ASSOCIATION for PUBLIC ECONOMIC THEORY

Piracy, Entry Deterrence and Intellectual Property Rights (IPR) Protection

Sougata Poddar
National University of Singapore and Hanyang University

Abstract

In this paper, we address the issue of piracy or illegal copying or counterfeiting of the original product and Intellectual Property Right (IPR) protections. The original product developer makes costly investment to deter piracy in given a regime of IPR protection. In this environment, we first characterize completely the entry deterrence and entry accommodation equilibrium in the presence of a commercial pirate. We find that it is profitable for the original producer to accommodate the pirate when there is weak IPR protection, while deterring is profitable when the IPR protection is strong. However, we find there is a non-monotonic relationship between the optimal level of deterrence (chosen by the original producer) and the degree of IPR protection in the economy. The relationship between the rate of piracy and IPR protection is found to be monotonically decreasing whereas the relationship between the rate of piracy and the quality of the pirated product turns out to be non-monotonic. In our welfare analysis, we find that the total welfare of the society decreases as the degree of IPR protection increases.

Submitted: March 22, 2008.

Piracy, Entry Deterrence and Intellectual Property Rights (IPR) Protection

Yuanzhu Lu
China Economics and Management Academy
Central University of Finance and Economics
No 39 College South Road, Beijing
China, 100081
Email: yuanzhulu@cufe.edu.cn
Fax: (86) 10 6228 8376

Sougata Poddar*
Department of Economics
National University of Singapore
10 Kent Ridge Crescent
Singapore 119260
E-mail: ecssp@nus.edu.sg
Fax: (65) 6775 2646

(Preliminary and Incomplete Version)

March 2008

Abstract

In this paper, we address the issue of piracy or illegal copying or counterfeiting of the original product and Intellectual Property Right (IPR) protections. The original product developer makes costly investment to deter piracy in given a regime of IPR protection. In this environment, we first characterize completely the entry deterrence and entry accommodation equilibrium in the presence of a commercial pirate. We find that it is profitable for the original producer to accommodate the pirate when there is weak IPR protection, while deterring is profitable when the IPR protection is strong. However, we find there is a non-monotonic relationship between the optimal level of deterrence (chosen by the original producer) and the degree of IPR protection in the economy. The relationship between the rate of piracy and IPR protection is found to be monotonically decreasing whereas the relationship between the rate of piracy and the quality of the pirated product turns out to be non-monotonic. In our welfare analysis, we find that the total welfare of the society decreases as the degree of IPR protection increases.

Keywords: Piracy, Copyright violations, Raising rival's cost, Deterrence, Accommodation, Product quality

JEL Classifications: D23, D43, L13, L86

---

* Address for correspondence.

1. Introduction

The issue of copyright violations and intellectual property rights (IPR) protection is presently receiving a great deal of attention in various economic analyses. Copyright violations take place when there is piracy or illegal copying or counterfeiting of the original product. These products can be digital products (like software, music CDs, movie DVDs, video games etc.) or non-digital products i.e. regular items (like cloth, shoes, books, bags etc.).[1] In this paper, we provide an economic analysis on the implications of piracy or illegal copying of products (digital or non-digital) under a given regime of IPR protection in an economy. In particular, in our main model, there is an original product developer and a commercial pirate (i.e. who sells pirated goods for profits). The original product developer makes costly investment to stop or limit piracy under the given regime/environment of IPR protection. The basic assumption we use here is stopping piracy is a costly activity, but if such costly activity is actively undertaken, it raises the cost of piracy to the pirate. Thus, our approach to deter/limit piracy is different from the standard approach of monitoring the pirate by a central authority or the local government and impose a fine if caught. The monitoring approach is widely studied in the literature of digital piracy, in particular, software piracy (see Chen and Png (2003), Banerjee (2003, 2006)). In our framework, the local government *per se* is not monitoring illegal piracy, but there is a general anti-piracy law that exists in the economy, and this is what we define as IPR protections. The original product developer takes the level/degree of (IPR protections in the economy as given and then optimally invests to raise the cost of piracy of the pirate. IPR protections can be weak or strong and the original developer adjusts its deterrence level (hence the costly investment) accordingly in an optimal manner.

In this environment, we first characterize completely the entry deterrence and entry accommodation equilibrium. We find that it is profitable for the original producer to accommodate the pirate when there is weak IPR protection, while deterring is profitable when the IPR protection is strong. However, in the comparative statics analysis, we find that there is a non-monotonic relationship between the optimal level of

[1] Globally counterfeiting activities have risen to 5-7% of world trade, or about $200 billion to $300 billion in lost revenue, according to recent estimates for the European Union. (see Time Magazine 2001).

deterrence (chosen by the product developer) and the degree/strength of IPR protection in the economy. The relationship between the rate of piracy and IPR protection is found to be monotonically decreasing whereas the relationship between the rate of piracy and the quality of the pirated product turns out to be non-monotonic. In our welfare analysis, we find that the total welfare of the society decreases as the degree of IPR protection increases.

Our results also give an explanation to the varying rates of piracy across countries and regions.[2] There exists empirical studies (see Gopal and Sanders (1998, 2000), Husted (2000), Donald and Steel (2000), Holm (2003), Banerjee et. al. (2005), Fischer and Rodriguez (2005)) to explain the varying (software) piracy rates across countries and regions, but to the best of our knowledge no theoretical framework has been used so far to explain the same phenomenon. In our model, we find that the piracy rate depends on the consumers' willingness to pay for the product, the quality of the pirated product and strength of IPR protections that prevails in the economy. It is the interaction of these three parameters that define the rate of piracy of an economy. One of our interesting findings is, for a profitable piracy, the optimal strategy of the commercial pirate would be to produce a pirated version with moderate reliability. A commercial pirate will not be inclined to produce a version which is too low in quality or which is too close to the original product in terms of quality/reliability even if it has the means to do so.

The plan of the paper is as follows. In the next section, we provide the basic framework. In section 3, we completely analyze the entry accommodation and entry deterrence equilibrium. We do the comparative statics analysis in section 4. In section 5, we have the welfare analysis. In section 6, we extend our model to analyze end-user piracy; and finally, we conclude in section 7.

2. The Model of Commercial Piracy

2.1 The Original Firm and the Pirate

[2] For example, the software piracy rates across countries varies a great deal, it can be as high as more than 90% in countries like Vietnam, China and can be as low as 25% as in USA. All other countries have piracy rates in between these two extremes. (Source: See BSA and IDC Global Software 2007 for a detailed survey on piracy rates in different countries).

Consider an original firm and a pirate. The pirate has the know-how or the technology to copy/counterfeit the original product. We assume the pirate produces copies, which are of lower quality than the original. The product quality of the pirated good (compared to original) is captured by the parameter q, $q \in (0,1)$. In the case of digital product, although the pirated copies are almost like original, however, they do not come with any guarantee or supporting services, thus making them inferior compared to the original.

We consider a two-period model, where in the first period $(t=1)$, the original product developer undertakes costly investment in order to deter piracy. It adopts the following entry deterring strategy. It tries to deter the pirate by increasing the cost of copying, in particular, raising the marginal cost of producing a copy of the original. The potential pirate appears in the market of the original product in the second time period $(t=2)$. We assume the higher the entry deterring investment made by the original product developer in the first period, the higher would be the marginal cost of copying by the pirate, hence higher would be the deterrence level. The pirate if survives, competes with the original developer in price by possibly producing a lower quality, *albeit* a cheaper product.

2.2 Costs and Profits

We assume at $t=1$, the cost of investment of the original product developer to increase the marginal cost of piracy by an amount of x is given by $c_O(x) = \frac{x^2}{2}$. Let us call x as the level of deterrence.

Thus, if the profit of the product developer at $t=2$ is denoted by $\pi_O^2 = p_O D_O$,[3] where p_O is the price charged by the product developer and D_O is the demand it faces, then the net profit of the developer at $t=1$ becomes $\pi_O = \pi_O^2 - c_O(x) = \pi_O^2 - \frac{x^2}{2}$ If the pirate is in the market at $t=2$ then its profit function becomes $\pi_P = (p_P - cx)D_P$, where

[3] For simplicity, we assuming the marginal cost of production for the original firm is constant and normalized to zero.

p_P is the price charged by the pirate. D_P is the pirate's demand and c is a parameter $(c > 0)$ *exogenously* given.

2.3 Interpretation of *c*

We would like to interpret c in the following way. Let's assume c is the degree of Intellectual Property Rights (IPR) protection in our model. In other words, c defines the strength of legal enforcement to stop piracy. If $c = 0$, piracy is costless or in other words, original firm's investment effort has no effect in deterring piracy. On the other hand, higher c increases the cost of piracy to the pirate, thus original firm's investment to stop piracy becomes more effective. c is important in our analysis as in this model, we study given an enforcement environment (i.e. given *c*) what would be the best entry deterrent strategy for the original product developer. It is understood that the local government or the regulatory authority can influence c. However, in our model, the government is not directly monitoring illegal piracy, but there is a general anti-piracy law that exists in the economy.[4]

2.4 Consumer Demand

Consider a continuum of consumers indexed by X, $X \in [0, \theta]$. A consumer's willingness to pay for the product depends on how much he/she values it – measured by X. A high value of X means higher valuation for the product and low value of X means lower valuation for the product. Therefore, one consumer differs from another on the basis of his/her valuation for the particular product. Valuations are uniformly with density $\frac{1}{\theta}$ distributed over the interval $[0, \theta]$. Each consumer purchases at most one unit of the good.

[4] For example, we can generally find a relatively high c in the developed countries where piracy is taken as a serious crime; hence it raises the cost of piracy significantly. On the other hand, in most of the developing countries, we will probably find c to be relatively low, because the enforcement policies against piracy may not be as strict, hence cost of piracy would remain relatively small.

A consumer's utility function is given as:

$$U = \begin{cases} X - p_O & \text{if buys original product} \\ q X - p_P & \text{if buys pirated product [5]} \\ 0 & \text{if buys none} \end{cases}$$

p_O and p_P are the prices of the original and pirated product respectively.

3. A Complete Characterization of Accommodation and Deterrence Equilibrium

3.1 Deriving Demands of the Product Developer and the Pirate

D_O and D_P can be derived from the distribution of buyers as follows.

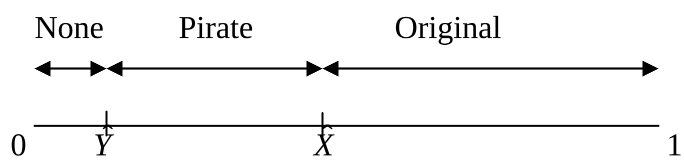

Figure 1: DISTRIBUTION OF BUYERS

Recall that consumers are heterogeneous with respect to their values towards the product. Thus, the marginal consumer, $\hat{X}$, who is indifferent between buying the original product and the pirated version is given by:

$$\hat{X} - p_O = q\,\hat{X} - p_P$$

$$\hat{X} = \frac{p_O - p_P}{1-q}$$

The marginal consumer, $\hat{Y}$, who is indifferent between buying the pirated product and not buying any product is given by:

$$q\hat{Y} - p_P = 0$$

[5] Note that $q = 0$ will eliminate the pirated product, while $q = 1$ will make the two products identical. In our model $q = 1$ is never possible as we have assumed that the pirated good is of lower quality. Also technically, $q \in (0,1)$ is needed so that demands, prices and profits are not indeterminate.

$$\hat{Y} = \frac{p_P}{q}$$

Thus, the demand for original product is: $D_O = \frac{1}{\theta}\int_{\hat{X}}^{\theta} dx \quad = \frac{(1-q)\theta - (p_O - p_P)}{(1-q)\theta}$

The demand for pirated product is: $D_P = \frac{1}{\theta}\int_{\hat{Y}}^{\hat{X}} dx = \frac{qp_O - p_P}{q(1-q)\theta}$

The Game

In the first period of the game, the original firm makes entry deterring investment, while in the second period if the pirate survives, both firms compete in price. We look for subgame perfect equilibrium of the two-period game and solve using the usual method of backward induction.

3.2 Price Competition in the Product Market

In the second period, if the pirate operates, the two firms engage in a Bertrand price competition and choose the profit maximizing prices of the respective products.

The profit function of the pirate is: $\pi_P = (p_P - cx)D_P = (p_P - cx)\,\frac{qp_O - p_P}{q(1-q)\theta}$

The profit function of the original firm is: $\pi_O^2 = p_O D_O = p_O\,\frac{(1-q)\theta - (p_O - p_P)}{(1-q)\theta}$

The reaction functions of the original firm and the pirate are as follows.

$$R_O(p_P) = \frac{p_P}{2} + \frac{(1-q)\theta}{2}\,; \quad R_P(p_O) = \frac{qp_O}{2} + \frac{cx}{2}$$

Notice that as the original firm increases investment effort in the first period, higher will be x in the second period, which means higher will be the marginal cost of copying to the pirate. Thus a increase in x (or an increase in the exogenous parameter c) will shift the reaction function of the pirate upward. This will result higher equilibrium prices for both the original firm and the pirate. It is easy to see that the original firm will gain from this change in the market competition stage as it is now charging higher price while its costs in that period remains the same. However, for the pirate since the total cost of piracy goes up for this change, the net effect in the change in total profit remains ambiguous. The possibility that there could be no real change in profit or even a decline in profit of the pirate cannot be ruled out.

The Nash equilibrium prices are given by

$$p_O = \frac{1}{4-q}\left[2(1-q)\theta + cx\right],\ p_P = \frac{1}{4-q}\left[q(1-q)\theta + 2cx\right]$$

Equilibrium demands are given by

$$D_O = \frac{1}{(4-q)(1-q)\theta}\left[2(1-q)\theta + cx\right];$$

$$D_P = \frac{1}{(4-q)q(1-q)\theta}\left[q(1-q)\theta - cx(2-q)\right]$$

The equilibrium profits can be worked out using the following expressions.

$\pi_O^2 = p_O.D_O$ and $\pi_P = (p_P - cx)D_p$

3.3 Pirate's Decision

The pirate will be in business as long as it can make positive profit.

Thus, equating $\pi_P = 0$, we get $\hat{x} = \dfrac{q(1-q)\theta}{c(2-q)}$

Thus for all $x \geq \hat{x}$, the profit of the pirate becomes non-positive hence, the pirate will not operate, and piracy will be deterred.

3.4 Choice of Optimal Level of Deterrence by the Original Firm

Now we move on to the first period of the game. In this period, original firm decides on its optimal choice on the level of x to deter piracy.

Thus it maximizes its net profit $\pi_O = \pi_O^2 - c_O(x) = \pi_O^2 - \dfrac{1}{2}x^2$ with respect to *x*.

Solving above, we get the optimal level of deterrence $x^* = \dfrac{4c(1-q)\theta}{(4-q)^2(1-q)\theta - 2c^2}$ [6]

[6] Note that if $c = 0$ i.e. when the original firm's investment effort has no effect in deterring piracy, the original firm will not choose any R&D investment in the first place, hence $x^* = 0$.

3.5 The Accommodation Case

If $x^* \le \hat{x}$, the original firm accommodates the pirate.

Now, $x^* \le \hat{x}$ requires $c^2 \le \dfrac{q(4-q)(1-q)\theta}{2}$

If the original producer chooses x^* in period 1, then we get the following:

$$p_o = \frac{2(1-q)^2(4-q)\theta^2}{(4-q)^2(1-q)\theta - 2c^2}, \quad D_o = \frac{2(1-q)(4-q)\theta}{(4-q)^2(1-q)\theta - 2c^2},$$

$$\pi_o = \frac{4(1-q)^2\theta^2}{(4-q)^2(1-q)\theta - 2c^2},$$

$$p_p = \frac{(1-q)\theta\left(q(4-q)(1-q)\theta + 2c^2\right)}{(4-q)^2(1-q)\theta - 2c^2}, \qquad D_p = \frac{q(1-q)(4-q)\theta - 2c^2}{q\left[(4-q)^2(1-q)\theta - 2c^2\right]},$$

$$\pi_p = \frac{(1-q)\theta\left[q(1-q)(4-q)\theta - 2c^2\right]^2}{q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2}.$$

Now when $x^* \ge \hat{x}$, we have $c^2 \ge \dfrac{q(4-q)(1-q)\theta}{2}$.

Then the original producer chooses $x = \hat{x}$ to deter the entry of the pirate. In the forthcoming analysis, we will call this deterrence case "Deterrence sub-case 4". (The other entry deterrence sub-cases will be derived in the next section.) In this particular deterrence sub-case, the original producer's profit is $\pi_o = \dfrac{(1-q)\theta\left(2c^2 - q^2(1-q)\theta\right)}{2(2-q)^2 c^2}$.

3.6 The Deterrence Cases

Besides the aforementioned deterrence sub-case 4, the original producer can deter the entry of the pirate by simply setting a price p_o such that $X - p_o \ge qX - cx$ and $X - p_o \ge 0$ or equivalently, $X > \max\left\{\dfrac{p_o - cx}{1-q}, p_o\right\}$. This is true since the pirate can never set a price lower than cx.

(a) When $qp_o \geq cx$ (i.e. $\frac{p_o - cx}{1-q} \geq p_o$),

The demand of the original product is: $D_o = \frac{\theta - \frac{p_o - cx}{1-q}}{\theta}$.

Profit maximization by the product developer yields

$$p_o = \begin{cases} \frac{(1-q)\theta + cx}{2} & if \quad x \leq \frac{(1-q)q\theta}{(2-q)c} \\ \frac{cx}{q} & otherwise \end{cases}$$

(b) When $qp_o \leq cx$ (i.e. $\frac{p_o - cx}{1-q} \leq p_o$),

The demand of the original product is: $D_o = \frac{\theta - p_o}{\theta}$.

Profit maximization by the product developer yields

$$p_o = \begin{cases} \frac{\theta}{2} & if \quad x \geq \frac{q\theta}{2c} \\ \frac{cx}{q} & otherwise \end{cases}$$

Therefore, we find the expression of the optimal price p_o depends on the range of *x* the deterrence level.

We also derive the original producer's profit in the second period. The expressions for the second period profits also depend on the range of *x*. We define the ranges as case (i), case (ii) and case (iii) as follows.

$$\pi_o^2 = \begin{cases} \frac{\left((1-q)\theta + cx\right)^2}{4(1-q)\theta} & if \quad x \leq \frac{(1-q)q\theta}{(2-q)c} & (case(i)) \\ \frac{cx(q\theta - cx)}{q^2\theta} & if \quad \frac{(1-q)q\theta}{(2-q)c} \leq x \leq \frac{q\theta}{2c} & (case(ii)) \\ \frac{\theta}{4} & if \quad x \geq \frac{q\theta}{2c} & (case(iii)) \end{cases}$$

Using different expression of π_o^2 for different ranges of x, we can finally characterize all the optimal levels of deterrence in period 1 of the original firm. The following proposition summarizes the finding.

Proposition 1

(a) *When* $c^2 \leq q(1-q)\theta$, *the original producer chooses* $x = \frac{(1-q)\theta c}{2(1-q)\theta - c^2}$, *or* $x = \frac{q\theta}{2c}$, *or* $x = \frac{(1-q)q\theta}{(2-q)c}$ *to deter entry.*

(b) *When* $c^2 \geq q(1-q)\theta$, *the original producer chooses* $x = \frac{(1-q)q\theta}{(2-q)c}$, *or* $x = \frac{q\theta}{2c}$, *or* $x = \frac{cq\theta}{q^2\theta + 2c^2}$ *to deter entry.*

***Proof**:* See appendix

Let us first follow up on the case (a) i.e. when $c^2 \leq q(1-q)\theta$. In the following analysis, when $x = \frac{(1-q)\theta c}{2(1-q)\theta - c^2}$ we will call this as "Deterrence sub-case 1"; when $x = \frac{q\theta}{2c}$ we call it as "Deterrence sub-case 2"; and when $x = \frac{(1-q)q\theta}{(2-q)c}$ we call it as "Deterrence sub-case 3" .

<u>Case (a)</u> When $c^2 \leq q(1-q)\theta$

Sub-case 1: $x = \frac{(1-q)\theta c}{2(1-q)\theta - c^2}$.

In this sub-case, we get $p_o = \frac{(1-q)^2\theta^2}{2(1-q)\theta - c^2}$, $\pi_o = \frac{(1-q)^2\theta^2}{2\left[2(1-q)\theta - c^2\right]}$.

Sub-case 2: $x=\dfrac{q\theta}{2c}$.

In this sub-case, we get $p_o=\dfrac{\theta}{2}$, $\pi_o=\dfrac{\left(2c^2-\theta q^2\right)\theta}{8c^2}$.

Sub-case 3: $x=\dfrac{(1-q)q\theta}{(2-q)c}$.

In this sub-case, we get $p_o=\dfrac{(1-q)\theta}{2-q}$, $\pi_o=\dfrac{(1-q)\theta\left[2c^2-q^2(1-q)\theta\right]}{2(2-q)^2c^2}$.

Note that there is no Deterrence sub-case 4 (see section 4.1) under case (a), i.e. when $c^2\le q(1-q)\theta$.

3.6.1 Profit Comparison

Now we can compare the original producer's profit in four cases; namely, one accommodation case and deterrence sub-cases 1-3. First comparing the profit in three deterrence sub-cases we get the following.

$$\pi_o(subcase1)-\pi_o(subcase3)=\frac{(1-q)^2\theta^2}{2\left[2(1-q)\theta-c^2\right]}-\frac{(1-q)\theta\left[2c^2-q^2(1-q)\theta\right]}{2(2-q)^2c^2}$$

$$=\frac{(1-q)\theta\left[q(1-q)\theta-c^2\right]^2}{(2-q)^2c^2\left[2(1-q)\theta-c^2\right]}>0,$$

$$\pi_o(subcase1)-\pi_o(subcase2)=\frac{(1-q)^2\theta^2}{2\left[2(1-q)\theta-c^2\right]}-\frac{\left(2c^2-\theta q^2\right)\theta}{8c^2}$$

$$=\frac{\theta\left[2c^4+2q^2(1-q)\theta^2-q(4-3q)c^2\theta\right]}{8c^2\left[2(1-q)\theta-c^2\right]}>0.$$

So the profit is the highest in sub-case 1.

Now comparing the profits in accommodation case and in deterrence sub-case 1:

$$\pi_o(accommodation)-\pi_o(subcase1)=\frac{4(1-q)^2\theta^2}{(4-q)^2(1-q)\theta-2c^2}-\frac{(1-q)^2\theta^2}{2\left[2(1-q)\theta-c^2\right]}$$

$$=\frac{(1-q)^2\theta^2\left[q(1-q)(8-q)\theta-6c^2\right]}{2\left[(4-q)^2(1-q)\theta-2c^2\right]\left[2(1-q)\theta-c^2\right]}>0.$$

Lemma 1

When $c^2 \le q(1-q)\theta$*, the original producer's best strategy is to accommodate the pirate and to choose the deterrence level* $x^* = \dfrac{4c(1-q)\theta}{(4-q)^2(1-q)\theta - 2c^2}$ *, price* $p_o = \dfrac{2(1-q)^2(4-q)\theta^2}{(4-q)^2(1-q)\theta - 2c^2}$ *and earn profit* $\pi_o = \dfrac{4(1-q)^2\theta^2}{(4-q)^2(1-q)\theta - 2c^2}$.

Now, let us first follow up on the case (b) i.e. when $c^2 \le q(1-q)\theta$. Here, as before when $x = \dfrac{(1-q)q\theta}{(2-q)c}$ we will call this as "Deterrence sub-case 1"; when $x = \dfrac{q\theta}{2c}$ we call it as "Deterrence sub-case 2"; and when $x = \dfrac{cq\theta}{q^2\theta + 2c^2}$ we call it as "Deterrence sub-case 3" .

<u>**Case (b)**</u> When $c^2 \ge q(1-q)\theta$

Sub-case 1: $x = \dfrac{(1-q)q\theta}{(2-q)c}$.

In this sub-case, we get $p_o = \dfrac{(1-q)\theta}{2-q}$, $\pi_o = \dfrac{(1-q)\theta^2\left(2c^2 - q^2(1-q)\theta\right)}{2(2-q)^2 c^2}$.

Sub-case 2: $x = \dfrac{q\theta}{2c}$.

In this sub-case, we get $p_o = \dfrac{\theta}{2}$, $\pi_o = \dfrac{\left(2c^2 - \theta q^2\right)\theta}{8c^2}$.

Sub-case 3: $x = \dfrac{cq\theta}{q^2\theta + 2c^2}$.

In this sub-case, we get $p_o = \dfrac{c^2\theta}{q^2\theta + 2c^2}$, $\pi_o = \dfrac{c^2\theta}{2\left(q^2\theta + 2c^2\right)}$.

Here note that there is no Deterrence sub-case 4 (see section 4.1) when $q(1-q)\theta \le c^2 < \frac{q(4-q)(1-q)\theta}{2}$, however, there is such a case when $c^2 \ge \frac{q(4-q)(1-q)\theta}{2}$.

3.6.2 Profit Comparison

Comparing the profits in deterrence sub-cases 1-3 we get:

$$\pi_o(subcase 1) - \pi_o(subcase 3) = \frac{(1-q)\theta^2\left(2c^2 - q^2(1-q)\theta\right)}{2(2-q)^2 c^2} - \frac{c^2\theta}{2\left(q^2\theta + 2c^2\right)}$$

$$= \frac{-\theta q^2\left[c^2 - q(1-q)\theta\right]^2}{2(2-q)^2 c^2\left(q^2\theta + 2c^2\right)} < 0,$$

$$\pi_o(subcase 3) - \pi_o(subcase 2) = \frac{c^2\theta}{2\left(q^2\theta + 2c^2\right)} - \frac{\left(2c^2 - \theta q^2\right)\theta}{8c^2} = \frac{\theta^3 q^4}{8c^2\left(q^2\theta + 2c^2\right)} > 0.$$

So the profit is the highest in sub-case 3.

Now comparing the profit in accommodation case and in deterrence sub-case 3 when $q(1-q)\theta \le c^2 < \frac{q(4-q)(1-q)\theta}{2}$.

$$\pi_o(accommodation) - \pi_o(subcase 3) = \frac{4(1-q)^2\theta^2}{(4-q)^2(1-q)\theta - 2c^2} - \frac{c^2\theta}{2\left(q^2\theta + 2c^2\right)}$$

$$= \frac{\theta\left[2c^4 - q(1-q)(8+q)\theta c^2 + 8q^2(1-q)^2\theta^2\right]}{2\left[(4-q)^2(1-q)\theta - 2c^2\right]\left(q^2\theta + 2c^2\right)} \ge 0 \quad if\ c^2 \le \frac{q(1-q)\left(8+q-\sqrt{q(16+q)}\right)\theta}{4}$$

$$\le 0 \quad if\ c^2 \ge \frac{q(1-q)\left(8+q-\sqrt{q(16+q)}\right)\theta}{4}.$$

Let us denote $d \equiv \frac{q(1-q)\left(8+q-\sqrt{q(16+q)}\right)\theta}{4}$.

Note that $d < \frac{q(4-q)(1-q)\theta}{2}$.

Lemma 2

When $q(1-q)\theta \le c^2 \le d$, the original producer's best strategy is accommodation and that when $d \le c^2 \le \frac{q(4-q)(1-q)\theta}{2}$, the original producer's best strategy is to deter and choose $x = \frac{cq\theta}{q^2\theta + 2c^2}$, price $p_o = \frac{c^2\theta}{q^2\theta + 2c^2}$ and earn profit $\pi_o = \frac{c^2\theta}{2(q^2\theta + 2c^2)}$ (i.e. sub-case 3).

Now we need to compare the profit in the deterrence sub-case 4 and deterrence sub-case 3 when $c^2 \ge \frac{q(4-q)(1-q)\theta}{2}$. However, note that the expression of the profit in deterrence sub-case 4 is actually the same as the one in deterrence sub-case 1 and we just showed that profit in deterrence sub-case 3 is higher than in deterrence sub-case 1. Thus, we get the following result.

Lemma 3

When $c^2 \ge \frac{q(4-q)(1-q)\theta}{2}$, the original producer's best strategy is to deter and choose $x = \frac{cq\theta}{q^2\theta + 2c^2}$, price $p_o = \frac{c^2\theta}{q^2\theta + 2c^2}$ and earn profit $\pi_o = \frac{c^2\theta}{2(q^2\theta + 2c^2)}$ (i.e. sub-case 3).

3.7 Summary

In the following proposition, we completely characterize the entry accommodation equilibrium and entry deterrence equilibrium in the whole parameter space of c, q and θ.

Proposition 2

(i) *When $c^2 \le d$, the original producer's best strategy is to accommodate the pirate and to choose deterrence level $x^* = \frac{4c(1-q)\theta}{(4-q)^2(1-q)\theta - 2c^2}$, price $p_o = \frac{2(1-q)^2(4-q)\theta^2}{(4-q)^2(1-q)\theta - 2c^2}$ and earn profit $\pi_o = \frac{4(1-q)^2\theta^2}{(4-q)^2(1-q)\theta - 2c^2}$*

(ii) *When* $c^2 \geq d$ *, the original producer's best strategy is to deter and choose deterrence level* $x = \dfrac{cq\theta}{q^2\theta + 2c^2}$ *, price* $p_o = \dfrac{c^2\theta}{q^2\theta + 2c^2}$ *and earn profit* $\pi_o = \dfrac{c^2\theta}{2\left(q^2\theta + 2c^2\right)}$ *(i.e. sub-case 3).*

An Example

Let’s take the following example where $\theta = 1$, $q = 0.5$. The original producer’s profits in different cases are represented in figure 2. The red curve represents the original producer’s profit in accommodation case when $c \leq \sqrt{\dfrac{(4-0.5)*0.5*(1-0.5)}{2}} = 0.6614$ (i.e. when $c^2 \leq \dfrac{q(4-q)(1-q)\theta}{2}$, see section 4.1). The blue curve represents the profit in deterrence sub-case 1 and when $c \geq 0.6614$ (i.e. when $c^2 \geq \dfrac{q(4-q)(1-q)\theta}{2}$) it also represents the profit in deterrence sub-case 4 since the profit in these two cases are exactly the same. The green one and the magenta one represent the profit in deterrence sub-case 2 and the profit in deterrence sub-case 3 respectively. From this figure, we can see that the original producer’s optimal strategy is to accommodate the pirate (when $c \leq \sqrt{\dfrac{0.5*(1-0.5)(0.5+8-\sqrt{0.25+16*0.5}}{4}} = 0.5931$) or deter the pirate as in deterrence sub-case 3 (when $c \geq 0.5931$). Note that we haven’t completely drawn the original producer’s profits in deterrence sub-cases 2 and 3 since the profits are relatively low (even negative for some range of c).

[Insert Figure 2]

4. Comparative Statics

4.1 The relationship between the optimal level of deterrence (*x*) and the degree of IPR protection (*c*)

When $c^2 \le d$, i.e. when the original firm always accommodates the pirate:

$$\frac{\partial x^*}{\partial c} = \frac{4(1-q)\theta\left[(4-q)^2(1-q)\theta + 2c^2\right]}{\left[(4-q)^2(1-q)\theta - 2c^2\right]^2} > 0 .$$

At the first instance it may seem surprising that the original producer chooses a higher x when the degree of IPR protection c increases since the intuition would tell us that the original producer would reduce x (to save its own cost) when c increases since a higher c anyway implies a higher cost of piracy. But thinking about the issue more carefully we find that the above finding is reasonable. A higher x will increase the cost of piracy, which is also equivalent to increasing the degree of product differentiation and thus resulting softer competition. However, the original producer cannot increase x too much since there is a cost associated with x as well.

When $c^2 \ge d$, i.e. when the original firm deters the pirate:

$$\frac{\partial x}{\partial c} = \frac{\partial}{\partial c}\left(\frac{cq\theta}{q^2\theta + 2c^2}\right) = \frac{q\theta\left(q^2\theta - 2c^2\right)}{\left(q^2\theta + 2c^2\right)^2} > 0 \quad if\ c^2 < \frac{q^2\theta}{2} .$$

We first compare d and $\frac{q^2\theta}{2}$:

Computations yield $d > \frac{q^2\theta}{2}$ when $q < 0.7239$; and $d < \frac{q^2\theta}{2}$ when $q > 0.7239$.

Thus we have the following result.

Proposition 3

When $q < 0.7239$, $\frac{\partial x}{\partial c} < 0$; and when $q > 0.7239$, $\frac{\partial x}{\partial c} > 0$ as long as $d < c^2 < \frac{q^2\theta}{2}$, and $\frac{\partial x}{\partial c} < 0$ when $c^2 > \frac{q^2\theta}{2}$. Thus, the relationship between optimal level of deterrence and the degree of IPR protection is non-monotonic.

Once the original producer decides to deter the entry, and when we see that the reliability of the pirated product is not sufficiently high, i.e., when the products are already very differentiated, the original producer will give less effort for deterring entry, thus will reduce x when c increases. On the other hand, when the reliability of the pirated product is sufficiently high, i.e. when the products are not too differentiated, the original producer will raise x before c increases to $q\sqrt{\frac{\theta}{2}}$ and after that, reduce x when c increase sufficiently. Thus, we find a non-monotonic relationship between the level of deterrence and the strength of IPR protection. Figures 3 and 4 illustrate the relationship between the level of deterrence and the strength of IPR protection when $q = 0.5$ and $q = 0.9$ respectively; and we set $\theta = 1$ in each figure.

[Insert Figure 3 and 4]

4.2 Rate of Piracy

We define the ratio of $\frac{D_p}{D_o + D_p}$ to measure the rate of piracy. Thus higher the ratio, higher will be the rate of piracy.

When $c^2 \leq d$, i.e. when the original firm accommodates the pirate: we know: $x^* = \frac{4c(1-q)\theta}{(4-q)^2(1-q)\theta - 2c^2}$.

In this case,

$D_o = \frac{2(1-q)(4-q)\theta}{(4-q)^2(1-q)\theta - 2c^2}$, $D_p = \frac{q(1-q)(4-q)\theta - 2c^2}{q\left[(4-q)^2(1-q)\theta - 2c^2\right]}$ and the ratio is:

$\frac{D_p}{D_o + D_p} = \frac{q(1-q)(4-q)\theta - 2c^2}{3q(1-q)(4-q)\theta - 2c^2}$, which is clearly decreasing in c.

When $c^2 \geq d$, entry is deterred, the rate of piracy is zero.

Proposition 4

When there is piracy, the rate of piracy is always decreasing in c.

This result just follows our intuition that increasing the strength of IPR protection unambiguously reduces the rate of piracy.

4.3 Rate of Piracy and Quality of the Pirated Product (*q*)

Since $\frac{\partial}{\partial q}\left(\frac{D_p}{D_o+D_p}\right)=\frac{3\left(3q^2-10q+4\right)c^2\theta}{\left[3q\left(1-q\right)\left(4-q\right)\theta-2c^2\right]^2}$, we have the following finding.

Proposition 5

When $q<0.465$ *(i.e. when q small), the rate of piracy is increasing in* q*, while it is decreasing in when* $q>0.465$ *(i.e. when q large). Thus, we find a non-monotonic relationship between the rate of piracy and the quality of the pirated product.*

The intuition for above result is as follows: When a consumer chooses between a pirated copy and original copy, she cares about both the reliability/quality and the price difference. When a pirated product becomes more and more reliable, the price competition between the pirate and the original producer becomes more and more intense, the price difference becomes smaller and smaller. This eventually leads to a non-monotonic relationship. When q is small, it is the reliability effect that dominates; whereas when q is large, the price difference effect dominates. Our interesting finding here is against the so-called conventional wisdom. Conventional wisdom would suggest that reliable pirated products means higher demand of the pirated good. However, in that logic the price effect is ignored. As products get less differentiated, lower will be the price difference between the pirated and original product. In such situations people will tend to buy the original product even if they have to pay little extra.

In the light of the above result, from the commercial pirate’s point of view we can conclude the following.

Corollary

For a profitable piracy, the optimal strategy of the commercial pirate would be to produce a pirated version with moderate reliability.

This implies, in general, a commercial pirate will not be inclined to produce a version which is too low in quality or which is too close to the original product in terms of quality/reliability even if it has the means to do so.

5. Welfare Analysis

(a) When $c^2 \le d$ (accommodation case), the original producer's best strategy is to accommodate the piracy and to choose $x^* = \dfrac{4c(1-q)\theta}{(4-q)^2(1-q)\theta - 2c^2}$.

The marginal consumer who is indifferent between buying an original product and buying a pirated copy is given by $\widehat{X} = \dfrac{p_o - p_p}{1-q} = \theta \dfrac{(1-q)(4-q)(2-q)\theta - 2c^2}{(4-q)^2(1-q)\theta - 2c^2}$ (by substituting the equilibrium values of p_o and p_p). The marginal consumer who is indifferent between buying a pirated product and not buying is given by $\widehat{Y} = \dfrac{p_p}{q} = \theta(1-q)\dfrac{q(1-q)(4-q)\theta + 2c^2}{q\left[(4-q)^2(1-q)\theta - 2c^2\right]}$.

Consumers' surplus is then $CS = \dfrac{1}{\theta}\left[\int_{\widehat{X}}^{\theta}(X - p_o)dX + \int_{\widehat{Y}}^{\widehat{X}}(qX - p_p)dX\right]$.

Straightforward computation yields

$$CS = \frac{\theta\left[q(5q+4)(1-q)^2(4-q)^2\theta^2 - 4c^2\left(3q(1-q)(4-q)\theta - c^2\right)\right]}{2q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2}.$$

Total welfare is then

$$W = CS + \pi_o + \pi_p = \frac{\theta\left[q(5q+4)(1-q)^2(4-q)^2\theta^2 - 4c^2\left(3q(1-q)(4-q)\theta - c^2\right)\right]}{2q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2}$$

$$+\frac{4(1-q)^2\theta^2}{(4-q)^2(1-q)\theta - 2c^2} + \frac{(1-q)\theta\left[q(1-q)(4-q)\theta - 2c^2\right]^2}{q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2}.$$

(b) When $c^2 \geq d$, (deterrence case) the original producer's best strategy is to choose $x = \frac{cq\theta}{q^2\theta + 2c^2}$, $p_o = \frac{c^2\theta}{q^2\theta + 2c^2}$ and get profit $\pi_o = \frac{c^2\theta}{2\left(q^2\theta + 2c^2\right)}$.

Consumers' surplus: $CS = \frac{1}{\theta}\int_{p_o}^{\theta}(X - p_o)dX$.

Computation yields $CS = \frac{\theta\left(q^2\theta + c^2\right)^2}{2\left(q^2\theta + 2c^2\right)^2}$.

Thus, the total welfare is then $W = CS + \pi_o = \frac{\theta\left(q^2\theta + c^2\right)^2}{2\left(q^2\theta + 2c^2\right)^2} + \frac{c^2\theta}{2\left(q^2\theta + 2c^2\right)}$.

5.1 Comparative Static

5.1.1 Price and IPR Protection

When $c^2 \leq d$, $p_o = \frac{2(1-q)^2(4-q)\theta^2}{(4-q)^2(1-q)\theta - 2c^2}$; when $c^2 \geq d$, $p_o = \frac{c^2\theta}{q^2\theta + 2c^2}$. It is obvious that p_o is increasing in c when $c^2 \leq d$ and when $c^2 \geq d$. To determine whether p_o is increasing in the whole support of c, we need to compare $p_o = \frac{2(1-q)^2(4-q)\theta^2}{(4-q)^2(1-q)\theta - 2c^2}$ and $p_o = \frac{c^2\theta}{q^2\theta + 2c^2}$ when evaluated at $c^2 = d$. According to our analysis, the original producer is indifferent between entry deterrence and entry accommodation when $c^2 = d$. Calculation yields $\frac{c^2\theta}{q^2\theta + 2c^2} > \frac{2(1-q)^2(4-q)\theta^2}{(4-q)^2(1-q)\theta - 2c^2}$ when evaluated at $c^2 = d$.

Lemma 4

Equilibrium price p_o *of the original producer is increasing in the whole support of* c .

5.1.2 Consumers' Surplus (CS) and IPR Protection

When $c^2 \le d$, $CS = \dfrac{\theta\left[q(5q+4)(1-q)^2(4-q)^2\theta^2 - 4c^2\left(3q(1-q)(4-q)\theta - c^2\right)\right]}{2q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2}$.

When $c^2 \ge d$, $CS = \dfrac{\theta\left(q^2\theta + c^2\right)^2}{2\left(q^2\theta + 2c^2\right)^2}$. It can easily be shown that CS is decreasing in c when $c^2 \le d$ and when $c^2 \ge d$. To determine whether CS is decreasing in the whole support of c , we need to compare

$$CS = \frac{\theta\left[q(5q+4)(1-q)^2(4-q)^2\theta^2 - 4c^2\left(3q(1-q)(4-q)\theta - c^2\right)\right]}{2q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2}$$

and $CS = \dfrac{\theta\left(q^2\theta + c^2\right)^2}{2\left(q^2\theta + 2c^2\right)^2}$ when evaluated at $c^2 = d$.

Calculation yields

$$\frac{\theta\left[q(5q+4)(1-q)^2(4-q)^2\theta^2 - 4c^2\left(3q(1-q)(4-q)\theta - c^2\right)\right]}{2q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2} > \frac{\theta\left(q^2\theta + c^2\right)^2}{2\left(q^2\theta + 2c^2\right)^2}$$ when

evaluated at $c^2 = d$.

Lemma 5

Consumer surplus is decreasing in the whole support of c *and it is maximized when* $c = 0$ *and* $CS_{\max} = \dfrac{(5q+4)\theta}{(4-q)^2}$.

5.1.3 Total welfare and IPR Protection

When $c^2 \le d$,

$$W = CS + \pi_o + \pi_p = \frac{\theta\left[q(5q+4)(1-q)^2(4-q)^2\theta^2 - 4c^2\left(3q(1-q)(4-q)\theta - c^2\right)\right]}{2q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2}$$

$$+\frac{4(1-q)^2\theta^2}{(4-q)^2(1-q)\theta - 2c^2} + \frac{(1-q)\theta\left[q(1-q)(4-q)\theta - 2c^2\right]^2}{q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2}.$$

Calculation yields $\frac{\partial W}{\partial\left(c^2\right)} = \frac{8(1-q)^2\theta^2\left[2c^2(2-q)(6-q) - \theta q(1-q)(3-q)(4-q)^2\right]}{q\left[(4-q)^2(1-q)\theta - 2c^2\right]^3} < 0.$

When $c^2 \ge d$, $W = CS + \pi_o = \frac{\theta\left(q^2\theta + c^2\right)^2}{2\left(q^2\theta + 2c^2\right)^2} + \frac{c^2\theta}{2\left(q^2\theta + 2c^2\right)}$.

Calculation yields $\frac{\partial W}{\partial\left(c^2\right)} = -\frac{q^4\theta^3}{2\left(q^2\theta + 2c^2\right)^3} < 0$.

To determine whether W is decreasing in the whole support of c, we need to compare the two expressions of total welfare when evaluated at $c^2 = d$.
Calculation yields

$$\frac{\theta\left[q(5q+4)(1-q)^2(4-q)^2\theta^2 - 4c^2\left(3q(1-q)(4-q)\theta - c^2\right)\right]}{2q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2}$$

$$+\frac{4(1-q)^2\theta^2}{(4-q)^2(1-q)\theta - 2c^2} + \frac{(1-q)\theta\left[q(1-q)(4-q)\theta - 2c^2\right]^2}{q\left[(4-q)^2(1-q)\theta - 2c^2\right]^2} > \frac{\theta\left(q^2\theta + c^2\right)^2}{2\left(q^2\theta + 2c^2\right)^2} + \frac{c^2\theta}{2\left(q^2\theta + 2c^2\right)}$$

when evaluated at $c^2 = d$.

Proposition 6

Total welfare decreases as the degree of IPR protection increases and is maximized when $c = 0$ *and* $W_{\max} = \frac{\left(12 - q - 2q^2\right)\theta}{2(4-q)^2}$.

Here we would like to qualify that this result is true in our model because of the nature of the utility functions of the consumers are considered here. In general, if IPR protection is weak, the available products (particularly the pirated ones) become very cheap so that almost everybody in the economy can afford to buy and use it. This unambiguously increases consumer surplus and welfare of the society. However, in this framework, we ignore one important dimension, namely, the innovation on quality. In our model the quality of the original product is assumed to be constant; and this is possibly consistent with a short-run situation. If the IPR protection is weak, then it is unlikely that the product developer would invest to improve upon the quality of the product. This would eventually reduce the utility and hence consumer surplus and welfare in a long-run situation.

6. Extension: End User Piracy

Under this situation, we assume there is no commercial pirate in the economy. The case of end-users piracy will be more likely for digital products which are easy to copy. Here the consumers (i.e. all potential product users) are the potential pirates.[7] As before, there is one original product developer (monopoly) and consumers' valuations are uniformly distributed over the interval $[0,\theta]$ with density $\frac{1}{\theta}$. Consumers have the choice to buy the original product from the product developer or they can pirate. The activity of the original product firm remains exactly the same as before, except that now it targets the end user pirates to stop piracy as opposed to commercial pirate that we have seen before. However, unlike before, here the original firm does not face any direct competition from anybody in the market; instead, it stands to lose its potential market because of end user pirates. Under this circumstance to limit/stop piracy, it invests to raise the cost of piracy to the end users.[8]

---

[7] An alternative explanation which is also consistent with this scenario would be when there is a competitive fringe of commercial pirates (i.e. larger number of identical commercial pirates) and each pirate makes zero profit due to perfect competition among them. Although the working for this case would be little different from the end-user piracy case, however, it can be easily verified that there will be no change in the final results (working is available upon request).

[8] Here, we do not need the two period time structure as before, everything can be formulated within a single period without loss of generality. There is no strategic game here, it's a monopoly analysis.

Thus a consumer's utility function is given as:

$$U = \begin{cases} X - p & \text{if buys original product} \\ qX - cx & \text{if pirates the original product} \\ 0 & \text{otherwise} \end{cases}$$

where x is the level of deterrence for piracy from the original producer and $c > 0$ is the exogenous cost parameter as before measuring the degree of IPR protection.

6.1 Deriving Demand of the Original and Pirated Product

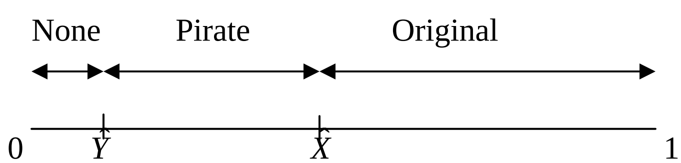

Figure 5: DISTRIBUTION OF BUYERS

The marginal consumer, $\hat{X}$, who is indifferent between buying the original product and pirating is given by:

$$\hat{X} - p = q\hat{X} - cx$$

$$\hat{X} = \frac{p - cx}{1 - q}$$

The marginal consumer, $\hat{Y}$, who is indifferent between pirating the product and not buying any product is:

$$q\hat{Y} - cx = 0$$

$$\hat{Y} = \frac{cx}{q}$$

Thus, the demand for the original firm is: $D_O = \frac{1}{\theta}\int_{\hat{X}}^{\theta} dx \;= \frac{(1-q)\theta - (p - cx)}{(1-q)\theta}$

Demand for the pirated product is: $D_P = \frac{1}{\theta}\int_{\hat{Y}}^{\hat{X}} dx = \frac{qp - cx}{q(1-q)\theta}$

6.2 Choice of Optimal Price and Level of Deterrence by the Product Developer

When we derive the demand for the original firm and for the pirated product, we have implicitly assumed $pq \geq cx$ so that the demand for the pirate product is nonnegative. The developer maximizes its net profit subject to this constraint. So the developer's profit maximization problem is

$$\max_{p,x} \pi_O = pD_O - c_O(x) = p\left(\frac{(1-q)\theta - (p - cx)}{(1-q)\theta}\right) - \frac{1}{2}x^2 .$$

$$s.t. \quad pq \geq cx$$

Solving, above we get the following result.

Proposition 7

(a) When $c^2 \leq q(1-q)\theta$ *, the optimal monopoly price* $p^* = \frac{(1-q)^2\theta^2}{2(1-q)\theta - c^2}$ *and the optimal level of deterrence* $x^* = \frac{c(1-q)\theta}{2(1-q)\theta - c^2}$

(b) When $c^2 \geq q(1-q)\theta$ *, the optimal monopoly price* $p^{**} = \frac{c^2\theta}{q^2\theta + 2c^2}$ *and the optimal level of deterrence* $x^{**} = \frac{cq\theta}{q^2\theta + 2c^2}$.

6.3 Deterrence and Non-Deterrence of Piracy

To deter piracy completely, the developer can also choose a price sufficiently low such that $pq < cx$. But this is not profitable. Given that piracy is completely deterred, the demand for the product firm is $\frac{\theta - p}{\theta}$, which is independent of *x* and decreases in *p*, the developer will choose a price such that $pq = cx$. Thus we state the condition for no piracy in the following result.

Proposition 8

When the pirates are the end users and stopping piracy is a costly activity to the product firm, the piracy will actually be stopped if $c^2 \geq q(1-q)\theta$. *Otherwise there will be piracy.*

Economic Interpretation

7. Conclusion

In this paper, we address the issue of piracy or illegal copying or counterfeiting of the original product and Intellectual Property Right (IPR) protections. The original product developer makes costly investment to deter piracy in given a regime of IPR protection. In this environment, we first characterize completely the entry deterrence and entry accommodation equilibrium in the presence of a commercial pirate. We find that it is profitable for the original producer to accommodate the pirate when there is weak IPR protection, while deterring is profitable when the IPR protection is strong. However, we find there is a non-monotonic relationship between the optimal level of deterrence (chosen by the original producer) and the degree of IPR protection in the economy. The relationship between the rate of piracy and IPR protection is found to be monotonically decreasing whereas the relationship between the rate of piracy and the quality of the pirated product turns out to be non-monotonic. In our welfare analysis, we find that the total welfare of the society decreases as the degree of IPR protection increases.

References

(To be inserted)

Appendix 1

Case (i)

In the first period, the original producer's profit maximization problem is

$$\max_x \frac{\left((1-q)\theta+cx\right)^2}{4(1-q)\theta}-\frac{1}{2}x^2 \qquad s.t.\ \ x\le\frac{(1-q)q\theta}{(2-q)c}.$$

Solving this problem gives us

$$x=\begin{cases}\dfrac{(1-q)\theta c}{2(1-q)\theta-c^2} & if \quad c^2\le q(1-q)\theta \\ \dfrac{(1-q)q\theta}{(2-q)c} & if \quad c^2\ge q(1-q)\theta\end{cases}.$$

Case (ii)

In the first period, the original producer's profit maximization problem is

$$\max_x \frac{cx(q\theta-cx)}{q^2\theta}-\frac{1}{2}x^2 \qquad s.t.\ \ \frac{(1-q)q\theta}{(2-q)c}\le x\le\frac{q\theta}{2c}.$$

Solving this problem gives us

$$x=\begin{cases}\dfrac{(1-q)q\theta}{(2-q)c} & if \quad c^2\le q(1-q)\theta \\ \dfrac{q\theta c}{q^2\theta+2c^2} & if \quad c^2\ge q(1-q)\theta\end{cases}.$$

Case (iii)

In the first period, the original producer's profit maximization problem is

$$\max_x \frac{\theta}{4}-\frac{1}{2}x^2 \qquad s.t.\ \ x\ge\frac{q\theta}{2c}.$$

Clearly, the original producer will choose $x=\frac{q\theta}{2c}$.

Appendix 2 (Can be deleted since it is straightforward)

We will compare d and $\frac{q^2\theta}{2}$ in this appendix.

$$d-\frac{q^2\theta}{2}=\frac{q\theta}{4}\left((1-q)\left(8+q-\sqrt{q(16+q)}\right)-2q\right)\begin{cases}>0 & when\ q<0.7239 \\ <0 & when\ q>0.7239\end{cases}.$$

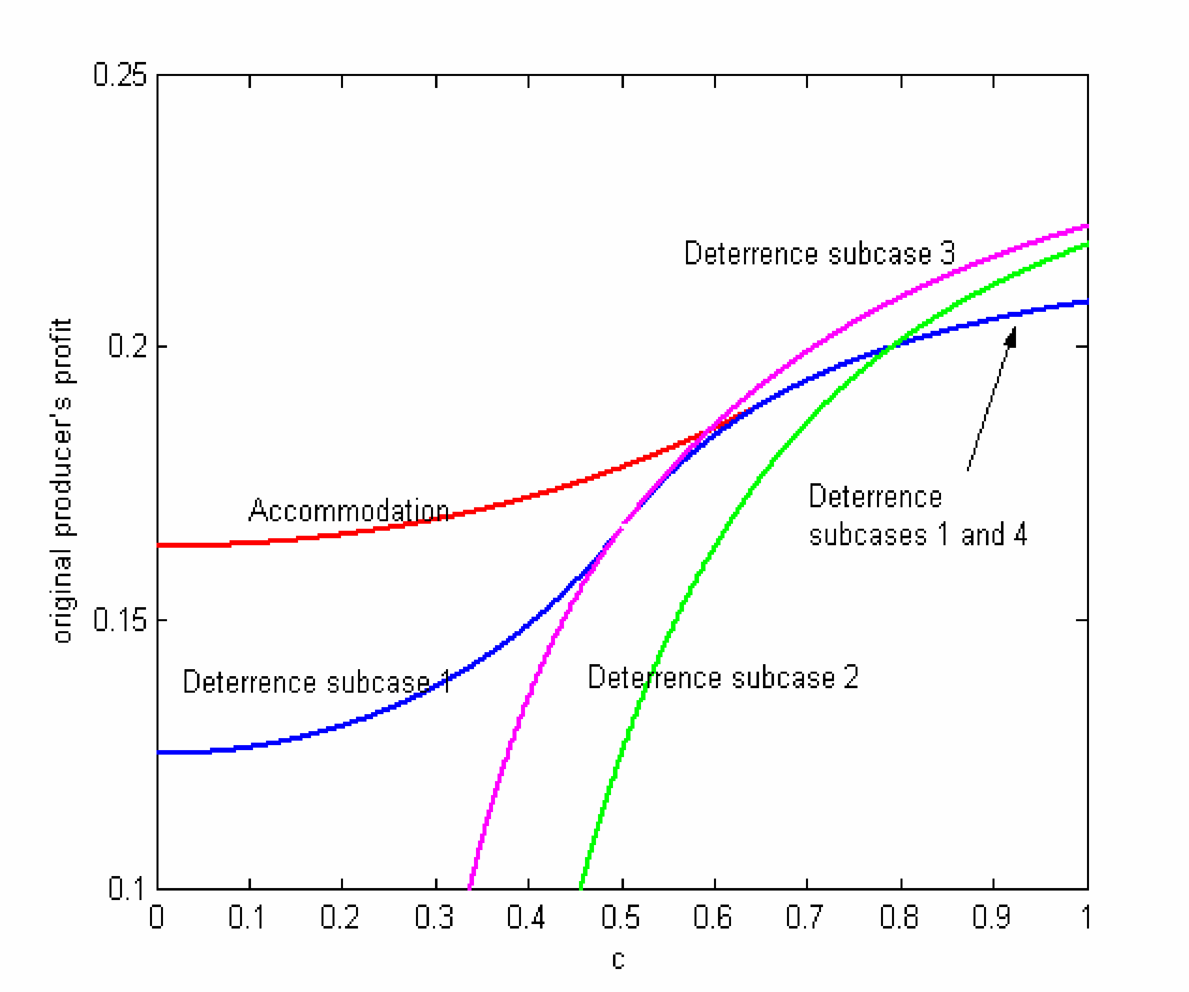

Figure 2 The Original Producer's Profits in Different Cases ($\theta = 1,\ q = 0.5$)

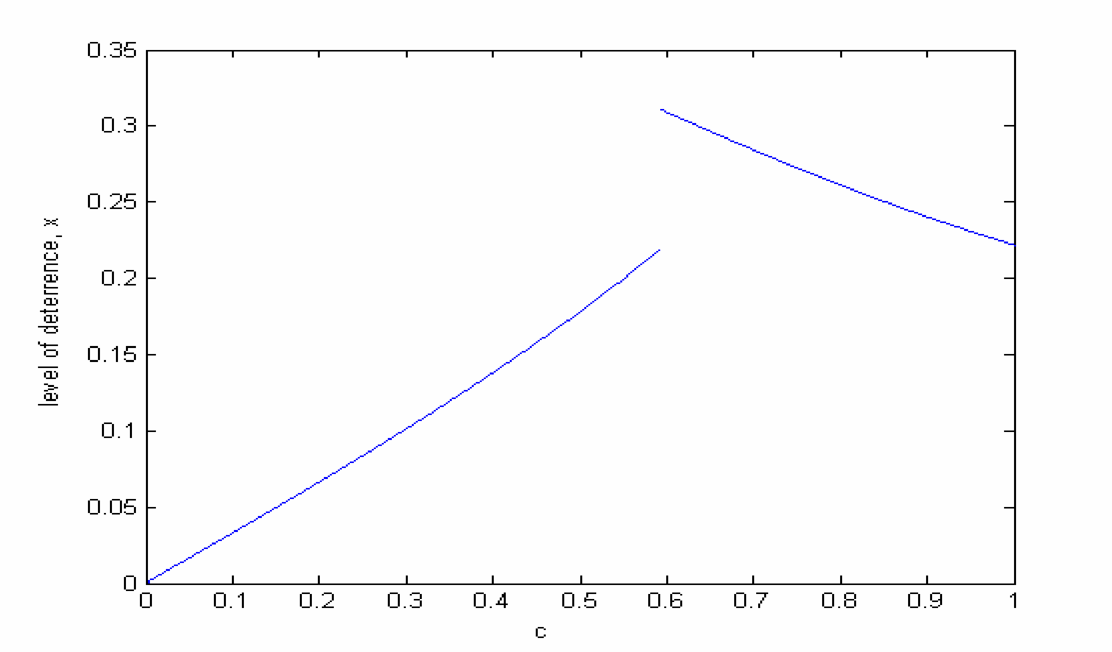

Figure 3 The relationship between the optimal level of deterrence *x* and *c*

($\theta = 1,\ q = 0.5$)

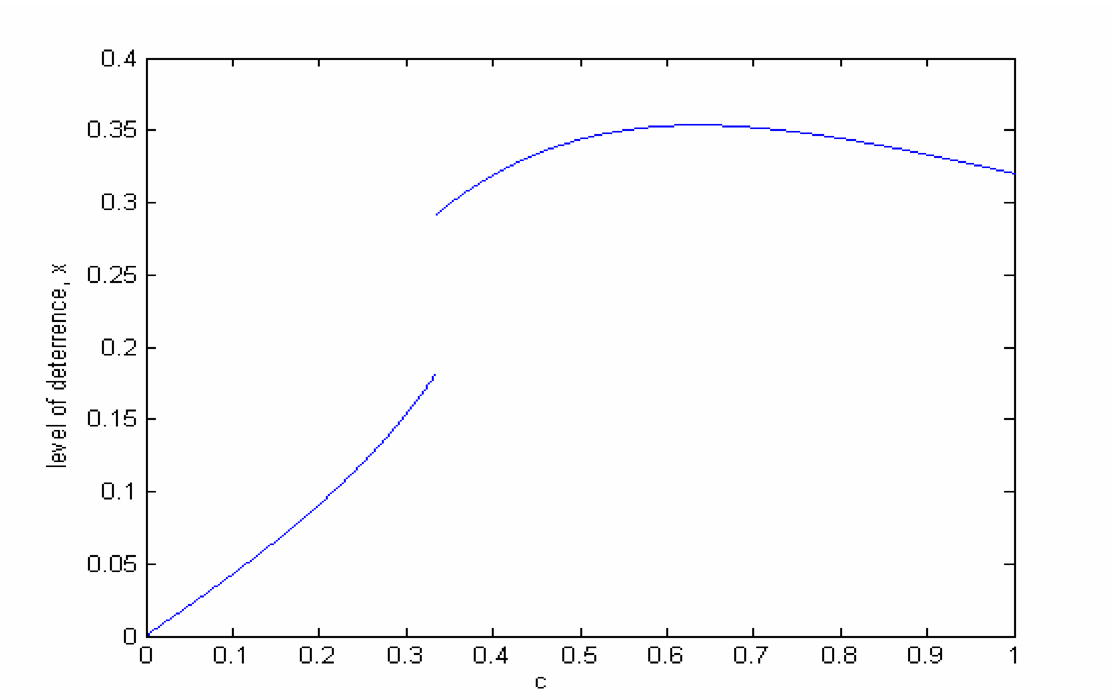

Figure 4 The relationship between the optimal level of deterrence *x* and *c*

($\theta = 1,\ q = 0.9$)