Endogenous Mergers under Multi-Market Competition

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Abstract

This paper examines a simple model of strategic interactions among firms that face at least some of the same rivals in two related markets (for goods 1 and 2). It shows that when firms compete in quantity, market prices increase as the degree of multi-market contact increases. However, the welfare consequences of multi-market contact are more complex and depend on how two fundamental forces play themselves out. The first is the selection effect, which works towards increasing welfare as shutting down the more inefficient firm is beneficial. The second opposing effect is the internalisation of the Cournot externality effect reducing the production of good 2 allows firms to sustain a higher price for good 1. This works towards increasing prices and, therefore, decreasing consumer surplus (but increasing producer surplus). These two effects are influenced by the degree of asymmetry between markets 1 and 2 and the degree of substitutability between goods 1 and 2.

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1   Introduction

Multi-market competition refers to the situation in which a firm faces at least some of the same rivals in multiple markets.[1] More specifically, under multi-market competition a firm that is active across multiple markets might find itself competing with firms that are also present in multi-markets and firms that are only present in a given market. This is a pervasive phenomenon in modern economies and the subject of this paper.

Examples of multi-market competition can be found in the telecommunications, banking, and air transportation industries. Telecommunication carriers compete in mobile and fixed telephony, voice and data services with companies that provide the full gamut of services and companies that only provide a subset of the services (e.g., data services). Commercial banks offering a full portfolio of financial products such as insurance, home loans, personal loans and credit cards compete not only with other full service banks but also with providers that offer only home loans or personal loans or insurance. Full service airlines (and their discount airline subsidiaries) compete with other full service airlines and with discount airlines.

Economic conventional wisdom once suggested[2] that when firms compete against the same rivals in multiple markets, the intensity of competition may suffer. The mechanism(s) through which competition would be softened were not, however, well understood. Bernheim and Whinston's (1990) seminal paper suggested a mechanism through which competition would suffer with multi-market contact: concerted or coordinated effects. By considering a supergame model where firms repeatedly compete with each other over time, these authors show that when firms interact in multiple markets, the opportunities for punishing deviations from collusive outcomes are enhanced. As punishing deviators becomes easier under multi-market competition, it is easier to sustain cooperative outcomes.[3] We should stress that

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[1] See Chen and Ross (2007).

[2] See, for example, Edwards (1955).

[3] Scott (1982, 1983) uses cross-industry data and find a positive link between multi-market contact and profits. Additional support for the hypothesis that multimarket contact leads to higher prices is also found in several single-indusry studies. Examples include Parker and Roller (1997) in telecommunications and Pilloff (1999) in banking.

this theoretical work is by no means conclusive. The collusive equilibrium identified by Bernheim and Whinston is one of the infinite many equilibria that result from the application of the folk theorem to infinitely-repeated games.

Our emphasis is, however, on unilateral effects. We are interested in the short-run strategic interactions that arise when firms compete across different markets. We provide a direct mechanism through which increased multi-market contact leads to higher prices.

We also explore two related questions in this paper. The first question is normative in nature. We examine different market structures – from no multi-market contact to full multi-market contact – and investigate market outcomes (e.g., prices and quantities) and welfare. We identify the socially optimal market structure contingent on the degree of asymmetry between the markets. The second question is positive in nature. We then ask the following question, if we allow firms to merge, which mergers would be profitable and what would be the likely resulting market structure.

Our paper is closely related to Chen and Ross (2007). They focused on the effects of multi-market competition on prices and welfare when firms serve two different markets with a single production facility and an increasing marginal cost technology. Although the demand functions are independent in their model, the link between the markets arises as the larger the production in one market, the higher the marginal cost in the other market. These authors then use this framework to explain phenomena that are not fully understood in competition analysis: the issues of recoupment (lower prices in one market are compensated by higher prices in other markets) and retaliatory entry. In contrast, our model considers interdependent demands and focuses on the impacts of mergers on prices, welfare and market structure with constant marginal costs.

Our analysis of multi-market competition has potentially important implications for competition law and policy. Standard merger analysis is concerned with price and welfare effects in the relevant market – a market that is defined essentially by the substitution possibilities. Our analysis suggests that although goods might not be in the same relevant market from the point of view of competition law, under multi-market competition a merger

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might have more complex effects (both positive and negative) beyond the immediate relevant market. It stands to reason that competition analysis should take such effects into consideration.

2   The Basic Setup

We consider preferences for goods 1 and 2 represented by the following social welfare function:

$$U = m + \alpha_1 Q_1 + \alpha_2 Q_2 - \frac{1}{2} \left(Q_1^2 + 2\gamma Q_1 Q_2 + Q_2^2 \right), \tag{1}$$

where m represents all other goods in the economy. The inverse demand curves for the two goods are given by:

$$P_1 = \alpha_1 - (Q_1 + \gamma Q_2) \tag{2}$$

and

$$P_2 = \alpha_2 - (\gamma Q_1 + Q_2). \tag{3}$$

The parameter γ measures the degree of product differentiation. If $\gamma = 0$, the demand for the two goods are independent. If $\gamma > 0$ the two goods are substitutes, and the two goods are complements if $\gamma < 0$. The analysis in this paper focuses on the case of substitutes. We assume that the marginal costs of production in markets 1 and 2 are equal to c_1 and c_2, respectively, and that there are no fixed costs. Under this framework, the total surplus (denoted by TS) is derived from the utility function given in Equation 1. Consumer surplus (CS) is defined as $TS - \Pi$, where Π is the sum of firms' profits.

The following definition is helpful in keeping our notations as simple as possible:

Definition 1 *Let $a \equiv (\alpha_1 - c_1) - (\alpha_2 - c_2)$ and $\alpha_1 - c_1 = 1$. Without loss of generality, assume $a \geq 0$.*

The index a summarises the asymmetry between the two markets. For $a = 0$, the two markets are symmetric.

We discuss three market structures in this paper. In our benchmark market structure (see Figure 1), there are two firms (A and B) that produce

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good 1 and two firms (C and D) that produce good 2. Firms compete by setting quantities. This simple framework allows us to capture both closer intra-market competition (e.g., between firms A and B) and also more distant inter-market competition (e.g., between firms in market 1 and firms in market 2). Moreover, it also allows us to investigate the consequences of changes in the market structure that affect intra and inter-market competition. In particular, we will consider a market structure where firms A and C and are allowed to offer both goods while facing different rivals in each market (see Figure 2). We refer to this market structure as partial multi-market contact. In the last structure we consider, both A and C and B and D have merged so that these two firms compete with each other in both markets (see Figure 3). We refer to this as full market contact (F).
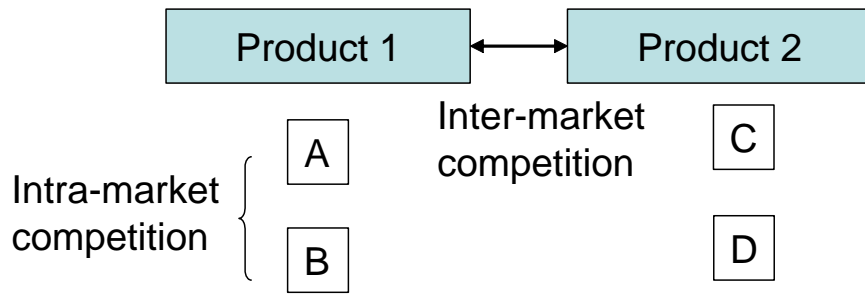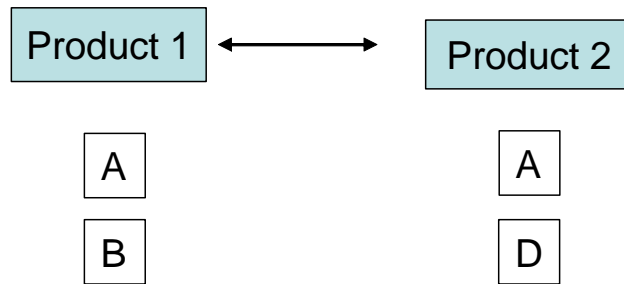

Figure 1: The Benchmark (B).

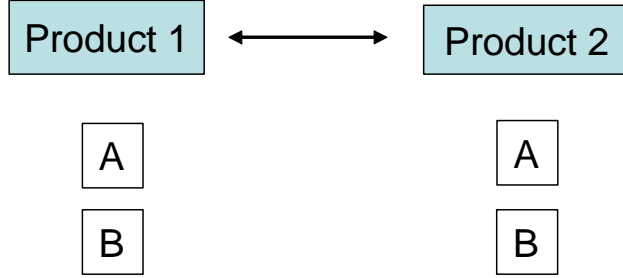Figure 2: Partial Multi-Market Contact (P).

Figure 3: Full Multi-Market Contact (F).

2.1 Market Equilibrium

In this subsection we characterise the market equilibria under the various market structures. This is presented in Table 1 below. As the degree of asymmetry, a, increases, firms cease offering product 2. Different market structures have different critical a values for corner solutions to eventuate. Since multi-market firms have more incentives to exit market 2 to internalise the externality between the two markets, the critical value a is the the lowest in market structure F ($a \geq 1 - \gamma$) and the highest in market structure B ($a \geq \frac{3-2\gamma}{3}$). For market structure P, the critical value of a is in between the other two cases, $a \geq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$. Since for $a \geq \frac{3-2\gamma}{3}$, all market structures give the same market outcome with $Q_2 = 0$, we present the analysis for the case $a < \frac{3-2\gamma}{3}$. All proofs are in the appendix. Omitted for simplicity, in all cases, the consumer surplus is computed as:

$$CS = TS - \Pi = \gamma Q_1 Q_2 + \frac{1}{2}Q_1^2 + \frac{1}{2}Q_2^2. \tag{4}$$

In the next section, we illustrate how the two fundamental effects – the selection and the internalisation of the Cournot externality effects – drive the strategic interaction among firms in the three market structures.

2.2 Prices and Welfare

In this subsection we present price and welfare comparison across the three market structures. The proposition below shows that prices for both goods

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	$a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$	$\frac{(3-\gamma)(1-\gamma)}{3+\gamma^2} \leq a \leq 1-\gamma$	$1-\gamma \leq a$
B	$q_A = q_B = \frac{3-2\gamma(1-a)}{(3+2\gamma)(3-2\gamma)};\ q_C = q_D = \frac{3(1-a)-2\gamma}{(3+2\gamma)(3-2\gamma)};\ Q_1 = 2q_A;$ $Q_2 = 2q_C;\ \pi_A = \pi_B = \frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2};\ \pi_C = \pi_D = \frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}.$		
P	$q_{A1} = \frac{4a\gamma - 4\gamma + \gamma^2 + 3}{(\gamma+1)(3-\gamma)(\gamma+3)(1-\gamma)};$ $q_{A2} = \frac{(3+\gamma^2)(1-a)-4\gamma}{(1+\gamma)(1-\gamma)(3-\gamma)(3+\gamma)};$ $q_B = \frac{3-\gamma(1-a)}{(3+\gamma)(3-\gamma)};\ q_D = \frac{3(1-a)-\gamma}{(3+\gamma)(3-\gamma)};$ $Q_1 = q_{A1} + q_B;\ Q_2 = q_{A2} + q_D;$ $\pi_A = \frac{2(1-\gamma)(3-\gamma)^2(1-a)+9a^2+7a^2\gamma^2}{(3+\gamma)^2(3-\gamma)^2(1+\gamma)(1-\gamma)};$ $\pi_B = \frac{(3-\gamma(1-a))^2}{(3-\gamma)^2(\gamma+3)^2};\ \pi_D = \frac{(3-3a-\gamma)^2}{(\gamma+3)^2(3-\gamma)^2}.$	$q_{A1} = q_B = \frac{2-\gamma(1-a)}{2(3-\gamma^2)};$ $q_{A2} = 0;\ q_D = \frac{3(1-a)-2\gamma}{2(3-\gamma^2)};$ $Q_1 = \frac{2-\gamma(1-a)}{(3-\gamma^2)};\ Q_2 = \frac{3(1-a)-2\gamma}{2(3-\gamma^2)};$ $\pi_A = \pi_B = \frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2};$ $\pi_D = \frac{(3a+2\gamma-3)^2}{4(3-\gamma^2)^2}.$	
F	$q_{A1} = q_{B1} = \frac{1-\gamma(1-a)}{3(\gamma+1)(1-\gamma)};\ q_{A2} = q_{B2} = \frac{1-a-\gamma}{3(1-\gamma)(\gamma+1)}$ $Q_1 = 2q_{A1}, Q_2 = 2q_{A2};\ \pi_A = \pi_B = \frac{2-2\gamma(1-a)-2a+a^2}{9(1+\gamma)(1-\gamma)}$		$q_{A1} = q_{B1} = \frac{1}{3};$ $q_{A2} = q_{B2}^* = 0;$ $Q_1 = \frac{2}{3}, Q_2 = 0;$ $\pi_A = \pi_B = \frac{1}{9}.$

Table 1: Summary of the market equilibrium.

are always higher under full multi-market contact. This is intuitive since the multi-market firm has less incentives to expand its output. Output expansion in one market hurts not only its profitability in the given market, but also the profitability in the other market. In the tables, we use superscript B (P, F) to denote variables for market structure B (P, F).

Proposition 1 *The ranking of market prices is summarised in Table 2 below.*

	$a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$	$\frac{(3-\gamma)(1-\gamma)}{3+\gamma^2} \leq a \leq 1-\gamma$	$1-\gamma \leq a$
P_1	$P_1^B \leq P_1^P \leq P_1^F$	$P_1^B \leq P_1^P \leq P_1^F$	$P_1^B \leq P_1^P \leq P_1^F$
P_2	$P_2^B \leq P_2^P \leq P_2^F$	$P_2^B \leq P_2^P \leq P_2^F$	$P_2^B \leq P_2^P$ P_2^F not defined

Table 2: Price rankings.

Proof. With market outputs given in Table 1, market prices are computed through Equations 2 and 3. The price comparison is straightforward, and the proof is not included here. The proof is available upon request. ∎

While Proposition 1 is perhaps not surprising, the welfare comparison is less straightforward as indicated in the following proposition:

Proposition 2 *The welfare ranking is summarised in Table 3.*

$a \leq \frac{\left(8\gamma^3-20\gamma^2-24\gamma+45\right)(3-2\gamma)}{135-12\gamma^2-16\gamma^4}$	$TS^B > TS^P > TS^F$
$\frac{\left(8\gamma^3-20\gamma^2-24\gamma+45\right)(3-2\gamma)}{135-12\gamma^2-16\gamma^4} \leq a \leq \frac{\left(9-3\gamma-4\gamma^2\right)(3-2\gamma)}{3(9-2\gamma^2)}$	$TS^P > TS^B > TS^F$
$\frac{\left(9-3\gamma-4\gamma^2\right)(3-2\gamma)}{3(9-2\gamma^2)} \leq a \leq \frac{16\gamma^3-12\gamma^2-78\gamma+81}{81-12\gamma^2}$	$TS^P > TS^F > TS^B$
$a \geq \frac{16\gamma^3-12\gamma^2-78\gamma+81}{81-12\gamma^2}$	$TS^F \geq TS^P \geq TS^B$

Table 3: Welfare rankings.

Proof. See the Appendix. ∎

The results in Proposition 2 reflects the tension between the selection effect (shutting down the inefficient firms is beneficial) and the internalisation of the Cournot externality effect (reducing the production of good 2 allows firms to sustain a higher price for good 1). These two effects are influenced by the degree of asymmetry (a) between markets 1 and 2 and the degree of substitutability (γ) between goods 1 and 2.

From the consumer's point of view, market structure B always yields the highest surplus since prices are the lowest. However, as a increases, the asymmetry between the two markets increases, and social welfare may increase with the presence of multi-market firms since there is more efficiency gain from reducing the production of good 2. Therefore, with a low a, the social welfare is the highest in market structure B. As a gets very large, market structure F dominates. Market structure P is the best for intermediate values of a. Note that all the critical a values listed in Table 3 decrease as γ increases. The band for market structure P to maximise the social welfare is the widest for intermediate values of γ.

3   Endogenous Mergers

This section examines two related questions. First, we ask what mergers are profitable in each market structure. For the full multi-market structure, there is only one merger possible – a merger from two firms producing the

two goods to a single firm producing two goods. Such merger to monopoly is clearly profitable. The determination of the profitability of mergers for the two other market structures is more complex and it is summarised by Propositions 3 and 4 below. These propositions also allow us to answer a second question: what market structure is more likely to arise in an environment where the benchmark firms were allowed to pursue any profitable mergers? We present the key factors in these two propositions, and the detailed conditions are available in the appendix.

Proposition 3 *Conditions for profitable mergers in the benchmark market structure are summarised in Table 4.*

	$a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$	$\frac{(3-\gamma)(1-\gamma)}{3+\gamma^2} \leq a \leq 1 - \gamma$	$1 - \gamma \leq a$
Inter-market (e.g., A & C)	not profitable	not profitable	large a
Intra-market (e.g., A & B)	$\gamma \leq 0.66$	$\gamma \leq 0.66$	$\gamma \leq 0.66$
Inter + intra (e.g., A B C)	large a or $\gamma \leq \sqrt{\frac{1}{2}}$ and small a	large a	$\gamma \leq 0.89$ or large a

Table 4: Profitable mergers under market structure B.

Proof. See the Appendix. ∎

We should note that for the merger between three firms – for example, firms A, B, and C – the merger profitability analysis is undertaken against the pre merger profits, π_A, π_B, and π_C. This is the standard approach. Different answers may be obtained if the reference point is a two-firm merger first – for example, firms A and B – followed by the profitability analysis of adding another firm – for example, firm C – into this coalition.

Proposition 3 suggests that whether or not an intra-market merger is profitable depends only on γ. For $\gamma = 0$, the two markets are independent, and an intra-market merger is simply a merger between duopolists to form a monopolist. Such a merger is always profitable. This suggests that under our set-up with both inter and intra competition, a merger of the two firms within one market is only profitable if the two markets are relatively isolated.

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Proposition 3 also shows that an inter-market merger is profitable for large a. In particular, a two-firm inter-market merger is only profitable in the parameter range where the merged entity ceases production in market 2. The merged entity produces more of good 1 and shuts down the production of good 2. This yields higher profits in market 1 when a is large. In this case it is also easier for this merger to satisfy the incentive compatibility constraints as the firm in market 2 would have lower pre-merger profit.

For a merger between three firms (inter- plus intra-market merger), the condition required is typically a large a. A lower γ reduces the threshold a required. It is possible for a three-firm merger to be profitable for small a and relatively isolated markets ($\gamma \leq \sqrt{\frac{1}{2}}$). In this parameter range ($a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$), the asymmetry between the markets is small and efficiency gains are therefore low. The merged entity continues to produce both goods. For a merger to be profitable, it must then involve firms with a large combined output in the market. This result is analogous to the classic result of Salant, Switzer and Reynolds (1983) that a merger among symmetric firms is not profitable unless it involves 80% of the firms in the industry.

The proposition below summarises the profitability analysis for mergers under partial multi-market contact.

Proposition 4 *Conditions for profitable mergers with partial multi-market contact are summarised in Table 5.*

	$a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$	$\frac{(3-\gamma)(1-\gamma)}{3+\gamma^2} \leq a \leq 1-\gamma$	$1-\gamma \leq a$
Inter-market B & D	Not profitable	$\gamma \leq 0.77$ and large a	$\gamma \leq 0.77$ or large a
Intra-market A & B	$\gamma \leq 0.77$ and large a	$\gamma \leq 0.77$	$\gamma \leq 0.77$
Intra-market A & D	$\gamma \leq 0.6$ and small a	$\gamma \leq 0.77$ and large a	$\gamma \leq 0.77$ or large a
$AB \succ AD$	for large a	if both mergers are profitable	

Table 5: Profitable mergers under market structure P.

Proof. See the Appendix. ∎

Since a merger in this market structure involves both intra- and inter-market merger, in general, profitable mergers require γ to be small and a to be large. An exception is the profitable AD merger for the parameter range, $a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$. In this case, the asymmetry is small between the two markets, there is no corner solution and the merged entity would continue to produce both products. Thus, A and D would only have the incentives to merge if a is small and firm D also has significant pre merger output share.

These propositions also allow us to consider the following question. Starting with the benchmark, if firms A and C were to merge, would firms B and D find it profitable to merge (resulting in structure F) or would B and D prefer to stay separate (resulting in structure P)? From the benchmark, firms A and C only have the incentive to merge for high values of a. Furthermore, the critical a value for profitable AC merger in market structure B is higher than the critical a value for profitable BD merger in market structure P. Therefore, if AC merger is profitalbe, firms B and D would always have the incentive to merge. For high values of a, market dynamics might naturally result in a market structure where firms operate in multiple markets. Importantly, for high values of a, market structure F yields the highest social welfare.[4] Note that for intermediate values of a, structure P maximises social welfare but this structure is unlikely to emerge given that the associated merger is not profitable.

With the inclusion of both inter- and intra-market competition, first, there exists endogenous mergers. Even with the presence of the outsider firms, some firms would still have the incentives to merge. The optimal market structure depends on both a and γ. The welfare effects of merger thus also depend on both a and γ.

4 Conclusion

This paper examines a simple model of multi-market competition. It shows that when firms compete in quantity, although full multi-market contact might lead to higher prices, the welfare consequences are more complex

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[4]Recall that this follows from the portfolio effect (that is, producing less of the inefficient good 2) rather than from lower prices.

and depend on how two fundamental forces play themselves out. The first is the selection effect, which works towards increasing welfare as shutting down the inefficient firm is beneficial. The second opposing effect is the internalisation of the Cournot externality effect; reducing the production of good 2 allows firms to sustain a higher price for good 1. This works towards increasing prices and, therefore, decreasing the consumer surplus (but increasing the producer surplus). These two effects are influenced by the degree of asymmetry between markets 1 and 2 (a) and the degree of substitutability (γ) between goods 1 and 2. The higher a is, the more relatively inefficient market 2 is, and the stronger the selection effect. The higher γ is, the more closely linked the two markets are and the stronger the externality effect would be. A merger would internalise the effects more when γ is large. This would make the merged entity a lot less aggressive and hence unlikely to raise profits for the merged entity. On top of this, a lower γ would imply more isolated markets and would make intra-market merger more profitable. Therefore, the general result is that merger is more likely to be profitable when γ is low.

This analysis should be viewed as a preliminary step towards understanding the dynamics of multi-market competition. It simply illustrates that mergers can increase welfare under multi-market competition. Although this result is not *per se* new[5], its novelty arises from the fact the increase in welfare might not originate from the market (as strictly defined from a competition analysis perspective) where the merger takes place but instead from a related market. This raises important issues for merger analysis under competition law.

This framework, however, can be generalised in a number of directions. There are four major areas that deserve further examination. First, it is important to understand how the two effects identified in the paper – the selection and internalisation of externality effects – play themselves out when there are more than two firms in both markets. It is important to understand how an increase in the number of competitors affects their impacts on both inter- and intra-markets competition. Second, one can explicitly consider

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[5]See, for example, Perry and Porter (1985); and Farrell and Shapiro (1990).

the existence of common fixed costs across markets (synergies). This will strengthen the selection effect and may also mean greater gains under full multi-market contact. Third, we can extend the framework to consider other pricing schemes. For example, we can allow firms that offer the two products to compete by offering bundles. We conjecture that this can lead to very fierce competition under full multi-market contact. Fourth, we can use this simple framework to consider the scope for a firm that offers the two goods to behave anti-competitively in order to exclude rivals from one of the markets.

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5   Appendix

Market equilibrium in three market structures: For a firm i operating in market j only, the optimisation problem is $\max_{q_i} (P_j - c_j) q_i$. For a multi-product firm i operating in both markets, the optimisaiton problem is $\max_{q_{i1}, q_{i2}} (P_1 - c_1) q_{i1} + (P_2 - c_2) q_{i2}$, where q_{i1} and q_{i2} are the multi-product firm's quantity choices in the two markets.

(1) Market structure B: For firm i, $i \in \{A, B\}$, in market 1, the best responses are $q_i = \frac{1 - q_j - \gamma(q_C + q_D)}{2}$, where $i, j \in \{A, B\}$ and $i \neq j$. For firms C and D: $q_C = \frac{1 - a - \gamma(q_A + q_B) - q_D}{2}$ and $q_D = \frac{1 - a - \gamma(q_A + q_B) - q_C}{2}$. Solving the four best responses simultaneously gives the interior solutions: $q_A = q_B = \frac{3 - 2\gamma(1-a)}{(3 + 2\gamma)(3 - 2\gamma)}$ and $q_C = q_D = \frac{3(1-a) - 2\gamma}{(3 + 2\gamma)(3 - 2\gamma)}$, with $\pi_A = \pi_B = \frac{(3 - 2\gamma(1-a))^2}{(2\gamma + 3)^2 (3 - 2\gamma)^2}$ and $\pi_C = \pi_D = \frac{(2\gamma - 3(1-a))^2}{(2\gamma + 3)^2 (3 - 2\gamma)^2}$. Since market 2 is relatively inefficient, the corner solution involves $q_C^* = q_D^* = 0$. This gives $q_A^* = q_B^* = \frac{1}{3}$. The corner solution would be an equilibrium if $BR_C \left(q_A^* = q_B^* = \frac{1}{3} \right) \leq 0$ and $BR_D \left(q_A^* = q_B^* = \frac{1}{3} \right) \leq 0$. This holds for $a \geq \frac{3 - 2\gamma}{3}$.

(2) Market structure P: Solving for the three firms' optimisation problems respectively gives the following best responses: $q_{A1} = \frac{1 - q_B - 2\gamma q_{A2} - \gamma q_D}{2}$, $q_{A2} = \frac{1 - a - 2\gamma q_{A1} - \gamma q_B - q_D}{2}$, $q_B = \frac{1 - q_{A1} - \gamma(q_{A2} + q_D)}{2}$, and $q_D = \frac{1 - a - \gamma(q_{A1} + q_B) - q_{A2}}{2}$. This gives the interior solutions: $q_{A1} = \frac{4a\gamma - 4\gamma + \gamma^2 + 3}{(\gamma + 1)(3 - \gamma)(\gamma + 3)(1 - \gamma)}$, $q_{A2} = \frac{(3 + \gamma^2)(1 - a) - 4\gamma}{(\gamma + 1)(3 - \gamma)(\gamma + 3)(1 - \gamma)}$, $q_B = \frac{a\gamma - \gamma + 3}{(\gamma + 3)(3 - \gamma)}$, and $q_D = \frac{3(1-a) - \gamma}{(\gamma + 3)(3 - \gamma)}$.

With production in both markets, firm A has more incentive to exit market 2 as a gets large. For $q_{A2} = 0$, $q_{A1} = \frac{1 - q_B - \gamma q_C}{2}$, $q_B = \frac{1 - q_{A1} - \gamma q_C}{2}$, and $q_D = \frac{1 - a - \gamma(q_{A1} + q_B)}{2}$. This gives $q_{A1} = q_B = \frac{a\gamma - \gamma + 2}{2(3 - \gamma^2)}$ and $q_D = \frac{3(1-a) - 2\gamma}{2(3 - \gamma^2)}$. These quantities indeed gives $q_{A2} = 0$ if $a \geq \frac{(3 - \gamma)(1 - \gamma)}{3 + \gamma^2}$. Finally, as in the

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benchmark case, $q_{A2} = q_D = 0$ if $a \geq \frac{3-2\gamma}{3}$. It can be verified that the combinations $q_{A2} = q_B = 0$ and $q_{A1} = q_D = 0$ can never be supported as an equilibrium.

(3) Market structure F: The symmetric firms' best responses are $q_{A1} = \frac{1-2\gamma q_{A2}-q_{B1}-\gamma q_{B2}}{2}$, $q_{A2} = \frac{1-a-2\gamma q_{A1}-\gamma q_{B1}-q_{B2}}{2}$, $q_{B1} = \frac{1-2\gamma q_{B2}-q_{A1}-\gamma q_{A2}}{2}$, and $q_{B2} = \frac{1-a-2\gamma q_{B1}-\gamma q_{A1}-q_{A2}}{2}$. Solving the four best responses simultaneously gives the interior solutions: $q_{A1} = q_{B1} = \frac{1-\gamma(1-a)}{3(\gamma+1)(1-\gamma)}$ and $q_{A2} = q_{B2} = \frac{1-a-\gamma}{3(1-\gamma)(\gamma+1)}$. From the best responses, $q_{A2} = q_{B2} = 0$ if $a \geq 1-\gamma$. The market equilibrium in this case is $q_{A1} = q_{B1} = \frac{1}{3}$. It can be verified that this is the only corner solution in this market structure.

Proof. of Proposition 2: From Equation 1, the total surplus is represented by $TS = Q_1 + (1-a)Q_2 - \frac{1}{2}\left(Q_1^2 + 2\gamma Q_1 Q_2 + Q_2^2\right) = CS + \Pi$.

Case 1 $(1-\gamma \leq a)$: Given $\{Q_1^B, Q_2^B, Q_1^F, Q_2^F\}$, $TS^B \geq TS^F$ if $a \leq \frac{(9-3\gamma-4\gamma^2)(3-2\gamma)}{3(9-2\gamma^2)}$. Note that $1-\gamma \leq \frac{(9-3\gamma-4\gamma^2)(3-2\gamma)}{3(9-2\gamma^2)} \leq \frac{3-2\gamma}{3}$. Given $\{Q_1^B, Q_2^B, Q_1^P, Q_2^P\}$, $TS^B \geq TS^P$ if $a \leq \frac{(8\gamma^3-20\gamma^2-24\gamma+45)(3-2\gamma)}{135-12\gamma^2-16\gamma^4}$. Note that $\frac{(8\gamma^3-20\gamma^2-24\gamma+45)(3-2\gamma)}{(135-12\gamma^2-16\gamma^4)} \leq \frac{3-2\gamma}{3}$. Finally, $TS^F \geq TS^P$ if $a \geq \frac{16\gamma^3-12\gamma^2-78\gamma+81}{81-12\gamma^2}$. Note that

$$1-\gamma \leq \frac{16\gamma^3 - 12\gamma^2 - 78\gamma + 81}{3\left(27 - 4\gamma^2\right)} \leq \frac{3-2\gamma}{3}.$$

Also, note that

$$
\begin{aligned}
\frac{16\gamma^3 - 12\gamma^2 - 78\gamma + 81}{3\left(27 - 4\gamma^2\right)} &\geq \frac{\left(9 - 3\gamma - 4\gamma^2\right)\left(3 - 2\gamma\right)}{3\left(9 - 2\gamma^2\right)} \\
&\geq \frac{\left(8\gamma^3 - 20\gamma^2 - 24\gamma + 45\right)\left(3 - 2\gamma\right)}{\left(135 - 12\gamma^2 - 16\gamma^4\right)}.
\end{aligned}
$$

Case 2 $(\frac{(3-\gamma)(1-\gamma)}{3+\gamma^2} \leq a \leq 1-\gamma)$: Given $\{Q_1^B, Q_2^B, Q_1^F, Q_2^F\}$ in this case, $TS^B \geq TS^F$ if

$$a \leq \frac{-\left(\gamma+3\right)\left(1-\gamma\right)\left(3-2\gamma\right)^2 + \sqrt{\left(1-\gamma\right)\left(\gamma+3\right)\left(3-\gamma\right)\left(\gamma+1\right)\left(3-2\gamma\right)^2\left(2\gamma+3\right)^2}}{2\gamma\left(27-2\gamma^2\right)}.$$

Note that $\frac{-(\gamma+3)(1-\gamma)(3-2\gamma)^2+\sqrt{(1-\gamma)(\gamma+3)(3-\gamma)(\gamma+1)(3-2\gamma)^2(2\gamma+3)^2}}{2\gamma(27-2\gamma^2)} \geq 1-\gamma$.

Similarly, $TS^P \geq TS^F$ if

$$\frac{\left(1-\gamma\right)\left(45 - 45\gamma + 42\gamma^2 - 4\gamma^4\right) - \sqrt{16\gamma^2\left(1-\gamma\right)\left(1+\gamma\right)\left(15-\gamma^2\right)\left(3-\gamma^2\right)^2}}{45 + 87\gamma^2 - 4\gamma^4}$$

$$\leq a \leq \frac{\left(1-\gamma\right)\left(45 - 45\gamma + 42\gamma^2 - 4\gamma^4\right) + \sqrt{16\gamma^2\left(1-\gamma\right)\left(1+\gamma\right)\left(15-\gamma^2\right)\left(3-\gamma^2\right)^2}}{45 + 87\gamma^2 - 4\gamma^4}.$$

Note that $\frac{(1-\gamma)\left(45-45\gamma+42\gamma^2-4\gamma^4\right)-\sqrt{16\gamma^2(1-\gamma)(1+\gamma)(15-\gamma^2)(3-\gamma^2)^2}}{45+87\gamma^2-4\gamma^4} \le \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$
and $\frac{(1-\gamma)\left(45-45\gamma+42\gamma^2-4\gamma^4\right)+\sqrt{16\gamma^2(1-\gamma)(1+\gamma)(15-\gamma^2)(3-\gamma^2)^2}}{45+87\gamma^2-4\gamma^4} \ge 1-\gamma$. Finally,
note that $\frac{\left(8\gamma^3-20\gamma^2-24\gamma+45\right)(3-2\gamma)}{(135-12\gamma^2-16\gamma^4)} \ge \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.

Case 3 $(a \le \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2})$: Given $\left\{Q_1^B, Q_2^B, Q_1^P, Q_2^P\right\}$ in this case, $TS^B \ge TS^P$ if

$$a \le \frac{-(\gamma+2)(1-\gamma)(2\gamma-3)^2(\gamma-3)^2}{\gamma\left(405-32\gamma^4+27\gamma^2\right)}$$
$$+\frac{\sqrt{(1-\gamma)(2-\gamma)(\gamma+2)(\gamma+1)(2\gamma+3)^2(\gamma+3)^2(\gamma-3)^2(2\gamma-3)^2}}{\gamma\left(405-32\gamma^4+27\gamma^2\right)}.$$

Note that

$$\frac{-(\gamma+2)(1-\gamma)(2\gamma-3)^2(\gamma-3)^2}{\gamma\left(405-32\gamma^4+27\gamma^2\right)}$$
$$+\frac{\sqrt{(1-\gamma)(2-\gamma)(\gamma+2)(\gamma+1)(2\gamma+3)^2(\gamma+3)^2(\gamma-3)^2(2\gamma-3)^2}}{\gamma\left(405-32\gamma^4+27\gamma^2\right)}$$
$$\ge \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}.$$

Given $\left\{Q_1^P, Q_2^P, Q_1^F, Q_2^F\right\}$ in this case, $TS^P \ge TS^F$ if

$$a \le \frac{-2(\gamma+6)(1-\gamma)(\gamma-3)^2 + \sqrt{4(1-\gamma)(\gamma+6)(6-\gamma)(\gamma+1)(\gamma-3)^2(\gamma+3)^2}}{2\gamma(9-\gamma)(\gamma+9)}.$$

Note that $\frac{-2(\gamma+6)(1-\gamma)(\gamma-3)^2+\sqrt{4(1-\gamma)(\gamma+6)(6-\gamma)(\gamma+1)(\gamma-3)^2(\gamma+3)^2}}{2\gamma(9-\gamma)(\gamma+9)} \ge \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.

∎

Proof. of Proposition 3: **Case 1** $(\frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)} \le a)$: (i) A merges with C: If
A merges with C, and B and D remain separated, the market structure be-
comes that of market structure P. Firms A and C would have the incentive
to merge if $\frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2} \ge \frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2} + \frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}$. This holds for $a \ge$
$\frac{(3-2\gamma)\left(4\gamma^3-25\gamma^2-12\gamma+4\gamma^4+36\right)}{(4\gamma^4-51\gamma^2+108)}$. Note that $\frac{3-2\gamma}{3} \ge \frac{(3-2\gamma)\left(4\gamma^3-25\gamma^2-12\gamma+4\gamma^4+36\right)}{(4\gamma^4-51\gamma^2+108)} \ge$
$1-\gamma$.

(ii) A merges with B: The merged firm has the best response: $q_{AB} = \frac{1-\gamma(q_C+q_D)}{2}$. For firm i in market 2, the best response is $q_i = \frac{1-a-\gamma q_{AB}-q_j}{2}$. This gives the interior solutions: $q_{AB} = \frac{3-2\gamma(1-a)}{2(3-\gamma^2)}$ and $q_C = q_D = \frac{2(1-a)-\gamma}{2(3-\gamma^2)}$.

Firms A and B would have the incentive to merge if $\frac{(3-2\gamma(1-a))^2}{4(3-\gamma^2)^2} \geq 2\frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2}$.
This holds for $\gamma \leq \sqrt{\frac{6-\sqrt{18}}{4}} \approx 0.66$.

(iii) Compare the AC merger and AB merger: For the parameter range where both mergers are profitable, AB merger always gives higher profit.

(iv) C mergers with D: The best responses are $q_A = \frac{1-q_B-\gamma q_{CD}}{2}$, $q_B = \frac{1-q_A-\gamma q_{CD}}{2}$, and $q_{CD} = \frac{1-a-\gamma(q_A+q_B)}{2}$. The interior solution is $q_A = q_B = \frac{2-\gamma(1-a)}{2(3-\gamma^2)}$ and $q_{CD} = \frac{3(1-a)-2\gamma}{2(3-\gamma^2)}$. Firms C and D would have incentives to merge if $\frac{(3(1-a)-2\gamma)^2}{4(3-\gamma^2)^2} \geq 2\frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}$. This holds for $\gamma \leq \sqrt{\frac{6-\sqrt{18}}{4}} \approx 0.66$.

(v) A merges with B and C: The best responses are $q_{ABC1} = \frac{1-2\gamma q_{ABC2}-\gamma q_D}{2}$, $q_{ABC2} = \frac{1-a-2\gamma q_{ABC1}-q_D}{2}$, and $q_D = \frac{1-a-\gamma q_{ABC1}-q_{ABC2}}{2}$. For the given parameter range, the firm ABC would cease to offer good 2. In equilibrium, $q_{ABC2} = 0$, $q_{ABC1} = \frac{2-\gamma(1-a)}{4-\gamma^2}$ and $q_D = \frac{2(1-a)-\gamma}{4-\gamma^2}$. Firms A, B, and C would have incentives to merge if

$$\frac{(2-\gamma(1-a))^2}{(\gamma+2)^2(2-\gamma)^2} \geq 2\frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2} + \frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}.$$

This holds for

$$a \geq \frac{-(2-\gamma)(3-2\gamma)(-4\gamma^2+7\gamma-7\gamma^3-4\gamma^4+24)}{25\gamma^2-17\gamma^4+8\gamma^6-144}$$
$$+\frac{\sqrt{2(1-\gamma)(\gamma+1)(2-\gamma^2)(2\gamma+3)^2(2-\gamma)^2(\gamma+2)^2(3-2\gamma)^2}}{25\gamma^2-17\gamma^4+8\gamma^6-144}.$$

Note that $\frac{-(2-\gamma)(3-2\gamma)(-4\gamma^2+7\gamma-7\gamma^3-4\gamma^4+24)+\sqrt{2(1-\gamma)(\gamma+1)(2-\gamma^2)(2\gamma+3)^2(2-\gamma)^2(\gamma+2)^2(3-2\gamma)^2}}{25\gamma^2-17\gamma^4+8\gamma^6-144} \leq \frac{3-2\gamma}{3}$ and $\frac{-(2-\gamma)(3-2\gamma)(-4\gamma^2+7\gamma-7\gamma^3-4\gamma^4+24)+\sqrt{2(1-\gamma)(\gamma+1)(2-\gamma^2)(2\gamma+3)^2(2-\gamma)^2(\gamma+2)^2(3-2\gamma)^2}}{25\gamma^2-17\gamma^4+8\gamma^6-144} \leq 1-\gamma$ for $\gamma \leq 0.89$. For most of the parameter range in this case, firms A, B, and C would have the incentive to merge.

Case 2 $(a \leq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)})$: (i) A and C merge: Firms A and C would have the incentive to merge since in this case

$$\frac{2(1-\gamma)(3-\gamma)^2(1-a)+9a^2+7a^2\gamma^2}{(3+\gamma)^2(3-\gamma)^2(1+\gamma)(1-\gamma)} \geq \frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2} + \frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}.$$

(ii) A and B merge: Firms A and B would have incentives to merge if $\gamma \leq \sqrt{\frac{6-\sqrt{18}}{4}}$. Note that $\frac{(2-\gamma)(1-\gamma)}{2+\gamma^2} \leq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}$. for $a \geq \frac{(2-\gamma)(1-\gamma)}{2+\gamma^2}$, as analysed in Case 1, firms A, B, and C would have incentives to merge if

$max\{\frac{(2-\gamma)(1-\gamma)}{2+\gamma^2}, \frac{-(2-\gamma)(3-2\gamma)\left(-4\gamma^2+7\gamma-7\gamma^3-4\gamma^4+24\right)+\sqrt{2(1-\gamma)(\gamma+1)(2-\gamma^2)(2\gamma+3)^2(2-\gamma)^2(\gamma+2)^2(3-2\gamma)^2}}{25\gamma^2-17\gamma^4+8\gamma^6-144}\} \leq$
$a \leq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}$.

For $a \leq \frac{(2-\gamma)(1-\gamma)}{2+\gamma^2}$, the equilibrium output levels are $q_{ABC1} = \frac{1-\gamma(1-a)}{2(1+\gamma)(1-\gamma)}$, $q_{ABC2} = \frac{\gamma^2-3\gamma-2a-a\gamma^2+2}{6(1-\gamma)(1+\gamma)}$, and $q_D = \frac{1-a}{3}$. Firms A, B, and C would have the incentive to merge if

$$\frac{\left(18a\gamma - 18\gamma - 8a + 4a^2 + 5\gamma^2 - 10a\gamma^2 + 5a^2\gamma^2 + 13\right)}{36\left(1+\gamma\right)\left(1-\gamma\right)}$$
$$\geq 2\frac{\left(3 - 2\gamma\left(1-a\right)\right)^2}{\left(3+2\gamma\right)^2\left(3-2\gamma\right)^2} + \frac{\left(3\left(1-a\right)-2\gamma\right)^2}{\left(3+2\gamma\right)^2\left(3-2\gamma\right)^2}.$$

This holds for $\gamma \leq \sqrt{\frac{1}{2}} \approx 0.7$. For $\gamma > \sqrt{\frac{1}{2}}$, A, B, and C would have incentives to merge if

$$a \geq \frac{\left(1-\gamma\right)\left(3-2\gamma\right)\left(6\gamma + 28\gamma^2 + 40\gamma^3 - 27\right) + \sqrt{72\left(1-\gamma\right)\left(\gamma+1\right)\left(2\gamma^2-1\right)\left(2\gamma-3\right)^2\left(2\gamma+3\right)^2}}{\gamma\left(80\gamma^4 - 8\gamma^2 + 153\right)}.$$

■

Proof. of Proposition 4: **Case 1** $(1-\gamma \leq a)$: (i) A merge with B: The post merger market structure is the same as the merger between A, B, and C in the benchmark analysed above. Firms A and B would have the incentive to merge if $\frac{(2-\gamma(1-a))^2}{(\gamma+2)^2(2-\gamma)^2} \geq 2\frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2}$. This holds for $\gamma \leq \sqrt{2-\sqrt{2}} \approx 0.77$.

(ii) A merge with D: The merged firm has best responses $q_{AD1} = \frac{1-q_B-2\gamma q_{AD2}}{2}$ and $q_{AD2} = \frac{1-a-2\gamma q_{AD1}-\gamma q_B}{2}$. For firm B, the best response is $q_B = \frac{1-q_{AD1}-\gamma q_{AD2}}{2}$. This gives the interior solution: $q_{AD1} = \frac{2-3\gamma(1-a)+\gamma^2}{6(1+\gamma)(1-\gamma)}$, $q_{AD2} = \frac{1-a-\gamma}{2(1-\gamma^2)}$, and $q_B = \frac{1}{3}$. It can be verified that the merged entity would never cease production in market 1. For market 2, if $q_{AD2} = 0$, $q_{AD1} = q_B = \frac{1}{3}$. These quantities would indeed induce $q_{AD2} = 0$ if $a \geq 1-\gamma$. This holds in this case. Therefore, firms A and D would have the incentive to merge if $\frac{1}{9} \geq \frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2} + \frac{(3a+2\gamma-3)^2}{4(3-\gamma^2)^2}$. This holds for $a \geq \frac{-30\gamma+3\gamma^2+2\gamma^3+27}{3(\gamma^2+9)}$. Note that $\frac{-30\gamma+3\gamma^2+2\gamma^3+27}{3(\gamma^2+9)} \leq 1-\gamma$ if $\gamma \leq \sqrt{\frac{3}{5}} \approx 0.77$.

(iii) It is more profitable for A to merge with D rather than B if $\frac{1}{9} \geq \frac{(2-\gamma(1-a))^2}{(\gamma+2)^2(2-\gamma)^2}$. This holds for $a \leq \frac{3\gamma-\gamma^2-2}{3\gamma}$. Note that $\frac{3\gamma-\gamma^2-2}{3\gamma} \leq 1-\gamma$.

(iv) B merges with D: If firms B and D merge, the market structure is the same as market structure F. Both firms A and BD do not offer good 2.

Firms B and D would have the incentive to merge if

$$\frac{1}{9} \geq \frac{(2 - \gamma(1-a))^2}{4(3-\gamma^2)^2} + \frac{(3a + 2\gamma - 3)^2}{4(3-\gamma^2)^2}.$$

The conditions are the same as the ones for profitable AD merger.

Case 2 ($\frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)} \leq a \leq 1-\gamma$): (i) A and B merge: As analysed in Case 1, firms A and B would have the incentive to merge if $\gamma \leq \sqrt{2 - \sqrt{2}} \approx 0.77$.

(ii) A and D merge: In this parameter range, the merged entity would continue to produce in both markets. Firms A and D would have the incentive to merge if

$$\frac{18a\gamma - 18\gamma - 18a + 9a^2 + 5\gamma^2 + 13}{36(1+\gamma)(1-\gamma)} \geq \frac{(2 - \gamma(1-a))^2}{4(3-\gamma^2)^2} + \frac{(3a + 2\gamma - 3)^2}{4(3-\gamma^2)^2}.$$

This holds for $a \geq \frac{-3(1+\gamma^2)(1-\gamma)^2 + \sqrt{(1-\gamma)(\gamma+1)(\gamma^2+1)(3-\gamma^2)^2}}{6\gamma(\gamma^2+1)}$. Note that

$$\frac{-3(1+\gamma^2)(1-\gamma)^2 + \sqrt{(1-\gamma)(\gamma+1)(\gamma^2+1)(3-\gamma^2)^2}}{6\gamma(\gamma^2+1)} \geq 1 - \gamma$$

if $\gamma \geq \sqrt{\frac{3}{5}} \approx 0.77$. Note also that $\frac{-3(1+\gamma^2)(1-\gamma)^2 + \sqrt{(1-\gamma)(\gamma+1)(\gamma^2+1)(3-\gamma^2)^2}}{6\gamma(\gamma^2+1)} \geq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.

(iii) It is more profitable for A to merge with D rather than B if

$$\frac{18a\gamma - 18\gamma - 18a + 9a^2 + 5\gamma^2 + 13}{36(1+\gamma)(1-\gamma)} \geq \frac{(2 - \gamma(1-a))^2}{(\gamma+2)^2(2-\gamma)^2}.$$

This holds for $a \leq \frac{3(1-\gamma)(2-\gamma)(8+8\gamma+2\gamma^2-\gamma^3) - \sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4 - 12\gamma^2 + 16)}$ or $a \geq \frac{3(1-\gamma)(2-\gamma)(8+8\gamma+2\gamma^2-\gamma^3) + \sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4 - 12\gamma^2 + 16)}$. Note that $\frac{3(1-\gamma)(2-\gamma)(8+8\gamma+2\gamma^2-\gamma^3) - \sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4 - 12\gamma^2 + 16)} \leq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}$ and $\frac{3(1-\gamma)(2-\gamma)(8+8\gamma+2\gamma^2-\gamma^3) + \sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4 - 12\gamma^2 + 16)} \geq 1 - \gamma$.

(iv) B merges with D: The market structure is the same as market structure F. In this parameter range, both firms offer both goods. Firms B and D would have the incentive to merge if

$$\frac{(2a\gamma - 2\gamma - 2a + a^2 + 2)}{9(1+\gamma)(1-\gamma)} \geq \frac{(2 - \gamma(1-a))^2}{4(3-\gamma^2)^2} + \frac{(3a + 2\gamma - 3)^2}{4(3-\gamma^2)^2}.$$

For $\gamma \geq \sqrt{\frac{-24+\sqrt{1161}}{13}} \approx 0.88$, the inequality holds if

$$a \geq \frac{-(1-\gamma)\left(-39\gamma^2 + 9\gamma + 9\gamma^3 - 4\gamma^4 + 45\right) + \sqrt{8\gamma^2\left(1-\gamma\right)\left(\gamma+1\right)\left(3-2\gamma^2\right)\left(3-\gamma^2\right)^2}}{\left(13\gamma^4 + 48\gamma^2 - 45\right)}.$$

For $\gamma \geq \sqrt{\frac{-24+\sqrt{1161}}{13}}$, $\frac{-(1-\gamma)\left(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45\right)+\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(13\gamma^4+48\gamma^2-45)} \geq$ $1-\gamma$ and firms B and D would not have the incentive to merge.

For $\gamma < \sqrt{\frac{-24+\sqrt{1161}}{13}}$, firms B and D would have incentives to merge if

$$\frac{(1-\gamma)\left(-39\gamma^2 + 9\gamma + 9\gamma^3 - 4\gamma^4 + 45\right) - \sqrt{8\gamma^2\left(1-\gamma\right)\left(\gamma+1\right)\left(3-2\gamma^2\right)\left(3-\gamma^2\right)^2}}{\left(45 - 13\gamma^4 - 48\gamma^2\right)}$$

$$\leq \quad a \leq \frac{(1-\gamma)\left(-39\gamma^2 + 9\gamma + 9\gamma^3 - 4\gamma^4 + 45\right) + \sqrt{8\gamma^2\left(1-\gamma\right)\left(\gamma+1\right)\left(3-2\gamma^2\right)\left(3-\gamma^2\right)^2}}{\left(45 - 13\gamma^4 - 48\gamma^2\right)}.$$

Note that $\frac{(1-\gamma)\left(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45\right)+\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(45-13\gamma^4-48\gamma^2)} \geq 1-\gamma$
and $\frac{(1-\gamma)\left(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45\right)-\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(45-13\gamma^4-48\gamma^2)} \geq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.
$\frac{(1-\gamma)\left(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45\right)-\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(45-13\gamma^4-48\gamma^2)} \leq 1-\gamma$ if $\gamma \leq$
$\sqrt{\frac{3}{5}} \approx 0.77$.

Case 3 $\left(a \leq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}\right)$: (i) A merges with B: Firms A and B have the
incentive to merge if

$$\frac{\left(2-\gamma\left(1-a\right)\right)^2}{\left(\gamma+2\right)^2\left(2-\gamma\right)^2}$$

$$\geq \quad \frac{2\left(1-\gamma\right)\left(3-\gamma\right)^2\left(1-a\right) + 9a^2 + 7a^2\gamma^2}{\left(\gamma+3\right)^2\left(3-\gamma\right)^2\left(\gamma+1\right)\left(1-\gamma\right)} + \frac{\left(3-\gamma\left(1-a\right)\right)^2}{\left(3-\gamma\right)^2\left(\gamma+3\right)^2}.$$

This holds for

$$\frac{(1-\gamma)(2-\gamma)\left(5\gamma + 3\gamma^2 + 8\right) - \sqrt{2\left(1-\gamma\right)\left(1+\gamma\right)\left(2-3\gamma^2\right)\left(2-\gamma\right)^2\left(\gamma+2\right)^2}}{\left(3\gamma^4 - \gamma^2 + 16\right)}$$

$$\leq \quad a \leq \frac{(1-\gamma)(2-\gamma)\left(5\gamma + 3\gamma^2 + 8\right) + \sqrt{2\left(1-\gamma\right)\left(1+\gamma\right)\left(2-3\gamma^2\right)\left(2-\gamma\right)^2\left(\gamma+2\right)^2}}{\left(3\gamma^4 - \gamma^2 + 16\right)}.$$

This would never hold if $\gamma \geq \sqrt{\frac{2}{3}} \approx 0.82$. For $\gamma < \sqrt{\frac{2}{3}} \approx 0.82$,
$\frac{(1-\gamma)(2-\gamma)\left(5\gamma+3\gamma^2+8\right)-\sqrt{2(1-\gamma)(1+\gamma)(2-3\gamma^2)(2-\gamma)^2(\gamma+2)^2}}{(3\gamma^4-\gamma^2+16)} \geq \frac{(3-\gamma)(1-\gamma)}{(3+\gamma^2)}$ if $\sqrt{2-\sqrt{2}} \approx$
$0.77 \leq \gamma \leq \sqrt{2+\sqrt{2}} \approx 1.85$. Note that

$$\frac{(1-\gamma)(2-\gamma)\left(5\gamma + 3\gamma^2 + 8\right) + \sqrt{2\left(1-\gamma\right)\left(1+\gamma\right)\left(2-3\gamma^2\right)\left(2-\gamma\right)^2\left(\gamma+2\right)^2}}{\left(3\gamma^4 - \gamma^2 + 16\right)} \geq \frac{(3-\gamma)(1-\gamma)}{(3+\gamma^2)}.$$

Also $\frac{(1-\gamma)(2-\gamma)\left(5\gamma+3\gamma^2+8\right)-\sqrt{2(1-\gamma)(1+\gamma)(2-3\gamma^2)(2-\gamma)^2(\gamma+2)^2}}{(3\gamma^4-\gamma^2+16)} \geq \frac{(1-\gamma)(2-\gamma)}{2+\gamma^2}$ if $\gamma \geq$ $\sqrt{\frac{1}{2}} \approx 0.7$. For $\sqrt{\frac{1}{2}} \leq \gamma < \sqrt{2-\sqrt{2}}$, firms A and B would have the incentive to merge if

$$\frac{(1-\gamma)(2-\gamma)\left(5\gamma+3\gamma^2+8\right)-\sqrt{2(1-\gamma)(1+\gamma)(2-3\gamma^2)(2-\gamma)^2(\gamma+2)^2}}{(3\gamma^4-\gamma^2+16)}$$
$$\leq \quad a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}.$$

For $\gamma < \sqrt{\frac{1}{2}}$, firms A and B would have incentives to merge if $\frac{(1-\gamma)(2-\gamma)}{2+\gamma^2} \leq$ $a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.

For $a \leq \frac{(1-\gamma)(2-\gamma)}{2+\gamma^2}$, the merged firm AB produces both goods. Firms A and B would have the incentives to merge if

$$\frac{\left(18a\gamma-18\gamma-8a+4a^2+5\gamma^2-10a\gamma^2+5a^2\gamma^2+13\right)}{36(1+\gamma)(1-\gamma)}$$
$$\geq \quad \frac{2(1-\gamma)(3-\gamma)^2(1-a)+9a^2+7a^2\gamma^2}{(3+\gamma)^2(3-\gamma)^2(1+\gamma)(1-\gamma)} + \frac{(3-\gamma(1-a))^2}{(3-\gamma)^2(\gamma+3)^2}.$$

This holds for $a \geq \frac{5\gamma-3}{5\gamma}$. $\frac{5\gamma-3}{5\gamma} \leq \frac{(1-\gamma)(2-\gamma)}{2+\gamma^2}$ if $\gamma \leq \sqrt{\frac{1}{2}} \approx 0.7$. Therefore, for $\gamma \leq \sqrt{\frac{1}{2}}$, firms A and B would have the incentives to merge for $a \geq \frac{5\gamma-3}{5\gamma}$.

(ii) A merges with D: Firms A and D would have the incentives to merge if

$$\frac{18a\gamma-18\gamma-18a+9a^2+5\gamma^2+13}{36(1+\gamma)(1-\gamma)}$$
$$\geq \quad \frac{2(1-\gamma)(3-\gamma)^2(1-a)+9a^2+7a^2\gamma^2}{(3+\gamma)^2(3-\gamma)^2(1+\gamma)(1-\gamma)} + \frac{(3-3a-\gamma)^2}{(\gamma+3)^2(3-\gamma)^2}.$$

This holds for $a \leq \frac{3-5\gamma}{3}$. Note that $\frac{3-5\gamma}{3} \geq 0$ if $\gamma \leq \frac{3}{5}$.

(iii) B and D merger: B and D would have incentives to merge if

$$\frac{\left(2a\gamma-2\gamma-2a+a^2+2\right)}{9(1+\gamma)(1-\gamma)} \geq \frac{(3-\gamma(1-a))^2}{(3-\gamma)^2(\gamma+3)^2} + \frac{(3-3a-\gamma)^2}{(\gamma+3)^2(3-\gamma)^2}.$$

This holds for $a \geq \frac{-(1-\gamma)(3-\gamma)^3+\sqrt{(1-\gamma)(\gamma+1)(3-\gamma)^3(\gamma+3)^3}}{2\gamma(5\gamma^2+27)}$. Note that

$$\frac{-(1-\gamma)(3-\gamma)^3+\sqrt{(1-\gamma)(\gamma+1)(3-\gamma)^3(\gamma+3)^3}}{2\gamma(5\gamma^2+27)} \geq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}.$$

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(iv) Comparison of AB and AD mergers: In this parameter range, both merged firms would continue to offer both products. AB merger would give higher profits compared with AD merger if

$$\frac{(2 - \gamma(1 - a))^2}{(\gamma + 2)^2(2 - \gamma)^2} \geq \frac{18a\gamma - 18\gamma - 18a + 9a^2 + 5\gamma^2 + 13}{36(1 + \gamma)(1 - \gamma)}.$$

This holds for

$$\frac{3(1 - \gamma)(2 - \gamma)\left(8 - \gamma^3 + 2\gamma^2 + 8\gamma\right) - \sqrt{16(5 - \gamma^2)(1 - \gamma)^2(2 - \gamma)^2(\gamma + 2)^2(\gamma + 1)^2}}{3(5\gamma^4 - 12\gamma^2 + 16)}$$
$$\leq a \leq \frac{3(1 - \gamma)(2 - \gamma)\left(8 - \gamma^3 + 2\gamma^2 + 8\gamma\right) + \sqrt{16(5 - \gamma^2)(1 - \gamma)^2(2 - \gamma)^2(\gamma + 2)^2(\gamma + 1)^2}}{3(5\gamma^4 - 12\gamma^2 + 16)}.$$

Note that $\frac{3(1-\gamma)(2-\gamma)\left(8-\gamma^3+2\gamma^2+8\gamma\right)+\sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4-12\gamma^2+16)} \geq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}.$

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