ASSOCIATION for PUBLIC ECONOMIC THEORY

Optimal environmental policy in developing economies

Koichi Saito
Graduate School of Systems and Information Enginieering, University of Tsukuba

Akira Yakita
Graduate School of Systems and Information Engineering, University of Tsukuba

Abstract

We investigate the optimal policy in a green-Solow-type model in which the government can allocate the tax revenue between productive capital formation and pollution abatement. It is shown that it is optimal to appropriate the tax revenue exclusively to productive capital accumulation in the transition to the long-term optimum, starting with poor productive capital and pristine environment, and that the tax revenue will be allocated to pollution abatement as well as capital formation at the long-term optimum. The Environmental Kuznets Curve, although inverse-V shaped in the present model, may reflect the optimal development and environmental policy.

Submitted: April 01, 2008.

Optimal environmental policy in developing economies

Koichi Saito and Akira Yakita*
Graduate School of Systems and Information Engineering, University of Tsukuba
1-1-1 Tennodai, Tsukuba 305-8573, Japan

Abstract
We investigate the optimal policy in a green-Solow-type model in which the government can allocate the tax revenue between productive capital formation and pollution abatement. It is shown that it is optimal to appropriate the tax revenue exclusively to productive capital accumulation in the transition to the long-term optimum, starting with poor productive capital and pristine environment, and that the tax revenue will be allocated to pollution abatement as well as capital formation at the long-term optimum. The Environmental Kuznets Curve, although inverse-V shaped in the present model, may reflect the optimal development and environmental policy.

Keywords: Green Solow Model; Environmental Kuznets Curve; Pollution abatement; Public investment
JEL Classification: E62; O13; O21; Q28

* Correspondence: yakita@sk.tsukuba.ac.jp (A. Yakita)

1. Introduction

Recently Brock and Taylor (2004) showed that the Environmental Kuznets Curve may be a by-product of convergence to a sustainable growth path in a “Green” Solow model. Thus, at earlier stages of economic development, pollution emissions increase with income growth but near the steady state these emissions decrease with income.[1] Technological progress especially in pollution abatement is primarily responsible to the inverse U-shape in their model. They tested their view using historical evidence of various countries, and suggested that there is considerable evidence of convergence in pollution emission measures. However, they did not obtain the policy implications, assuming that the abatement expenditure/GDP ratio is kept constant along the converging path to the long-term equilibrium.

In this short note, we investigate the optimal policy in a green-Solow-type model in which the government can allocate the tax revenue between productive capital formation and pollution abatement. Among others, a study somewhat close to ours is Economides and Philippopoulos (2008), who investigated the optimal allocation between growth-enhancing public capital formation and pollution abatement. However, they are concerned only with the stationary states. Our focus is on the optimal policy along the developing path of an economy starting with poor stock of productive capital and a pristine natural environment. We show that it is not optimal to appropriate the tax revenue exclusively to pollution abatement on the transition to the long-term optimum.

2. Model

We extend the Solow model by incorporating the quality of environment, in a way similar to Brock and Taylor (2004). The aggregate production technology of goods is assumed constant-returns-to-scale, in an intensive form, $y = f(k)$, where y is per

---

[1] For the Environmental Kuznets Curve, see, for example, Grossman and Krueger (1995).

capita output and k is the capital labor ratio. For simplicity we assume that there is no technological progress in output production and that the labor population remains constant over time. We normalize the population size to unity. Pollution is emitted as a by-product of output in proportion to the level of output: $P = py$ where $p > 0$.[2]

Assuming as in the standard Solow model that the savings rate out of the after-tax income is constant, consumption of a representative agent is given as $c = (1-s)(1-\tau) f(k)$ where s is the savings rate and τ denotes the proportional income tax rate ($0 < s, \tau < 1$).

The government collects income tax and allocates the revenue between productive capital accumulation and pollution abatement. Denoting the expenditures on capital formation and pollution abatement by G and A, respectively, the budget constraint of the government is

$$\tau f(k) = G + A \tag{1}$$

Since our purpose in the present analysis is to investigate the optimal allocation of the revenue, we denote the allocation ratios β and $1-\beta$, respectively; i.e., $G = \beta\tau f(k)$ and $A = (1-\beta)\tau f(k)$.

The evolution of the quality of the environment can be written as

$$\dot{Q} = \eta Q - P + \theta A \tag{2}$$

where Q is the quality of environment (stock), η is the assimilation rate of the nature and θ denotes the efficiency of the abatement expenditure. The dot on a variable means the time derivative of the variable. Assuming for simplicity that public productive capital and private productive capital are perfect substitutes, the evolution of productive capital stock in the economy is then given as

$$\dot{k} = s(1-\tau) f(k) - \delta k + G \tag{3}$$

where δ is the depreciation rate.

The social objective is assumed as the discounted instantaneous utility of a

[2] We do not assume a choice of production technologies of producers à la Stokey (1998).

representative individual $u(c,Q)$: $\int_0^T u(c,Q)e^{-\rho t}dt$ where ρ is the discount rate and T is the planning time horizon. We assume that T is sufficiently great. For expositional simplicity, we also assume the instantaneous utility, $u(c,Q)$, is additively separable.[3] Given the initial conditions $k(0)=k_0$ and $Q(0)=Q_0$, the problem for the government is to choose the tax rate τ and the allocation ratio β so as to maximize the social objective and the end-point conditions $k(T)=k^*$ and $Q(T)=Q^*$.

In the present study we start with a sufficiently small stock of productive capital and a pristine environment, while the end-point conditions of the economic policy are specified such that the capital labor ratio, the quality of environment and the optimal policies satisfy the following two conditions:

$$\rho=(1-\frac{p}{\theta})f'(k^*)-\delta \tag{4a}$$

$$\frac{u_c(c^*,Q^*)}{u_Q(c^*,Q^*)}=\frac{\theta}{\rho-\eta} \tag{4b}$$

where subscripts denote partial derivatives with respect to the variable, $f'(k)=df/dk$, $c^*=(1-s)(1-\tau^*)f(k^*)$, and τ^* is the optimal tax rate. From $\dot{Q}=\dot{k}=0$, we have $\beta^*=1-[p-\eta Q^*/f(k^*)]/(\theta\tau^*)$.[4] We must have $1-(p/\theta)>0$ and $\rho-\eta>0$ for conditions (4) to be economically meaningful. The left-hand side of (4a) is the marginal cost of productive investment and the right-hand side is the marginal net-of-depreciation benefit of investment, taking into account pollution abatement costs. Condition (4b) implies that the marginal benefit of the quality of

[3] The additively separable specification can often be seen in the literature, e.g., Tahvonen and Salo (2001). The complementarity (or substitutability) tends to make the long-term quality of environment greater than the one which would be obtained when $u_{Qc}>0$ (or $u_{Qc}<0$, respectively).

[4] Although the possibility that $\beta^*=1$ can not be ruled out a priori, we will have $\beta^*<1$ since the problem is trivial when $\eta Q-pf(k)>0$, i.e., when natural assimilation is greater than pollution emissions.

environment owing to an additional pollution abatement, $u_Q/(\rho-\eta)$, is equal to the marginal cost in terms of the marginal utility of consumption, u_c/θ. These conditions are obtained at the social optimum (see Appendix).

The current-value Hamiltonian can be given as

$$H = u(c,Q) + \lambda[s(1-\tau)f(k) - \delta k + \beta\tau f(k)] + \sigma[\eta Q - pf(k) + \theta(1-\beta)\tau f(k)]$$

where λ and σ are the shadow prices of productive capital and quality of environment, respectively. The optimal conditions are as follows:

$$H_\tau = [-u_c(1-s) - \lambda(s-\beta) + \sigma\theta(1-\beta)]f = 0 \tag{5a}$$

$$H_\beta = \tau f(\lambda - \sigma\theta) \tag{5b}$$

$$\dot{\lambda} = \lambda\rho - u_c(1-s)(1-\tau)f' \\ - \lambda\{[s(1-\tau)+\beta\tau]f' - \delta\} + \sigma[p - \theta(1-\beta)\tau]f' \tag{5c}$$

$$\dot{\sigma} = \sigma(\rho-\eta) - u_Q \tag{5d}$$

Assuming the existence of the optimal plan at the end-point of which $\lambda = \sigma\theta$, we investigate the properties of the optimal plan. From (5a) to (5d), we can show that the conditions (4a) and (4b) hold when $\lambda = \sigma\theta$.

The dynamic system of the model is that of non-linear four-dimensional differential equations of state variables, k and Q, and co-state variables, λ and σ, i.e., (2), (3), (5c) and (5d). Since it is difficult to solve, we investigate the optimal path to the end point in terms of the shadow prices of productive capital and environmental quality in this and the next sections, while the relation between the shadow prices and the state variables will be examined in Section 4. In the following two sections, therefore, we implicitly consider the state variables in examining the phases of the shadow prices. Since, as is well known, the Solow model does not necessarily lead to the long-term optimum (4) even in infinite time, we may assume that there is an optimal path of the tax rate, $\tau > 0$, satisfying (5a) without loss of generality. Because of the constraint $0 \le \beta \le 1$, form (5b), we have the following three cases: (i) $\beta = 1$ as $\lambda > \sigma\theta$; (ii) $\beta = 0$ as $\lambda < \sigma\theta$; and (iii) $\beta \in (0,1)$ as $\lambda = \sigma\theta$, where the long-term optimum is case (iii).

Case (i) $\lambda > \sigma\theta$

From (5b) we have $\beta = 1$, and, from (5a), $u_c = \lambda$. Condition (5c) can be rewritten as

$$\dot{\lambda} = \lambda[(\rho+\delta) - f'] + \sigma\theta(p/\theta)f' \qquad (6)$$

At this stage, we assume that the (initial) productive capital is sufficiently small, i.e., $(\rho+\delta) - f' < 0$. Therefore, the slope of line $\dot{\lambda} = 0$ is positive but less than one in the $(\lambda, \sigma\theta)$ plane, and $\dot{\lambda} < 0$ as illustrated in Figure 1.[5] On the other hand, the line $\dot{\sigma} = 0$ is represented by a vertical line through $\sigma\theta = \theta u_Q/(\rho - \eta)$.[6] Therefore, we have a phase diagram for $\lambda > \sigma\theta$ as in Figure 1.

Case (ii) $\lambda < \sigma\theta$

From (5b) we have $\beta = 0$ and from (5a) we obtain $-u_c(1-s) = \lambda s - \sigma\theta$. Using it we rewrite (5c) as

$$\dot{\lambda} = \lambda(\rho+\delta) + \sigma\theta[(p/\theta) - 1]f' \qquad (7)$$

From (7) we have the $\dot{\lambda} = 0$ line, i.e., $\lambda = \{[1-(p/\theta)]f'/(\rho+\delta)\}\sigma\theta$ whose slope is greater than 1 when $k < k^*$. Therefore, we have $\dot{\lambda} < 0$ when $\lambda < \sigma\theta$. The line $\dot{\sigma} = 0$ is obtained from (5d) as in the previous case. Therefore, we obtain the phase diagram as depicted in Figure 1 for $\lambda < \sigma\theta$.

3. Optimal policy

Now we consider the optimal policy in the transition to the long-term optimum. Poor physical capital stock and a pristine environment will be reflected in a higher shadow price of physical capital and a lower shadow price of the quality of the

---

[5] We can show that when the slope of line $\dot{\lambda} = 0$ is greater than one, there is no optimal path converging to the end-point. The $\dot{\lambda} = 0$ does not appear in the part of $\lambda > \sigma\theta$ in Figure 1.

[6] We can show that when $u_{Qc} > 0$ (or $u_{Qc} < 0$), the $\dot{\sigma} = 0$ line is downward sloping (or upward sloping, respectively).

environment, say, point I in Figure 2. At the initial point in time, the relatively high quality of the environment will make the marginal utility from the environmental quality very low, that is, u_Q is very small. The $\dot{\sigma}=0$ line is close to the origin. The line is depicted by a dotted line. An optimal transition path of the economy starting from I can be depicted as in Figure 2. It seems plausible that the shadow price of the environment is very small at the initial point in time, and that it increases along the development path. Thus, from (5d), we will have $\sigma\theta > \theta u_{Qc}/(\rho-\eta)$ at the initial point, that is, $\sigma\theta$ lies on the right-hand side of the $\dot{\sigma}=0$ line.[7] On the other hand, when the capital labor ratio is very small, its shadow price will be very high.

Near the initial point, the productive capital accumulation is accelerated by the tax revenue allocation policy since $\lambda > \sigma\theta$. The accumulation of productive capital decreases its shadow price. Given that the initial point I is on the right-hand side of the $\dot{\sigma}=0$ line, the shadow price of the environmental stock rises as the environmental quality deteriorates. However, the accumulated capital makes the allocation policy less pro-capital-accumulation, and thereby brings about greater consumption relative to output. The deceleration of productive capital accumulation tends to set back the deterioration of the environmental quality. In other words, while the $\dot{\sigma}=0$ line shifts rightward as the quality of environment deteriorates owing to pollution emissions caused by high productive capital accumulation, the rightward shift becomes sufficiently rapid, and the path of the shadow prices $(\sigma\theta,\lambda)$ will be on the intersection of the 45 degree line and the $\dot{\sigma}=0$ line, i.e., the long-term optimum (E in Figure 2), by the end of the planning period, T.[8]

In the transition, therefore, the shadow price of productive capital declines and the shadow price of environment quality increases monotonically. The optimal

[7] We can not rule out the possibility that the shadow price of the environmental quality satisfies $\sigma\theta = \theta u_{Qc}/(\rho-\eta)$, i.e., on the $\dot{\sigma}=0$ line. Even in this case, our result is not altered essentially.

[8] If the path continues to be on the right-hand side of the $\dot{\sigma}=0$ line, it soon crosses the 45 degree line and will then diverge to the southeast, as depicted by the dotted arrow in Figure 2.

transitional path is above the 45 degree line, that is, in the range of $\lambda > \sigma\theta$. Finally, at the long-term optimum where $\lambda = \sigma\theta$, the tax revenue will be allocated to pollution abatement as well. Therefore, the policy specified in pollution abatement ($\beta = 0$) can not be optimal on the transition to the long term optimum.

Therefore, we have the following results:

Result 1

On the transition to the long-term optimum, the resources should not be allocated to pollution abatement, while, once the long-term optimum is attained, the environmental quality should be supported optimally by the environmental investment policy.

The abatement investment policy will be undertaken only after the optimal level of capital stock is attained, that is, the optimal allocation to pollution abatement moves from a corner solution to an interior solution. On the long-term optimum, the positive environment investment prevents the environment from deteriorating with output production.[9]

4. Productive capital and environmental quality

The optimal paths of state variables, k and Q, are depicted in Figure 3. As mentioned in the text, when $k > k^*$, we have

$$\dot{\lambda} = \lambda[(\rho+\delta) - f'] + \sigma\theta(p/\theta)f' < \lambda\{(\rho+\delta) - [1-(p/\theta)]f'\} < 0$$

i.e., the shadow price of capital stock per capita monotonically decreases along the optimal path. The path is illustrated in the (k,λ) plane of the second quadrant of Figure 3. It should be noted that the $\dot{\lambda} = 0$ line in the (k,λ) plane shifts as the

[9] In the present model, environmental quality will not improve even with pollution abatements, but should be kept at the "threshold" level in contrast to John and Pecchenino (1994), who suggested that environmental quality will begin improving with economic growth after the "threshold" point.

shadow price of the environmental quality changes.

On the other hand, the $\dot{\sigma} = 0$ line in the $(Q, \sigma\theta)$ plane of the fourth quadrant of Figure 3, is given as the combinations Q and $\sigma\theta$ satisfying $\sigma\theta = \theta u_Q(c,Q)/(\rho - \eta)$. When the environmental quality is at the pristine level and the consumption level is low due to poor capital stock, u_Q is very low. At earlier stages in which productive capital accumulation is politically accelerated, the quality of environment declines due to pollution emissions, while the shadow price, σ, increases. Then, since capital accumulation brings about greater output and thereby consumption, the marginal utility of the environmental quality rises together with degradation of the environment. At the long-term optimum point, we have $\dot{\sigma} = 0$, or equivalently, $\sigma\theta = \theta u_Q(c,Q)/(\rho - \eta)$.

So far we are not concerned with the path of the tax rate since our purpose in the present paper is to analyze how the tax revenue should be allocated between growth-encouraging and environment-enhancing policies. Although we can not solve explicitly the path of the tax rate, we can conjecture it from the above analysis. In the earlier stages, the tax rate is likely to be high in order to accelerate and finance (public) capital formation. However, as it draws near to the long-term optimum, the government will seek to improve the welfare by increasing consumption through lower tax rates.

5. Concluding remarks

We have showed that it may be optimal to spend on productive capital formation but not on pollution abatement along the transition path toward the long-term optimum, and that the tax revenue is also optimally allocated to pollution abatement, but only after the long-term optimum is attained. The relationship between income and pollution in the present model therefore has a skewed inverted-V shape, where pollution is measured in net-of-abatement terms. The Environmental Kuznets Curve may reflect the optimal development and environmental policy.

So far, in contrast to Brock and Taylor (2004), we have not considered the effects of technological progress.[10] Endogenizing pollution-abatement technologies and the policy effects on the technological progress along the development path are interesting issues for future research.

[10] Andreoni and Levinson (2001) suggest that increasing return in abatement is crucial to the inverse-U-shaped Environmental Kuznets Curve. Copeland and Taylor (2003) classified the mechanisms of the Curve as involving either income effects, threshold effects or increasing return to abatement.

Acknowledgements

The authors wish to thank Makoto Hirazawa and the seminar participants at Nagoya Macroeconomics Workshop for their incisive comments.

References

Andreoni, J., Levinson, A., 2001. The simple analytics of the Environmental Kuznets Curve. Journal of Public Economics 80, 269-286.

Brock, W. A., Taylor, M. S., 2004. The Green Solow Model. NBER Working Paper 10557.

Brock, W. A., Taylor, M. S., 2005. Economic growth and the environment: A review of theory and empirics. In: P. Aghion and S. N. Durlauf (eds.) Handbook of Economic Growth. North-Holland; Amsterdam, pp. 1749-1821.

Copeland, B. R., Taylor, M. S., 2003. International trade and the environment: theory and evidence. Princeton University Press; Princeton, NJ.

Economides, G., Philippopoulos, A., 2008. Growth enhancing policy is the means to sustain the environment. Review of Economic Dynamics 11, 207-219.

Grossman, G., Krueger, A., 1995. Economic growth and the environment. Quarterly Journal of Economics 101, 353-377.

John, A., Pecchenino, R., 1994. An overlapping generations model of growth and the environment. Economic Journal 104, 1393-1410.

Stokey, N. L., 1998. Are there limits to growth? International Economic Review 39, 1-31.

Tahvonen, O., Salo, S., 2001. Economic growth and transitions between renewable and nonrenewable energy resources. European Economic Review 45, 1379-1398.

Appendix : First best optimum

Consider the optimization problem of the social planner:

$$\underset{c,A,k,Q}{Max} \int_0^{\infty} u(c,Q)e^{-\rho t}dt$$

subject to $\dot{k} = f(k) - c - A - \delta k$

$$\dot{Q} = \eta Q - pf(k) + \theta A$$

and $k(0) = k_0$; $Q(0) = Q_0$

where A denotes output allocated to pollution abatement. The current-value Hamiltonian is $H = u(c,Q) + \lambda[f(k) - c - A - \delta k] + \sigma[\eta Q - pf(k) + \theta A]$. The optimal conditions are as follows:

$$H_c = u_c - \lambda = 0 \tag{A1}$$

$$H_A = -\lambda + \sigma\theta = 0 \tag{A2}$$

$$\dot{\lambda} = \lambda\rho - \lambda(f' - \delta) + \sigma p f' \tag{A3}$$

$$\dot{\sigma} = \sigma\rho - u_Q - \sigma\eta \tag{A4}$$

and the transversality conditions. We consider here that case $A > 0$ at the optimum is plausible. Letting $\dot{\lambda} = \dot{\sigma} = 0$ in (A3) and (A4) and making use of (A1) and (A2), we obtain (4a) and (4b) in the text.

Figure 1　Phase diagram

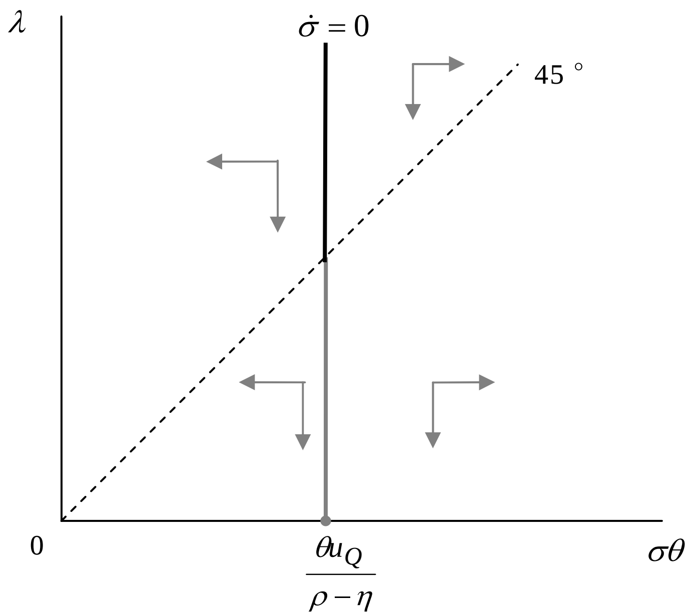

Figure 2　Optimal path

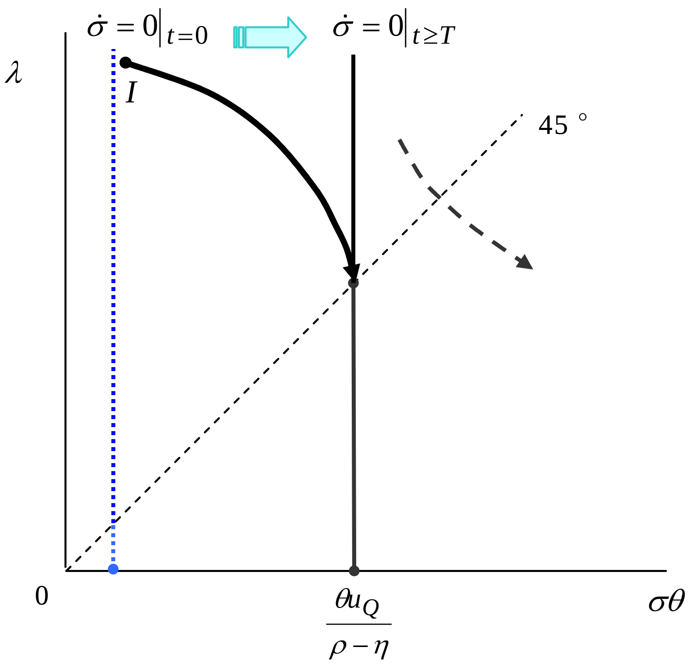

Figure 3