delta \lambda (1 - A^*(R)) V \cdot \mu(\theta_R, k_R^*) \} \mathbf{1}_{\mathcal{B}(k_R^*, k_N^*)} \right] - \int_{\underline{c}}^{k_R^*} c_R dF(c_R) \\ & \geq E \left[\delta \lambda (1 - A^*(R)) V \cdot \mu(\theta_R, k_R'') \right] - \int_{\underline{c}}^{k_R''} c_R dF(c_R). \end{aligned}$$

This shows that for any one-stage equilibrium (k_R'', \underline{c}) ,

$$\begin{aligned} & E \left[\{ (1 - A'(R)) V \cdot \mu(\theta_R, k_R') - \lambda (1 - A'(R)) V \cdot \hat{\mu}(\theta_R) \} \mathbf{1}_{\hat{\mathcal{B}}} \right] \\ & + E \left[\{ (1 - A'(N)) V \cdot \mu(\theta_N, k_N') - \lambda (1 - A'(N)) V \cdot \hat{\mu}(\theta_N) \} \mathbf{1}_{\Omega/\hat{\mathcal{B}}} \right] \\ & > E \left[(1 - A^*(R)) V \cdot \mu(\theta_R, k_R'') - \lambda (1 - A^*(R)) V \cdot \mu(\theta_R, k_R'') \right]. \end{aligned}$$

By the symmetry, this implies that for any one-stage equilibrium (\underline{c}, k_N'') ,

$$\begin{aligned} & E \left[\{ (1 - A'(R)) V \cdot \mu(\theta_R, k_R') - \lambda (1 - A'(R)) V \cdot \hat{\mu}(\theta_R) \} \mathbf{1}_{\hat{\mathcal{B}}} \right] \\ & + E \left[\{ (1 - A'(N)) V \cdot \mu(\theta_N, k_N') - \lambda (1 - A'(N)) V \cdot \hat{\mu}(\theta_N) \} \mathbf{1}_{\Omega/\hat{\mathcal{B}}} \right] \\ & > E \left[(1 - A^*(N)) V \cdot \mu(\theta_N, k_N'') - \lambda (1 - A^*(N)) V \cdot \mu(\theta_N, k_N'') \right]. \end{aligned}$$

■

Proof of Corollary 1. The proof follows from (5) and Proposition 4, so it is omitted. ■

References

- Adams, G. (1982), *Politics of Defense Contracting*. New Brunswick, N.J.: Transaction Publishers.
- Arrow, K. J. (1973), The theory of discrimination, in Orley Ashenfelter and Albert Rees, eds., *Discrimination in Labor Markets*, Princeton, NJ: Princeton University Press, 3-33.
- Brezis, E. and Weiss, A. (1997), Conscientious regulation and post-regulatory employment restrictions, *European Journal of Political Economy* **13**, 517-536.
- Coate, S. and Loury, G. C. (1993), Will affirmative-action policies eliminate negative stereotypes?, *American Economic Review* **83**, 1220-1240.

- Che, Y. (1995), Revolving door and the optimal tolerance for agency collusion, *Rand Journal of Economics* **26**, 378-397.
- Eckert, R.D. (1981), The life cycle of regulatory commissioners, *Journal of Law and Economics* **24**, 113-120.
- Gely, R. and Zardkoochi, A. (2001), Measuring the effects of post-government-employment restrictions, *American Law and Economics Review* **3**, 288-301.
- Hadar, J. and Russell, W. (1971), Stochastic dominance and diversification, *Journal of Economic Theory* **3**, 288-305.
- Martimot, D. (1999), The life cycle of regulatory agencies: dynamic capture and transaction costs, *The Review of Economic Studies* **66**, 929-947.
- Moro, A. and Norman, P. (2004), A general equilibrium model of statistical discrimination, *Journal of Economic Theory* **114**, 1-30.
- Phelps, E. S. (1972), The statistical theory of racism and sexism, *American Economic Review* **62**, 659-661.
- Salant, D. (1995), Behind the revolving door: a new view of public utility regulation, *Rand Journal of Economics* **26**, 362-377.
- Tirole, J. (1996), A theory of collective reputation, *Review of Economic Studies* **63**, 1-22.