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Progressive Taxation, Wealth Distribution, and Macroeconomic Stability*

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Using the standard neoclassical growth model with two types of agents, we examine how the presence of heterogenous agents affects the stabilization role of progressive income taxation. We first show that if the marginal tax payment of each agent increases with her relative income, the steady state satisfies local saddlepoint stability so that the equilibrium is determinate. However, unlike the representative agent models with progressive taxation, our model with heterogeneous agents may have the possibility of equilibrium indeterminacy. The indeterminacy conditions depend not only on the property of tax functions but also on production and preference structures.

Keywords: heterogeneous agents, progressive taxation, wealth distribution, aggregate stability

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1 Introduction

It has been widely acknowledged that progressive income taxation under the balanced-budget rule is one of the most effective tools for establishing macroeconomic stability. In fact, Guo and Lansing (1998) demonstrate that progressive income taxation may eliminate sunspot-driven economic fluctuations caused by equilibrium indeterminacy even in the presence of strong degree of external increasing returns. Guo and Harrison (2002 and 2004) also claim that equilibrium indeterminacy obtained in a model with regressive income taxation with a fixed government spending shown by Schmitt-Ghro and Uribe (1998) does not exist when the government spending is adusted to keep a fixed level of income tax.

Although those findings are intuitively appealing, they are obtained in the context of representative agent models. The purpose of this paper is to reconsider the stabilization effect of progressive income taxation in a model with heterogeneous agents. For analytical clarity, we use a simple neoclassical growth model with fixed labor supply in which there are only two types of agents. Both groups of agents are infinitely lived and have the same time discount rate. Each group, however, may have different utility functions and hold different level of initial wealth. Our main concern is to investigate how the presence of heterogeneous agents affects the stabilization effect of progressive income taxation under the balanced-budget discipline. We first examine the case in which the same rate of tax applies to both labor and capital incomes. Following Guo and Lansing (1998), we assume that the rate of tax is assumed to increase with the private income relative to the average income in the economy at large. We then consider the model with factor specific income taxation: different rates of tax apply to labor and capital incomes, respectively. Given each taxation scheme, we characterize the steady state equilibrium and explore its local stability.

We obtain three main results. First, if the marginal rate of tax is a monotonic function of the relative income, the economy has a unique steady state equilibrium where all the agents hold an identical amount of capital. Second, if the marginal tax payment of each agent increases with her relative income, then the steady state satisfies local saddlepoint stability so that the equilibrium is determinate. Third, if the marginal tax payment decreases with the relative income, then the steady state equilibrium is either unstable or locally indeterminate. In the latter, there may exist a continuum of converging paths around the steady state. It is also shown that indeterminacy of equilibrium tends to emerge when the elasticity of intertemporal substitution in consumption of each types of agents is sufficiently different from each other.

The present study is closely related to some of the existing investigations on wealth distribution in the neoclassical growth model with heterogenous agents and non-linear income taxation. Sarte (1997) first demonstrates that introducing progressive income taxation may yield a unique interior steady state even though every agent's time discount rate is different from each other.[1] Soger (2002) re-examines Srate's (1997) model and presents numerical examples in which converging equilibrium path is indeterminate around the steady state. distributed back to the households as transfers. Carroll and Young (2007) analyze stationary wealth distribution under progressive taxation when each agent's labor supply is heterologous. The authors mentioned above assume that the income tax depends on the absolute level of individual income. Due to this assumption, income and wealth of each agent may not be equalized in the steady state even if every agent has a common time discount rate. Such an asymmetry in the steady state could be a source of complex behavior of the model economy. In contrast, our assumption that the income tax depends on the relative level of income always holds the symmetric steady as long as the time discount rate is common for all agents. In addition, except for numeral experiments conducted by Sorger (2002), the existing literature on wealth distribution and income taxation have focus on the steady state equilibrium alone. In this paper we inspect the relation between tax functions, preference structure and the dynamic behavior of the economy near the steady state without considering the inequality of wealth distribution in the steady state equilibrium.[2] Finally, it is to be pointed out that Li

[1] As is well known, in the standard neoclassical growth model with heterogenous households, the agent who has the lowest time discount rate ultimately owns the entire stock of capital: see Becker (1980). The presence of non-linear income taxation avoids yielding such an extreme conclusion.

[2] In addition, Soger (2000) treats the model with elastic labor supply. García-Peñalosa, C. and Turnovsky (2006) examines an endogenous growth version of Soger's setting.

and Sarte (2004) consider an endogenous growth model with heterogenous agents in which the taxation rule is assumed to be the same as that used in our study. Due to the assumption of Ak technology, the model economy in Li and Sarte (2004) always stays at the balanced growth path. Thus the stability analysis is not discussed in their study.

The next section constructs an analytical framework. Section 3 characterizes the steady state equilibrium and investigate equilibrium dynamics under the uniform income tax, while Section 4 discusses the model with factor-specific taxation. Section 5 presents numerical examples. Concluding remarks are given in Section 6.

2 The Base Model

2.1 Households

There are two groups of infinitely-lived agents who have the same time discount rate. Two types of agent have different levels of initial wealth and their utility functions could be different each other. For simplicity, population in the economy is assumed to be constant over time so that the number of agents in each group will not change. The economy is closed and the government does not issue interest bearing bonds. Thus the stock of capital is the only asset held by the households. The representative agent in group i $(i = 1, 2)$ supplies one unit of labor in each moment and maximizes a discounted sum of utility

$$U_i = \int_0^\infty e^{-\rho t} u_i(c_i) dt, \qquad \rho > 0, \qquad i = 1, 2, \tag{1}$$

over an infinite horizon subject to her flow budget constraint such that

$$\dot{k}_i = \hat{r} k_i + \hat{w} - c_i + T_i, \quad i = 1, 2. \tag{2}$$

Here, k_i is capital stock owned by an agent in group i, c_i consumption, $\hat{r}$ after-tax rate of return to asset, $\hat{w}$ the after-tax real wage rate and T_i expresses a transfer from the government. The initial holding of capital, $k_i(0)$, is given. The instantaneous utility function of each type of agent, $u_i(c_i)$, is monotonically increasing and strictly concave in c_i.

2.2 Production

The representative firm produces a single good according to a constant-returns-to-scale technology expressed by

$$\bar{Y} = F\left(\bar{K}, N\right),$$

where $\bar{Y}$, $\bar{K}$ and N denote the total output, capital and labor, respectively. Using the homogeneity assumption, in what follows, we write the production function in such a way that

$$Y = f\left(K\right),$$

where $Y \equiv \bar{Y}/N$ and $K \equiv \bar{K}/N$. The productivity function, $f\left(K\right)$, is assumed to be monotonically increasing and strictly concave in the capital-labor ratio, K, and fulfills the Inada conditions. The commodity market is competitive so that the before-tax rate of return to capital and real wage are respectively determined by

$$r = f'(K), \quad w = f(K) - Kf'(K). \tag{3}$$

For simplicity, we assume that capital does not depreciate.

If we denote the number of agents in group i by N_i $(i = 1, 2)$, then the full-employment condition for labor and capital are:

$$N_1 + N_2 = N,$$

$$N_1 k_1 + N_2 k_2 = \bar{K}.$$

Letting $\theta_i = N_i/N$, we can the full-employment conditions as follows:

$$K = \theta_1 k_1 + \theta_2 k_2, \ , \quad 0 < \theta_i < 1, \quad \theta_1 + \theta_2 = 1. \tag{4}$$

For notational simplicity, in the following we normalize the total population, N, to one. Thus θ_i represents the mass of agents of type i as well as the population share of that type.

2.3 Fiscal Rules

The government levies discretionary income taxes and distributes back its tax revenue as a transfer to each agent. In the main part of the paper, we assume that the same rate of tax applies to both capital and labor incomes. The rate of tax applied to an agent in group i is given by

$$\tau_i = \tau\left(\frac{y_i}{Y}\right), \qquad i = 1, 2,$$

where τ_i is the rate of tax and y_i $(= rk_i + w_i)$ denotes the total income of an agent in group i. Namely, the rate of tax depends on agent's individual income relative to the average income in the economy at large. The tax function $\tau(y_i/Y)$: $\Re_+ \rightarrow \Re_+$ is continuous, monotonically increasing, a twice differentiable function and satisfies $0 < \tau(y_i/Y) < 1$.

Given such a taxation rule, the marginal tax payment is

$$\frac{\partial\left(\tau_i y_i\right)}{\partial y_i} = \tau\left(\frac{y_i}{Y}\right) + \tau'\left(\frac{y_i}{Y}\right)\frac{y_i}{Y}.$$

Since the average tax rate is $\tau\left(y_i/Y\right)$, we obtain

$$\frac{\partial\left(\tau_i\left(y_i/Y\right)y_i\right)/\partial y_i}{\tau\left(y_i/Y\right)} = 1 + \frac{\tau'\left(y_i/Y\right)}{\tau\left(y_i/Y\right)}\frac{y_i}{Y}\left(>1\right), \tag{5}$$

which measure the degree of progressiveness of taxation. Notice that the 'marginal progressiveness' of taxation is

$$\frac{\partial^2\left(\tau_i y_i\right)}{\partial y_i^2} = \frac{1}{Y}\left[2\tau'\left(\frac{y_i}{Y}\right) + \frac{y_i}{Y}\tau''\left(\frac{y_i}{Y}\right)\right]. \tag{6}$$

If the above has a positive value, the marginal tax payment increases with the relative income. In contrast, if $\partial^2\left(\tau_i y_i\right)/\partial y_i^2 < 0$ (so $\tau''\left(y_i/Y\right) < 0$), then the marginal tax payment decreases with the relative income. In what follows, we see that the sign of (6) may play a pivotal role in determining macroeconomic stability of the economy.

The after-tax rate of return and real wage are respectively written as

$$\hat{r} = \left[1 - \tau\left(\frac{y_i}{Y}\right)\right]r, \qquad \hat{w} = \left[1 - \tau\left(\frac{y_i}{Y}\right)\right]w.$$

As a result, the flow budget constraint for the household (2) is rewritten as

$$\dot{k}_i = \left[1 - \tau\left(\frac{y_i}{Y}\right)\right]y_i - c_i + T_i, \qquad i = 1, 2.$$

We assume that the government follows the balanced-budget rule and, therefore, its flow budget constraint (in per-capita term) is.

$$\theta_1 T_1 + \theta_2 T_2 = \theta_1\tau\left(\frac{y_1}{Y}\right)y_1 + \theta_2\tau\left(\frac{y_2}{Y}\right)y_2.$$

In addition, if we assume that the government pay back an identical amount of transfer to each agent, the lump-sum transfers of the group 1 and the group 2 are given by

$$T_1 = T_2 = \theta_1 \tau \left(\frac{y_1}{Y} \right) y_1 + \theta_2 \tau \left(\frac{y_2}{Y} \right) y_2. \tag{9}$$

2.4 Consumption and Capital Formation

Under the fiscal rules shown above, the type i agent's flow budget constraint is expressed as

$$\dot{k}_i = \left[1 - \tau \left(\frac{y_i}{Y} \right) \right] (rk_i + w) - c_i + T_i, \quad i = 1, 2, \tag{10}$$

where T_i is determined by (9). Following Guo and Lansing (1998), we assume that the households perceive the rule of progressive taxation on private income, but she takes the transfer payment, T_i, as given. Therefore, taking anticipated sequences of $\{r(t), w(t), Y(t), T_i(t)\}_{t=0}^{\infty}$ and the initial holding of capital, $k_i(0)$, as given, the household of type i maximizes (1) subject to (10).

Using the optimization conditions and (3), we find that the optimal consumption in each moment satisfies the Euler equation such that

$$\dot{c}_i = \frac{c_i}{\sigma_i (c_i)} \left\{ \left[1 - \tau \left(\frac{y_i}{Y} \right) - \frac{y_i}{Y} \tau' \left(\frac{y_i}{Y} \right) \right] f'(K) - \rho \right\}, \quad i = 1, 2, \tag{11}$$

where $\sigma_i(c_i) = -u_i''(c_i)c_i / u_i'(c_i) (> 0)$. The optimal level of consumption should also fulfill the transversality condition: $\lim_{t \to \infty} q_1(t) k_1(t) e^{-\rho t} = 0$.

Equations (9) and (10) yield

$$\dot{k}_i = \left[1 - \tau \left(\frac{y_i}{Y} \right) \right] y_i - c_i + \theta_1 \tau \left(\frac{y_1}{Y} \right) y_1 + \theta_2 \tau \left(\frac{y_2}{Y} \right) y_2 \tag{12}$$

Summing up the flow budget constraint (10) over all of the households and dividing the both sides by N , we obtain

$$\theta_1 \dot{k}_1 + \theta_2 \dot{k}_2 = \theta_1 y_1 + \theta_2 y_2 - \theta_1 c_1 - \theta_2 c_2.$$

Thus, in view of $y_i = rk_i + w$ and (4), we obtain the final-good market equilibrium condition for the entire economy:

$$\dot{K} = f(K) - C,$$

where $C = \theta_1 c_1 + \theta_2 c_2$. For notational simplicity, we ignore capital depreciation.

3 Macroeconomic Stability

3.1 Dynamic System

By use of (3) and (4), we obtain:

$$y_i = rk_i + w = f(K) + (k_i - K)f'(K),$$

implying that

$$\frac{y_i}{Y} = 1 + \frac{(k_i - K)f'(K)}{f(K)}, \quad i = 1, 2, \tag{13}$$

where $K = \theta_1 k_1 + (1 - \theta_1)\, k_2$. Substituting (13) into (11) and (12) , we obtain a complete dynamic system with respect to (k_1, k_2, c_1, c_2) . The solution of this dynamic system that fulfills the initial conditions on $k_1\,(0)$ and $k_2\,(0)$ as well as the transversality conditions for the households' optimization problem, $\lim_{t\to\infty} u_i'\,(c_i\,(t))\, e^{-\rho t} k_i\,(t) = 0$, presents the perfect-foresight competitive equilibrium of our model economy.

3.2 Steady-State Equilibrium

In the steady-state equilibrium, k_i and c_i $(i = 1, 2)$ stay constant over time. In view of (11) and (12), we see that the steady-state conditions are given by

$$c_i^* = y_i^* + \theta_j \left[\tau\left(\frac{y_j^*}{Y^*}\right) y_j^* - \tau\left(\frac{y_i^*}{Y^*}\right) y_i^* \right], \qquad i, j = 1, 2, \quad i \neq j. \tag{14}$$

$$\rho = f'(K^*) \left[1 - \tau\left(\frac{y_i^*}{Y^*}\right) - \frac{y_i^*}{Y^*}\tau'\left(\frac{y_i^*}{Y^*}\right) \right], \qquad i, j = 1, 2, \tag{15}$$

where c_i^* and ki^* denote steady-state levels of k_i and c_i.

To simplify analytical argument, we make the following assumption:

Assumption 1 $\tau\left(\frac{y}{Y}\right) + \frac{y_i}{Y}\tau'\left(\frac{y_i}{Y}\right)$ $(i = 1, 2)$ *is a monotonic function of the relative income,* y_i/Y.

Since the derivative of the above function with respect to y_i/Y is $2\tau'\,(y_i/Y) + (y_i/Y)\,\tau''\,(y_i/Y)$, from (6) this assumption means that the marginal tax payment, $\partial^2\,(\tau y_i)\,/\partial y_i^2$, has the same sign for all feasible levels of y_i/Y. Given Assumption 1, it is easy to confirm the following fact:

Proposition 1. *There is a unique, symmetric steady state in which* $k_1^* = k_2^*$ and $c_1^* = c_2^*$ for $i = 1.2$.

Proof. Conditions (15) yield

$$\tau\left(\frac{y_1^*}{Y^*}\right)+\frac{y_1^*}{Y^*}\tau'\left(\frac{y_1^*}{Y^*}\right)=\tau\left(\frac{y_2^*}{Y^*}\right)+\frac{y_2^*}{Y^*}\tau'\left(\frac{y_2^*}{Y^*}\right).$$

By Assumption 1, the above equation holds if and only if $y_1^*=y_2^*$. Thus from (14) it holds that $c_1^*=c_2^*$. ■

Note that $y_1^*=y_2^*=Y^*$ and $k_1^*=k_2^*=K$ in the symmetric steady state, so that the rate of income tax in the steady state equilibrium is a given constant, $\tau(1)$. To make the steady state feasible, from (15) we should assume the following:

Assumption 2 *Tax function $\tau(y_i/Y)$ satisfies*

$$1-\tau(1)-\tau'(1)>0. \tag{16}$$

3.3 Stability

We are now ready to examine the local stability condition of the steady state equilibrium defined above. Linear approximation of dynamic system, (11) and (12), around the steady state equilibrium yields the following:

$$\begin{bmatrix}\dot{c}_1\\ \dot{c}_2\\ \dot{k}_1\\ \dot{k}_2\end{bmatrix}=\underbrace{\begin{bmatrix}0 & 0 & \partial\dot{c}_1/\partial k_1 & \partial\dot{c}_1/\partial k_2\\ 0 & 0 & \partial\dot{c}_2/\partial k_1 & \partial\dot{c}_2/\partial k_2\\ -1 & 0 & f'(k^*)[1-\theta_2(\tau(1)+\tau'(1))] & \theta_2 f'(k^*)[\tau(1)+\tau'(1)]\\ 0 & -1 & \theta_1 f'(k^*)[\tau(1)+\tau'(1)] & f'(k^*)[1-\theta_1(\tau(1)+\tau'(1))]\end{bmatrix}}_{J}\begin{bmatrix}c_1(t)-c_1^*\\ c_2(t)-c_2^*\\ k_1(t)-k_1^*\\ k_2(t)-k_2^*\end{bmatrix}.$$

Heere, $\partial\dot{c}_i/\partial k_j$ $(i,j=1,2)$ evaluated at the steady state are given by

$$\begin{aligned}\frac{\partial\dot{c}_1}{\partial k_1}&=\frac{c^*}{\sigma_1(c^*)}\frac{f'(k^*)^2}{f(k^*)}[\theta_1\Gamma(k^*)(1-\tau(1)-\tau'(1))-\theta_2(\tau''(1)+2\tau'(1)],\\ \frac{\partial\dot{c}_1}{\partial k_2}&=\frac{c^*}{\sigma_1(c^*)}\frac{f'(k^*)^2}{f(k^*)}[\theta_2\Gamma(k^*)(1-\tau(1)-\tau'(1))+\theta_2(\tau''(1)+2\tau'(1)],\\ \frac{\partial\dot{c}_2}{\partial k_1}&=\frac{c^*}{\sigma_2(c^*)}\frac{f'(k^*)^2}{f(k^*)}[\theta_1\Gamma(k^*)(1-\tau(1)-\tau'(1))+\theta_1(\tau''(1)+2\tau'(1)],\\ \frac{\partial\dot{c}_1}{\partial k_1}&=\frac{c^*}{\sigma_2(c^*)}\frac{f'(k^*)^2}{f(k^*)}[\theta_2\Gamma(k^*)(1-\tau(1)-\tau'(1))-\theta_1(\tau''(1)+2\tau'(1)],\end{aligned} \tag{17}$$

where

$$\Gamma(k^*)\equiv\frac{f''(k^*)f(k^*)}{f'(k^*)^2}<0.$$

Let us write the characteristic equation of J in such a way that

$$\lambda^4 - \text{Tr}J\lambda^3 + \text{W}J\lambda^2 - \text{Z}J\lambda + \text{Det}J = 0, \tag{18}$$

where

$$\text{Tr}J = f'(k^*)[2 - \tau(1) - \tau'(1)], \tag{19a}$$

$$\text{W}J = f'(k^*)\rho + \frac{\partial \dot{c}_1}{\partial k_1} + \frac{\partial \dot{c}_2}{\partial k_2}, \tag{19b}$$

$$\begin{aligned} \text{Z}J = f'(k^*)\Bigg\{ & [1 - \theta_1(\tau(1) + \tau'(1))]\frac{\partial \dot{c}_1}{\partial k_1} + [1 - \theta_2(\tau(1) + \tau'(1))]\frac{\partial \dot{c}_2}{\partial k_2} \\ & - (\tau(1) + \tau'(1)) \left[\theta_1 \frac{\partial \dot{c}_1}{\partial k_2} + \theta_2 \frac{\partial \dot{c}_2}{\partial k_1}\right] \Bigg\}, \end{aligned} \tag{19c}$$

$$\text{Det}J = -\frac{f(k^*)f'(k^*)f''(k^*)\rho}{\sigma(c_1)\sigma(c_2)}[2\tau'(1) + \tau''(1)]. \tag{19d}$$

Since our dynamic system involves two jumpable variables, c_1 and c_2, and two predetermined variables, k_1 and k_2, the local stability condition requires that the dynamic system exhibits a regular saddlepoint property at least around the steady state equilibrium. Inspecting the characteristic equation given above, we find one of the main results of this paper:

Proposition 2. *Given Assumptions 1 and 2, if* $2\tau'(1) + \tau''(1) > 0$, *then the steady state is uniquely given and it satisfies local determinacy.*

Proof. Let us denote roots of the characteristic equation by $\lambda_s \, (s = 1, 2, 3, 4)$. Assumption 2 means that the trace of J, which equals $\Sigma_{s=1}^4 \lambda_s$, is strictly positive, so that at least one of the characteristic roots has positive real part. In addition, if $2\tau'(1) + \tau''(1) > 0$, the determinant of J $\left(= \Pi_{s=1}^4 \lambda_s\right)$ is positive and, hence, the number of characteristic roots with positive real parts is either two or four. Note that using (17), we may write $\text{Z}J$ in $(19c)$ as

$$\begin{aligned} \text{Z}J = \frac{(f')^3}{\sigma_1(c^*)\sigma_2(c^*)} & \left\{\Gamma(k^*)\Delta^2(\theta_1\sigma_2(c^*) + \theta_2\sigma_1(c^*)) \right. \\ & \left. -(\theta_1\sigma_1(c^*) + \theta_2\sigma_2(c^*))(2\tau'(1) + \tau''(1))\right\}. \end{aligned}$$

Since $\text{Z}J$ has a negastive value under our assumptions and since $\text{Z}J = \lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\lambda_4 + \lambda_3\lambda_4\lambda_1 + \lambda_4\lambda_1\lambda_2$, at least one root should be negative. Consequently, there are two stable roots, implying that the competitive equilibrium path converging to the steady state is uniquely determined. ■

The above result means that if the marginal tax payment of each agent increases with the individual income, then the economy (at least locally) converges to the symmetric steady state equilibrium where wealth is equally distributed to each agent, regardless of the initial distribution of wealth and form of the utility function of each type of agents. In this sense, the specific form of progressive income taxation assumed in this paper may contribute to establishing income equality in the long run.

If $2\tau'(1) + \tau''(1) < 0$, the dynamic system may not exhibit a regular saddlepoint property. In this case, from $(19d)$ the determinant of J is negative, and, therefore, the number of characteristic root with negative real part is either one or three. If there is only one stable root, the steady state is locally unstable. If matrix J has three stable roots, there is a continuum of converging paths around the steady state equilibrium. equilibrium pathsSince at least one of the charcteristic root is positive, the equilibrium path is indeterminate if and only if (18) has three roots with negative real parts. In this case, we may first observe the following fact:

Proposition 3. *Suppose that* $2\tau'(1) + \tau''(1) < 0$. *Then if agents in both groupes have an identical utility function, the steady state equilibrium is asymptotically unstable.*

Proof. See Appendix Appendix A. ■

Consequently, if $2\tau'(1) + \tau''(1) < 0$, the existence of multiple equilibrium paths converging to the steady state requires that agents in each group have different forms of utility functions. Since it is hard to obtain the analytical expression of sufficient conditions for the presence of three roots with negative real parts, in Section 4.1 we investigate neumerical examples to inspect the possibility of equilibrium indeterminacy in the case of $\sigma_1(c_1) \neq \sigma_2(c_2)$.

4 Alternative Fiscal Rules

4.1 Factor-Specific Taxation

So far, we have assumed that the income tax applies to the total revenue of an individual agent. In this section we consider a more general case where the different tax scheme may apply to labor and capital income, respectively. To make the argument parallel to the previous discussion, we assume that the rate of tax levied on each factor income is given by

$$\tau_k = \tau_k\left(\frac{rk_i}{rK}\right), \qquad \tau_w = \tau_w\left(\frac{wl_i}{wL}\right), \quad i = 1, 2,$$

where τ_k and τ_w respectively denote the rates of tax on capital and labor income. As before, these tax functions are assumed to be monotonically increasing, at least twice differentiable and satisfies, $0 < \tau_k\left(rk_i/rK\right), \tau_w\left(wl_i/wL\right) < 1$. Since we have assumed that an individual household supplies one unit of labor in each moment, it holds that $l_1 = l_2 = L\left(= \theta_1 l_1 + \theta_2 l_2\right)$. Therefore, the rates of tax are determined by $\tau_k = \tau_k\left(k_i/K\right)$ and $\tau_w = \tau_w\left(1\right)$ (= a constant).

Then, the modified capital accumulation constraint in group i is

$$\dot{k}_i = \left[1 - \tau_k\left(\frac{k_i}{K}\right)\right] rk_i + (1 - \tau_w)w - c_i + T_i, \quad i = 1, 2, \tag{20}$$

where T_i represents the government transfer in this model. The government collects the tax revenue by the progressive income tax and returns the lump-sum transfer that amount to the share of each group. Then, the modified flow budget constraint is

$$\theta_1 T_1 + \theta_2 T_2 = \theta_1\left[rk_1\tau_k\left(\frac{k_1}{K}\right) + \tau_w w\right] + \theta_2\left[rk_2\tau_k\left(\frac{k_2}{K}\right) + \tau_w w\right].$$

Assuming that the government pay back an identical amount of transfer to each agent, the lump-sum transfers of each group is

$$T_1 = T_2 = \theta_1\left[rk_1\tau_k\left(\frac{k_1}{K}\right) + \tau_w w\right] + \theta_2\left[rk_2\tau_k\left(\frac{k_2}{K}\right) + \tau_w w\right]. \tag{21}$$

It is easy to see that under the factor-specific taxation, the Euler equation for the optimal consumption of the type i agent is given by

$$\dot{c}_i = \frac{c_i}{\sigma(c_i)}\left\{\left[1 - \tau_k\left(\frac{k_i}{K}\right) - \frac{k_i}{K}\tau_k{}'\left(\frac{k_1}{K}\right)\right] f'(K) - \rho\right\}, \quad i = 1, 2, \tag{22}$$

where $\sigma_i = -u''(c_i)c_i/u'(c_i)(> 0)$. From equations (20) and (21), the dynamic behavior of capital stock held by the type i agents is

$$\dot{k}_i = y_i - c_i + \theta_j \left[rk_j\tau_k\left(\frac{k_j}{K}\right) - rk_i\tau_k\left(\frac{k_i}{K}\right)\right], \quad i, j = 1, 2, \quad i \neq j. \tag{23}$$

Here, K and y_i in (22) and (23) are defined by

$$K = \theta_1 k_1 + \theta_2 k_2, \quad \theta_1 + \theta_2 = 1,$$

$$y_i = f(K) + (k_i - K)f'(K).$$

The steady-state conditions under which $\dot{c}_i = \dot{k}_i = 0$ $(i = 1, 2,)$ are the following:

$$c_i^* = f(K^*) + \theta_j f'(K^*)\left[k_j^*\tau_k\left(\frac{k_j^*}{K^*}\right) - k_i^*\tau_k\left(\frac{k_i^*}{K^*}\right)\right], \quad i, j = 1, 2, \quad i \neq j,$$

$$\rho = f'(K^*)\left[1 - \tau_k\left(\frac{k_1^*}{K^*}\right) - \frac{k_i^*}{K^*}\tau_k'\left(\frac{k_i^*}{K^*}\right)\right], \quad i = 1, 2.$$

If $\tau_k(.)$ function satisfies the same property given in Assumption 1, there is a unique, symmetric steady state where the, $k_1^* = k_2^* = K^*$. Consequently, the steady state conditions reduce to

$$c_1^* = c_2^* = f(K^*), \tag{24}$$

$$\rho = f'(K^*)\left[1 - \tau_k(1) - \tau_k'(1)\right]. \tag{25}$$

As before, (25) requires that.

$$1 - \tau_k(1) - {\tau_k}'(1) > 0. \tag{26}$$

We can inspect local stability of dynamic system consisting of (22) and (23) in the same way as done in the previous section. The linearized system is given by

$$\begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{k}_1 \\ \dot{k}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & \partial\dot{c}_1/\partial k_1 & \partial\dot{c}_1/\partial k_2 \\ 0 & 0 & \partial\dot{c}_2/\partial k_1 & \partial\dot{c}_2/\partial k_2 \\ -1 & 0 & f'(k^*)[1 - \theta_2(\tau_k(1) + {\tau_k}'(1))] & f'(k^*)\theta_2[\tau_k(1) + {\tau_k}'(1)] \\ 0 & -1 & f'(k^*)\theta_1[\tau_k(1) + \tau_k'(1)] & f'(k^*)[1 - \theta_1(\tau_k(1) + {\tau_k}'(1))] \end{bmatrix}}_{M} \begin{bmatrix} c_1 - c_1^* \\ c_2 - c_2^* \\ k_1 - k_1^* \\ k_2 - k_2^* \end{bmatrix}.$$

In this case $\partial \dot{c}_i/\partial k_j$ $(i,j=1,2)$ is given by

$$\begin{aligned}
\frac{\partial \dot{c}_1}{\partial k_1} &= \frac{c^* f'(k^*)}{\sigma_1(c^*)k^*}[\theta_1\Pi(k^*)(1-\tau_k(1)-\tau_k'(1))-\theta_2(2\tau_k{}'(1)+\tau_k{}''(1))], \\
\frac{\partial \dot{c}_1}{\partial k_2} &= \frac{c^* f'(k^*)}{\sigma_1(c^*)k^*}[\theta_2\Pi(k^*)(1-\tau_k(1)-\tau_k'(1))+\theta_2(2\tau_k{}'(1)+\tau_k''(1))], \\
\frac{\partial \dot{c}_2}{\partial k_1} &= \frac{c^* f'(k^*)}{\sigma_2(c^*)k^*}[\theta_1\Pi(k^*)(1-\tau_k(1)-\tau_k'(1))+\theta_1(2\tau_k'(1)+\tau_k''(1)], \\
\frac{\partial \dot{c}_2}{\partial k_2} &= \frac{c^* f'(k^*)}{\sigma_2(c^*)k^*}[\theta_2\Pi(k^*)(1-\tau_k(1)-\tau_k'(1))-\theta_1(2\tau_k{}'(1)+\tau_k{}''(1))],
\end{aligned} \tag{27}$$

where

$$\Pi(k^*) \equiv \frac{f''(k^*)k^*}{f'(k^*)} < 0.$$

The characteristic equation of M is given by

$$\lambda^4 - \mathrm{Tr}M\lambda^3 + \mathrm{W}M\lambda^2 - \mathrm{Z}M\lambda + \mathrm{Det}M = 0,$$

where

$$\mathrm{Tr}M = f'(k^*)[2-\tau_k(1)-\tau_k{}'(1)], \tag{28a}$$

$$\mathrm{W}M = f'(k^*)^2[1-\tau_k(1)-\tau_k{}'(1)] + \frac{\partial \dot{c}_1}{\partial k_1} + \frac{\partial \dot{c}_2}{\partial k_2}, \tag{28b}$$

$$\begin{aligned}
\mathrm{Z}M = f'(k^*)\Bigg\{&[1-\theta_1(\tau_k(1)+\tau'(1))]\frac{\partial \dot{c}_1}{\partial k_1} + [1-\theta_2(\tau_k(1)+\tau_k{}'(1))]\frac{\partial \dot{c}_2}{\partial k_2} \\
&- [\tau_k(1)+\tau_k{}'(1)]\left(\theta_2\frac{\partial \dot{c}_2}{\partial k_1} + \theta_1\frac{\partial \dot{c}_1}{\partial k_2}\right)\Bigg\},
\end{aligned} \tag{28c}$$

$$\mathrm{Det}M = -\frac{f''(k^*)f'(k^*)f(k^*)^2\sigma_1(c^*)\sigma(c^*)[1-\tau_k(1)-\tau_k{}'(1)]}{k^*}[2\tau_k{}'(1)+\tau_k{}''(1)]. \tag{28d}$$

We find that $\mathrm{Z}M$ given above is written as

$$\begin{aligned}
\mathrm{Z}M = \frac{(f')^2 f}{k^*\sigma_1(c^*)\sigma_2(c^*)}\Bigg\{&\Pi(k^*)\Omega^2(\sigma_2(c^*)\theta_1+\sigma_1(c^*)\theta_2) \\
&- (\sigma_1(c^*)\theta_1+\sigma_2(c^*)\theta_2)(2\tau_k{}'(1)+\tau_k{}''(1))\Bigg\},
\end{aligned}$$

It is easy to show that if we replace $\tau(y_i/Y)$ function with $\tau_k(k_i/K)$, then Proposition 2 also holds for the case of factor-specific taxation. First, if $2\tau_k{}'(1)+\tau_k{}''(1)>0$, there is a unique, symmetric steady state. In addition, given our assumptions, we see that Tr $M>0$, Det $M>0$ and $\mathrm{Z}M<0$. Therefore, as shown by the proof for Proposition 3, we may claim the following results:

Proposition 4. *Under the factor-specific income taxation, the steady-state equilibrium is uniquely given and satisfies local determinacy, if the marginal tax payment from capital income monotonically increases with relative capital holding, k_i/K.*

4.2 Government Consumption

It is to be noted that in our setting the transfer payment to the household plays a critial role to detertmine stability of the steady-state equilibrium. To see this, suppose that all the tax revenue is spent for consumption by the government. If this is the case, the flow budget constraint for the government is given by

$$\theta_1 \tau \left(\frac{y_1}{Y}\right) y_1 + \theta_2 \tau \left(\frac{y_2}{Y}\right) y_2 = G,$$

where G denotes the government consumption of the final goods. Since there is no transfer from the government, the budget constraint for type i agent is

$$\dot{k}_i = \left[1 - \tau\left(\frac{rk_i + w}{Y}\right)\right](rk_i + w) - c_i, \quad i = 1, 2,$$

and the aggregate dynamics of capital is

$$\dot{K} = f(K) - C - G.$$

Here, we again assume that the income tax is levied on capital and labor income uniformely.

In this case it is easy to see that the linearized dynamic system can be written as follows:

$$\begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{k}_1 \\ \dot{k}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & \partial \dot{c}_1/\partial k_1 & \partial \dot{c}_1/\partial k_2 \\ 0 & 0 & \partial \dot{c}_2/\partial k_1 & \partial \dot{c}_2/\partial k_2 \\ -1 & 0 & f'(k^*)[1-(\tau(1)+\tau'(1))] & 0 \\ 0 & -1 & 0 & f'(k^*)[1-(\tau(1)+\tau'(1))] \end{bmatrix}}_{N} \begin{bmatrix} c_1(t) - c_1^* \\ c_2(t) - c_2^* \\ k_1(t) - k_1^* \\ k_2(t) - k_2^* \end{bmatrix}.$$

Inspecting this system immediately presents the results shown below:

Proposition 5. *If the governemt consumes its tax revenue and if the tax function satisies $2\tau'(1) + \tau''(1) > 0$, then the steady state equilibrium satisfies regular* *saddle point.stability.*

Proof. See Appendix Appendix B. ■

5 Numerical Examples

In Sections 3.3 and 4.1 we have confirmed that if the marginal tax payments decreases with the relative income, the steady state equilibrium is either locally indeterminate or totally unstable. Uless the two groups of agents have an identical utility function, it is hard to obtain analytical conditions that determine whether or not the steady state equilibrium is indeterminate. Thus we examine numerical examples to obtain intuitive implications of the dynamics behavior of our model economy. We first consider the case of uniform taxation, using the following tax function:.

$$\tau\left(\frac{y_i}{Y}\right) = \frac{(y_i/Y)^{\xi}}{b + m\,(y_i/Y)^{\xi}}. \tag{29}$$

where

$$b + m > 0, \quad b\xi > 0, \quad \text{and} \quad (b+\xi)^2 > b(1+\xi) + m.$$

Note that this functional form satisfies all of our assumptions on the tax function including Assumption 1.[3] Given this tax function, the key values evaluated at the steady state equilibrium are given by the following:

$$\tau(1) = \frac{1}{b+m} > 0.$$

[3]Guo and Lansing (1998) and Li and Sarte (2004) specify the tax function in such a way that

$$\tau\left(\frac{y_i}{Y}\right) = \tau_0\left(\frac{y_i}{Y}\right)^{\phi}, \quad 0 < \tau_0 < 1, \quad \phi < 1.$$

.Usint this functional form, we obtain

$$\frac{\partial\left(\tau\left(y_i/Y\right)y_i\right)/\partial y_i}{\tau\left(y_i/Y\right)} = 1 + \phi,$$

$$2\tau'(1) + \tau''(1) = \phi(\phi+1).$$

Therefore, in this case $2\tau'(1) + \tau''(1)$ cannot have a negative sign, unless income taxation is regressive, i.e. $\phi < 0$. In addition, the above function monotonically increases with y_i/Y, it may violates $\tau(y_i/Y) < 1$. Function (29) is free from those problems.

$$\tau'(1) = \frac{b\xi}{(b+m)^2} > 0.$$

$$\tau''(1) = \frac{b\xi\{b(\xi-1) - m(1+\xi)\}}{(b+m)^3},$$

$$1 - \tau(1) - \tau'(1) = \frac{(b+m)^2 - b(1+\xi) - m}{(b+m)^2} > 0.$$

We also assume that the utility function is a CES function specified by (**??**) .

As for the bench mark case, we set:

$$\alpha = 0.3,\ b = 0.58,\ m = 2.2,\ \xi = 5.8,\ \rho = 0.02.$$

Then the before-tax rate of return to capital, r, is 0.9756 and the rate of the income tax is 0.3579 so that $1 - \tau(1) - \tau'(1)$ has a positive value. [4] In what follows, we focus on the elasticity of intertemporal substitution in consumption, $1/\sigma_i$, as well as on the population share of each group, θ_i,in order to explore the possibility of equilibrium indeterminacy around the steady state. In so doing, we depict the region that satisfies (**??**) in (σ_1, σ_2) under alternative values of θ_1: see Figure 1.[5] In view of this figure and numerical experiments, we notice the following facts. First, when σ_1 and σ_2 are close to each other, equilibrium indeterminacy is not likely to be observed. In contrast, if σ_2 is relatively larger than σ_1, then the possibility of indeterminacy increases. Second, if we raise α from 0.3 to a higher value such as 0.8, then indeterminacy tends to disappear.

In the model with factor specific taxation, (29) is replaced with

$$\tau_k\left(\frac{rk_i}{rK}\right) = \frac{(k_i/K)^{\xi}}{b + m\,(k_i/K)^{\xi}},$$

$$\tau_w\left(\frac{wN_i}{wN}\right) = \frac{(\theta_i)^{\varepsilon}}{b' + m'\,(\theta_i)^{\varepsilon}}.$$

Using those tax functions, we conduct numerical experiments to obtain the results displayed in Figure 2. As the figure shows, the results are similar to the case of uniform taxation: indeterminacy tends to emerge when σ_2 is sufficiently larger than σ_1 (or σ_1 is sufficiently larger than σ_2).It is also to be noted that the region of

[4] Since we have ignore capital depreciation, the before tax rate of return to capital in the steady state has a rather high value.

[5] To depict the graphs in Figure 1, we change σ_i from 0.1 to 5.0 with an intervals of 0.05.

(σ_1, σ_2) in which indeterminacy holds is smaller than that in the case of uniform tax: see Figure 1 again. Therefore, as far as in our setting, the factor-specific taxation may reduces the possibility of expectations-deriven economic fluctuations caused by multiplicity of perfect-foresight competitive equilibrium.

Inspecting the numerical examples shown above suggests the following. First of all, some form of heterogeneity of agents should be present to hold equilibrium indeterminacy. This result looks plausible, because the representative agent economy with our taxation scheme will not exhibit multiple converging paths. If two groups are identical, the tax rate is always fixed at $\tau(1)$ even out of the steady state and the government budget satisfies $T = \tau Y$. Thus the aggregate dynamic system under our fiscal rule may be summarized as

$$\dot{C} = \frac{C}{\sigma(C)}\left[\left(1 - \tau(1) - \tau'(1)\right) f'(K) - \rho\right],$$
$$\dot{K} = f(K) - C,$$

so that the regular saddlepoint stability is guaranteed for all $\tau(1) \in [0, 1)$.

Similarly, suppose that one of the capital stocks, say k_2, stays constant. Then the linearized dynamic equations for the group 1 agent is

$$\begin{bmatrix} \dot{c}_1 \\ \dot{k}_1 \end{bmatrix} = \begin{bmatrix} 0 & \partial \dot{c}_1 / \partial k_1 \\ -1 & f'(k^*)[1 - \theta_2(\tau(1) + \tau'(1))] \end{bmatrix} \begin{bmatrix} c_1 - c^* \\ k_1 - k^* \end{bmatrix}.$$

The coefficient matrix of this system has a positive trace $(= f'(k^*)[1 - \theta_2(\tau(1) + \tau'(1))])$, so that indeterminacy never emerges. Hence, equilibrium indeterminacy critically depends on the fact that k_2 is adjusted simultaneously out of the steady state equilibrium. Finally, the linearized dynamic system always has at least one characteristic root with positive real part. Indeterminacy thus emerges when there are three roots with negative real parts.

6 Conclusion

This paper has studied equilibrium dynamics of a Ramsey economy with heterogenous agents in which income taxation is progressive. We have assumed that the rate of income tax depends on an individual income relative to the average income of the economy at large and that the tax payments are equally distributed back to each household. In this setting, it is shown that under weak restrictions on the tax function, the steady-state equilibrium is uniquely given and there exists a unique converging path at least around the steady state unless the marginal tax payment of each household increases with its relative income. Otherwise, the steady state is either unstable or locally indeterminate. If the latter holds, there is a continuum of converging path around the steady state, so that expectatinos-deriven fluctuations may be present. Using numerical examples,we have confirmed that the presence of equilibrium indeterminacy requires that the elasticity of intertemporal substitution in consumption of each type of agent is sufficiently different from each other. The central message of our study is that the stabilizing power of progressive income taxation demonstrated in representaive-agent models may not be always effective if there are heterogenous agents with different preferences.

The analytical framework of this paper is one of the simplest settings. We have assumed that there are only two types of agents and each agent supplies a fixed level of labor. In addition, we have focused on the symmetric steady state equilibrium in which all the agents hold the identical levels of wealth and income. Among the possible extensions of our discussion, an argent task is to introduce endogenous labor-leisure choice of the households. Such a generalization would be particularly interesting for comparing uniform taxation with factor-specific taxation discussed in Sections 3 and 4, because the factor-specific taxation may play a more prominent role when labor supply is flexible. In addition, it is also worth studying the case that the stabilization effect of taxation in the asymmetric steady state due to, for example, the presence of difference in the rate of time preference.

Appendicies

Appendix A

Letting I be 4×4 unit matrix, the characteristic equqtion matrix J is expressed in the following manner:{

$$\det\left[I\lambda - J\right] = \det\begin{bmatrix} \lambda & 0 & -\frac{\omega}{\sigma_1}\left[\theta\Gamma\Delta - (1-\theta)\,T\right] & -\frac{\omega}{\sigma_1}(1-\theta)\left[\Gamma\Delta + T\right] \\ 0 & \lambda & -\frac{\omega}{\sigma_2}\theta\left[\Gamma\Delta + T\right] & -\frac{\omega}{\sigma_2}\left[(1-\theta)\,\Gamma\Delta - \theta T\right] \\ 1 & 0 & \lambda - f'[1-(1-\theta)(1-\Delta)] & -(1-\theta)\,f'[1-\Delta] \\ 0 & 1 & -\theta f'[1-\Delta] & \lambda - f'[1-\theta(1-\Delta)] \end{bmatrix}$$

$$= \det\begin{bmatrix} \begin{matrix}-\frac{\omega}{\sigma_1}\left[\theta\Gamma\Delta - (1-\theta)\,T\right] - \lambda^2 \\ +f'[1-(1-\theta)(1-\Delta)]\lambda\end{matrix} & \begin{matrix}-\frac{\omega}{\sigma_1}(1-\theta)\left[\Gamma\Delta + T\right] \\ +(1-\theta)\,f'[1-\Delta]\lambda\end{matrix} \\ -\frac{\omega}{\sigma_2}\theta\left[\Gamma\Delta + T\right] + \theta f'[1-\Delta]\lambda & \begin{matrix}-\frac{\omega}{\sigma_2}\left[(1-\theta)\,\Gamma\Delta - \theta T\right] \\ -\lambda^2 + f'[1-\theta(1-\Delta)]\lambda\end{matrix} \end{bmatrix}$$

In the above, we define:

$$\theta = \theta_1 = 1 - \theta_2 \quad \text{so } \theta_2 = 1 - \theta$$

$$\Gamma = \frac{f''(k^*)\,f(k^*)}{f(k^*)^2} < 0, \qquad \Delta = 1 - \tau(1) - \tau'(1) > 0,$$

$$T = 2\tau'(1) + \tau''(1), \quad \omega = \frac{c^* f'(k^*)^2}{f(k^*)} > 0.$$

It is now easy to confirm that, if $\sigma_1 = \sigma_2 = \sigma$, then the characterristic equation can be expressed as

$$\begin{aligned} \det\left[I\lambda - J\right] &= -\left(\lambda^2 - f'\lambda + \frac{\omega}{\sigma}\Gamma\Delta\right)\Big\{-\frac{\omega}{\sigma}\left[(1-\theta)\,\Gamma\Delta - \theta T\right] - \lambda^2 \\ &\quad + f'[1-\theta(1-\Delta)]\lambda + \frac{\omega}{\sigma}(1-\theta)\left[\Gamma\Delta + T\right] - (1-\theta)\,f'[1-\Delta]\lambda\Big\} \\ &= \left[\lambda^2 - f'\lambda + \frac{\omega}{\sigma}\Gamma\Delta\right]\left[\lambda^2 - \Delta f'\lambda - \frac{\omega}{\sigma}T\right]. \end{aligned}$$

Thus the characteristic equation, $\det\left[I\lambda - J\right] = 0$, is given by the following:

$$\left[\lambda^2 - f'\lambda + \frac{c^* f' f''}{\sigma}\left(1 - \tau - \tau'\right)\right]\left[\lambda^2 - \left(1 - \tau - \tau'\right)f'\lambda - \frac{c^* f'^2}{\sigma f}\left(2\tau' + \tau''\right)\right] = 0$$

Notice that equation

$$\lambda^2 - f'(k^*)\,\lambda + \frac{c^* f'(k^*)\,f''(k^*)}{\sigma(c^*)}\left(1 - \tau(1) - \tau'(1)\right) = 0$$

has one positive and one negative roots, while both roots of

$$\lambda^2 + \left(1 - \tau(1) - \tau'(1)\right) f'\lambda + \frac{c^* f'^2(k^*)}{\sigma(c^*) f(k^*)} \left(2\tau'(1) + \tau''(1)\right) = 0$$

have positive real parts under the assumption of $2\tau'(1) + \tau''(1) < 0$. Therefore, J has one negative and three characteristic roots with positive real parts, which means that there is no converging path around the steady state when the initianl values of k_1 and k_2 diverge from their steady state values of k_1^* and k_2^*.

Appendix B

The characteristic equation of matrix N is

$$\det[\lambda I - N] = \begin{bmatrix} \lambda & 0 & -\frac{\omega}{\sigma_1}[\theta\Gamma\Delta - (1-\theta)T] & -\frac{\omega}{\sigma_1}(1-\theta)[\Gamma\Delta + T] \\ 0 & \lambda & -\frac{\omega}{\sigma_2}\theta[\Gamma\Delta + T] & -\frac{\omega}{\sigma_2}[(1-\theta)\Gamma\Delta - \theta T] \\ 1 & 0 & \lambda - f'(k^*)[1 - (\tau(1) + \tau'(1))] & 0 \\ 0 & 1 & 0 & \lambda - f'(k^*)[1 - (\tau(1) + \tau'(1))] \end{bmatrix}$$

$$= \det\begin{bmatrix} \begin{array}{c} -\frac{\omega}{\sigma_1}[\theta\Gamma\Delta - (1-\theta)T] - \lambda^2 \\ +f'[1 - (1-\theta)(1-\Delta)]\lambda \end{array} & -\frac{\omega}{\sigma_1}(1-\theta)[\Gamma\Delta + T] \\ -\frac{\omega}{\sigma_2}\theta[\Gamma\Delta + T] & \begin{array}{c} -\frac{\omega}{\sigma_2}[(1-\theta)\Gamma\Delta - \theta T] \\ -\lambda^2 + f'[1 - \theta(1-\Delta)]\lambda \end{array} \end{bmatrix},$$

where T, Γ, Δ, ω and θ are the same defined in Appendix A. Thus the characteristic equqtion, $\det[\lambda I - N] = 0$, is given by

$$\begin{aligned} &\left[\lambda^2 - f'(1 - (1-\theta)(1-\Delta))\lambda + \frac{\omega}{\sigma_1}[\theta\Gamma\Delta - (1-\theta)T]\right] \\ &\times \left[\lambda^2 - f'(1 - \theta(1-\Delta))\lambda + \frac{\omega}{\sigma_1}(1-\theta)(\Gamma\Delta + T)\right] - \frac{\omega^2}{\sigma_1\sigma_2}(\Gamma\Delta + T)^2 \\ &= 0 \end{aligned}$$

Applying the same logic used in the proof of Proposition 1, we can confirm that this equation has two roots with negative real parts if $T = 2\tau'(1) + \tau''(1) > 0$..

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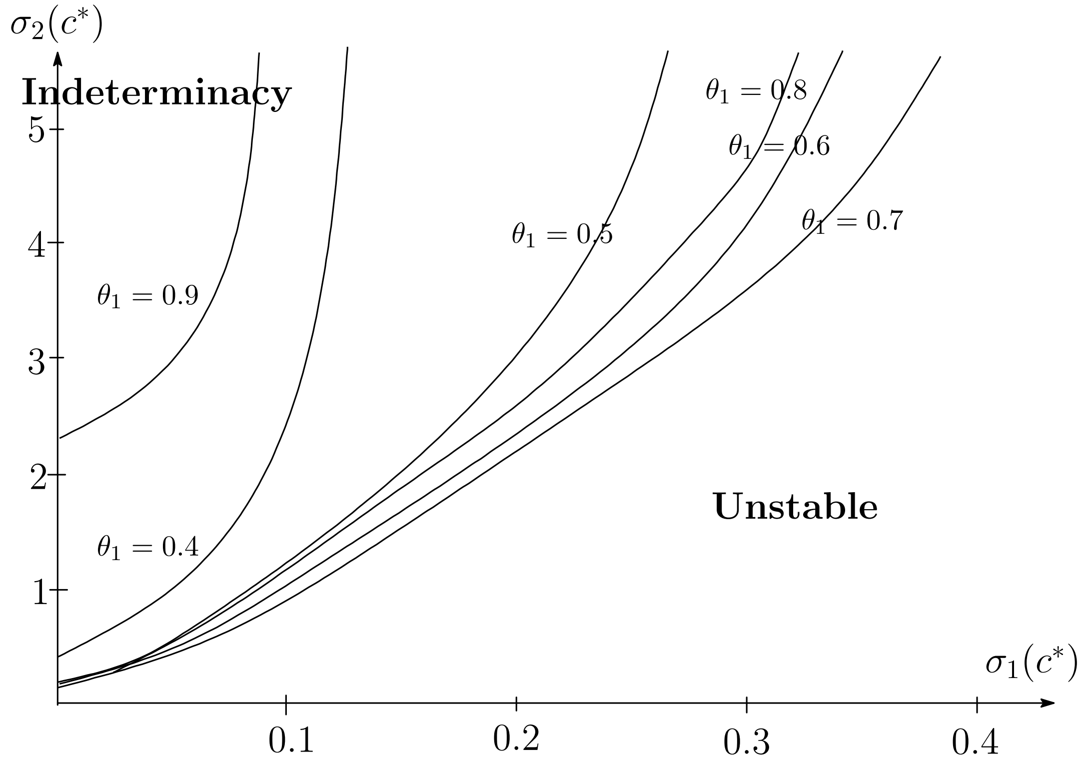
The case of non-separable tax function



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