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Gyoung-Gyu Choi
Dongguk University

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SEQUENTIAL EQUILIBRIUM IN LEGISLATIVE BARGAINING

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Abstract

In this study, a simple model of legislative bargaining under asymmetric information in the discounting factors is constructed. To emphasize the importance of uncertainty and heterogeneity, we consider three-person legislative game with asymmetric information and heterogeneity among members of the legislature.

If the relatively patient player reveals her type, other players may not include her in the coalition due to the high price to buy her vote. Therefore, there may exist the incentive not to reveal the true type, and it is possible to have no fully separating equilibrium. Pooling equilibrium may be realized in which oversized coalitions can be rationalized due to the bargaining position of the coalition partner.

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[1] Research Professor, Techno-Economic Policy Program, Engineering School, Seoul National University.

Ⅰ. Introduction

In distributive policy making, predictions of legislative bargaining theories about the characteristics of legislative outcome have been widely ranged. Classical tradition argues that the majority will adopt distributive policies that benefit themselves at the expense of the minority. Majorities will be of the barest possible size, since a minimum winning coalition (MWC) maximizes the per capita gains for the winners (Riker, 1962; Shepsle, 1974). As a result, adopted policies tend to be ones with concentrated benefit but dispersed costs.

Such theories, however, are not general enough to be consistent with many different institutional environments. Particularly, predictions of such theories are heavily dependent on the decision making or voting procedure employed by legislative members, the size of legislative body, information structure, and discount factor. Using noncooperative bargaining theory, Baron and Ferejohn(1989a) show that under a closed rule, a majoritarian outcome results; minimal winning coalition will enjoy monopoly benefit while costs are distributed among all members. And this model is generalized and applied to a variety of issues in economics and political science.[2]

Under an open rule, however, universalism could arise if the legislature is small. In large legislature, the size of winning coalition is not minimal but inbetween minimal and universalistic size, leading to more equal distribution of benefit to members than under a closed rule. Weingast (1979), contends that legislators prefer to adopt a norm of "universalism" based on cooperative incentives to maximize the ex-ante expected

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[2] Baron and Ferejohn (1989b) apply the model to analyze the role of committees. Chari et al. (1997) use the model in the analysis of split-ticket voting, McKelvey and Reizman (1992) use it to discuss seniority in legislatures, and Merlo(1997) studies legislative bargaining in a stochastic setting.

payoffs. Norman (2000) argues an equilibrium with oversized coalitions can be supportable if players use history dependent strategies. Groseclose and Snyder (1996) also get supermajority coalitions in a vote buying model, and discuss surveys of empirical studies on oversized coalitions.

In a zero-sum majority rule, the coalitions will be minimal size, or $\frac{N+1}{2}$ of legislatutors. If a fixed amount of benefit is to be divided up, increasing the numbers in the coalition will serve only to decrease the payoff to some or all of the winning coalition. If a coalition forms that is bigger than the minimum size, then a subset of the original coalition can increase their own payoff by excluding some members of the larger coalition.

In the present analysis, a simple model of legislative bargaining under asymmetric information is constructed and a separating equilibrium is derived. The constructed model is designed to support the hypothesis that minimal wining coalition will enjoy benefits while cost is dispersed among all members.

The ideal model should, of course, be able to explain the observed phenomena under very weak assumptions. There is a long list of characteristics that an ideal model should have. An ideal model should incorporate informational as well as strategic uncertainties, heterogeneity among members of legislative body, generalized voting of decision making procedure, and many more.[3] To emphasize the importance of

[3] There are a few studies in bargaining with incomplete information infinite-horizon games. Models with one-sided uncertainty are discussed in Sobel and Takahashi (1983) with one-side offers and continuous type space, and Grossman and Perry (1986) with alternating offers. For two-side uncertainty models, see Cramton (1984, 1992) with one-sided offers and continuous type space and Chaterjee and Smauelson (1987, 1988) with alternating offers and binary type space. Fudenberg and Tirole (1983) provide a two-period game with one-sided offers and binary space.

uncertainty and heterogeneity, and to be more realistic, we consider three person (n=3) legislative game with asymmetric information and heterogeneity among members of the legislature. A simple asymmetric information structure is assumed about the size of discount factor of each player. As is well known, discount factor (or reelection probability) is an important determinant of the set of subgame perfect equilibria in models with no uncertainty and perfect information.

In the next chapter, we discuss the basic set-up of the model. In Chapters III and IV, the sequential equilibrium under complete information and the sequential equilibrium under incomplete information are analyzed, respectively. And we conclude in Chapter V.

II. The Model

The model in this paper is a modification of the Rubinstein's bargaining model,[4] which the basic setup of the legislative bargaining is a three-member legislature equipped with a majority voting scheme. The task of the legislature is to allocate one unit of divisible benefits among three members (A, B and C) through majority rule. In each session, the proposal maker is selected randomly with probability $1/3$. For tractability, the model assumes a closed rule in which once a proposal is made by the proposal maker, it is voted without amendment. If the proposal is passed, then the legislature adjourns. Otherwise, the legislature continues to the next session.

The model assumes that each member is characterized by her reelection probability, δ_i, i = A, B and C, where $0 < \delta_i < 1$.[5] For simplicity, we assume that

[4] See Osborne and Rubinstein (1990).

[5] The reelection probability of each member can be interpreted as her discount factor.

either $\delta_i = \delta_H$ or $\delta_i = \delta_L$ where $0 < \delta_L < \delta_H < 1$. Member i's type is H if $\delta_i = \delta_H$ while it is L if $\delta_i = \delta_L$. The preference of member i depends only on the share received by the member i. The bargaining in the model occurs in the process of forming a winning coalition under the majority voting scheme.

III. Sequential Equilibrium under Complete Information

Complete information structure of the model implies that all members know the institutional setting described in section II and that each member's true type is common knowledge. Under complete information and majority voting with closed rule, the legislature concludes its task in the first session ($t = 0$). If $\delta_i = \delta$ for all i, then any proposal maker will form a minimal winning coalition which consists of two members by trivially randomizing with probability $1/2$. In this case, the proposer will propose $\delta/3$ and receive $1-\delta/3$ in equilibrium. Each member's ex-ante continuation value of the bargaining game is simply $1/3$.

If the legislative members are not of homogeneous type, however, the possibility of nontrivial randomization arises depending on the size of δ_L and δ_H. In this section, we will identify members by their types (H, L) rather than their names (A, B, C), since under complete information all members are identical except their types. There are two cases in which all members' types are not identical; (H, H, L) and (L, L, H). Since the two cases are very similar, we only provide the results of the case (H, H, L).

Consider the case (H, H, L). Basically, L has an intrinsic advantage relative to H as a coalition member; Due to the smaller reelection probability, L is more likely to be chosen as a coalition member than H is. Such an intrinsic advantage, however, could lead to higher continuation value for L than H, making L less attractive as a coalition member ceteris paribus. Depending on the size of reelection probabilities of H and L, this kind of trade-off could induce H to randomize rather than to employ the pure strategy of always choosing L as a coalition member.

Member L, if recognized, proposes to one of two H's with equal probability $1/2$. This is a trivial randomization case. On the other hand, if one of two H's becomes a proposal maker, she will randomize between H and L so that H and L will be chosen as a coalition member with probabilities r and $1-r$, respectively, where $0 \leq r < 1$. The pure strategy equilibrium in which H always proposes to L or H corresponds to the special case $r = 0$ or $r = 1$, respectively.

With this structure of the bargaining game, consider first the case in which L is recognized. Member L's proposal is given by

$$1 - Z_L = \delta_H V_H \tag{1}$$

$$V_H = \frac{1}{3} Z_H + \frac{1}{6} \delta_H Z_H + \frac{1}{3} r(1 - Z_H)$$

where Z_L and Z_H are the share received by the proposer L and H, respectively, when the proposal is passed. V_H is the continuation value of H in the next session. The first term in V_H is the expected payoff to H when recognized. The second term reflects the fact that if L is recognized, H will become a coalition member with probability $1/2$. The last term is the expected payoff of H when other H is recognized.

If H is selected as the proposal maker, then H's proposal $1-Z_H$ to either H or L is represented by

$$1-Z_H = \max[\delta_L V_L, \delta_H V_H] \qquad (2)$$

$$V_L = \frac{1}{3}Z_L + \frac{2}{3}(1-r)(1-Z_H)$$

where V_L is the continuation value of L. The first equality in (2) implies that the proposal made by H is equal to the larger between $\delta_L V_L$ and $\delta_H V_H$ If $r=0$ in equilibrium, this equality reduces to $1-Z_H = \delta_L V_L$ in which $\delta_L V_L > \delta_H V_H$ while it reduces to $1-Z_H = \delta_H V_H > \delta_L V_L$ if $r=1$. If $0<r<1$, this equality implies that under the randomization strategy(i, e. $\delta_L V_L = \delta_H V_H$), the proposal of H is same regardless of the coalition member's type so that (i) the proposer's own payoff is same in either case and (ii) the proposal is accepted for sure. Note that the legislative bargaining game ends in the first session $t=0$ under complete information.

Combining (1) and (2), and solving for Z_H lead to the following equality given by

$$1-Z_H = \max\left[\frac{(2-\delta_H)\delta_L}{6-\delta_H-4(1-r)\delta_L}, \frac{2\delta_H}{6+\delta_H-2r\delta_H}\right] \qquad (3)$$

where the first term in the square bracket comes from the inequality $1-Z_H \geq \delta_L V_L$ while the second term from $1-Z_H \geq \delta_H V_H$ Note that the first term is a decreasing function of r while the second term is an increasing function of r. It can be easily shown that the first term is larger than or equal to the second term only if $r \leq r_1$ where

$r_1 = (1/2) - (\delta_H - \delta_L)/(\delta_L \delta_H < 1/2)$. What is an optimal r^* for H when she randomizes? The optimal r^* for H is the one that maximizes Z_H. Using r_1 and comparing the first and the second term in the square bracket in (3), it can be easily shown that $r^* = r_1$. Consequently, $\delta_L V_L = \delta_H V_H$ as long as $0 < r^* = r_1 < 1$. Since $r^* < 1/2$, H is more likely to choose L as a coalition member.

Depending on the relative values of δ_L and δ_H, however, r_1 is not always strictly positive. Since it is strictly less than $1/2$ for all δ_L and δ_H such that $0 < \delta_L < \delta_H < 1$, optimal r^* cannot equal 1. This implies that the pure strategy equilibrium in which H proposes to other H with probability 1 cannot occur. On the other hand, $r_1 \leq 0$ if $\delta_H \geq 2\delta_L/(2-\delta_L)$. In this case, optimal $r^* = 0$ and H, if recognized, proposes to L always. If $r^* = 0$, however, we do not need to consider $\delta_H V_H$ in (2). In this case, H's proposal is simply $1 - Z_H = \delta_L V_L$. By replacing the first equality in (2) by $1 - Z_H = \delta_L V_L$ and by solving (1) and (2) together, the pure strategy equilibrium is given by

$$1 - Z_H^P = \delta_L V_L^P, \qquad V_L^P = (2-\delta_H)/(6 - \delta_H - 4\delta_L) \tag{4}$$

$$1 - Z_L^P = \delta_H V_H^P, \qquad V_H^P = 2/(6 + \delta_H)$$

where superscript P is used to denote the pure strategy equilibrium.

Alternatively, if $\delta_H < 2\delta_L/(2-\delta_L)$, r_1 is strictly positive and the nontrivial randomization by H occurs. Given $0 < r^* < 1$, the randomization strategy equilibrium is constructed as

$$r^* = 1/2 - (\delta_H - \delta_L)/(\delta_L \delta_H) \tag{5}$$

$$1 - Z_H^R = \delta_L V_L^R, \qquad V_L^R = \delta_H / (2\delta_L + \delta_H)$$

$$1 - Z_L^R = \delta_H V_H^R, \qquad V_H^R = \delta_L / (2\delta_L + \delta_H)$$

where superscript R is used to denote the randomization strategy equilibrium. Note that $Z_H^R = Z_L^R$ in this case.

The sequential equilibria under complete information in (4) and (5) are summarized in [Table 1].6

[Table 1]. Sequential Equilibrium with (H, H, L)

An interesting aspect of the sequential equilibria shown in Table 1 is that the nature of equilibrium depends not only on relative size of δ_L and δ_H but also on the absolute size of δ_L.

[Figure 1] shows the region of tuple (δ_L, δ_H) for equilibrium with pure and randomization strategies. In region I, H employs a pure strategy so that L is chosen as a coalition member with probability 1. In region II, H will randomize by proposing to H and L with probabilities r^* and $1-r^*$, respectively. Figure 1 shows that if $2/3 < \delta_L < 1$, the pure strategy equilibrium cannot occur regardless of the relative size of δ_L and δ_H. Also consider point A and B in region I. The ratio δ_H / δ_L is much smaller at point A than at point B, but in both cases, pure strategy equilibrium arises.

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[6]Under complete information, the sequential equilibrium in Table 1 is simply a subgame perfect equilibrium.

[Figure 1]

IV. Sequential Equilibrium under Incomplete Information

In this section, we will investigate the possibility of sequential equilibrium under incomplete information. The incomplete information structure of the legislative bargaining game is that member i's reelection probability δ_i, i = A, B, C, is private information and that $\delta_i = \delta_H$ (type H) or δ_L (type L), with probabilities p and $1-p$, respectively, where p is common knowledge. We assume $0 < p < 1$. As in the previous section, only a closed rule is considered.

As will be shown below, recognition is valuable in this game, and this fact suggests us that the magnitude of p is an important factor that determines the strategies of the member recognized. In each session, the proposer has two strategies called S and PL. Under strategy S, the member recognized risks the probability of rejection by making a proposal that will be passed only if the chosen coalition member is of type L. Alternatively, under strategy PL, the member recognized makes a proposal that will be passed for sure. Note that under such a strategy profile, the three-member legislature can continue at most to session $t = 3$. The game tree under such a strategy profile is depicted in [Figure 2].

[Figure 2]

If strategy S is employed at session t, the legislature will complete its task in session t with probability $1-p$ whereas the proposal is rejected and the legislature will continue to session $t+1$ with probability p. The rejection of the proposal at t will reveal information about the coalition member's type so that other members will update their belief about the coalition member's true type. By construction, other members will believe the coalition member to be of type H upon the rejection of the proposal under strategy S. In sequential equilibrium, such belief should be correct and consistent with the information structure in each session.

It is important to note that the bargaining game can continue to session $t=1$ only if strategy S is employed at $t=0$ and the proposal is rejected. Therefore, only one member's type must be revealed to be of H at the beginning of session $t=1$. By the same logic, the game continues to session $t=2$ only if strategy S is employed at $t=0$ and 1, and both proposals are rejected. Since the member whose type is already revealed to be of H at $t=1$ cannot be chosen as a coalition member if strategy S is taken at $t=1$, exactly two members' types must be revealed to be H at the beginning of session $t=2$. Obviously, if the game continues to $t=3$, then all members' types are revealed to be H.

For simplicity, we assume that the proposal maker employs strategy PL when she is indifferent between strategies S and PL, and that the coalition member will vote for the proposal if she is indifferent between voting for and against the proposal. Without loss of generality, we construct a sequential equilibrium by assuming throughout the analysis that if the game continues to session $t=1$ it is member B whose type is already revealed at $t=1$, and that if the game continues to session $t=2$, types of both A and B are already revealed at $t=2$. This combination of information revealing across sessions is general enough because other combinations can be covered simply by reshuffling the names of members.

To proceed, we introduce the following notation: $V_t^i(\delta_i;d)$, where $d=H,L$ and U. $V_t^i(\delta_i;d)$ represents the continuation value of the member i at session t when she is believed by other members to be of type d at session t. Type $d=U$ means "unknown". We will solve this legislative bargaining game backward from the ultimate session $t=3$.

1. Session $t=3$

Consider the last session $t=3$. Since the legislature will continue to session $t=3$ only if all proposers at $t<3$ take strategy S and all proposals are rejected, every member is believed to be of type H by other members at $t=3$. In other words, $d=H$ at $t=3$ for all members. In equilibrium, such belief should be correct so that all members are truly of type H. As in the case of complete information with $\delta_i=\delta_H$ for all i, the continuation value at $t=3$ is

$$V_3^i(\delta_i;H)=1/3 \qquad \text{for all } i \tag{6}$$

In fact, the continuation value of all members at $t=3$ is $1/3$ regardless of their true reelection probability δ_i. To see this, suppose that member A's true reelection probability is δ_L while members B and C believe her to be of type H. Also suppose that each member believes other members to be of type H. If member A is recognized at

$t=3$, she will propose $(1/3)\delta_H$ to either B and C based on her belief and will receive $1-(1/3)\delta_H$. If either B or C is recognized, they will propose $(1/3)\delta_H$ upon their belief about other members' true type. As a result, the ex-ante continuation value of member A is $1/3$

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2. Session $t=2$

In session $t=2$, types of members A and B are revealed and believed to be H by assumption. In equilibrium, such belief should be correct so that types of member A and B are truly H. Suppose that member C, whose type is not revealed at $t=2$, is recognized. Then C will propose $\delta_H V_3^i(\delta_H;H)=(1/3)\delta_H$ to either member A or member B because both members A and B are believed to be of type H, and the proposal will be passed and the game ends at $t=2$ in equilibrium. Note that member C's proposal does not depend on her true type.

Next, consider the case in which member A is recognized. If A pursues strategy S, she will propose only to member C because member B is known to be of type H. Her proposal to member C is $\delta_L V_3^C(\delta_L;H)=(1/3)\delta_H$, and, in equilibrium, the proposal will be passed with probability $1-p$ if member C is of type L and fail to be passed with probability p otherwise. If rejected, the legislature continues to session $t=3$, and the continuation value of member A at $t=2$ is given by $\delta_H V_3^A(\delta_H;H)=(1/3)\delta_H$ in equilibrium. Therefore, upon the recognition, member A's expected payoff R (S) under strategy S is given by

$$R_2^A(S) = (1-p)\left(1 - \frac{1}{3}\delta_L\right) + p\frac{1}{3}\delta_H \qquad (7)$$

Alternatively, if member A employs strategy PL, then she proposes $\delta_H V_3^i(\delta_H; H) = (1/3)\delta_H$ to either member B or C, and this proposal will be passed with probability 1. Obviously, member A's expected payoff $R_2^A(PL)$ under strategy PL is

$$R_2^A(PL) = 1 - (1/3)\delta_H \qquad (8)$$

Upon the recognition at $t = 2$, member A will prefer S to PL only if $R_2^A(S) > R_2^A(PL)$. Substituting from (7) and (8), this inequality can be transformed to yield

$$p < p_2 \qquad (9)$$

where $p_2 = (\delta_H - \delta_L)/(3 - \delta_H - \delta_L)$ and $0 < p_2 < 1/2$ for all δ_L and δ_H. The inequality in (9) implies that member A, if recognized at $t = 2$, will risk the probability of rejection only if the rejection probability p is relatively small. Otherwise, $R_2^A(S) \le R_2^A(PL)$ so that A will not take chance and the game ends at $t = 2$. In any case, A's payoff upon the recognition is strictly higher than $2/3$. This result reflects the fact that recognition is valuable.

The case in which member B is recognized is same as the case in which member A is recognized simply because both members A and B are known to be of type H before recognition at $t = 2$. As a result, the inequality in (9) is equally appropriate even if member B instead of member A is recognized at $t = 2$. Therefore, member B's equilibrium strategy is S only if $p < p_2$ while it is PL otherwise. [Table 2] summarizes the sequential equilibrium at $t = 2$.

[Table 2]. Sequential Equilibrium at t=2

We need to construct ex-ante continuation value of each member before the recognition at $t=2$. The ex-ante continuation value of each member depends on the probability p because different strategy will be employed depending on the size of p. Consider first the case in which $p \geq p_2$. In this case, as shown in [Table 2], all members will pursue strategy PL at $t=2$ by proposing $(1/3)\delta_H$ to the coalition member, and the proposal will be passed. It can be easily shown that all members' continuation value at $t=2$ under PL is $1/3$. For example, $V_2^A(\delta_H;H)$ under PL is given by

$$V_2^A(\delta_H;H)=\frac{1}{3}\left(1-\frac{1}{3}\delta_H\right)+\frac{1}{18}\delta_H+\frac{1}{18}\delta_H=\frac{1}{3}$$

The first term reflects the case in which member A is recognized at $t=2$ while the second and the third terms reflect the case in which member A is chosen as a coalition member and receives $(1/3)\delta_H$.

Note that the continuation value of each member under PL at $t=2$ does not depend on her true type because the proposal is passed always. In other words, all members propose $(1/3)\delta_H$ to their coalition member upon the recognition, and such proposal is passed no matter what the coalition member's true type. So, $V_2^A(\delta_L;H)=V_2^A(\delta_H;H)$ for example.

If $p < p_2$, member A and B will pursue strategy S upon the recognition. Taking into account this fact, we have

$$V_2^A(\delta_H;H) = \frac{1}{3}\left[(1-p)\left(1-\frac{1}{3}\delta_L\right)+p\frac{1}{3}\delta_H\right]+\frac{1}{9}p\delta_H+\frac{1}{18}\delta_H \qquad (10)$$

$$V_2^B(\delta_{H;};H) = V_2^A(\delta_H;H)$$

$$V_2^{CL}(\delta_L;U) = \frac{1}{3}\left(1-\frac{1}{3}\delta_H\right)+\frac{2}{9}\delta_L$$

$$V_2^{CH}(\delta_H;U) = \frac{1}{3}\left(1-\frac{1}{3}\delta_H\right)+\frac{2}{9}\delta_H$$

where superscripts CH and CL refer to member C of type H and L, respectively.

The first term in $V_2^A(\delta_H;H)$ is equal to $(1/3)R_2^A(S)$ where $R_2^A(S)$ is as given in (7). The second term is the expected payoff of member A when member B is recognized at $t=2$. Since member B will make proposal only to member C at $t=2$ under strategy S, member A's payoff is zero if B's proposal is passed with probability $1-p$. If not passed with probability p, member A's payoff is $(1/3)\delta_H$ which is simply her discounted continuation value at $t=3$. The last term reflects the case in which member C is recognized and proposes $(1/3)\delta_H$ to member A with probability $1/2$.

The first terms in V $V_2^{CL}(\delta_L;U)$ and $V_2^{CH}(\delta_H;U)$ reflect the case in which member C is selected as a proposal writer at $t=2$. The second terms represent the expected payoff of member C when member A or B is selected as a proposal maker and proposes $(1/3)\delta_L$ to member C under strategy S. If C is of type L, she votes for the proposal. If C is of type H, however, she votes against the proposal and the game continues to session $t=3$. The term $(2/9)\delta_H$ in $V_2^{CH}(\delta_H;U)$ is constructed from the fact that C's continuation value at $t=3$ is $1/3$ and that her true type is H.

Unlike the case in which $p \geq p_2$, the continuation value of each member depends not only on other member's belief but also on her own type. As shown in (10), member C's continuation value depends on whether she is CH or CL. We provide only $V_2^A(\delta_H;H)$ and $V_2^B(\delta_H;H)$ in (4.5) because of the assumption that types of member A and B are already revealed to be H at $t=2$ and the requirement that the revealed information should be correct in a sequential equilibrium. As will become clear later, however, we need to specify $V_2^A(\delta_H;H)$ and $V_2^B(\delta_H;H)$ to construct a sequential equilibrium, although such off-equilibrium continuation values are not relevant in equilibrium by definition. In the next section, we construct those off-equilibrium continuation values at $t=2$.

3. Session $t=1$

At session $t=1$, only one member's type is revealed to be H. By assumption, it is member B whose type is revealed to be H while member A and C's types are not yet revealed. In equilibrium, such belief of A and C about B's true type should be correct so that member B is truly of type H. Since different strategies are employed by members at $t=2$ depending on the size of probability p, we need to consider two cases separately.

A. Case 1: $p \geq p_2$

In this case, it is important to remember that, at $t=2$, strategy PL will be employed by all members upon the recognition, and the proposal will be passed for sure. Hence, once the game proceeds to $t=2$, it ends at $t=2$.

Suppose that member B, whose type is already revealed to be H at $t=1$, is selected as a proposal maker at $t=1$. If she employs strategy S, she will propose $\delta_L V_2^i(\delta_L;H)=(1/3)\delta_L$ to member i, where i = A or C. Note that the continuation value of each member at $t=2$ is $1/3$ iirrespective of their types as long as $p \geq p_2$. Member B's proposal under strategy S will be passed with probability $1-p$, the case in which the coalition member happens to be of type L. If the proposal is rejected with probability p, then the expected payoff of member B is $\delta_H V_2^B(\delta_H;H)=(1/3)\delta_H$, which is simply her discounted continuation value at $t=2$ under strategy PL. Therefore, under strategy S, the expected payoff $R_1^B(S)$ of member B upon the recognition is

$$R_1^B(S)=(1-p)\left(1-\frac{1}{3}\delta_L\right)+p\frac{1}{3}\delta_H \qquad (11)$$

Alternatively, if member B employs strategy PL, she proposes $(1/3)\delta_H$ to either member A or C, which is simply A or B's continuation values discounted at δ_H. This proposal will be passed with probability 1. Therefore, member B's expected payoff $R_1^B(PL)$ under strategy PL at $t=1$ is

$$R_1^B(PL)=1-\frac{1}{3}\delta_H \qquad (12)$$

Upon the recognition at $t=1$, member B's equilibrium strategy is S only if $R_1^B(S)\geq R_1^B(PL)$. Combining (11) and (12), this inequality leads to the same inequality $p<p_2$ as in (9). However, this inequality cannot hold because it is constructed given

the assumption that $p_2 \le p < 1$. Therefore, $R_1^B(S) \le R_1^B(PL)$ as long as $p \ge p_2$. In other words, if $p \ge p_2$ and member B is recognized at $t = 1$, then B's equilibrium strategy at $t = 1$ is PL.

Next, consider the case in which member C is selected as a proposal writer. We do not consider separately the case in which member A is recognized because it leads to the same conclusion as that of the case of member C's recognition.7 Under strategy S, member C proposes only to member A because member B is already believed to be of type H at $t = 1$ by assumption. As in the case in which member B is a proposer, member C will propose $(1/3)\delta_L$, and this proposal will be passed with probability $1 - p$ and rejected with probability p. If rejected, member C's expected payoff is $(1/3)\delta_C$ where $\delta_C = \delta_H$ or δ_L, which is her continuation value discounted at her true reelection probability. Note that member C's true reelection probability is not revealed at $t = 1$ by assumption.

As before, let CH and CL refer to member C of type H and L, respectively. Then the expected payoffs of member CH and CL under strategy S at t = 1 are given by

$$R_1^{CH}(S) = (1 - p)\left(1 - \frac{1}{3}\delta_L\right) + p\frac{1}{3}\delta_H \qquad (13)$$

$$R_1^{CL}(S) = (1 - p)\left(1 - \frac{1}{3}\delta_L\right) + p\frac{1}{3}\delta_L$$

Alternatively, if member C pursues strategy PL, she proposes $(1/3)\delta_H$ to either member A or B, and this proposal will be passed with probability 1 because, when

7 Since we assume that, at $t = 2$, members A and B are known to be of type H while member C is not known yet, we choose the case in which member C, rather than member A, is recognized at $t = 1$. The case in which member A is recognized at $t = 1$ can be equally analyzed by reshuffle the names of members.

$p_2 \le p < 1$, all members' continuation value at $t = 2$ is $1/3$ irrespective of their true types. As a result, member C's payoff under PL is

$$R_1^{CH}(PL) = R_1^{CL}(PL) = 1 - \frac{1}{3}\delta_H \tag{14}$$

Note that C's payoff does not depend on her true type because C's proposal under PL is passed for sure.

For member CH, S is an equilibrium strategy only if $R_1^{CH}(S) > R_1^{CH}(PL)$. Similar inequality $R_1^{CL}(S) > R_1^{CL}(PL)$ needs to be satisfied for strategy S to be an equilibrium strategy for member CL. Substituting from (13) and (14), these inequalities produce

(i) $p < p_2$ if C = CH

(ii) $p < p_1$ if C = CL

where $p_1 = (\delta_H - \delta_L)/(3 - 2\delta_L) > 0$. Clearly, p_1 is strictly less than p_2 for all δ_L and δ_H. Since $p_1 < p_2$, neither (i) nor (ii) can hold under the assumption that $p_2 \le p < 1$. Consequently, $R_1^{CH}(S) \le R_1^{CH}(PL)$ and $R_1^{CL}(S) \le R_1^{CL}(PL)$ as long as $p_2 \le p < 1$, which in turn implies that strategy PL is an equilibrium strategy for member C at $t = 1$ as long as $p \ge p_2$.

In summary, all members upon the recognition at $t = 1$ will take strategy PL as long as $p \ge p_2$. This fact implies that once the legislature continues to session $t = 1$, it completes its task at $t = 1$ if $p \ge p_2$. As a result, the legislative bargaining game can proceed to session $t = 2$ only if $p < p_2$. When $p < p_2$, however, only S is an equilibrium strategy for all members at $t=2$ given that the bargaining game already continued to session $t=2$. Consequently, only a sequential equilibrium with strategy S emerges at $t=2$.

The ex-ante continuation values of legislative members at $t=1$ are simply $1/3$ for all members regardless of their true types. All members propose $(1/3)\delta_H$ upon the recognition, and such proposal is passed with probability 1. Since the bargaining game does not continue to session $t=2$ in this case, true reelection probabilities of members are not of any significance in constructing the ex-ante continuation values.

B. Case 2: $p < p_2$

Note that if $p < p_2$, strategy S is an equilibrium strategy for members A and B at $t=2$. The sequential equilibrium of the legislative bargaining game turns out to critically depend on who is recognized at $t=1$. Due to such dependence, we consider two cases separately; (ⅰ) the case in which member B, whose type is already revealed to be H at $t=1$ by assumption, is recognized and (ii) the case in which member C is recognized at $t=1$. The case in which member A is recognized at $t=1$ leads to the same conclusion as in the case of member C's recognition.

(i) Member B is a proposal maker at $t=1$

Suppose that member B is selected as a proposal maker at $t=1$. Note that member B's type is already revealed at $t=1$. Under strategy S, member B will make proposal to either member A or member C. Without loss of generality, assume that member B selects member A as a coalition member under strategy S.[8]

Member B will propose $\delta_L V_2^A(\delta_L;H)$ to member A, hoping that member A is of type L. $\delta_L V_2^A(\delta_L;H)$ is the continuation value of member A whose type is L but believed to be of type H at $t=2$ upon the rejection of the proposal made by member B. Note that this proposal will be passed with probability $1-p$ and will not be passed with probability p. If not passed, proposer B's expected payoff is simply $\delta_H V_2^B(\delta_H;H)$ where $V_2^B(\delta_H;H)$ is as given in (10). Note from Case 1 that only S is an equilibrium strategy at $t=2$ because the legislature can continue to session $t=2$ only if $p<p_2$. Consequently, proposer B's expected payoff under strategy S is given by

$$R_1^B(S)=(1-p)\left[1-\delta_L V_2^A(\delta_L;H)\right]+p\delta_H V_2^B(\delta_H;H) \tag{15}$$

Given the assumption that members A and B are believed to be of type H at $t=2$ and the fact that only an equilibrium with S emerges at $t=2$, we construct $V_2^A(\delta_L;H)$ as follows:.

$$V_2^A(\delta_L;H)=\frac{1}{3}\left[(1-p)\left(1-\frac{1}{3}\delta_L\right)+p\frac{1}{3}\delta_L\right]+\frac{1}{9}p\delta_L+\frac{1}{18}\delta_H \tag{16}$$

The expression of $V_2^A(\delta_L;H)$ is only slightly different from that of $V_2^A(\delta_H;H)$ in (10). This is because the different reelection probabilities for member A matter only if the game continues to session $t=3$.

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[8] The case in which B proposes to C to the same result as in the case in which B proposes to A, simply because the type of neither A nor C is revealed at $t=1$ so that B is indifferent between proposing to A and to C. We choose the former case, however, because it is consistent with our assumption that it is A and B whose types are already revealed at $t=2$.

Under strategy PL, on the other hand, member B will propose $\delta_H V_2^A(\delta_H;H)$ to member A, which is the continuation value of A at $t=2$ discounted at high reelection probability δ_H. This proposal will be passed for sure. Therefore, proposer B's payoff at $t=1$ under PL is simply,

$$R_1^B(PL)=1-\delta_H V_2^A(\delta_H;H). \qquad (17)$$

Using (15)-(17), the inequality $R_1^B(S)>R_1^B(PL)$ can be reduced to the following inequality for p given by,

$$p<\tilde{p}_1 \qquad (18)$$

where (a) $0<\tilde{p}_1<p_2$ if $\delta_L \geq 1/4$ or if $\delta_L<1/4$ and $\delta_H<\tilde{\delta}_H$

(b) $p_2 \leq \tilde{p}_1<1$ if $\delta_L<1/4$ and $\delta_H \geq \tilde{\delta}_H$

$\tilde{p}_1$ and $\tilde{\delta}_H$ are defined in Definition 1 of Appendix. $\tilde{p}_1$ is a function of δ_L and δ_H, and $0<\tilde{p}_1<1$ for all δ_L and δ_H. $\tilde{\delta}_H$ is also a function of δ_L, and is shown in [Figure 3].

If δ_L and δ_H satisfy the conditions in (a), $\tilde{p}_1$ is strictly less than p_2 so that member B's equilibrium strategy at $t=1$ is S if $p<\tilde{p}_1$ whereas it is PL if $\tilde{p}_1 \leq p<p_2$. Since PL is always an equilibrium strategy at $t=1$ as long as $p \geq p_2$, strategy PL is in fact an equilibrium strategy for member B at $t=1$ as long as $p \geq \tilde{p}_1$. On the other hand, if δ_L and δ_H satisfy the conditions in (b), $\tilde{p}_1$ is greater than or equal to p_2. In this case, member B's equilibrium strategy at $t=1$ is S only if $p<p_2$ because, by assumption, p must be strictly less than p_2. In other words, strategy PL,

rather than strategy S, is an equilibrium strategy for proposer B if $p_2 \le p < \tilde{p}_1$ as shown in Case 1.

(ii) Member C is a proposal maker at $t=1$

The main difference between member B and member C as a proposal maker at $t=1$ is the fact that member C's type is not revealed to other members while member B's type is known. Such difference is important particularly in case of strategy S because member C's type will not be revealed at $t=2$ even if member C's proposal is rejected at $t=1$. Consequently, her continuation value at $t=2$ will be different from that of member B whose type is known at $t=2$. Different continuation values at $t=2$ in turn will lead to different strategies for member C at $t=1$ from those for member B.

Under strategy S, member C will propose $\delta_L V_2^A(\delta_L;H)$ to member A, a proposal which is identical to that made by member B to member A upon B's recognition. If the proposal is passed with probability $1-p$, C will receive $1-\delta_L V_2^A(\delta_L;H)$. If rejected with probability p, her expected payoff upon the rejection is simply $\delta_L V_2^C(\delta_L;U)$ if $\delta_C=\delta_L$ and $\delta_H V_2^C(\delta_H;U)$ if $\delta_C=\delta_H$, where $V_2^C(\delta_L;U)$ and $V_2^C(\delta_H;U)$ are as given in (10). Note that member C's true type is not revealed at session $t=2$ by assumption.

Similarly as before, let CH and CL refer to member C of type H and L, respectively. The expected payoff of member C under strategy S is given by

$$R_1^{CH}(S)=(1-p)\left[1-\delta_L V_2^A(\delta_L;H)\right]+p\delta_H V_2^{CH}(\delta_H;U) \tag{19.a}$$

$$R_1^{CL}(S) = (1-p)\left[1-\delta_L V_2^A(\delta_L;H)\right] + p\delta_L V_2^{CH}(\delta_L;U) \qquad (19.b)$$

The expressions in (19.a) and (19.b) are same except the last terms because the different reelection probabilities of member C matter only if the proposal fails to be passed. Both CH and CL proposes $\delta_L V_2^A(\delta_L;H)$ to member A.

Alternatively, under strategy PL, member C will attract the vote from either A or B by proposing $\delta_H V_2^i(\delta_H;H)$, i = A or B. 9In fact, member C's proposal is same whether the chosen coalition member is A or B because $\delta_H V_2^A(\delta_H;H) = \delta_H V_2^B(\delta_H;H)$ as shown in (10), and C's proposal will be passed regardless of types of A or B. As a result, proposer C's payoff at $t=1$ under PL is simply,

$$R_1^{CH}(PL) = R_1^{CL}(PL) = 1-\delta_H V_2^A(\delta_H;H) \qquad (20)$$

Using (19) and (20), the inequalities $R_1^{CH}(S) > R_1^{CH}(PL)$ and $R_1^{CL}(S) > R_1^{CL}(PL)$ can be transformed into inequalities for p. Consider first the case in which member C is CH. The inequality $R_1^{CH}(S) > R_1^{CH}(PL)$ yields the following inequality for p :

$$p < p_1^* \qquad (21)$$

where (c) $0 < p_1^* < p_2$ if $\delta_H < \delta_H^*$

(d) $p_2 \le p_1^* < 1$ if $\delta_H \ge \delta_H^*$

p_i^* and δ_H^* are defined in Definition 2 of Appendix. Like $\tilde{p}_1$, p_i^* is a function of δ_L and δ_H, and $0 < p_i^* < 1$. It can be shown that δ_H^* is a nondecreasing function of δ_L and that $\delta_H^* \to 3/5$ as $\delta_L \to 0$ and $\delta_H^* \to 1$ as $\delta_L \to 1$. The function δ_H^* is

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9 $\delta_H V_2^i(\delta_H;H)$ is as given in (10)

depicted in [Figure 3].

[Figure 3]

Under the conditions for δ_L and δ_H specified in (c), p_1^* is strictly less than p_2 so that an equilibrium strategy for member CH at $t=1$ is strategy S if $p < p_1^*$ while it is PL if $p_1^* \le p < p_2$. Utilizing the results of Case (1), it immediately follows that PL is in fact an equilibrium strategy at $t=1$ for member CH as long as $p \ge p_1^*$. On the other hand, if the conditions for δ_L and δ_H in (d) are satisfied, then p_1^* is greater than or equal to p_2. This fact implies in turn that an equilibrium strategy for member CH is S only if $p < p_2$ because the probability p must be strictly less than p_2 by assumption.

When member C is CL, the inequality $R_1^{CL}(S) > R_1^{CL}(PL)$ can be reduced to the following inequality given by,

$$p < \hat{p}_1 \qquad (22)$$

where $0 < \hat{p}_1 < p_2$ for all δ_L and δ_H

$\hat{p}_1$ is defined in Definition 3 of Appendix. Like $\tilde{p}_1$ and p_1^*, $\hat{p}_1$ is also a function of δ_L and δ_H. Since $\hat{p}_1 < p_2$ for all δ_L and δ_H, an equilibrium strategy for proposer CL at $t=1$ is S if $p < \hat{p}_1$ while it is PL if $\hat{p}_1 \le p < p_2$. Note that p must be strictly less than p_2 by assumption.

Equilibrium strategies of legislative members at $t=1$ depend on the model parameters δ_L, δ_H and p as well as the information structure at $t=1$. To be more concrete, we need to determine the relative magnitudes of $\tilde{p}_1$, p_1^* and $\hat{p}_1$, which are complicated functions of δ_L and δ_H. After a lot of algebra, it can be shown that p_1^* is strictly greater than $\tilde{p}_1$ and $\hat{p}_1$ for all δ_L and δ_H such that $0<\delta_L<\delta_H<1$. It can be shown that the relative magnitudes of $\tilde{p}_1$ and $\hat{p}_1$ are given by

$\hat{p}_1 \le \tilde{p}_1$ if $\delta_H \ge \bar{\delta}_H$ (23)

$\hat{p}_1 > \tilde{p}_1$ if $\delta_H \ge \bar{\delta}_H$

where $\bar{\delta}_H$ is as defined in Definition 4 of the Appendix. As shown in [Figure 3], $\bar{\delta}_H$ is a nondecreasing function of δ_L, and that $\bar{\delta}_H \to 0$ as $\delta_L \to 0$.

Given the assumption that $p<p_2$ and the results in (18), (21), (22) and (23), we can partition the feasible set of (δ_L,δ_H) into five subsets, each of which has different ordering for p_2, $\tilde{p}_1$, $\hat{p}_1$ and p_1^*. [Figure 3] presents such partition and ordering. For example, in subset D2 of [Figure 3], the ordering is given by $\hat{p}_1<\tilde{p}_1<p_2<p_1^*$. In this case, S is an equilibrium strategy for members B, CH and CL if $p<\hat{p}_1$ while it is only for members B and CH if $\hat{p}_1<p<\tilde{p}_1$. If $\tilde{p}_1<p<p_2$, S is an equilibrium strategy only for member CH while PL is an equilibrium strategy for members B and CL. Remember that PL is always an equilibrium strategy at $t=1$ for all members as long as $p \ge p_2$. [Table 3] summarizes the sequential equilibrium in session $t=1$ depending on δ_L, δ_H and p. Close inspection of [Table 3] reveals that

five different types of sequential equilibria can emerge in session $t=1$ depending on the values of δ_L, δ_H and p: (i) all members employ S, (ii) all members employ PL, (iii) B and CH employ S, while CL employs PL, (iv) CH employs S, while B and CL employ PL, and (v) CH and CL employ S, while B employs PL.

As shown in [Table 3], subsets D4 and D5 involve the possibility that all members -- B, CH and CL -- pursue strategy PL even if $p < p_2$. Such possibility does not arise in subsets D1 through D3. This result is closely related to the fact that, for given δ_L, δ_H is smaller in subsets D4 and D5 than in subsets D1, D2 and D3. Such a smaller δ_H tends to induce the member recognized not to risk the probability of rejection if p is not too far from p_2. This is because the higher possible payoff from taking risk is not large enough due to the relatively small difference between δ_L and δ_H.

[Table 3]. Sequential Equilibrium at t=1

4. Session $t=0$

In session $t=0$, no member's type is revealed. As in the previous section, we consider two cases separately -- the case in which $p \geq p_2$ and the case in which $p < p_2$ -- because the equilibrium strategies of members at $t=1$ are different, depending on the size of probability p.

A. Case (1): $p \geq p_2$

The last row of [Table 3] shows that PL is an equilibrium strategy for all members at $t=1$ as long as $p \geq p_2$. In addition, all members' continuation value is simply $1/3$ regardless of their true types as shown in section 3.A. Given these facts, it can be easily shown that PL is also an equilibrium strategy for all members at $t=0$ as long as $p \geq p_2$ and that the ex-ante continuation value at $t=0$ is simply $1/3$ for all members as long as $p \geq p_2$.10 Upon the recognition, all members propose $(1/3)\delta_H$ to the chosen coalition member and such proposal is passed with probability 1.Therefore, the legislature concludes its task in the first session $t=0$ as long as $p \geq p_2$. Consequently, the equilibrium with PL at $t=1$ would not occur at least for $p \geq p_2$. Since $p_2 < 1/2$ for all δ_L and δ_H, the legislature concludes at $t=0$ regardless of δ_L and δ_H if $p \geq 1/2$.

B. Case (2): $p < p_2$

In this case, without loss of generality we assume that member C is recognized at $t=0$, and she selects member B as a coalition member under strategy S.11 This assumption is consistent with our informational assumption that it is member B whose type is revealed to be H at $t=1$. As shown in [Table 3], the nature of sequential equilibrium at $t=1$ depends on the size of δ_L, δ_H, and p as well as the information structure at

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[10] Showing that is an equilibrium strategy at $t=0$ for all members as long as $p \geq p_2$ exactly parallels the analysis in section Ⅳ.3.A.

[11] As before, the other cases in which member A or B is recognized at $t=0$ can be fully covered by reshuffling the names of members.

$t=1$. To be complete, of course, we need to construct a sequential equilibrium at $t=0$ for all possibilities shown in [Table 3]. Since the basic intuition is very similar, however, we consider only the case in which $(\delta_L, \delta_H) \in D4$ and $p_1^* < p < p_2$.

Let us assume that $(\delta_L, \delta_H) \in D4$ and $p_1^* < p < p_2$. Suppose that member C is recognized at $t=0$. Under strategy S, member C will propose $\delta_L V_1^B(\delta_L; H)$ to member B. If this proposal is not passed with probability p, then C's expected payoff is simply $\delta_C V_1^C(\delta_C; U)$ where $\delta_C = \delta_L$ or δ_H depending on her true type. Note that C's true type is not revealed even at $t=1$. Alternatively, if member C pursues strategy PL, she proposes $\delta_H V_1^B(\delta_H; H)$ to member B (or member A), and this proposal will be passed with probability 1. Upon the recognition at $t=0$, member C's expected payoffs under S and PL are given by

$$R_0^{CH}(S) = (1-p)\left[1-\delta_L V_1^B(\delta_L; H)\right] + p\delta_H V_1^C(\delta_H; U) \tag{24}$$

$$R_0^{CL}(S) = (1-p)\left[1-\delta_L V_1^B(\delta_L; H)\right] + p\delta_L V_1^C(\delta_L; U)$$

$$R_0^{CH}(PL) = R_0^{CL}(PL) = 1-\delta_H V_1^B(\delta_H; H)$$

Similarly as before, S is an equilibrium strategy at $t=0$ for member CH if $R_0^{CH}(S) > R_0^{CH}(PL)$ while it is for member CL if $R_0^{CL}(S) > R_0^{CL}(PL)$. When $p_1^* < p < p_2$, all the ex-ante continuation values in (24) are simply $1/3$ because all members will employ strategy PL at $t=1$ as shown in [Table 3]. Using this fact, the inequalities $R_0^{CH}(S) > R_0^{CH}(PL)$ and $R_0^{CL}(S) > R_0^{CL}(PL)$ can be simply reduced to $p < p_2$ and $p < p_1$, respectively, where p_2 and p_1 are as defined before in sections

2 and 3.A. Therefore, S is an equilibrium strategy for member CH at $t = 0$.

To determine the equilibrium strategy for member CL, however, we need to compare p_1^* and p_1 because both p_1^* and p_1 are less than p_2 for all $(\delta_L, \delta_H) \in D4$. After a lot of algebra, we can show that

$$p_1^* \geq p_1 \text{ if } \delta_H \geq \delta_H' \qquad (25)$$

$$p_1^* < p_1 \text{ if } \delta_H < \delta_H'$$

where δ_H' is a nondecreasing function of δ_L such that $Z(p_1(\delta_L, \delta_H')) = 0$. It can be shown that $\delta_H' < \delta_H^*$ for all $(\delta_L, \delta_H) \in D4$, and that $\delta_H' \to 0$ as $\delta_L \to 0$ and $\delta_H' \to 1$ as $\delta_L \to 1$.

[Figure 4] shows the partition of D4 into two subsets D4A and D4B depending on $\delta_H \geq \delta_H'$ or $\delta_H < \delta_H'$. When $(\delta_L, \delta_H) \in D4A$, member CL's equilibrium strategy at $t = 0$ is PL. Alternatively, when $(\delta_L, \delta_H) \in D4B$, member CL's equilibrium strategy is S if $p_1^* < p < p_1$, while it is PL if $p_1 \leq p < p_2$. Hence, PL is an equilibrium strategy at $t = 0$ for member C regardless of her true type as long as $p > p_1^*$ if $(\delta_L, \delta_H) \in D4A$.[12] Similarly, if $(\delta_L, \delta_H) \in D4B$, PL is an equilibrium strategy for all members as long as $p > p_1$ upon the recognition at $t = 0$.

[Figure 4]

C. Discussion

[12] Note that PL is an equilibrium strategy for all members as long as $p \geq p_2$.

Based on the constructed sequential equilibria in the previous sections, we can examine the expected number of sessions, denoted by EN, until the legislature completes its task. Let N be the actual number of sessions until the legislature concludes. We assume $N \geq 1$ so that $N = 1$ if the game ends at $t = 0$. Note that $N \leq 4$ in the three-member legislature of the model.

As shown in the above, as long as $p \geq p_2$, the legislature completes its task at $t = 0$ with probability 1 regardless of the true type of the recognized. Therefore, $N = 1$ in this case. If $p < p_2$, the sequential equilibrium can take different sequence depending on the values of δ_L, δ_H and p as well as the true type of the recognized at $t = 0$. Suppose that $(\delta_L, \delta_H) \in D4$, and $p_1^* < p < p_2$. If $(\delta_L, \delta_H) \in D4A$, then $EN = N = 1$ for all $p \in (p_1^*, p_2)$ because the game ends at $t = 0$ regardless of the true type of the recognized at $t = 0$. Similarly, if $(\delta_L, \delta_H) \in D4B$ and $p_1 \leq p < p_2$, then $EN = N = 1$.

If $(\delta_L, \delta_H) \in D4B$ and $p_1^* < p < p_1$, however, the equilibrium strategy for the recognized at $t = 0$ is PL only if she is of type H. If she is of type L, strategy S is an equilibrium strategy for her. In the former case, $N = 1$. In the latter case, the proposal of the recognized will be passed with probability $1 - p$, the case in which the game ends at $t = 0$ so that $N = 1$. Therefore, $N = 1$ with probability $p + (1 - p)^2$. If the proposal is not passed with probability p, the game will continue to session $t = 1$. At $t = 1$, however, the sequential equilibrium in [Table 2] shows that all members employ strategy PL and hence the game ends at $t = 1$ with probability 1. This implies that

$N = 2$ with probability $(1-p)p$. Consequently the expected number of sessions $EN = p + (1-p)^2 + 2(1-p)p = 1 + p - p^2$.

The expected number of sessions until the conclusion of the legislature, of course, will be different depending on the nature of sequential equilibrium of the model, which in turn determined by model parameters δ_L, δ_H and p. However, it can be conjectured that EN would not be large in any case. This is because large N tends to be associated with small p and vice versa. For relatively large p, N is likely to be small as strategy PL is more likely to be an equilibrium strategy for the recognized. For relatively small p, N could be large as strategy S is likely to be an equilibrium strategy for the recognized. But even in this case, N is likely to be small because the proposal will be passed with high probability $1-p$.

V. Conclusion

[To be completed.]

Appendix

1. Definitions

A. Definition 1:

$0 < \tilde{p}_1 < 1, \qquad F(\tilde{p}_1) = 0$

$\tilde{\delta}_H = [(9 - 9\delta_L - 8\delta_L^2) + y(\delta_L)^{1/2}]/2(9 - 8\delta_L)$

where $F(p) = [6(1-\delta_L)(\delta_H + \delta_L) - 4\delta_H(\delta_H - \delta_L)]p^2 +$

$[2(3-\delta_L^2) - (\delta_H - \delta_L)(5\delta_H + 6\delta_L)]p - \quad (\delta_H - \delta_L)(6 - 2\delta_L + \delta_H)$

$y(\delta_L) = (9 - 9\delta_L - 8\delta_L^2)^2 + 48(9 - 8\delta_L)\delta_L^2$

The function F(p) is constructed from the inequality $R_1^B(S) > R_1^B(PL)$ in section Ⅳ.3.B(i) so that $F(P) < 0$ implies $R_1^B(S) > R_1^B(PL)$. $\tilde{\delta}_H$ is a function of δ_L, which is constructed from the equality $F(p_2(\delta_L, \tilde{\delta}_H)) = 0$ where p_2 is as defined in (9). It can be shown that $\delta_L < \tilde{\delta}_H < 1$ if $\delta_L < 1/4$ while $\tilde{\delta}_H \geq 1$ if $\delta_L \geq 1/4$.

B. Definition 2:

$0 < p_1^* < 1, \qquad Z(p_1^*) = 0$

$\delta_L < \delta_H^* < 1, \qquad X(\delta_H^*, \delta_L) = 0$

where $X(\delta_H, \delta_L) = 5\delta_H^3 - 9(2 - \delta_L)\delta_H^2 + (9 - 9\delta_L - 8\delta_L^2)\delta_H + 12\delta_L^2$

$Z(P) = 6\delta_L(1-\delta_L)p^2 + [8\delta_L^2 - 3\delta_L(4 + \delta_H) + 18 - 6\delta_H^2]p$

$-(\delta_H - \delta_L)(6 - 2\delta_L + \delta_H)$

The function $Z(p)$ is constructed from the inequality $R_1^{CH}(S) > R_1^{CH}(PL)$ in section Ⅳ.3.B(ⅱ) so that $Z(p) < 0$ implies $R_1^{CH}(S) > R_1^{CH}(PL)$. δ_H^* is constructed from the equality $Z(p_2(\delta_L, \delta_H^*)) = 0$ where p_2 is as defined in (9). δ_H^* is a nondecreasing function of δ_L and that $\delta_H^* \to 3/5$ as $\delta_L \to 0$ and $\delta_H^* \to 1$ as $\delta_L \to 1$.

C. Definition 3:

$0 < \tilde{p}_1 < 1,$ $\qquad Q(\hat{p}_1) = 0$

where $Q(p) = 6\delta_L(1-\delta_L)p^2 + [4\delta_L^2 - \delta_L(18+\delta_H) + 18 + 6\delta_H - 4\delta_H^2]p$

$-(\delta_H - \delta_L)(6 - 2\delta_L + \delta_H)$

The function $Q(p)$ is constructed from the inequality $R_1^{CL}(S) > R_1^{CL}(PL)$ in section Ⅳ.3.B(ii) so that $Q(p) < 0$ implies $R_1^{CL}(S) > R_1^{CL}(PL).$

D. Definition 4:

$h_0(\delta_L) < \delta_H < h_1(\delta_L),$ $\qquad F(k(\overline{\delta}_H, \delta_L)) = Q(k(\overline{\delta}_H, \delta_L)) = 0$

where $h_0(\delta_L) = -3 + (9 + 6\delta_L + 4\delta_L^2)^{1/2}$ for $\delta_L \in (0, \delta_{L0})$

$h_1(\delta_L) = [-\delta_L + (3\delta_L(10 + 7\delta_L))^{1/2}]/5$ for $\delta_L \in (0, \delta_{L1})$

$$k(\delta_H, \delta_L) = \frac{\delta_H^2 + 6\delta_H - \delta_L(6 + 4\delta_L^2)}{2\delta_H(3 - \delta_L - 2\delta_H)}$$

$\delta_{L0} = \left(-3 + \sqrt{37}\right)/4$ and $\delta_{L1} = \left(-1 + \sqrt{6}\right)/4$ F(.) and Q(.) are as defined in Definition

1 and 3, respectively. $F(p) = Q(p)$ at $p = k(\delta_H, \delta_L)$. $h_0(\delta_L)$ and $h_1(\delta_L)$ satisfy the equalities $k(h_0(\delta_L), \delta_L) = 0$ and $k(h_1(\delta_L), \delta_L) = 1$, respectively, so that $k(\delta_H, \delta_L) < 0$ if $\delta_H \le h_0(\delta_L)$ and $k(\delta_H, \delta_L) \ge 1$ if $\delta_H \le h_1(\delta_L)$. $\bar{\delta}_H$ is a nondecreasing function of δ_L, and $\bar{\delta}_H \to 0$ as $\delta_L \to 0$.

2. Tables and Figures

[Table 1]. Sequential Equilibrium with (H, H, L)

(δ_L, δ_H)	Equilibrium Strategy	
	Pure	Randomize
$\delta_H \geq 2\delta_L/(2-\delta_L)$	H	L
$\delta_H < 2\delta_L/(2-\delta_L)$		L, H

[Table 2]. Sequential Equilibrium at t=2

p	Equilibrium Strategy	
	S	PL
$p < p_2$	A, B	C
$p \geq p_2$		A, B, C

[Table 3]. Sequential Equilibrium at t=1

(δ_L, δ_H)	p	Equilibrium Strategy	
		S	PL
D1	$p < \hat{p}_1$	B, CH, CL	
	$\hat{p}_1 < p < p_2$	B, CH	CL
D2	$p < \hat{p}_1$	B, CH, CL	
	$\hat{p}_1 < p < \tilde{p}_1$	B, CH	CL
	$\tilde{p}_1 < p < p_2$	CH	B, CL
D3	$p < \tilde{p}_1$	B, CH, CL	
	$\tilde{p}_1 < p < \hat{p}_1$	CH, CL	B
	$\hat{p}_1 < p < p_2$	CH	B, CL
D4	$p < \hat{p}_1$	B, CH, CL	
	$\hat{p}_1 < p < \tilde{p}_1$	B, CH	CL
	$\tilde{p}_1 < p < p_1^*$	CH	B, CL
	$p_1^* < p < p_2$		B, CH, CL
D5	$p < \tilde{p}_1$	B, CH, CL	
	$\tilde{p}_1 < p < \hat{p}_1$	B, CH	CL
	$\hat{p}_1 < p < p_1^*$	CH	B, CL
	$p_1^* < p < p_2$		B, CH, CL
D1-D5	$p_2 \leq p$		B, CH, CL

Fig. 1.

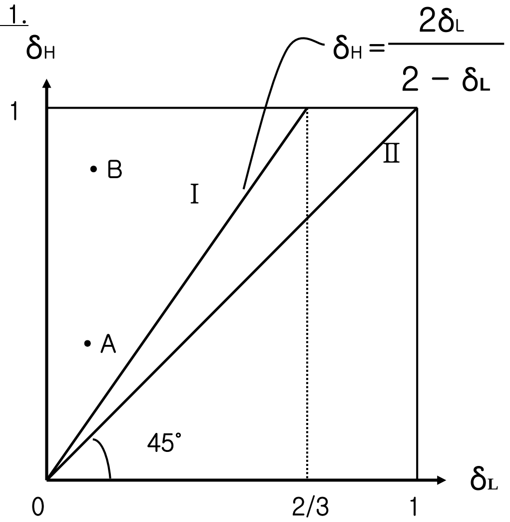
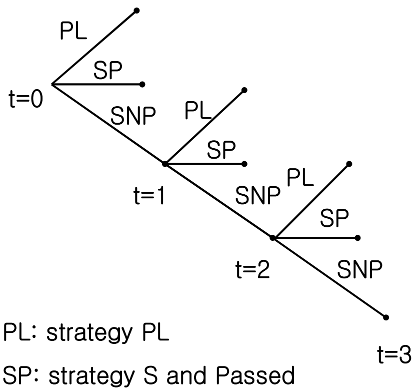


Fig. 2.

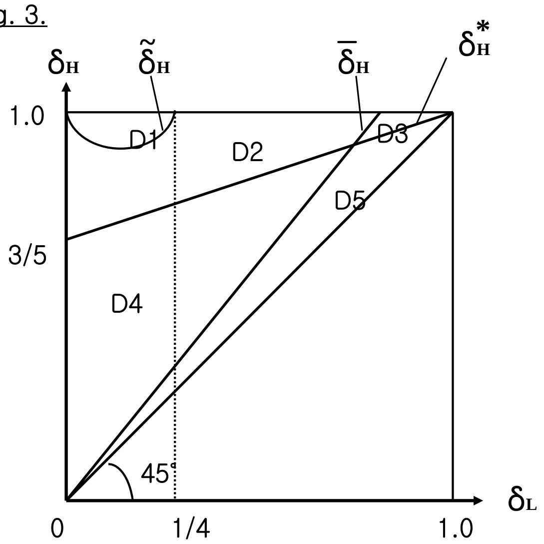


PL: strategy PL

SP: strategy S and Passed

SNP: strategy S and Not Passed

Fig. 3.



$D1: \hat{p}_1 < p_2 < \tilde{p}_1 < p_1^*$

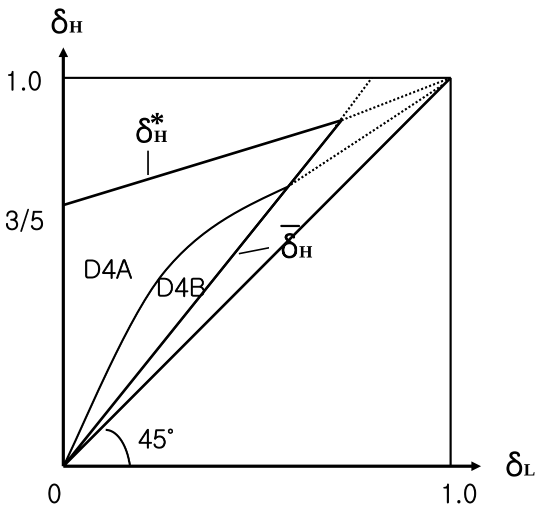
$D2: \hat{p}_1 < \tilde{p}_1 < p_2 < p_1^*$

$D3: \tilde{p}_1 < \hat{p}_1 < p_2 < p_1^*$

$D4: \hat{p}_1 < \tilde{p}_1 < p_1^* < p_2$

$D5: \tilde{p}_1 < \hat{p}_1 < p_1^* < p_2$

Fig. 4.



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