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Regulating the Anti-commons: Insights from Public Expenditure Theory

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This paper investigates the pricing strategies and regulation of multiple monopolies that produce products which consumers view as perfect complements. We show under general conditions that collusion by the firms increases total welfare and that the collusion problem can be reinterpreted as a public good provision problem from the point of view of the firms. We take this insight further and derive the familiar concepts of the Samuelson marginal condition for Pareto efficiency for the firms and the Ratio equilibrium. We compare these outcomes to the first best solution and then apply incentive compatible mechanisms to strategically implement the Pareto superior Ratio equilibrium outcome and the optimal marginal cost pricing outcome.

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Regulating the Anticommons: Insights from Public Expenditure Theory

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Abstract

This paper investigates the pricing strategies and regulation of multiple monopolies that produce products which consumers view as perfect complements. We show under general conditions that collusion by the firms increases total welfare and that the collusion problem can be reinterpreted as a public good provision problem from the point of view of the firms. We take this insight further and derive the familiar concepts of the Samuelson marginal condition for Pareto efficiency for the firms and the Ratio equilibrium. We compare these outcomes to the first best solution and then apply incentive compatible mechanisms to strategically implement the Pareto superior Ratio equilibrium outcome and the optimal marginal cost pricing outcome.

Keywords: Anticommons, Complementary Monopoly, Public Goods, Ratio Equilibrium, Mechanism Design

1 Introduction

In this paper, we investigate equilibrium and regulation of two monopolies which compete in prices to attract consumers that view the firms' products

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as perfect complements. This is an old problem first introduced by Cournot (1838) and further studied by others.[1] For illustration, Cournot considers a problem where a producer of copper and a producer of zinc sell their input goods to producers of brass. Zinc and copper, from the point of view of the brass producer are perfect complements (in some proportion).[2] He shows: first, the price of the composite commodity is higher when monopolists act in their own interest compared to when they collude; second, as the number of input producers increases, the inefficiency problem becomes worse.[3] In other words, the competing input producer model yields the opposite results of the standard Cournot model of competing producers. Cournot's insights extend to a variety of situations outside traditional industrial organization including patent law, land assembly, collective bargaining, markets for broadband spectrum, and operating systems. Recently, these problems, defined by their highly fragmented ownership of input goods, have been popularized by Michael Heller as the "Tragedy of the Anticommons," a term used to reflect the symmetries of this problem with the commons problems.[4],[5] These symmetries are explored by Buchanan and Yoon (2000) who develop a simple model of anticommons to illustrate the symmetry of the commons and the anticommons.

The goal of this paper is not to repeat this analysis or to provide more examples of anticommons, but rather to use the theory of public goods to gain a new perspective into the problem. By re-interpreting the problem slightly, we are able to draw intuition from the immense public goods and regulation literature in public finance. This insight is valuable when identifying the incentive problems as well as considering how to mitigate the tragedy of anticommons. Our results show the existence of a non-trivial Nash equilibrium for general environments and provide conditions under which collusion by the

[1] For example see, Sonnenschein (1968) and Bergstrom (1978).

[2] See Cournot, *Mathematical Principles of the Theory of Wealth*, Chapter 9 entitled "of the mutual relations of producers."

[3] This is a similar policy prescription to Spengler's 1950 article on vertical integration.

[4] Heller has a number of papers and a book on this subject illustrating the problem. For instance, see the 1998 Harvard Law Review article, "The Tragedy of the Anticommons: Property in the Transition from Marx to Markets," the 1998 *Science* article with Rebecca Eisenberg on Biomedical research and patents, the 2008 paper in the Harvard Law Review with Hillis on Land Assembly, and Heller's book, *The Gridlock Economy* (2008), is an excellent, non-technical introduction to the large set of examples encompassed by the tragedy of the anticommons.

[5] Hardin (1969) is the classic article on the tragedy of the commons.

firms improves social welfare. As suggested by Cournot, the natural policy recommendation in this setting is to force the firms to merge. The merged firms internalize the externality they had imposed on one another and the reduction in price increases consumer welfare. This recommendation is not without problems. First, there is not necessarily an "obvious" solution for assignment of profit share between the two firms being merged. This lack of a mutually acceptable profit sharing solution presents a significant barrier to a successful merger. In other cases, we would like the firms to act as if they had merged without going through the legal steps to force such a step. For instance, we may wish zinc and copper producers to collude when pricing to brass producers, but perhaps to compete in a market for wiring. This requires changes to the institutional structure of the individual markets rather than simply merging the firms together. Similar examples would extend to patent law, where the desire for collusive like behavior among patent holders is likely case dependent. Without changing the market structure we are left in a situation where we desire the firms to tacitly collude when it is in their individual interest to cheat. Thus, the second problem is with incentives.

We address both of these problems by appealing to the public expenditure theory literature. Reflecting on the problem, since the number of units sold by the firms is a non-rival and non-exludable good between the firms, the quantity of units sold can be defined as a public good. We formalize this intuition by representing the collusion problem of the firms as a public good provision problem where the demand function plays the role of the production technology (or more correctly a *reduction* technology). We derive the Samuelson marginal condition and the ratio equilibrium for this public good economy. The ratio equilibrium, due to Kaneko, is a particular cost sharing arrangement which yields the optimal provision and financing of the public good.[6] Moreover, Ratio equilibria are always in the core (i.e., allocations are individually rational and Pareto optimal) and exist in a wider set of environments than Lindahl equilibria. In our model, for the special case of linear demand, the ratio equilibrium exactly corresponds to the Lindahl equilibrium of this economy. We show the Samuelson marginal condition is equivalent to the first order condition characterizing the set of collusive prices by the firms; give sufficient conditions for a ratio equilibrium to exist

[6] See Kaneko (1977) for the orginal article; also, for expansions and generalizations of ratio equilibria, see Mas-Colell and Silvestre (1989), Diamantaras and Wilkie (1992), and van den Nouweland, Tijs, and Wooders (2002).

in the firms' economy; and illustrate with several examples. The Ratio equilibrium, therefore, gives a Pareto superior recommendation to the firms (the firms benefit from collusion and consumers benefit from lower overall price) to the Nash equilibrium outcome.[7] Taking our inspiration from the public good mechanism design literature, we show there is an incentive compatible mechanism which can be used to regulate this problem achieving Pareto superior collusive outcomes. We tailor Corchón and Wilkie's 1996 Cost Share mechanism to fit our environment.[8] Finally, we compare the collusive solution to the Pareto optimal solution. The collusive solution is not Pareto optimal, and the natural regulatory solution of marginal cost pricing is not incentive compatible. However, we are able to design an alternative incentive compatible regulation concept (based on the Loeb-Magat mechanism for regulating natural monopoly) which implements the marginal cost pricing outcome.

There are several related papers on mitigating anticommons that take different approaches to ours. Kominers and Weyl (2010) is probably the closest in spirit to our paper. They consider an environment where there is one buyer with unit demand and multiple sellers. The buyer makes a take-it-or-leave it offer which gets accepted if and only if all of the sellers agree.[9] This is the familiar hold-out problem. They analyze the problem in a Bayesian setting and consider alternative auction like mechanisms for mitigating hold-up.[10] They are able to approach an efficient outcome in the limit by designing a interesting Lindahl-"esque" mechanism. We differ from Kominers and Weyl with our choice of information environment and the implementation concepts we apply. Our paper considers a oligopoly setting where firms have complete

[7] This same approach can be taken in the standard Cournot quantity competition where the firms treat the price as a public good. The Pareto set in this setting corresponds to the set of collusive outcomes and the Lindahl equilibrium, again, picks out an allocation in the core. The key difference in the current setting is that collusion increases social welfare as opposed to hurting overall welfare.

[8] In a related paper, Groves and Loeb (1975) have firms that use a public good as an input (like a stream), they design a demand revealing scheme for the firms that covers the cost of production. Their mechanism may however, violate individual rationality of the participants. Something a Lindahl, or Ratio, mechanism will not do.

[9] Bargaining mechanisms between multiple agents in an incomplete information setting are studied by Mailath and Postelwaite (1990).

[10] Other studies which have looked at mitigating the holdout problem include, but are not limited to, Shavell (2007), Heller and Hills (2008), Shoup (2008), and Grossman et al (2010).

information about the market structure. We, thus, appeal to dominant strategy and Nash implementation concepts to achieve our regulatory efficiency goals. Another related paper is due to Cornes and Hirokawa (2007) who investigate bargaining a la the Nash Bargaining solution between the government and firms as a means to mitigating or regulating the tragedy of the anticommons in a setting with linear demand and zero marginal cost. Our approach to the problem is different. First, we allow for a wide variety of demand and cost functions in our pricing game. Second, we examine the firm's collusion problem in more depth, but using Ratio equilibrium rather than the Nash bargaining solution.[11] Although not Pareto optimal, merger is a common policy prescription in this environment and therefore the collusive outcome is worth examining. Finally, we use non-cooperative mechanisms to achieve solutions recommended by either the Ratio or marginal cost pricing equilibrium.

2 Model

Consider two monopolists: Firm A and Firm B.[12] These firms make differentiated products that consumers view as perfect complements. The market structure is one where each firm i simultaneously announces a price $p_i \in [0, \bar{P}]$, where $\bar{P}$ is some upper bound.[13] Consumers can purchase the bundled product A and B at the price $p_A + p_B$, and choose to buy if and only if their maximum willingness to pay for the bundle exceeds this price. The total quantity demanded Q for the bundled product is represented by a marked demand $F : \mathbb{R}_+ \to \mathbb{R}_+$, where $Q = F(p_A + p_B)$. Firms A and B produce Q units according to cost functions $C^A(\cdot)$ and $C^B(\cdot)$ respectively where for all i, $C^i(0) = 0$.

Assumptions on Demand and Cost

A1. There is an aggregate price $\bar{P} < \infty$ such that $F(\bar{P}) = 0$; and for all $p_A + p_B \in (0, \bar{P})$, F is a twice continuously differentiable function that

[11] One can show that the ratio equilibrium profits are identical to the symmetric Nash bargaining solution in environments with linear demand and constant marginal costs. However, the two concepts diverge when Nash equilibrium profits are asymmetric.

[12] The restriction to two monopolists is purely to keep things simple and to be able to illustrate things in graphs. None of the results rely on this assumption.

[13] Amir et al. (2006) look at a sequential analog of the anticommons pricing game.

is strictly decreasing and weakly concave (i.e., $F_{11} \leq 0$).

A2. Each firm i's cost function is twice continuously differentiable, increasing, and weakly convex (i.e., $C^i_{11}(\cdot) \geq 0$).

A3. Each firm i's profit is strictly concave on $(0, \bar{P})$– i.e., $F_{11}p_i - F_{11}C^i_1 + 2F_1 - C^i_{11}[F_1]^2 < 0$.

A4. It is jointly profitable for the firms to price below the choke price $\bar{P}$ – i.e., $\bar{P} - C^A_1(0) - C^B_1(0) > 0$.

3 Three Families of Outcomes

There are three classes of outcomes which we study in this paper: the Nash equilibria outcomes; the collusive outcomes; and the Pareto optimal outcomes. As in the more standard Cournot model, we have assumed that each firm takes the other firm's price as parametric and chooses own price to maximize their profit. Firm i's best response problem then is to solve

$$\max_{p_i} \pi^i(p_i, p_{-i}) = F(p_A + p_B)p_i - C^i(F(p_A + p_B)).$$

The best response correspondence is the set of prices that maximize this expression for each p_{-i}. Under (A1)-(A4), the existence of a Nash equilibrium is trivial. The strategy profile where both firms announce the choke price $\bar{P}$ is a Nash equilibrium. We say this is a no-trade equilibrium since $F(p_A+p_B) = 0$ and there will typically be many such equilibria.[14] In the case where demand is $Q = \max\{0, a - b(p_A + p_B)\}$ and output is produced at a constant marginal cost c_i for each firm i such that $\frac{a}{b} > c_i + c_j$, we can solve for a closed form solution. Optimal behavior by each firm i given p_j is to price according to

$$p^*_i(p_j) = \begin{cases} \frac{a+bc_i}{2b} - \frac{1}{2}p_j, & \text{if } p_j \leq \frac{a-bc_i}{b} \\ [\frac{a}{b} - p_j, \frac{a}{b}] & \text{otherwise.} \end{cases}$$

The other firm's reaction correspondence is symmetric. The unique trade equilibrium where each i charges $p^{NE}_i = (1/3b)(\,a + 2bc_i - bc_j) > 0$. The aggregate trade Nash equilibrium price is $p^{NE}_A + p^{NE}_B = (1/3b)(2a + bc_A + bc_B)$, where equilibrium output is $Q^{NE} = \frac{1}{3}(a - bc_i - bc_j) > 0$. Equilibrium profit

[14] The existence of a Nash equilibrium where $F(p_A + p_B) > 0$ is slightly more involved. We postpone this proof until the next section.

for the two firms are equal at $\pi_A^{NE} = \pi_B^{NE} = (1/9b)(a - bc_A - bc_B)^2$. This outcome is neither best for the firms nor the best for society.

The second outcome we investigate is the one where firms get together to maximize aggregate profit. While this is usually deemed detrimental to social welfare, the anticommons environment has the unusual characteristic that collusive behavior by the firms actually increases social welfare. This property, realized first by Cournot, suggests merger as a natural policy recommendation. The collusive outcome is found by choosing prices to maximize total profits – i.e., choosing prices such that

$$\frac{p_A^* + p_B^* - C_1^A(F(p_A^* + p_B^*)) - C_1^B(F(p_A^* + p_B^*))}{F(p_A^* + p_B^*)} = -\frac{1}{F_1(p_A^* + p_B^*)}$$

is satisfied. The next proposition reviews some well estabilished properties of the Nash and collusive outcomes. We offer a proof for completeness.[15]

Proposition 1 *If (A1)-(A4), then: (I) there is a unique Nash equilibrium where production is positive; (II) the aggregate price for the collusive solution and the Nash equilibrium solution with production are uniquely defined; (III), the aggregate collusive price is strictly smaller than the aggregate Nash price; and (IV), there exists a profit sharing arrangement, under collusion, that yields a strict Pareto improvement for the economy.*

Proof. See Appendix. ■

To illustrate, consider the case where demand is linear and marginal costs are constant. The collusive outcome is found by choosing $p_A^* + p_B^* = (1/2b)(a + bc_A + bc_B)$ which yields an output of $Q^C = \frac{a - bc_A - bc_B}{2}$. It is straightforward to check that the aggregate collusive price is smaller than the aggregate Nash equilibrium price and that total welfare has increased.

While an improvement on the Nash equilibrium outcome, the collusive allocation itself is not Pareto optimal since consumer welfare is not been taken into account. This means that the fully efficient outcome cannot be obtained by simply merging the firms together. The Pareto problem is to choose Q to maximize total surplus (i.e., consumer plus producer surplus) – i.e.,

[15] These results are consistent with those found in Cournot (1838). The existence of a choke price and no-trade equilibria cause some differences in the method of proof.

$$\max_{Q} \int_{0}^{Q} P(x)dx - C^A(Q) - C^B(Q),$$

where $P(x)$ is the inverse demand function. The first order condition is $P(Q^*) = C_1^A(Q^*) + C_1^B(Q^*)$ – i.e., aggregate price should equal aggregate marginal cost.

4 A Public Good Interpretation of the Collusion Problem

The set of collusive outcomes represent *potential* for a Pareto superior improvement on the Nash equilibrium outcome (depending on the assignment of profit share to the firms). Thus, regulation which encourages collusion may be desired. It remains a question on the method in which the firms "should" divide the profit when they collude. While there are a number of competing bargaining solutions that could apply to this situation, we appeal to the public expenditure literature for insight into this problem.[16] The public goods approach yields an alternative, attractive solution.

We start by redefining the firms' collusion problem as a public good provision problem. Since the total number of bundled units sold to consumers, Q, is a non-rival and non-excludable good; from the perspective of the firms, Q is a public good. Each firm cares about their consumption of a public good Q and a private good p_i. The private good can be converted into the public good according to the production function $Q = F(p_A + p_B)$. If the firms supply no input goods then the public good is at its maximum. The "good" produced for the firms is therefore a reduction in the public good. Thus, the marginal product for the good is $-F_1$ and the amount of input firms must give to get a one unit increase in public good reduction (i.e., the real marginal cost of production) is $-\frac{1}{F_1}$. The Pareto problem for the firms is to maximize the profits of Firm A subject to resource constraints, production constraints, and the constraint that the profit of Firm B is at least as high as some benchmark k. Formally, we solve:

[16] The Nash or Kalai-Smorodinsky bargaining solutions are examples.

$$
\begin{aligned}
\max_{p_A, p_B, Q} \pi_A(Q, p_A) \qquad \text{st. } p_A, p_B, Q &\geq 0 \\
Q - F(p_A + p_B) &\leq 0 \;:[\sigma_Q] \\
\pi_B(Q, p_B) &\geq k \;:[\lambda_B]
\end{aligned}
$$

where σ_Q and λ_B are the Lagrangian multipliers and k is an arbitrary constant. The set of interior Kuhn-Tucker conditions for the problem are:

$$
\begin{aligned}
\frac{\partial \pi_A}{\partial p_A} &= -\sigma_Q F_1 &(1)\\
\lambda_B \frac{\partial \pi_B}{\partial p_B} &= -\sigma_Q F_1 &(2)\\
\frac{\partial \pi_A}{\partial Q} + \lambda_B \frac{\partial \pi_B}{\partial Q} &= \sigma_Q &(3)
\end{aligned}
$$

Putting (1), (2), and (3) together yield the Samuelson Marginal Condition $MRS^A_{p_A Q} + MRS^B_{p_B Q} = MC$, where $-\frac{1}{F_1} > 0$ is the *real* marginal cost of the public good.[17]

Proposition 2 *If an allocation (Q^*, p_A^*, p_B^*) satisfies the Samuelson marginal condition for Pareto efficiency and adding up condition, then that allocation will also satisfy the collusive first order condition.*

Proof. See Appendix. ■

Suppose demand is linear and A and B have constant marginal costs. The *real* marginal cost of production is $\frac{1}{b}$ and the marginal rate of substitution for each firm i is $MRS^i_{p_i Q} = \frac{p_i - c_i}{Q}$. Using the Samuelson marginal condition the set of Pareto optimal outcomes (in the interior) satisfy $(p_A - c_A)/Q + (p_B - c_B)/Q = 1/b$. Rearranging this expression and substituting in the value of Q in from the demand function to get $p_A + p_B = (1/2b)(a + bc_A + bc_B)$ which is identical to the collusive first order condition derived earlier.

[17] Note that the real marginal cost in this setting is not the marginal cost of production, but rather the negative of the inverse marginal rate of transformation – i.e., the total price firms have to give up to reduce Q by one unit.

In summary, the set of Pareto optimal outcomes coincides with the set of collusive outcomes for the two firm economy. This set is large and does not really offer much of a recommendation since there is not an *obvious way* that the firms should divide the total profits. We solve this dilemma by appealing the Ratio equilibrium as a solution concept in the next section.

4.1 A Procedure for Finding an Acceptable Collusive Outcome

Since we are considering a public good provision problem, it is natural to look for a Lindahl equilibrium of the economy. Lindahl equilibria, under some conditions, possess desirable welfare properties such as Pareto efficiency and individual rationality. However, these conditions are not general enough to cover many of the demand functions we want to consider.[18] A better alternative is the Ratio equilibrium concept which possesses these same welfare properties, exists in a wider set of environments, and coincides with Lindahl equilibrium allocation when MRT is constant. In this section, we define the Ratio equilibrium for the firms' public good provision problem, give conditions for it to exists, and then work through several examples.

We first require the definition of a cost share system.

Definition 1: A *cost share system* is defined by a pair of ratios $r = (r_A, r_B)$ such that $r_i \geq 0$ and $r_A + r_B = 1$, where each individual i's cost share is given by $r_i c(Q)$, where $c(Q) \equiv -F^{-1}(Q)$ is the real cost of the public good (in terms of Q).

A ratio equilibrium is an allocation that, given a cost share system, maximizes each person's utility subject to a "budget constraint."

Definition 2: A feasible allocation (Q^*, p_A^*, p_B^*) is a *ratio equilibrium*, given the ratio vector $r = (r_A, r_B)$, if and only if for each i: $p_i^* = w_i - r_i c(Q^*)$ and $u^i(Q, p_i) > u^i(Q^*, p_i^*)$ implies $p_i + r_i c(Q) > w_i$, where $w_i \in \mathbb{R}$ is the value of i's initial endowment –i.e., if $(\mathring{Q}, \mathring{p}_i)$ is i's initial endowment, then $w_i \equiv r_i c(\mathring{Q}) + \mathring{p}_i$.

[18] Example 2 below illustrates how Lindahl equilibria can fail to exist in an environment with a reasonable demand function.

In a Ratio equilibrium, each firm sets $MRS_i = r_i MC$ and demands the same Q. Adding up these conditions gives us the Samuelson marginal condition for efficiency – i.e., Q is efficient for the firms. Also, the cost of the "production" is exactly covered by the cost share arrangement. Finally, since each firm can always "demand" their initial allocation bundle, the equilibrium allocation is individually rational. The next proposition identifies a sufficient conditions for a ratio equilibrium to exist.[19]

Proposition 3 *If A1 and both firms produce their outputs at constant marginal cost, then there exists a Ratio equilibrium for the two firm public good economy.*

Proof. See Appendix. ■

To illustrate we present several examples.

Example 1 The first example is an environment where the demand faced by the firms is linear, marginal costs are constant, and the Nash equilibrium outcome is our initial endowment.[20] The firm's problem, in this case, is maximizing a Cobb-Douglas utility function subject to a linear budget constraint– i.e., $\max\limits_{Q,m_i} \pi_i(Q, m_i)$ subject to $\frac{r_i}{b}Q + w_i = \frac{r_i}{b}Q^{NE} + w_i^{NE}$, where w_i is the firm's chosen mark-up $p_i - c_i$ and w_i^{NE} is firm i's Nash equilibrium mark-up. Applying standard methods, Firm A's demand for Q is

$$Q_A(\frac{r_A}{b}) = \frac{\frac{r_A}{b}Q^{NE} + w_A^{NE}}{2(\frac{r_A}{b})}.$$

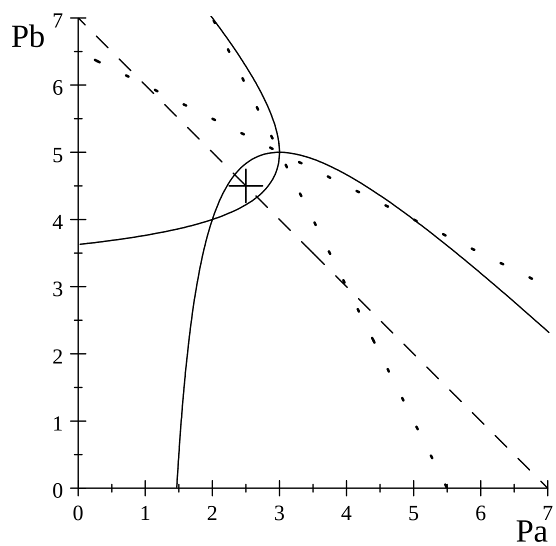
In a similar fashion we determine Firm B's demand for Q. Since Q is a public good, our goal is to find a ratio for each firm so $Q_A = Q_B$. Using $r_A + r_B = 1$, we write both demands in terms of firm A's cost share r_A and plugging in the values for (Q^{NE}, p_i^{NE}), which were determined in an earlier example, we solve for A's personal price share $r^A = 1/2$. Inputting this value into the demand equations we arrive at the Ratio allocation $(Q^{RE}, p_A^{RE}, p_B^{RE})$ where

[19] Note the restriction to constant marginal cost. An existence theorem with general cost functions is more problematic as the public good is not always a "good" for the firms. This is a key assumption in Kaneko's proof of existence.

[20] Recall, we want an allocation that is preferred to the Nash equilibrium outcome.

$Q^{RE} = (1/2)(a - bc_A - bc_B)$ and, for each i, $p_i^{RE} = (1/4b)(a + 3bc_i - bc_{-i})$. Each firm earns $\pi_A^{RE} = \pi_B^{RE} = (1/8b)(a - bc_A - bc_B)^2$ at the Ratio equilibrium which is greater than the profit earned in the unique interior Nash equilibrium allocation.[21]

To illustrate, suppose the parameters are $a = 10$, $b = 1$, $c_A = 1$ and $c_B = 3$. Firm A and B's reaction curves, represented by dotted lines in the figure below, intersect at the Nash equilibrium set of prices $(3, 5)$. The corresponding production level is $Q^{NE} = 2$. For reference, we draw the firms' iso-profit lines through the Nash equilibrium. The "lens" that the iso-profit lines forms the set of individually rational improvements. The set of collusive prices are where $p_A + p_B = 7$ (i.e., $Q^C = 3$), indicated by the dashed line in our figure. The collusive outcomes contained in the lens comprise the core. Last, the Ratio allocation, indicated with a (+), is $Q^L = 3$, $p_A^L = \frac{10}{4}$, $p_B^L = \frac{18}{4}$.



Ratio Equilibrium for the Collusive Problem with Constant MRT

Example 2 This is an example where a Lindahl equilibrium does not exist, but there is a ratio equilibrium. Suppose Q is produced according to $\max\{0, 10 - \frac{1}{4}\sqrt{p_A + p_B}\}$ and $c_A = c_B = 0$. The unique interior Nash equilibrium of the pricing game is $(p_A^{NE}, p_B^{NE}) = (\sqrt{5}, \sqrt{5})$ with corresponding

[21] This is also the symmetric Nash Bargaining solution when the Nash equilibrium outcome is used as the disagreement point.

output $Q^{NE} = 5$. The set of collusive prices is $p_A + p_B = \sqrt{\frac{40}{3}}$ and corresponding quantity $Q = \frac{20}{3}$. In a Lindahl equilibrium the "producer" of a public good must be profit maximizing and the profit is distributed to the owners of the firm. In this case, the producer takes the firms prices as inputs and produces Q at cost $-F^{-1}(Q)$. The cost of Q in this example is $c(Q) = -2\sqrt{10 - Q}$. The firm's profit maximizing decision is $\max_z \pi(z) = (p_A^L + p_B^L)Q + 2\sqrt{10 - Q}$, where p_A^L, p_B^L are A and B's Lindahl prices. Since marginal cost of production is decreasing, for any positive $p_A^L + p_B^L$, there is no profit maximizing choice for the firm and therefore no Lindahl equilibrium. There is a ratio equilibrium where $r_A = r_B = \frac{1}{2}$. Setting $r_A = \frac{1}{2}$, Firm A chooses p_A and Q to maximize Qp_A subject to budget constraint $-\sqrt{10 - Q} + p_A = -\sqrt{10 - Q^{NE}} + p_A^{NE}$. Optimal choices by the two firms requires $\frac{p_A}{Q} = \frac{1}{2}(10 - Q)^{-\frac{1}{2}}$ and $\frac{p_B}{Q} = \frac{1}{2}(10 - Q)^{-\frac{1}{2}}$. Adding these two marginal conditions together we get $\frac{p_A + p_B}{Q} = (a - Q)^{-\frac{1}{2}}$, the first order condition for efficiency. The ratio equilibrium allocation is found to be $(Q, p_A, p_B) = (\frac{20}{3}, \sqrt{\frac{10}{3}}, \sqrt{\frac{10}{3}})$.

Example 3 A Ratio equilibrium may still exist in environments with non-constant marginal cost functions.[22] Suppose $Q = \max\{0, 15 - (p_A + p_B)\}$ and $c_A(Q) = \frac{1}{2}Q^2$ and $c_B(Q) = 0$. The collusive set of prices are where $p_A + p_B = 10$ (i.e., $Q = 5$). At the interior Nash equilibrium, (p_A^{NE}, p_B^{NE}) must satisfy both: $p_A^{NE} = 10 - \frac{2}{3}p_B^{NE}$ and $p_B^{NE} = \frac{15}{2} - \frac{1}{2}p_A^{NE}$. This occurs at $(p_A^{NE}, p_B^{NE}) = (\frac{15}{2}, \frac{15}{4})$, where the corresponding Nash output is $Q^{NE} = \frac{15}{4}$ and $\pi_A^{NE} = 21.09375$ and $\pi_B^{NE} = 14.0625$. The Ratio equilibrium is solved in the standard way. Given ratio r_A, Firm A's problem is to maximize his utility subject to his budget constraint – i.e., $\max\limits_{p_A, Q} p_A Q - \frac{1}{2}Q^2$ subject to $-r_A(15 - Q) + p_A = -r_A(\frac{45}{4}) + \frac{15}{2}$. Firm B's problem is to similarly set-up. The cost share system that clears the market is $r = (r_A, r_B) = (\frac{2}{5}, \frac{3}{5})$.[23] The ratio equilibrium allocation is $(Q, p_A, p_B) = (5, 7, 3)$ with corresponding profits $\pi_A = 22.5$ and $\pi_B = 3 * 5 = 15$.

[22] This example also shows that the Ratio equilibrium allocation need not be symmetric and differs from the Nash bargaining solution which the last two examples did not.

[23] Note: $r_A = \frac{(p_A - p_A^{NE})}{(p_A - p_A^{NE}) + (p_B - p_B^{NE})}$. This is the proportion of the decrease in price (i.e., the cost) consumer A makes.

5 Regulation of the Anticommons

This environment where monopolies whose products are perfect complements remains relatively unexplored and regulatory solutions untested. One of our main goals in this paper is to analyze several Pareto improving regulatory institutions or mechanisms. A *mechanism* specifies: (1) the choices or messages that each of the agents can send; and, (2), how those choices get mapped into an outcome. The set of messages is called a message space, and the functions that take messages into allocations are called outcome functions. Sometimes a mechanism is called a game form. When preferences over outcomes are added we have a game. In this section we specify two mechanisms that induce games whose Nash equilibria have "nice" properties. Specifically, one of the mechanisms enforces a Pareto superior collusive outcome as a Nash equilibrium and one mechanism which enforces a Pareto optimal outcome.[24]

5.1 Implementing Ratio Equilibria Outcomes with a Cost Share Mechanism

In this section, we demonstrate how a public good mechanism can be adopted to achieve the Ratio outcome of the anticommons game. Specifically, we use the *Cost Share Mechanism* due to Corchón and Wilkie (1996), hereafter CSM. This mechanism is a market-like mechanism that Nash implements the Ratio correspondence. In order to fit our economic environment, we slightly adapt their mechanism to accommodate a positive endowment of the public good by letting the mechanism accommodate a potential decrease of this initial level.

Suppose $(\mathring{Q}, \mathring{p}_A, \mathring{p}_B) \in \mathbb{R}^3_{++}$ is the strictly positive initial endowment. In the CSM, the message each firm sends is 2 dimensional. Let $M_i = [0,1] \times \mathbb{R}$ be i's message space, with generic element $m_i = (r_i, q_i)$, where r_i is interpreted as a cost share proposal and q_i as a vote for the level of the public good. The outcome functions are defined as follows:

[24] As a disclaimer, the mechanisms we propose are not necessarily ones that we would recommend for *actual* regulation. We are simply demonstrating there are institutions capable of reaching the Pareto superior collusive outcome or the Pareto optimal outcome in a decentralized fashion. The degree to which one mechanism, or process, is better suited for dealing with a problem in practice is an empirical question. It is something well posed for experimental testing.

$$Q(m) = \begin{cases} \sum_k q_k + \mathring{Q} & \text{-if } \sum r_k = 1 \text{ and } \sum_k q_k \geq -\mathring{Q} \\ \mathring{Q} & \text{-otherwise} \end{cases}$$

$$P_i(m) = \begin{cases} \mathring{p}_i - r_i[c(Q(m)) - c(\mathring{Q})] & \text{-if } \sum r_k = 1 \text{ and } \sum_k q_k \geq -\mathring{Q} \\ \mathring{p}_i - \epsilon & \text{-otherwise} \end{cases}.$$

In words, this mechanisms works as follows: The firms submit a message to a planner.[25] The planner checks the messages to make sure that the cost is *exactly* covered (i.e., $r_A + r_B = 1$) and that the final amount of the public good to be produced makes sense (i.e., $Q \geq 0$), if these conditions are met then the public good is changed to meet demand and the cost of production is distributed between firms. If these conditions are not met, the mechanism reverts back to the initial endowment minus a small penalty ϵ (for example $\epsilon = \frac{1}{N}\min\{\mathring{p}_1, ..., \mathring{p}_N\}$ could be used).

Proposition 4 *The CSM Nash implements the Ratio equilibria of the economy defined by the profit functions of Firm A and Firm B, the production technology F, and the initial endowment* $(\mathring{Q}, \mathring{p}_A, \mathring{p}_B) \in \mathbb{R}^3_{++}$.

We illustrate the proof using Example 2, where $Q = \max\{0, 10 - \frac{1}{4}(p_A + p_B)^2\}$ and $c_A = c_B = 0$. Recall, the Nash equilibrium allocation is $(Q^{NE}, p_A^{NE}, p_B^{NE}) = (5, \sqrt{5}, \sqrt{5})$ and $[(r_A, r_B), (Q, p_A, p_B)] = [(\frac{1}{2}, \frac{1}{2}), (\frac{20}{3}, \sqrt{\frac{10}{3}}, \sqrt{\frac{10}{3}})]$ is the unique ratio equilibrium associated with the Nash equilibrium initial endowment. First we demonstrate that there is a Nash equilibrium whose allocation is the same as the above specified Ratio equilibrium. Second, we demonstrate that any Nash equilibrium allocation corresponds to a Ratio equilibrium allocation.[26]

Consider the profile of messages m, where each firm i sends $m_i = (\frac{1}{2}, \frac{5}{6})$. The outcome specified by the mechanism is:

$$Q(m) = q_A + q_B + 5 = \frac{20}{3}$$

$$P^i(m) = \sqrt{5} - \frac{1}{2}(c(\frac{20}{3}) - c(5)) = \sqrt{5} - \frac{1}{2}(-2\sqrt{\frac{10}{3}} + 2\sqrt{5}) = \sqrt{\frac{10}{3}}.$$

[25] Recall c is commonly known by the firms since it is derived from the market demand.

[26] The general proof of this proposition closely follows Corchón and Wilkie (1996) and is therefore omitted.

Firm A's profit is $\pi^A(\frac{20}{3}, \sqrt{\frac{10}{3}})$. Since a Ratio equilibrium allocation is individually rational it is weakly preferred to the initial endowment – i.e., $\pi^A(\frac{20}{3}, \sqrt{\frac{10}{3}}) \geq \pi^A(5, \sqrt{5})$. Now consider a deviation by Firm A to $m'_A = (r'_A, q'_A)$. If $r'_A \neq \frac{1}{2}$, then the outcome yielded by the mechanism is $Q(m'_A, m_B) = 5$ and $P^A(m'_A, m_B) = \sqrt{5} - \epsilon$. Since, for $Q > 0$, profit is strictly increasing in p_i, we have $\pi^A(5, \sqrt{5}) > \pi^A(5, \sqrt{5} - \epsilon)$. Therefore, an optimal deviation is of the form $m'_A = (\frac{1}{2}, q'_A)$. For a similar reason, $q'_A \geq -(5 + q_B)$. Finally, since any level of Q can be achieved by appropriate choice of q'_A, A's best response problem is to choose q_A to

$$\max_{q_A} \pi^A(q_A + \frac{5}{6} + 5, w^i - \frac{1}{2}c(q_A + \frac{5}{6} + 5)).$$

However, because the $(Q, p_A, p_B) = (\frac{20}{3}, \sqrt{\frac{10}{3}}, \sqrt{\frac{10}{3}})$ is a ratio equilibrium, we know $q_A = \frac{5}{6}$ is a solution to this best response problem. The argument for B is similar. Therefore m is a Nash equilibrium.

Now we demonstrate the second part. Suppose $\hat{m}$ is a Nash equilibrium. First, $r_A + r_B = 1$ and $q_A + q_B \geq -5$. Suppose not. Then the NE profit is $\pi^A(5, \sqrt{5} - \epsilon)$, but Firm A could deviate to $r_A = 1 - r_B$ and $q'_A = -q_B$ unilaterally achieving the initial endowment and a profit of $\pi^A(5, \sqrt{5})$ which is strictly preferred to the Nash profit. This is a contradiction. Since $\hat{m}$ is Nash, $\hat{m}_i$ is a best response to $\hat{m}_{-i}$. We have $r_A + r_B = 1$ in any best response. Thus, each i takes $r_i = 1 - r_j$ as given and chooses a q_i to maximize

$$\max_{q'_i} u^i(q'_i + q_j + \mathring{Q},\ \mathring{p}_i - r_i[c(q'_i + q_j + \mathring{Q}) - c(\mathring{Q})]).$$

However, this is just the Ratio equilibrium problem. The two equilibrium allocations, therefore, coincide.

5.2 Implementing a Pareto Optimal Outcome with an Augmented Loeb-Magat Mechanism

Marginal cost pricing, while Pareto optimal, is not in firms' interest. Moreover, if we make the plausible assumption that the government does not know the cost structure of the firm, then enforcing marginal cost pricing may seem unrealistic. However, the government can design an alternative environment for the firms to interact where the incentives are aligned with efficiency. This is done, as in the regulation of natural monopoly (see Loeb and Magat (1979)), by choosing an appropriate subsidy scheme $\tau_i(p_A, p_B)$ for each firm which is added to the profit function. Specifically, define the Augmented Loeb-Magat mechanism (LM) as follows: The message space for each firm is $M_i = \mathbb{R}_+$ with a generic element just being a price announcement p_i. The public good outcome function is the demand function F and the subsidy outcome function τ_i is $\tau_i : \mathbb{R}^2_+ \to \mathbb{R}$ defined according to

$$\tau_i(p_A, p_B) = \int_0^{F(p_A+p_B)} P_i(x, p_{-i})dx - p_i F(p_A + p_B) + K_i,$$

where K_i is a constant and $P_i(x, p_{-i}) = \max\{0, P(x) - p_i\}$.

Proposition 5 *Given the mechanism* $LM = \{(M_A, M_B), (\tau_A, \tau_B)\}$*, if* (p^*_A, p^*_B) *is a Nash equilibrium, then the equilibrium allocation is Pareto optimal.*

The proof is by construction. Applying preferences to this mechanism, given the message of the other firm, each firm faces a best response problem – i.e., he chooses a price to maximize the objective function $\max\limits_{p_i} \pi_i(p_A, p_B) + \tau_i(p_A, p_B)$. Given a price announcement p_B, Firm A's best response problem in this mechanism is to choose a p_A to maximize

$$\max_{p_A} \int_0^{F(p_A+p_B)} P_A(x, p_B)dx - C^A(F(p_A + p_B)) + K_i.$$

Applying Leibniz's Rule, the first order condition yields $P_A(F(p^*_A + p_B)) = C^A_1(F(p^*_A + p_B))$. Thus, given B's strategy, A should choose his price to equal marginal cost. A similar subsidy applied to B yields the symmetrical result. When both firms act in this manner, the outcome is Pareto optimal.

Equilibrium yields $P_A(F(p^*_A + p^*_B), p^*_B) = C^A_1(F(p^*_A + p^*_B))$ and $P_B(F(p^*_A + p^*_B), p^*_A) = C^B_1(F(p^*_A + p^*_B))$. Adding the two conditions together we have the condition for Pareto efficiency. The constant term K_i can be used to extract money from the firms and return it to the consumers. Thus, a wide variety of Pareto allocations can be achieved through appropriate redistribution.

In general, a firm's best response depends on the price of the other firm. However, if demand is linear, we get a stronger result.

Corollary 1 *Given mechanism LM, if demand is linear and Firms A and B have constant marginal costs of production, then marginal cost pricing is always a best response.*

The proof is straightforward. If we apply the Augmented Loeb-Magat subsidy, A's problem is

$$\max_{p_A} \int_0^{a-b(p_A+p_B)} P_A(x, p_B)dx - c_A(a - b(p_A + p_A)) + K_i,$$

where $P_A(x, p_B) = \frac{a}{b} - \frac{1}{b}x - p_B$ .The first order condition for the problem yields $p_A^* = c_A$. Thus, A should price the same no matter what B prices!

6 Conclusion

In this paper, we have shown the existence of a non-trivial Nash equilibrium and give conditions under which collusion improves social welfare. By re-interpreting the collusion problem as a public good provision problem, we can make a recommendation to the firms on how to collude resulting in a Pareto superior movement from the interior Nash equilibrium outcome. While this outcome is not incentive compatible under the pricing game, we have shown institutions (or mechanisms) exist which induce games whose Nash equilibria outcomes coincide with the Ratio equilibria. Finally, we compare the collusive outcome with the Pareto outcome and demonstrate by augmenting the Loeb-Magat procedure it is possible to implement the first best solution of the problem where both firms price at marginal cost. Interestingly, the Loeb-Magat procedure, in a natural monopoly setting has been expanded on by Finsinger and Vogelsang (1981). They derive an alternative procedure which approximates the Loeb-Magat outcome, but requires less observable information. Cox and Isaac (1987) provide an alternative to the Finsinger-Vogelsang procedure and test all three procedures in a laboratory environment finding their procedure outperforms the other two. One area for further study would be to investigate whether these procedures can augmented to fit the anti-commons environment and, if so, whether the mechanisms' performances are consistent with the results from natural monopoly.

7 Appendix

Proof of Proposition 1. Define the continuous function $A(z) = zF_1(z) + F(z) - C_1^A(F(z))F_1(z) - C_1^B(F(z))F_1(z)$. Note, $A = 0$ at the collusive optimum. From the definition of $\bar{P}$, (A1), (A2), and (A4), there exists a $\epsilon > 0$ such that $A(\bar{P} - \epsilon) = F_1(\bar{P} - \epsilon)[\frac{F(\bar{P}-\epsilon)}{F_1(\bar{P}-\epsilon)} + (\bar{P} - \epsilon) - C_1^A(F(\bar{P} - \epsilon)) - C_1^B(F(\bar{P} - \epsilon))] < 0$. Similarly, there exists a $\hat{\epsilon} > 0$ such that $A(\hat{\epsilon}) = -F_1(\hat{\epsilon})[-\frac{F(\hat{\epsilon})}{F_1(\hat{\epsilon})} - \hat{\epsilon} + C_1^A(F(\hat{\epsilon})) + C_1^B(F(\hat{\epsilon}))] > 0$. From (A3), $A_1 < 0$. So, from the Intermediate Value Theorem, there is a unique $z = p_A^C + p_B^C$ such that $A(p_A^C + p_B^C) = 0$ and $0 < p_A^C + p_B^C < \bar{P}$. Now, define $B(z) = A(z) + F(z)$. Note, $B = 0$ at any trade Nash equilibrium. Since, $0 < F(p_A^C + p_B^C) < \infty$, $B(p_A^C + p_B^C) = 0 + F(p_A^C + p_B^C) > 0$. Again, from the definition of $\bar{P}$, (A1), (A2), and (A4), we have there is $\epsilon > 0$ that $B(\bar{P} - \epsilon) = F_1(\bar{P} - \epsilon)[\frac{F(\bar{P}-\epsilon)}{F_1(\bar{P}-\epsilon)} + 2(\bar{P} - \epsilon) - C_1^A(F(\bar{P} - \epsilon)) - C_1^B(F(\bar{P} - \epsilon))] < 0$. Since $A_1 < 0$ and $F_1 < 0$, then $B_1(p_A^C + p_B^C) < 0$. Thus, by the Intermediate Value Theorem, there exists a unique $p_A^{NE} + p_B^{NE} \in (p_A^C + p_B^C, \bar{P} - \epsilon)$ such that $B(p_A^{NE} + p_B^{NE}) = 0$. Let $P^{NE} = p_A^{NE} + p_B^{NE}$. From the first order condition, we back out the unique individual equilibrium prices $p_A^{NE} = C_1^A(F(P^{NE})) - \frac{F(P^{NE})}{F_1(P^{NE})}$ and $p_B^{NE} = C_1^B(F(P^{NE})) - \frac{F(P^{NE})}{F_1(P^{NE})}$. It follows directly that $p_A^C + p_B^C < p_A^{NE} + p_B^{NE} < \bar{P}$. The proof of the last part of the theorem is obvious from (III). ■

Proof of Proposition 2. The Samuelson marginal condition for the firms is

$$\frac{p_A^* - C_1^A(Q^*)}{Q^*} + \frac{p_B - C_1^B(Q^*)}{Q^*} = -\frac{1}{F_1(p_A^* + p_B^*)}.$$

Plugging $Q^* = F(p_A^* + p_B^*)$ into the equation we have

$$\frac{p_A^* - C_1^A(F(p_A^* + p_B^*))}{F(p_A^* + p_B^*)} + \frac{p_B^* - C_1^B(F(p_A^* + p_B^*))}{F(p_A^* + p_B^*)} = -\frac{1}{F_1(p_A^* + p_B^*)}$$

which is exactly the first order condition for the collusive problem. ■

Proof of Proposition 3. Our proof verifies that the components of the firm's environment under (A1) and constant marginal costs of production satisfy the conditions of Kaneko's existence theorem (see Theorem 1, p. 127). First, each firm i's decision problem, given $r_i \in [0, 1]$, is to choose a public good quantity $Q \geq 0$ and mark-up $w_i \equiv p_i - c_i \geq 0$ to maximize their utility $\pi(Q, w_i) = Qw_i$ subject to the budget constraint $w_i + r_i[c(Q) - c(\mathring{Q})] = \mathring{w}_i$, where $\mathring{w}_i = p_i^{NE} - c_i > 0$. Utility, π, is continuous, monotonically increasing, and quasi-concave. Second, cost is increasing and convex since $\frac{\partial c(Q)}{\partial Q} = -1/\frac{\partial F}{\partial Q}(F^{-1}(Q)) > 0$ and $\frac{\partial^2 c(Q)}{\partial Q^2} = \frac{\partial^2 F}{\partial Q^2} \frac{1}{\frac{\partial F}{\partial Q}} \frac{1}{[\frac{\partial F}{\partial Q}]^2} \geq 0$ and zero if production does not increase past the initial endowment – i.e., $Q = \mathring{Q}$. Thus, the two-firm economy satisfies Kaneko's theorem and there exists a ratio equilibrium. ■

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