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The Taxation of Life Annuities Under Adverse Selection

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This paper studies how annuities should be taxed in a model à la Mirelees (1971) in presence of adverse selection and a positive link between income and longevity. It is shown that the taxation can address the adverse selection problem by setting a progressive tax schedule on annuities. Moreover, as the rich are more likely to attain old age, a government can reduce lifecycle inequalities by taxing annuities insofar as they signal consumption by high incomes. Numerical simulations show that the last effect shifts upward the level of taxation while dampening the degree of progressivity. They also suggest that the level of taxation on annuities is significant, highly progressive and increases when annuitants get older.

1 Introduction

Concerns over the future of public pension systems have led many governments to promote the development of private life annuity products by means of tax incentives. Whitehouse (1999) shows that most developed countries exempt from income tax either the contributions during the accumulation period or the benefits during the payout phase. Both options alter the post-tax rate of return to annuities, albeit in a different way. Antolin et al. (2004) finds that the first option is the most common regime in 17 OECD countries. Moreover, as individuals generally pay a lower marginal income tax rate while retired than in work, this tax-deferral policy tends to pull up the rate of return. Yet observed tax treatments are difficult to assess on the ground of economic principles.

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Those fiscal exemptions raise a number of important policy issues which are more broadly related to the debate on the taxation of saving. Should the government tax or susidize the returns to annuities ? Should the taxation be progressive in order to redistribute incomes ? Considering that developed economies already achieve redistributive goals through a personal income tax, is there any complementary role for a tax on annuities ? As regard to saving, the literature generally concludes that the taxation should be avoided. This statement can be traced back to the influential paper of Atkinson and Stiglitz (1976). They show that indirect taxation is needless when a government can use a non-linear income tax and utility functions are weakly separable between goods and leisure. In particular, the objective of redistribution is better achieved by an income tax alone. Since a tax on saving is equivalent to a comodity taxation which varies over the life cycle of the agent, this result extends to the taxation of saving as well.

Few studies exist however which look at whether this result applies to life annuities. Private annuity markets are indeed a distinctive segment of capital market. The return involves the expected mortality rate of the annuitants. Since it is generally not observed by insurance companies, this leads to an adverse selection problem. Moreover, average longevity tends to increase with income. Both features justify why the analysis of annuity taxation deserves a separate analysis.

This paper studies how annuities should be taxed in a model à la Mirelees (1971) with a continuum of skills, one working period and many retirement periods. It presents two arguments in favor of a taxation of life annuities. First, the taxation should address the adverse selection problem that plagues the annuity market. Indeed, the impossibility to extract or exploit information about individual mortality rates leads insurance companies to offer a common rate of return to all their customers. Compared with a first best economy, it follows that the market price of annuities is too high for the short-lived agents and too low for the long-lived individuals. In this context, the government can restore actuarial fairness by setting a corrective tax schedule on annuities.

A second arguments for annuity taxation comes from redistribution puposes. It relies on the fact that, as the rich are more likely to attain old age, they benefit from a longer stream of annuities in average. A government can then reduce lifecycle inequalities by taxing annuities insofar as they signal consumption by high incomes.

The first argument considered in isolation implies a progressive taxation of annuities. The second argument (the luxury good argument) tells us that the marginal tax rate on annuities varies in the same direction than the optimum marginal tax rate on income. Since the latter is typically decreasing over the main part of the income distribution (except in some cases at the two extremes), the marginal tax rate on annuities should be decreasing as well, implying a regressive tax schedule. The model cannot therefore determine whether the overall effect leads to a progressive or a regressive tax. Next, I turn to a calibrated version of the model. Numerical results suggest that the level of taxation on annuities is significant, highly progressive and increases when annuitants get older.

This is not the first model which adresses the issue of annuity taxation. Saez (2002) assumes in a two period model that the discount rate is positively correlated with skills. As a higher discount rate produces the same effect on future marginal utility than a longer life expectancy, his rationale for taxing saving is close to the luxury good argument presented in this paper for the taxation of annuities. Contrary to Saez, the paper by Bruner and Pech (2008) explicitly focuses on the annuity market in a model with two types of productivity and a single retirement period. They find that the sign of the taxation is undetermined for the less productive and positive for the most productive. With a continuum of workers, the question of the sign of the taxation translates into the issue of the progressivity or the regressivity of the taxation. To this regard, the present model exhibits two opposite functions of the taxation and provides quantitative insights about their respective strength.

The paper proceeds as follows. Section 2 lays out the basic setup of the economy and Section 3 presents some properties of the income and annuity taxation system. In Section 4, the parameters of the model are calibrated and quantitative results are provided. Section 5 concludes.

2 The model

Let us consider an economy with n periods and a continuum of consumers whose productivities (or skills) w are spread over the continuum $W = [\underline{w}, \overline{w}[$ according to the distribution function $F(.)$. The first period is a working period during which agents choose their labor supply L. The remaining dates are retirement periods. Consumption $C = (c_1, c_2, ..., c_n)$ takes place at each date if the consumer survives until then. Let $\pi_i(w)$ denote the survival probability at age i of an individual w who is alive at date 1. It is assumed to be an increasing function of productivity: $\pi_i'(w) \geq 0$.

An individual is characterized by a utility function $U(C, L, w)$ which is additively separable between consumption and leisure ($U_{Li} = 0$, $i = 1, ..., n$ where U_{Li} is the cross derivative between consumption at date i and labor) but not between consumption and productivity since the latter affects the survival probability ($U_{wi} > 0$, $i = 2, ..., n$ and $U_{w1} =$

0). It is also assumed that the preferences for leisure do not vary with skill ($U_{wL} = 0$). In particular, most results will be illustrated with the standard time separable utility function:

$$U(C, L, w) = \sum_{i=1}^{n} \beta^{i-1} \pi_i(w) u(c_i) + v(L) \tag{1}$$

where β is the discount factor, u and v are respectively period utility and disutility of work with the usual concavity and continuity properties.

The uncertainty of survival calls for the purchase of annuities which deliver an income when the subscriber is still alive in exchange for her wealth upon death. In the absence of a bequest motive or uninsurable risk, agents fully annuitize their wealth (see Yaari, 1965). Hence agents simply consumes their annuity while retired. For simplicity, there is no minimal annuity provided by a state program.

Labor and annuities can be taxed by way of separate non-linear schedules in an economy à la Mirelees (1971). The key asumption is that the government cannot observe separately the labor supply and the wage rate. It is thus restricted to setting taxes as a function only of earnings or consumption/annuities. Let $T(wL)$ and $t_i(c_i)$ be repectively the earning tax and the annuity tax at age i. Only the structure of commodity taxes, and not their level, constitutes an independent policy instrument. A uniform commodity taxation can be replicated by an appropriate adjustment in the income tax schedule. The commodity tax rate on first period good is therefore set equal to zero: $t_1(c_1) = 0 \; \forall c_1$.

Let us define the (pooling) rate of return Q_i between date 1 and date i for a consumer still alive after i periods. In order to fund consumption at date i, the insurance company collects at date 1 from individual w the sum $(c_i + t_i(c_i))/Q_i$, so that it delivers the annuity $a_i = c_i + t_i(c_i)$ and the (net of tax) consumption c_i at the specified period. From the company's perpspective, it is liable to the expected sum $(c_i + t_i(c_i))\pi_i/R^{i-1}$. The zero profit condition in the insurance market with unobservable survival rates leads to the equality of the two expected cash flows for the whole market:

$$\int_W (c_i + t_i(c_i))/Q_i dF(w) = \int_W (c_i + t_i(c_i))\pi_i/R^{i-1} dF(w) \tag{2}$$

Given market rates of return Q_i, the programme of the consumer w is given by:

$$\max U(C, L, w)$$
$$\sum_{i=1}^{n} (c_i + t_i(c_i))/Q_i = wL - T(wL)$$

with $Q_1 = 1$. A consumer reaches a level of utility $\mathcal{U}$ which ultimately depends on her wage rate:

$$\mathcal{U}(w) = U(C(w), L(w), w)$$

where $C(w)$ and $L(w)$ maximize her utility given the resource constraint.

The government sets the income and annuity taxes $T(wL)$ and $t_i(c_i)$, $i = 2, ..., n$, by maximizing the integral over the population of a concave function of individual utilities $\int_W \Psi(\mathcal{U}(w))dF(w)$, subject to an aggregate budget constraint

$$\int_W \left[\sum_{i=1}^{n} \pi_i t_i(c_i)/R^{i-1} + T(wL) \right] dF(w) = B, \tag{3}$$

and subject to the constraint that individuals optimize in their choice of labor supply given the relationship between work and after-tax income.

3 Analytical results

I begin by presenting the optimum income tax formula (see Appendix 1 for the details):

$$\begin{aligned}
\frac{T'(wL)}{1 - T'(wL)} &= A(w)B(w)D(w) && (4)\\
A(w) &= 1 + 1/\varepsilon(w) \ ; \qquad \varepsilon(w) = U_L/LU_{LL} \\
B(w) &= \frac{1 - F(w)}{wf(w)} \\
D(w) &= \frac{U_1}{1 - F(w)} \int_w^{\overline{w}} \left(\frac{1}{U_1} - \frac{\Psi'}{E(\Psi')} E(1/U_1) \right) dF(z)
\end{aligned}$$

$\varepsilon(w)$ is the uncompensated elasticity of labor supply, U_1 the derivative with regard to consumption during the working period and $\Psi' = \Psi'(\mathcal{U}(w))$ is the marginal valuation of utility taken at the optimum. The intertemporal nature of the problem does not change how the factors usually found in the litterature enter the income tax formula in a static framework. The labor elasticity ε affects negatively the marginal tax rate because it reflects how much labor supply will be reduced following an increase of the marginal rate. The third term represents the benefit in terms of reduced inequalities and dispersion of marginal utilities from raising additional resources. The last term weighs the importance of those two effects (see also Diamond, 2003).

The marginal annuity tax at date $i > 1$ takes the following form at the optimum (see Appendix 2):

$$
\begin{aligned}
1+t_i'(c_i(w)) &= E(w)\left(1-\eta_i(w)B(w)D(w)\right)^{-1} \qquad (5)\\
E(w) &= \frac{\pi_i(w)Q_i}{R^{i-1}}\\
\eta_i(w) &= \frac{wU_{iw}}{U_i}
\end{aligned}
$$

where $B(w)$ and $D(w)$ are defined in Equations (4). U_i is the derivative with regard to consumption at age i, and U_{iw} the cross derivative with respect to annuity and wage. η_i indicates how much expected marginal utility at retirement is affected by a shift of skill. Its interpretation is straghtforward when $U(C, L, w)$ takes the time seprarable form (1). In that case, $\eta_i(w) = w\pi_i'/\pi_i$ is simply the elasticity of the survival probability at age $i > 1$ with regard to skill.

Eq. (5) shows that the tax rate is the product of two terms $E(w)$ and $(1-\eta_i(w)B(w)D(w))^{-1}$. The first term $E(w)$ satisfies the following property:

Proposition 1. Let us define the level of skill $\widetilde{w} \subset]\underline{w}, \overline{w}[$ satisfying:

$$
\pi_i(\widetilde{w}) = \frac{\int_W (c_i(z) + t_i(c_i(z)))\pi_i(z)dF(z)}{\int_W (c_i(z) + t_i(c_i(z)))dF(z)}
$$

Then $w \geq \widetilde{w}$ ($w < \widetilde{w}$) implies $E(w) \geq 1$ ($E(w) < 1$).

Proof of Proposition 1. Because $\pi_i(\widetilde{w})$ is a weighted average of survival rates over the whole population, it is obvious that $\pi_i(\widetilde{w}) \subset]\pi_i(\underline{w}), \pi_i(\overline{w})[$ and consequently that $\widetilde{w} \subset]\underline{w}, \overline{w}[$. Next, we have from Definition (2) :

$$
E(w) = \frac{\pi_i(w) \int_W (c_i(z) + t_i(c_i(z)))dF(z)}{\int_W (c_i(z) + t_i(c_i(z)))\pi_i(z)dF(z)}
$$

By definition of $\widetilde{w}$, $E(\widetilde{w}) = 1$. Hence $w \geq \widetilde{w}$ ($w < \widetilde{w}$) implies $\pi_i(w) \geq \pi_i(\widetilde{w})$ ($\pi_i(w) < \pi_i(\widetilde{w})$) and consequently $E(w) \geq 1$ ($E(w) < 1$).

Taken in isolation, the first term $E(w)$ implies that the marginal tax rate on annuities received at date i, $t_i'(c_i)$, is positive for agents whose skills is greater than the threshold $\widetilde{w}$. It is negative for agents whose skills is less than $\widetilde{w}$, which means that saving is subsidized fot those agents.

The term $E(w)$ actually deals with the adverse selection problem. Asymmetric information on survival rates leads insurance companies to offer a unique rate of return Q_i to the whole population. The private rate of return is therefore an increasing function of the probability of survival. The term $E(w)$ offsets the distorsion by applying an increasing marginal tax rate on annuities. This compensating effect can be clearly displayed with time separable preferences displayed in Eq. (1). The hypothesized equality $1 + t_i' = E(w)$ would imply that the marginal rate of substitution $u'(c_1)/\pi_i\beta^{i-1}u'(c_i)$ is equal to the marginal rate of transformation R^{i-1}/π_i which would prevail in a first best economy. Hence, taken in isolation, this term equalizes the net of tax return accross all individuals, thereby suppressing the adverse selection problem.

The second term $(1 - \eta_i(w)B(w)D(w))^{-1}$ reflects the fact that the government cares about income inequality. Since the scope of the earning taxation is limited by disincentive effects on labor supply, the government uses annuity taxation as a complementary tool. The more productive agents tend to live longer and therefore value more retirement consumption ($U_{iw} > 0$). The government can then reduce life cycle inequalities by taxing annuities, identified as a luxury good. Hence the greater the elasticity $\eta_i(w)$, the higher the tax rate on annuities.

The annuity tax schedule can alternatively be expressed as a function of the income tax rate (see Appendix 2):

$$1 + t_i'(c_i(w)) = \frac{\pi_i(w)Q_i}{R^{i-1}}\left[1 - \frac{\eta_i(w)}{1 + 1/\varepsilon(w)}\frac{T'(wL)}{1 - T'(wL)}\right]^{-1} \tag{6}$$

The marginal tax rate of labor income carries information about the relevant characteristics of the economy, such as the redistributive tastes of the government and its budget constraint.

Overall, how the tax rate on annuities vary with skill ? The first term of the product in (6) is clearly increasing with skill whereas the second term (the luxury good effect) varies in the same direction than the marginal tax rate of income $T'(wL)$ (assuming that the elasticities η_i and ε do not vary with skill). Since the relevant litterature generally finds that $T'(wL)$ is decreasing over the main part of the income distribution (except in some cases at the two extremes, see for instance Diamond (1998)), the marginal tax rate on annuities is decreasing as well, implying a regressive tax schedule. The overall effect is therefore undetermined.

Eq. (6) is consistent with the results of Brunner and Pech (2008) who study a similar model with two levels of skills. They find that the sign of taxation is unedetermined for the less productive agents and unambiguously positive for the most productive ones. Eq. (6) shows that their last result is a very local one as it only applies to agents endowed with the greatest level of skill and for whom marginal taxation on income is zero.

Note finally that the absolute value of the marginal tax rate on annuities can potentially be very high as $(\eta_i / (1+1/\varepsilon))(T'/(1-T'))$ gets close to unity. This may entail an interior kink in the budget constraint of the consumer and consequently a gap in the distribution of consumptions. Some individual second-order conditions would break in this case. Such a possibility is not explored further at this stage. Instead, it will be checked in the calibration exercise that optimal solutions lead to increasing earnings, which is a necessary and sufficient condition for individual second-order conditions (Mirrlees, 1971).

4 Numerical simulations

The aim of this Section is to provide some quantitative assessments about the general shape of the annuity tax schedule. We would also like to know how much the rate of return is affected by taxation.

4.1 Calibration

Optimal rates simulations are performed using a time separable form for agents' utility and constant relative risk aversion:

$$\sum_{i=1}^{n} \beta^{i-1} \pi_i \frac{c_i^{1-\sigma}}{1-\sigma} - \frac{\gamma^{-1/\varepsilon}}{1+1/\varepsilon} L^{1+1/\varepsilon}$$

The intertemporal elasticity of substitution σ is taken to be 2. Despite a sizeable literature on the labor supply response to changes in the net-of-tax wage, a fair amount of uncertainty persists about the precise empirical value taken by the labor supply elasticity. Moreover, its actual value is likely to depend on individual characteristics like gender or age. Those contingencies will be simply ignored and a baseline value of 0.5 will be chosen for the economy-wide uncompensated labor supply elasticity ε.

The skills w are log-normally distributed, where the mean and the standard deviation are 0.7 and 0.9. The existence of an atom of non workers is also assumed at the bottom of the distribution. They represent 10% of the working force.

The riskless interest rate is taken to be 4%. The subjective discount rate is chosen such that $\beta R = 1$ implying that a constant consumption profile across time would prevail in a first best environment.

For simplicity, a utilitarian social welfare criterion is chosen for the government (that is $\Psi' = 1$). Redistribution takes place in the model through a guaranteed income level (equal to $-T(0)$) that is taxed away as earnings increase. It is set such that the ratio of government spending B to aggregate production is equal to 0.3.

A closer inspection of the government's and consumers' constraints shows that the income taxes and the annuity taxes are not independent tools of redistribution (even though marginal taxes are). Only the intertemporal sum of taxes matters in intertemporal budget constaints. Hence, without loss of generality, the overall tax transfer of the less skilled agents is restricted to the guaranteed income level, meaning that annuity taxes are all set equal to zero: $t_i(c_i(\underline{w})) = 0$, $i > 1$.

The model uses mortality tables by socioeconomic groups provided by Robert-Bobée and Monteil (2005) for the French population who deceased in the middle of the 90s. Two complementary assumptions are made to fit their data into the model. First, the two extreme mortality tables of the less skilled $\underline{w}$ and of the most skilled $\overline{w}$ are identified to the male unskilled workers' and the male executives' tables respectively. Second, the mortality tables for intermediate skills are filled in by assuming that the survival probability proportionally increases with wage between the two extreme tables.

The elasticity $\eta_i(w)$ reflects to what extent the survival probability at date i is shifted when the skill increases by one percent. It is a central parameter for the determination of the marginal tax rate on annuities. Two recent studies analyze this empirical issue for the French population. Jusot (2006) matches two fiscal databases and shows that a strong correlation exists between the mortality risk and the level of income and that this relationship holds across the whole range of income distribution. She also finds that the income elasticity of mortality risk is about -0.50 for the population under 65s and -0.34 for the individuals over 65s. Overall, her results suggest that the elasticity do not vary much across incomes. Accordingly, a constant elasticity assumption will be made. Bommier et al. (2005) use administrative files to study the mortality of several cohorts of retirees between 1997 and 2001. They find elasticity estimates which range from -0.62 for 67 year old men to -0.14 for 91 year old men.

Figure 1 extrapolates those estimates at various ages. Income elasticities of *mortality* risk are first transformed into income elasticities of *survival* risk. If the former is denoted θ_i at age i and the latter η_i, the following formula applies:

$$\eta_i = -\frac{1 - \pi_i}{\pi_i}\theta_i \ , \quad i > 2 \tag{7}$$

Next, the graph for Jusot (2006) in Figure 1 is plotted by assuming that the elasticity of mortality risk remains constant between age 60 and age 82 and equal to $-0,34$. Note that according to Eq. (7), the asumption of a constant elasticity of mortality risk translates into an increasing elasticity of survival risk, as survival rates decrease with age. The graph for Bommier et al. (2005) takes into account two values (-062 for age 67 and $-0,14$ for age) and extrapolates elasticities at other ages by means of a linear combination around these two points.

It is possible to approximate the elasticities of the model by taking its two extreme mortality tables, the one of the unskilled workers and the one of the excutives :

$$\eta_i \sim \frac{\pi_i(\overline{w}) - \pi_i(\underline{w})}{\pi_i(\underline{w})} / \frac{\overline{w} - \underline{w}}{\underline{w}}$$

The last graph plots the corresponding estimates of elasticities where it is assumed that the executives' income is four times greater than the workers' income, which is closed to the empitrical magnitude. We can see that the elasticities found are broadly consistent with their empirical counterparts. Finally, Appendix 3 provides a sketch of the numerical method of simulation.

4.2 Numerical results

Optimal marginal rates on income are plotted in Figure 2 where average income is normalized to unity. Due to a bounded distribution of skills, marginal rate is zero for the most productive agents. It is strictly positive for the less productive ones because of the existence of nonworkers (Seade, 1977). Compared to what is generally found in the litterature (Mirelees, 1971, or Tuomala, 1990), marginal rates do not decrease fast with income. This feature essentially comes from the intertemporal nature of the utility function where income effects seems to be stronger than usually assumed.

Figure 3 shows the optimal marginal tax rates on annuities at three different ages. The tax schedule is increasing with the size of annuities, indicating a progressive mode of taxation. Recall from the analytical section that the tax profile is the product of two opposite patterns. The first one serves to restore actuarial fairness by taxing more heavily agents who live longer in average. The second one reflects the luxury good argument. With a decreasing marginal tax rate on income, it entails a positive, yet regressive, annuity tax. Numerical simulations show that the first argument prevails here. To better assess how these two opposite effects work, Figure 4 compares the marginal tax rate on annuities at age 75 with the "actuarial" tax rate which only compensates for mortality differentials without taking into account the luxury good effect. As expected, the actuarial tax rate is more progressive. It also starts from negative values, reflecting a subsidized rate of return for the small annuities as indicated in Proposition 1.

We have seen that the income elasticity of survival rate η_i is increasing with age in the data. This makes the annuity marginal tax rate shifting upward at later ages. Figure 4 shows this pattern for three different levels of skill, the first quartile, the median skill and the last decile of the distribution. The magnitude of variation is significant. For instance the top ten group faces approximately the same level of marginal taxation at age 70 that the first quartile at age 87.

5 Conclusion

Most governments in developped countries promote retirement saving by offering tax exemptions. On the contrary, this paper concludes that annuity products should be taxed rather than subsidized and that the tax schedule should be progressive. The main argument relies on the well documented fact that the rich live longer in average than the poor and therefore benefit from a higher acturial rate of return. This source of inequality can be corrected by taxation insofar as large annuities reveal consumption by the rich. Moreover, the rich represent an increasing fraction of their age class as they get older. Annuities can therefore be assimilated to a luxury good that the government can tax in order to reduce life-cycle inequalities.

However, the present paper does not aim at presenting a definitive answer to the problem of life annuity taxation. Several improvements which are left for future work could well alter its conclusions. First, a deterministic link has been assumed between income and longevity. Replacing such a link by a mere positive correlation should attenuate the size of the tax rates. Second, it is often argued that individuals do not annuitize as often as theory would predict (Davidoff et al., 2005). This can be due to many reasons like a strong bequest motive, the presence of uncertain medical expenditures or some annuitization through state social security. Non-rational explanations have also been invoked (Brown, 2007). Insofar as a lack of annuitization is a public concern, incorporating those features in a more comprehensive model is also likely to reduce the general level of taxation.

6 Apendices

6.1 Apendix 1: The income tax

Most of the proof is standard and the main lines can be found in Salanié (2003) or Atkinson and Stiglitz (1980). The present proof differs by the existence of a pooling annuity market, nonlinear comodity taxes and a link between the consumer's preference and skills through the probability of survival. The usual caveats concerning the lack of generality of the proof applies. In particular, it is assumed that the first-order condition does indeed characterize an optimum. I exclude the possibility that the distribution of skills results in a distribution of incomes that either has bunching at some income level or a gap in the distribution of incomes.

The first order conditions (FOC) of the consumer's program are given by:

$$\frac{U_i}{U_1} = \frac{1 + t_i'(c_i)}{Q_i} \qquad i = 2, ..., n$$
$$\frac{U_L}{U_1} = -w(1 - T')$$

The FOC and the Enveloppe theorem lead to:

$$\mathcal{U}'(w) = U_w - \frac{LU_L}{w}$$

The budget constraint of the govenment can be purged from the tax schedules in the following way. Insert into the integral the resource constraint of the consumer:

$$\int_W \begin{bmatrix} wL - \sum_{i=1}^n (c_i + t_i(c_i))/Q_i - T(wL) \\ + \sum_{i=1}^n \pi_i t_i(c_i)/R^{i-1} + T(wL) \end{bmatrix} dF(w) = B$$

The constraint simplifies to:

$$\int_W \left[wL - \sum_{i=1}^n (c_i + t_i(c_i))/Q_i + \sum_{i=1}^n \pi_i t_i(c_i)/R^{i-1} \right] dF(w) = B$$

Or, according to the definition (2) of Q_i to:

$$\int_W (wL - \sum_{i=1}^n \pi_i c_i/R^{i-1}) dF(w) = B$$

Hence, the government's program can be stated as:

$$\max \int \Psi(\mathcal{U}(w)) dF(w)$$
$$\int_W (wL - \sum_{i=1}^n \pi_i c_i/R^{i-1}) dF(w) = B$$
$$\mathcal{U}'(w) = -LU_L/w + U_w$$

The c_i, $i = 2, ..., n$ and L are the control variables and $\mathcal{U}$ is the state variable of the Hamiltonian:

$$\mathcal{H} = \Psi(\mathcal{U}) f + \lambda(wL - \sum_{i=1}^n \pi_i c_i/R^{i-1}) f + \mu(-\frac{LU_L}{w} + U_w) \qquad (8)$$

The first date consumption c_1 is determined as an implicit function of $(c_2, ..., c_n, L, \mathcal{U})$ through the definition of $\mathcal{U}$. $L(w)$ maximizes the Hamiltonian (asuming $U_{wL} = 0$):

$$\frac{\partial \mathcal{H}}{\partial L} = \lambda f(w + \frac{U_L}{U_1}) + \mu U_L(-\frac{LU_{LL}}{U_L} - 1)/w = 0$$

Then U_L is replaced by $-w(1-T')U_1$ (FOC of the consumer's problem) and the elasticity of labor U_L/LU_{LL} by ε:

$$\lambda f w T' = -\mu(1-T')U_1\,(1+1/\varepsilon)) \tag{9}$$

or:

$$\mu/\lambda = -\frac{T'}{1-T'} wf \frac{1}{U_1(1+1/\varepsilon)} \tag{10}$$

From the definition of the Hamiltonian (5), μ varies with the wage according to (with $U_{w1} = U_{L1} = 0$):

$$\begin{aligned}\frac{d\mu}{dw} &= -\frac{\partial \mathcal{H}}{\partial \mathcal{U}} = -f\Psi' - \lambda f(-\frac{dc_1}{d\mathcal{U}}) \\ &= (\frac{\lambda}{U_1} - \Psi')f\end{aligned}$$

μ satisfies the two transversality conditions $\mu(\underline{w}) = \mu(\overline{w}) = 0$. Integrating between w and $\overline{w}$:

$$\mu(w)/\lambda = -\int_w^{\overline{w}} (\frac{1}{U_1} - \frac{\Psi'}{\lambda})dF(z) \tag{11}$$

The expression at $w = \underline{w}$ gives:

$$\mu(\underline{w})/\lambda = \int_{\underline{w}}^{\overline{w}} \left(\frac{\Psi'}{\lambda} - \frac{1}{U_1}\right) dF(z) = 0$$

or:

$$\frac{1}{\lambda} = \frac{\int_{\underline{w}}^{\overline{w}} \frac{1}{U_1} dF(z)}{\int_{\underline{w}}^{\overline{w}} \Psi' dF(z)} = \frac{E(1/U_1)}{E(\Psi')} \tag{12}$$

Substituting the left hand term of (9) in (8) leads to:

$$\frac{T'}{1-T'} = (1+1/\varepsilon)\,\frac{U_1}{wf} \int_w^{\overline{w}} \left(\frac{1}{U_1} - \frac{\Psi'}{\lambda}\right) dF(z)$$

Substituting λ by its expression in (12) yields the desired result:

$$\frac{T'}{1-T'} = (1+1/\varepsilon)\,\frac{U_1}{wf} \int_w^{\overline{w}} \left(1/U_1 - \frac{\Psi'}{E(\Psi')} E(1/U_1)\right) dF(z)$$

6.2 Apendix 2: The annuity tax

The date i consumption of individual endowed with w maximizes the Hamiltonian (5), assuming that $U_{w1} = 0$:

$$\frac{\partial \mathcal{H}}{\partial c_i} = -\lambda f(-\frac{U_i}{U_1} + \pi_i/R^{i-1}) + \mu U_{wi} = 0$$

Replacing U_i/U_1 by $(1 + t_i')/Q_i$ (FOC of the consumer's problem) and rearranging the terms:

$$(1 + t_i')\frac{R^{i-1}}{\pi_i Q_i} = 1 - \frac{R^{i-1}}{\pi_i} U_{wi}\mu/\lambda f$$

Combining this equation with (**??**):

$$(1 + t_i')\frac{R^{i-1}}{\pi_i Q_i} = 1 + \frac{R^{i-1}}{\pi_i}\frac{U_i}{U_1}\frac{wU_{wi}}{U_i}\frac{1}{1 + 1/\varepsilon}\frac{T'}{1 - T'}$$

Substituting again U_i/U_1 by $(1 + t_i')/Q_i$ leads to the expression of the annuity tax rate in terms of the income tax rate :

$$1 + t_i' = \frac{\pi_i Q_i}{R^{i-1}}\left[1 - \frac{wU_{wi}}{U_i}\frac{1}{1 + 1/\varepsilon}\frac{T'}{1 - T'}\right]^{-1}$$

Last, the formula of the annuity tax rate can easily be derived from the expression of the income marginal rate.

6.3 Appendix 3: Numerical method

This Appendix presents a sketch of the numerical procedure employed. Numerical results are obtained by discretizing the interval of skills over a fine grid of points. A two-step estimation procedure is used, which is repeated until convergence. First, the consumer problem is solved for every skill of the grid and and for given tax schedules and market returns $\{T_w, T_w', t_{iw}, t_{iw}', Q_i\}$ which take a value for every point of the grid and at each date. The first order conditions are :

$$c_i = \left(\beta^{i-1}\pi_i Q_i\right)^{1/\sigma}(1 + t_{iw}')^{-1/\sigma} c_1$$
$$L = \gamma\left[w(1 - T')\right]^{\varepsilon} c_1^{-\varepsilon\sigma}$$

Replaced in the budget constraint of the consumer with skill w:

$$\gamma w^{1+\varepsilon}(1 - T_w')^{\varepsilon} c_1^{-\varepsilon\sigma} - \left[1 + \sum_{j=2}^{n}\left(\frac{\beta^{j-1}\pi_j}{1 + t_j'}\right)^{1/\sigma} Q_j^{(1-\sigma)/\sigma}\right] c_1 - \left[T_w + \sum_{j=2}^{n} t_{jw}/Q_j\right] = 0$$

This equation has one unknown c_1 and is numerically solved for each level of skill. Later date consumptions and labor supply are retrieved from first order conditions.

Next, those values are exploited to update the vectors $\{T'_w, t'_{iw}, Q_i, T_w, t_{iw}\}$ by using Eq. (4), (6), (2) and the government's budget constraint (3) respectively. Standard techniques of integration are utilized for estimating the integrals. An initial guess for the consumption and labor rules is obtained by assuming a zero value for the elasticity η.

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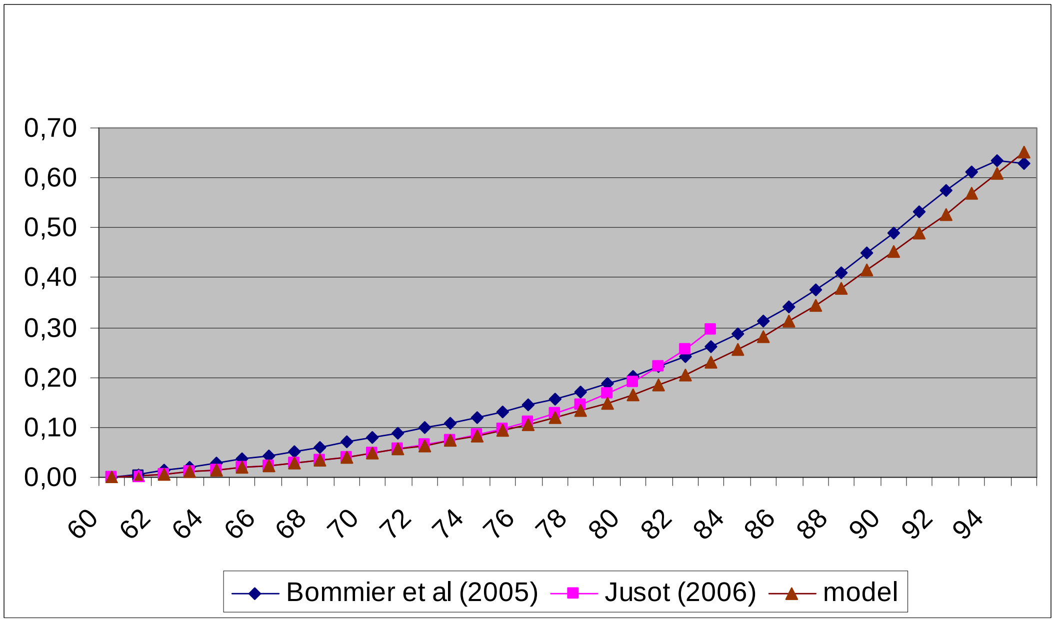

Fig. 1. Estimates of income elasticities of survival risk by ages

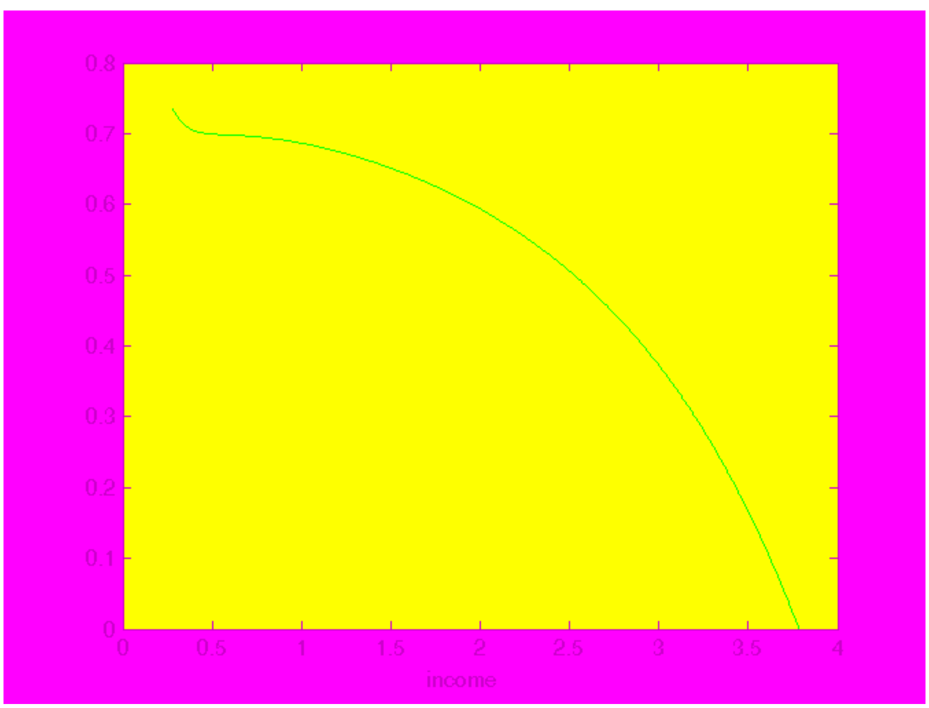

Fig. 2. The optimum marginal tax rate on income

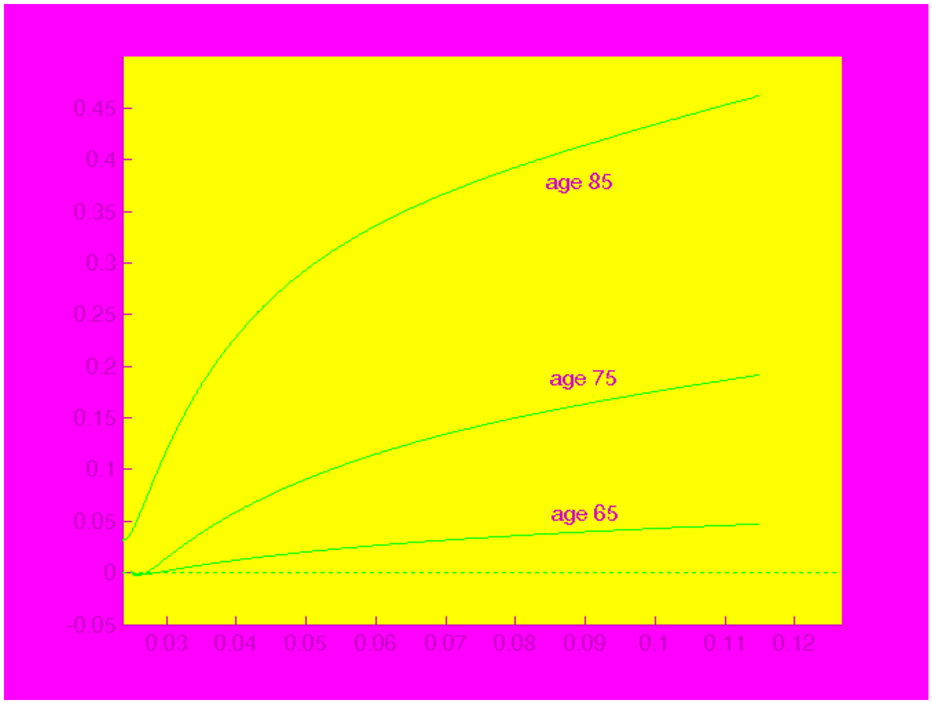

Fig. 3. Marginal tax rates on annuities at three different ages

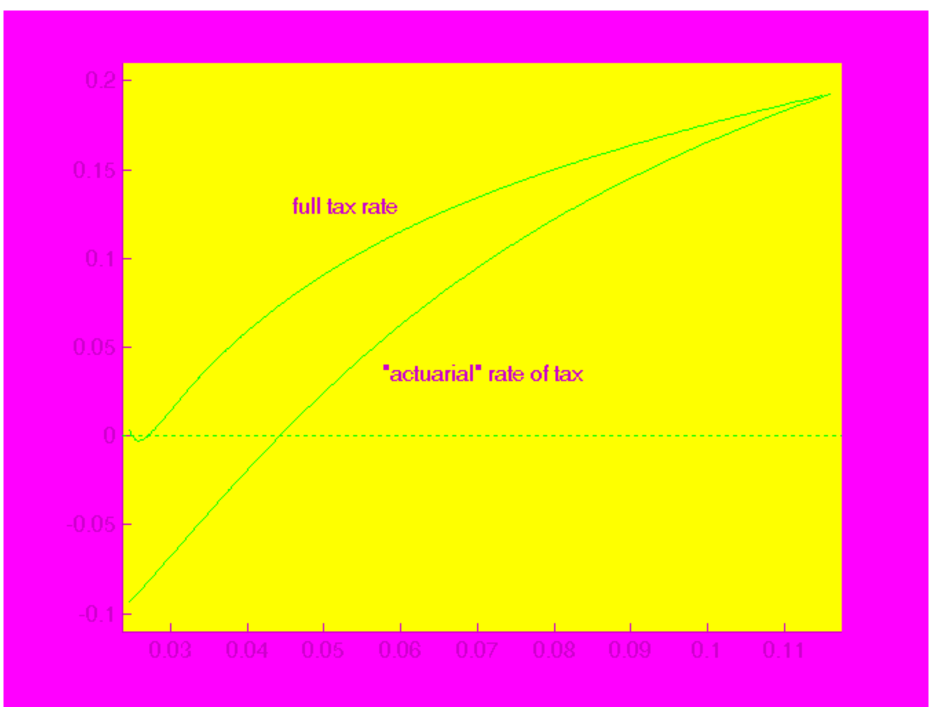

Fig. 4. Comparison with the "actuarial" tax rate (at age 75)

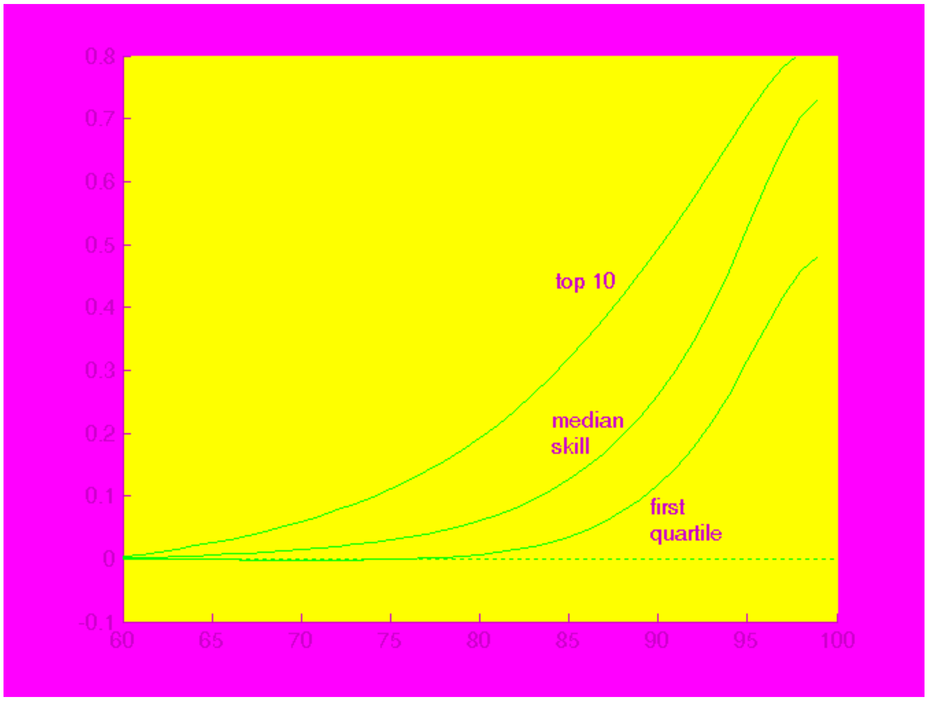

Fig. 5. Marginal tax rates on annuities and age