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# Public funding of higher education in small open economies 

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# Public Funding of Higher Education in Small Open Economies 

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#### Abstract

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## 1. Introduction

In each country the educational system is complex and depends on many relevant parameters. Achieving an efficient allocation of the educational resources for the various components of the system demands a sophisticated decision making process and a huge amount of information. Nowadays, educational policy can hardly be implemented without allowing for international aspects, even for decisions that are considered 'domestic' such as compulsory schooling. In most countries higher education generates a significant part of a country's stock of human capital. As a result, it affects the marginal returns to physical capital and channels the limited supply of foreign investments. Despite its importance there are very few studies that capture the way in which international market conditions influence individuals' decision-making in the acquisition of additional training and skills. This paper studies 'efficient' education policies in a non-stationary competitive equilibrium of a small open economy considering the impact of international capital mobility and the role of government in higher education. In this context, the paper develops an overlappinggenerations model with heterogeneous agents and examines significant issues such as: (i) the allocation process in equilibrium of individuals to low-skilled and skilled sets of workers in each generation; (ii) the evolution of growth-enhancing stock of human capital and the impact of public funding to higher education; (iii) the endogenous support for government policies generated within a political equilibrium.

There is a long-standing debate in the empirical literature regarding the effects of international markets on wages and the size of unskilled workforce. Some empirical studies have shown that international trade accounts for somewhere between 0 and 20 percent of any rising income inequality. Hence, the conclusion is that globalization has been a small contributor to growing wage inequalities in trading nations (see, e.g. Greenaway and Nelson, 2000; Winchester, 2008). In our theoretical framework we use a different approach
to the decisions whether to acquire skills or not: altruistic rational parents consider the ability of their child together with the family background and the foregone income due to the time spent acquiring higher education. Based on these considerations they decide on whether to invest in their child's higher education or just let him/her start working right after compulsory schooling and become low-skilled.

There is little disagreement about the presence of intergenerational transfers (between parents and their children) in developed and developing countries. These transfers arise from altruistic motives of parents and take the form of educational investments, of tangible transfers like inter vivos gifts and bequests (Viaene and Zilcha, 2002). The growing interest of the economic literature in altruistic behaviour lies in its key role in the analysis of optimal savings, income distribution, wealth dynamics, higher education and the optimal design of tax systems. Parental altruism in our model may give rise to two types of transfers: physical capital transfers and investment in the education of the offspring. When altruistic rational parents make forward-looking decisions between financial transfers and investments in attaining skills they compare the return on physical capital and the return on human capital taking into account the interest rate and the future wage rate respectively (see Zilcha, 2003). In our framework, due to free capital mobility, we find that intergenerational transfers are directly affected by international market conditions.

Government budget balance is an important constraint on education policy. A popular view is:

> "If you want to have a new program, figure out a way to pay for it without raising taxes" US Senate Majority Leader H Reid".

This quotation stresses the importance of including both sides of the government balance sheet when looking at the effects of new policies. This importance is also confirmed in the empirics on growth where the evidence concerning the effects of public education spending
${ }^{1}$ US Senator H. Reid on Face the Nation, CBS News Transcript, Nov 12, 2006.
on growth is generally mixed but becomes positive when the method of finance is properly accounted for (see, e.g., Bassanini and Scarpenta, 2001; Blankenau et al., 2007). But when resources are scarce is there a justification for public funding, at least in part, of higher education? To answer this question we must consider the net social benefits to public investments in higher education.

The social costs of education include all expenses incurred by society that performs the act of training and expenses by an individual that are necessary to acquire skills, as well as the foregone income that would have been earned otherwise. Low-skilled workers are important contributors to the government budget because taxes that are collected on their labor income serve to finance public expenses elsewhere (see Garrat and Marshall, 1994; Fernandez and Rogerson, 1995; Gradstein and Justman, 1995; Bevia and Iturbe-Ormaetxe, 2002). The social benefits include higher earnings enjoyed directly by individuals as well as the benefits that society derives from the human capital formation via higher education. The latter include, for example, a capacity to absorb new production technologies, a higher marginal return to physical capital which gives rise to inflows of foreign physical capital. On balance, a great portion of the student population will generate a benefit to society, but not all. Given this background, is a government funding policy, like a subsidy to all individuals who wish to attend higher education, going to lead to a net social benefit? In a world with limited resources other programs like improved basic education, or poverty reliefs might provide a higher social return to invested public funds (Johnson, 1984).

Our dynamic framework analyzes non-stationary equilibria and starting from some given initial conditions derives the impact of international factor prices on the allocation of the workforce between skilled and low-skilled workers. International market conditions affect also the voting behaviour of heterogenous agents when the public funding policy is concerned since they position the median voter in the distribution of individual
endowments. In addition, our framework determines how education policies act on the sets of skilled and low-skilled individuals. In particular it questions the use of public resources in higher education under certain circumstances related to the productivity and cost of the higher education system.

The paper is organized as follows. Section 2 outlines preferences and the multistage formation of human capital in an OLG economy together with the characterization of nonstationary competitive equilibria. Section 3 studies the allocation of the workforce between 'low-skilled' and 'skilled' workers and how it relates to international factors. Implications of international factors and of government funding policy for growth are further analyzed in Sections 4 and 5. Section 6 introduces political equilibrium in our model and examines majority voting to allocate government revenues according to the outcome of a political equilibrium. Section 7 contains concluding remarks.

## 2. The Dynamic Equilibrium

## Preferences and Hierarchical Education

Consider an overlapping generation economy with a continuum of consumers in each generation, each living for three periods. During the early stage each child is engaged in education/training, but takes no economic decisions. Individuals are economically active during the working period which is followed by the retirement period. At the beginning of the working period, each parent gives birth to one offspring, hence we assume no population growth. Each household is characterized by a family name $\omega \in[0,1]$ where $\Omega=[0,1]$ denotes the set of all families in each generation. We also denote by $\mu$ the Lebesgue measure on $\Omega$.

Consider generation $t$, denoted $G_{t}$, which consists of all individuals $\omega$ born at the outset of date $t$, and let $h_{t+1}(\omega)$ be the human capital of $\omega$ at the beginning of the
working period. We assume that $h_{t+1}(\omega)$ is achieved by a hierarchical production function for human capital like in Restuccia and Urrutia (2004): it consists of fundamental education (assumed to be compulsory) and higher education. ${ }^{2}$ A child obtains his general skills from the compulsory fundamental education and acquires eventually specialized skills from higher education. Innate ability of an individual $\omega$, which is assumed to be random and denoted by $\tilde{\theta}_{t+1}(\omega)$, is a draw from a timeindependent distribution. Namely, the random abilities are independent and identically distributed variable across individuals in each generation and over time.

The human capital of individual $\omega$ in $G_{t}$ acquired during compulsory schooling is given by the following process:

$$
\begin{equation*}
h_{1 t+1}(\omega)=\tilde{\theta}_{t+1}(\omega) h_{t}^{v}(\omega) X_{t}^{\xi} \tag{1}
\end{equation*}
$$

where $h_{t}(\omega)$ stands for parents' human capital level and $X_{t}$ represents early-life public investment via compulsory schooling. ${ }^{3}$ The above production function is a representation of the complex interaction between innate ability, family dynamics and public intervention. It stresses the key role played by the individual home environment that is specific to each individual $\omega$ via the parental human capital and public resources invested in public education that are common to all. The elasticities $v$ and $\xi$ represent how efficient and effective is parent's human capital in their efforts towards educating their child, and how efficient is public education in generating human capital respectively: $v$ is affected by home education and family background while $\xi$ is affected by the schooling system, size of classes, facilities, neighborhood, etc.

[^0]Enrollment in higher education is costly and, in most countries, requires the payment of a tuition fee at date $t$, denoted by $z_{t}^{*}>1$. Let $g_{t}$ denote the government (public) allocation to each student attending higher education. Thus, at each date $t$, $z_{t}(\omega)=z_{t}=z_{t}^{*}-g_{t}$ is the net payment that each individual pays to access higher education. ${ }^{4}$ Hence, the cost of higher education is the same for all individuals of the same generation. For simplicity, we assume that the tuition and public funding are denominated in dollars of the working period of the student (e.g., it can be financed by students loans). We assume that obtaining higher education augments each individual's basic skills by a factor $B>1$. If individual $\omega$ invests $z_{t}^{*}$ in tertiary education then his/her human capital accumulation process is given by:

$$
\begin{equation*}
h_{t+1}(\omega)=B h_{1++1}=B \tilde{\theta}_{t+1}(\omega) h_{t}^{v}(\omega) X_{t}^{\xi} \tag{2}
\end{equation*}
$$

$\mathrm{He} /$ she is then called a skilled worker. In contrast, if an agent $\omega$ does not enroll in higher education, his/her human capital is determined by compulsory schooling only, hence:

$$
\begin{equation*}
h_{t+1}(\omega)=h_{1 t+1}(\omega)=\tilde{\theta}_{t+1}(\omega) h_{t}^{\nu}(\omega) X_{t}^{\xi} \tag{3}
\end{equation*}
$$

In this case we call this agent a low-skilled worker. Instead of attending higher education, a low-skilled agent works during part of his youth using basic skills given in (3). We assume that all low-skilled individuals do work during part $m(0 \leq m<1)$ of their youth period. As they work during all period $t+1$ as well, then lifetime after-tax wage income earned by a low-skilled worker is:

$$
(1-\tau) h_{1 t+1}(\omega)\left[m w_{t}\left(1+r_{t+1}\right)+w_{t+1}\right]
$$

where $w_{t}$ is the wage rate per unit of effective labor at date $t$ and $\tau$ is the tax rate on

[^1]labor income. In contrast, a skilled worker's after-tax lifetime earnings are:
$$
(1-\tau) B h_{1 t+1}(\omega) w_{t+1}
$$

Given (2) and (3) it is straightforward to obtain the aggregate (or mean, as well, in our case) of human capital $H_{t}$ that is available to the economy at date $t$. Let $A_{t}$ denote the set of individuals who are skilled in $G_{t}$ and let $\sim A_{t}$ be the complement of $A_{t}$, namely the set of low-skilled individuals. Hence:

$$
\begin{equation*}
H_{t}=\int h_{t}(\omega) d \mu(\omega)+m \int_{\sim A_{t}} h_{t+1}(\omega) d \mu(\omega) \tag{4}
\end{equation*}
$$

We shall assume, throughout our analysis that the government keeps the education tax imposed on wage incomes constant at the rate $\tau$. Therefore, government tax revenues are simply $\tau w_{t} H_{t}$ where $H_{t}$ is defined in (4). On the other side of its balance sheet the government faces expenditures in each stage of the education process. Denote by $\mu\left(A_{t}\right)$ the measure of individuals who are skilled and receive some public funding. Then the government budget constraint at date $t$ is:

$$
\begin{equation*}
\tau w_{t}\left[\int h_{t}(\omega) d \mu(\omega)+m \int_{\sim A_{t}} h_{t+1}(\omega) d \mu(\omega)\right]=X_{t}+g_{t} \mu\left(A_{t}\right) \tag{5}
\end{equation*}
$$

We say that an education policy $\left\{\left(X_{t}, g_{t}\right)\right\}$ is feasible if at each date t : (a) given $X_{t}$ and $g_{t}$, the set of skilled $\mathrm{A}_{t}$ is determined by each individual's 'optimal choice' and (b) condition (5) holds in all periods $t$.

In this economy we assume that parents care about the future of their offspring and derive utility directly from the lifetime income of their child. ${ }^{5}$ In particular, lifetime preferences of $\omega \in G_{t}$ are represented by the Cobb-Douglas utility function:

[^2]$$
U_{t}(\omega)=\left(c_{t}^{y}(\omega)\right)^{\alpha_{1}}\left(c_{t}^{o}(\omega)\right)^{\alpha_{2}}\left(y_{t+1}(\omega)\right)^{\alpha_{3}}
$$
where
\[

$$
\begin{equation*}
\alpha_{1}+\alpha_{2}+\alpha_{3} \leq 1 \tag{6}
\end{equation*}
$$

\]

Consumption when young and old is denoted by $c_{t}^{y}(\omega)$ and $c_{t}^{o}(\omega)$ respectively; $y_{t+1}(\omega)$ is the offspring's lifetime income. Intergenerational transfers that arise from the altruistic motives represented by (A1) may take two forms: transfers via physical capital and transfers via human capital, i.e., investment in education of the child. Thus, the earning capacity of the younger generation is enhanced by taxes parents pay to finance the education budget, and as a result to enhance the human capital levels. Moreover, under the above preferences parents are willing to transfer tangible assets directly. Denote by $b_{t}(\omega)$ the transfer of physical capital by household $\omega \in G_{t}$ to his/her offspring. Given the return to capital and wages $\left\{r_{t}, w_{t}\right\}$, lifetime non-wage income of an offspring, whether skilled and low-skilled, is $\left(1+r_{t+1}\right) b_{t}(\omega)$. Then, lifetime income of $\omega \in G_{t+1}$ if he is a lowskilled worker is:

$$
\begin{equation*}
y_{t+1}^{l}(\omega)=(1-\tau) h_{t+1}(\omega)\left[m w_{t}\left(1+r_{t+1}\right)+w_{t+1}\right]+\left(1+r_{t+1}\right) b_{t}(\omega) \tag{7}
\end{equation*}
$$

If he/she is a skilled worker then:

$$
\begin{equation*}
y_{t+1}^{\mathrm{s}}(\omega)=(1-\tau) h_{t+1}(\omega) w_{t+1}+\left(1+r_{t+1}\right) b_{t}(\omega) \tag{8}
\end{equation*}
$$

Assume that, at date $t=0$, our economy is integrated into the rest of the world such that they form a single capital market but keep labor internationally immobile. In addition we assume that physical capital is perfectly mobile and that our economy is small compared to the rest of the world, implying that $\left\{r_{t}\right\}$ is equal to the foreign interest rate. Production is carried out by competitive firms that produce a single commodity that is both consumed and used as production input. Physical capital $K_{t}$
(assumed to fully depreciate) and aggregate effective human capital $H_{t}$ are inputs of a neo-classical production function that satisfies the following conditions: it exhibits constant returns to scale; it is strictly increasing, concave, continuously differentiable and all inputs are required for production.

## Competitive Equilibrium

Given the intergenerational transfers at date $\mathrm{t}, b_{t-1}(\omega)$, the tax rate $\tau$ and $K_{0}, H_{0}$, public education policy $\left\{\left(X_{t}, g_{t}\right)\right\}_{t=0}^{\infty}$ and the international prices of capital and labor, $\left\{r_{t}, w_{t}\right\}$, an agent $\omega$ at time $t$ chooses the levels of savings $s_{t}(\omega)$ and bequests $b_{t}(\omega)$ together with the financial investment in higher education, $z_{t}(\omega)$, so as to maximize:

$$
\begin{equation*}
\operatorname{Max} \quad U_{t}(\omega)=\left(c_{t}^{y}(\omega)\right)^{\alpha_{1}}\left(c_{t}^{o}(\omega)\right)^{\alpha_{2}}\left(y_{t+1}(\omega)\right)^{\alpha_{3}} \tag{9}
\end{equation*}
$$

subject to constraints,

$$
\begin{align*}
& z_{t}(\omega)=0 \quad \text { or } \quad z_{t}(\omega)=z_{t}^{*}-g_{t}  \tag{10}\\
& c_{t}^{y}(\omega)=y_{t}(\omega)-s_{t}(\omega)-b_{t}(\omega)-z_{t}(\omega) \geq 0  \tag{11}\\
& c_{t}^{o}(\omega)=\left(1+r_{t+1}\right) s_{t}(\omega) \geq 0 \tag{12}
\end{align*}
$$

where $y_{t}(\omega)$ and $y_{t+1}(\omega)$ are the corresponding incomes given by (7) or (8), while $h_{t+1}(\omega)$ is defined by (2) if $z_{t}(\omega)=0$, or by (3) if $z_{t}(\omega)=z_{t}^{*}-g_{t}$.

A competitive equilibrium (CE) is $\left\{\left(c_{t}^{y}(\omega), c_{t}^{o}(\omega), s_{t}(\omega), b_{t}(\omega), z_{t}(\omega)\right) ; w_{t}, r_{t}\right\}_{t=0}^{\infty}$ if:
(i) For each date $t$ and each household $\omega$, given factor prices $\left(r_{t}, w_{t}\right)$ and the bequest $b_{t-1}(\omega),\left(c_{t}^{y}(\omega), c_{t}^{o}(\omega), s_{t}(\omega), b_{t}(\omega), z_{t}(\omega)\right)$ is the optimum under conditions (9)-(12).
(ii) Given the aggregate production function, the wage rates $w_{t}$ are determined by the marginal product of (effective) human capital.
(iii) The education policy $\left\{\left(X_{t}, g_{t}\right)\right\}_{t=0}^{\infty}$ is feasible, hence the government budget constraint in (5) holds at each date $t$.

After substituting the constraints, first order conditions that lead to the necessary and sufficient conditions for an optimum are (assuming interior solution):

$$
\begin{align*}
& \frac{c_{t}^{y}(\omega)}{y_{t+1}(\omega)}=\frac{\alpha_{1}}{\alpha_{3}} \frac{1}{\left(1+r_{t+1}\right)}  \tag{13}\\
& \frac{c_{t}^{y}(\omega)}{c_{t}^{o}(\omega)}=\frac{\alpha_{1}}{\alpha_{2}} \frac{1}{\left(1+r_{t+1}\right)} \tag{14}
\end{align*}
$$

From (12), (13) and (14):

$$
\begin{equation*}
y_{t+1}(\omega)=\frac{\alpha_{3}}{\alpha_{2}}\left(1+r_{t+1}\right) s_{t}(\omega) \tag{15}
\end{equation*}
$$

Using (15) and the definitions of income in (7) and (8), we obtain an expression for bequest if the offspring turns out to become low skilled:

$$
\begin{equation*}
b_{t}(\omega)=\frac{\alpha_{3}}{\alpha_{2}} s_{t}(\omega)-\frac{(1-\tau)\left[m w_{t}\left(1+r_{t+1}\right)+w_{t+1}\right]}{\left(1+r_{t+1}\right)} h_{t+1}(\omega) \geq 0 \tag{16}
\end{equation*}
$$

Likewise for a skilled offspring:

$$
\begin{equation*}
b_{t}(\omega)=\frac{\alpha_{3}}{\alpha_{2}} s_{t}(\omega)-\frac{(1-\tau) w_{t+1}}{\left(1+r_{t+1}\right)} h_{t+1}(\omega) \geq 0 \tag{17}
\end{equation*}
$$

Comparing (16) to (17) the incidence of $m$ in (16), other things equal, decreases the transfer of tangible assets across generations.

## 3. Equilibrium Sets of Skilled and Low-Skilled Workers

From the first order conditions, assuming interior solutions, for the optimization of parents in (13) and (14) we obtain $C_{t}^{y}(\omega)=\left(\alpha_{1} / \alpha_{2}\right) y_{t+1}(\omega) /\left(1+r_{t+1}\right)$ and $C_{t}^{0}=\left(\alpha_{2} / \alpha_{3}\right) y_{t+1}(\omega)$. After inserting these expressions into (9) the utility function has the following reduced-form:

$$
\begin{equation*}
U_{t}(\omega)=\Phi\left(\frac{1}{1+r_{t+1}}\right)^{\alpha_{1}}\left[y_{t+1}(\omega)\right]^{\alpha_{1}+\alpha_{2}+\alpha_{3}} \tag{18}
\end{equation*}
$$

where parameter $\Phi$ is a constant independent of time and independent of $\omega$. Therefore (18) is an expression for utility that holds for both skilled and low-skilled agents. The reduced form utility of parents is now proportional to the lifetime income of their offspring where the term of proportionality is decreasing in the world interest factor at the future date. Thus, if education resources are allocated by a utilitarian social planner that maximizes the current aggregate of individual utilities, it maximizes at the same time next generation's aggregate income. Also, when looking at a political equilibrium (like in Section 6) parents establish their preferences regarding the allocation of education resources by maximizing utility and their child's income at the same time. Lastly, whether parents invest in higher education depends very much on their utility, which only depends on a comparison of future lifetime income of their child.

Table 1: Cross-Country Variation of the Skilled Workforce ${ }^{\text {a,b }}$

| OECD <br> Countries | Age Group 25-64 with <br> at least Upper Secondary <br> Education | Partner <br> Countries | Age Group 25-64 with <br> at least Upper Secondary <br> Education |
| :--- | :---: | :--- | :---: |
| Italy | 52 | Brazil | 37 |
| Korea | 78 | Chile | 50 |
| Mexico | 33 | Estonia | 89 |
| Netherlands | 73 | Israel | 80 |
| Portugal | 27 | Russian Fed | 88 |
| Turkey | 29 | Slovenia | 82 |

Notes: (a) The skilled workforce is approximated by the percentage of the population of age group 25-64 with at least upper secondary education; (b) In percentage, in 2007.

Source: OECD (2009, Table A1.2A, column 1)

Making use of (18), the next result defines the proportion of the population that will receive higher education and will become skilled at the future date. It sheds some light into the observed cross-country variations in the skill composition of workforces in both developed and developing countries. For example, data in Table 1 show the skill composition of workforces for a subset of OECD countries and for OECD's partner countries. The extent of a skilled workforce is approximated by the share of age group 25-

64 with at least upper secondary education. Shares in 2007 vary largely, between 29 percent in Turkey and 89 percent in Estonia.

Proposition 1: Let $A_{t}$ denotes the set of individuals who receive higher education at date $t$. Then: (a) $A_{t}$ is nonempty if and only if :

$$
\begin{equation*}
\frac{w_{t+1}}{1+r_{t+1}} \geq \frac{m}{B-1} w_{t} \tag{19}
\end{equation*}
$$

(b) Assume that $A_{t}$ is nonempty. Define, $\Lambda_{t}=\frac{1}{1-\tau}\left[\frac{1}{(B-1) \frac{w_{t+1}}{1+r_{t+1}}-m w_{t}}\right]\left(\frac{z_{t}}{X_{t}^{\xi}}\right)$, then:

$$
\begin{equation*}
A_{q}=\left\{\omega \mid \quad \tilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v} \geq \Lambda_{t}\right\} \tag{20}
\end{equation*}
$$

Namely, all individuals in this set $A_{t}$ become skilled workers at date $t+1$.

Proof: Consider the case where the child is skilled (denoted by superscript s). Substitute first order conditions in (11) and solve for $b_{t}(\omega)$. Making use of (17) we are able to solve for $y_{t+1}^{s}(\omega)$. Repeat the same steps for the case where the same child is low skilled (superscript $l$ ) to derive $y_{t+1}^{l}(\omega)$. Hence,

$$
y_{t+1}^{s}(\omega) \geq y_{t+1}^{l}(\omega) \Leftrightarrow U_{t}^{s}(\omega) \geq U_{t}^{l}(\omega)
$$

implies:

$$
(1-\tau) B \tilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v} X_{t}^{\xi} w_{t+1}-z_{t} \geq(1-\tau) \tilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v} X_{t}^{\xi}\left[w_{t+1}+\left(1+r_{t+1}\right) m w_{t}\right]
$$

Note that this inequality holds only if condition (19) holds. Moreover, it is easy to verify that when (19) holds the set of skilled individuals is given by (20).

Let us call $Z_{t+1}(\omega)=\tilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v}$ the initial endowment of $\omega$. Under the assumption that (19) holds, all $\omega \in G_{t+1}$ with an initial endowment larger than $\Lambda_{t}$ will invest in higher education and become skilled whereas individuals with an initial endowment lower than $\Lambda_{t}$ will not invest and, hence, become unskilled. Consider now the extreme case of fullpublic funding of higher education, namely, $\hat{g}_{t}=z_{t}^{*}$ holds for all t . Denote by $\hat{H}_{t}$ the corresponding stocks of human capital and $\hat{X}_{t}=\tau w_{t} \hat{H}_{t}-z_{t}^{*}$. In this case, $z_{t}=0$ for all, hence individual $\omega$ invests in higher education if and only if:

$$
(1-\tau) B \tilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v} \hat{X}_{t}^{\xi} w_{t+1} \geq(1-\tau) \tilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v} \hat{X}_{t}^{\xi}\left[w_{t+1}+\left(1+r_{t+1}\right) m w_{t}\right]
$$

which holds if and only if condition (19) is satisfied. Thus under full public funding inequality (19) implies that all young individuals become skilled, regardless of their initial endowments! Clearly, if the inequality in (19) is reversed all individuals will become lowskilled. Thus the exogenous factor prices play an important role in the formation of types of workers under full public funding of higher education. Since our analysis is relevant mainly when the higher education system operates, hence skilled individuals exist in each generation, we assume:
(A2) Given the exogenous wages and interest rates, the parameters $m$ and $B$, condition (19) holds at all dates $t, t=0,1,2, \ldots .$.

Some monotonicity results that can be verified from condition (20) are reported in Table 2 and should be interpreted as follows. Suppose that at date $t$ an increase occurs in one of the parameters in the first row, then the impact of this change on $\Lambda_{t}$ and $A_{t}$ is given in Table 2:

Table 2: Monotonicity Results for $\Lambda_{t}$ and $A_{t}$

|  | $w_{t+1} /\left(1+r_{t+1}\right)$ | $w_{t}$ | $X_{t}$ | $z_{t}$ | $g_{t}$ | $\tau$ | $B$ | $m$ | $\xi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Lambda_{t}$ | - | + | - | + | - | + | - | + | - |
| $A_{t}$ | + | - | + | - | + | - | + | - | + |

Table 2 offers an interpretation as to how globalization affects the process of skill formation. Upon capital market integration, physical capital flows from the low-return to the high-return country. Once the small open economy removes all capital controls physical capital will flow in if the economy is relatively less endowed in physical capital. As the marginal return decreases to the world interest rate the economy will experience an expansion of its skilled workforce. In contrast if the open economy has initially high levels of capital then capital market integration will bring about an increase in its unskilled workforce. Summarizing:

Proposition 2: In the above framework in equilibrium we obtain that: a higher wagerental ratio $w_{t+1} /\left(1+r_{t+1}\right)$ at a given date expands the set of skilled agents at that date, while a lower wage-rental ratio enlarges the set of low-skilled labor.

Proof: Let us rewrite the condition that defines the set of individuals $\omega \in G_{t+1}$ that choose to assume higher education:

$$
Z_{t+1}(\omega)=\tilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v} \geq \frac{1}{1-\tau}\left[\frac{1}{(B-1) \frac{w_{t+1}}{1+r_{t+1}}-m w_{t}}\right] \frac{z_{t}}{X_{t}^{\xi}}=\Lambda_{t}
$$

Assume that in date $t$ we have a higher interest rate $\left(1+r_{t+1}\right)$; this implies a lower wage-rental ratio $w_{t+1} /\left(1+r_{t+1}\right)$. As a result, note that condition (19) remains valid, examining the definition of $A_{t}$ we find that the value of $\Lambda_{t}$ increases since the
private investment $z_{t}$ and public investment in compulsory schooling $X_{t}{ }^{\xi}$ remain unchanged. Hence the set of skilled agents $A_{t}$ shrinks. Similarly, lowering the rate of interest will lower $\Lambda_{t}$, hence expanding the set of skilled workers $A_{t}$.

Corollary 1: The allocation of individuals at generation $t$ between the groups of skilled and low-skilled workers does not depend on the intensity of altruism $\alpha_{3}$. Likewise, the stock of human capital $H_{t}$ is independent of the altruism parameter.

Thus, in our model the intensity of altruism does not affect growth, as long as $\alpha_{3}>0$. This result is in contrast to the result obtained in dynastic models like in Armellini and Basu (2009).

Table 2 also identifies other model parameters that affect the set of skilled workers $A_{t}$. Among these, parameter $m$ is important since together with $w_{t}$ it represents lost earnings while studying and captures therefore the opportunity cost of higher education. Data in Table 3 provides estimates of the maximum value of $m$ as the difference between the average graduation date and the ending age of compulsory schooling relative to the number of years in a generation. Taking the ending age of compulsory schooling of Table 3 and assuming further that the average graduation age is 24 , while the first generation is 24 years long, we obtain estimates of $m$ in Table 3. As parameter $m$ is inversely related to the ending age of compulsory schooling it is determined largely by institutions. Data reveal that countries differ in their policies regarding the opportunity cost of higher education.

Table 3: Estimates of Parameter $m^{\text {a,b }}$

| OECD <br> Countries | Ending Age <br> of <br> Compulsory <br> Schooling | $m$ | Partner <br> Countries | Ending Age <br> of <br> Compulsory <br> Schooling | $m$ |
| :--- | :---: | :---: | :--- | :---: | :---: |
| Italy | 15 | 0.375 | Brazil | 14 | 0.417 |
| Korea | 14 | 0.417 | Chile | 18 | 0.250 |
| Mexico | 15 | 0.375 | Estonia | 15 | 0.375 |
| Netherlands | 18 | 0.250 | Israel | 15 | 0.375 |
| Portugal | 14 | 0.417 | Russian Fed | 15 | 0.375 |
| Turkey | 14 | 0.417 | Slovenia | 14 | 0.417 |

Notes: (a) Parameter $m$ is computed as the difference between 24 (the average graduation age) and the ending age of compulsory schooling divided by 24 (the number of years in the first generation); (b) In 2006, with no change in 2007.

Source: Authors' own computations and OECD (2009, Table C1.1).

## 4. Implications for Growth

From the analysis thus far, another important question arises: when does the expansion of the set of skilled workers lead to a higher stock of human capital that is available for production activities? It all depends on the causes of this expansion since variables and parameters of the model have a different status. For example, $w_{t}, m, \tau, g_{t}, X_{t}$ interfere directly with the government budget balance while the world factor prices $\left(w_{t+1} / 1+r_{t+1}\right)$ and technology parameters $(B, \xi)$ are exogenous to budget balance. The next proposition applies only to the former predetermined variables:

Proposition 3: Assume that $B>1+m$ holds. Output declines at the current date $t$ but expands in all periods $t+k, k \geq 1$ in each of the following cases: (a) an unexpected increase in the wage-rental ratio at date $t$; (b) a technological progress in the education sector (higher B or $\xi$ ) as of date $t$.

The proof is based on the result of Proposition 2 and on the next lemma.

Lemma 1: Assume that $B>1+m$. Expanding the set $A_{t}$ at date $t$ results in a lower $H_{t}$ but a higher $H_{t+k}, k \geq 1$.

Proof: Recall the definition of the stock of human capital at date $t$ :

$$
H_{t}=\int h_{t}(\omega) d \mu(\omega)+m \int_{\sim A_{t}} h_{t+1}(\omega) d \mu(\omega)
$$

As $A_{t}$ increases, the first term in this expression remains unchanged while the second decreases. Hence $H_{t}$ drops. Consider now later periods:

$$
\begin{aligned}
& H_{t+1}=\int h_{t+1}(\omega) d \mu(\omega)+m \int_{\sim A_{t+1}} h_{t+2}(\omega) d \mu(\omega) \\
& H_{t+1}=\int_{A_{t}} h_{t+1}(\omega) d \mu(\omega)+\int_{\sim A_{t}} h_{t+1}(\omega) d \mu(\omega)+m \int_{\sim A_{t+1}} h_{t+2}(\omega) d \mu(\omega)
\end{aligned}
$$

There are two effects. First, low-skilled workers join the skilled workforce: $A_{t}$ increases but $\sim A_{t}$ decreases by the same number. Second, low-skilled workers induce their child to be low-skilled workers as well but at date $t+1$ because of the endowment condition: $\sim A_{t+1}$ decreases (hence $A_{t+1}$ expands). Consider now two situations and denote the corresponding sets of skilled workers by: $A_{t}^{1}$ and $A_{t}^{0}$ with $\mu\left(A_{t}^{1}\right)>\mu\left(A_{t}^{0}\right)$. Since we transfer unskilled workers to skilled ones we obtain that $\int h_{t+1}(\omega) d \mu(\omega)$ increases. On the other hand, since $A_{t+1}$ expands we obtain that $\int h_{t+2}(\omega) d \mu(\omega)$ increases. Let us write:

$$
\begin{aligned}
& H_{t+1}^{0}=\int h_{t+1}^{0}(\omega) d \mu(\omega)+m \int_{\sim A_{t+1}^{0}} h_{t+2}^{0}(\omega) d \mu(\omega) \\
& H_{t+1}^{1}=\int h_{t+1}^{1}(\omega) d \mu(\omega)+m \int_{\sim A_{t+1}^{1}}^{h_{t+2}^{1}}(\omega) d \mu(\omega)
\end{aligned}
$$

Let us denote by $\Delta_{t}=\left[\sim A_{t}^{1}\right] /\left[\sim A_{t}^{0}\right]$, then for any $\omega \in \Delta_{t+1}$ we have by our assumptions: $h_{t+1}^{1}(\omega) \geq B h_{t+1}^{0}(\omega)$, hence

$$
\int_{\Delta_{t+1}} h_{t+1}^{1}(\omega) d \mu(\omega) \geq B \int_{\Delta_{t+1}} h_{t+1}^{0}(\omega) d \mu(\omega)>(1+m) \int_{\Delta_{t+1}} h_{t+1}^{0}(\omega) d \mu(\omega)
$$

This implies that $H_{t+1}^{1}-H_{t+1}^{0} \geq m \int_{\Delta_{t+1}} h_{t+1}^{0}(\omega) d \mu(\omega)$. This process can be continued for all coming dates since we obtained that $A_{t+1}^{0}$ also expands. Thus our claim is proved.

Data in Table 4 illustrate the sufficient condition of Lemma 1. Parameter B represents the productivity of higher education in generating human capital and depends therefore on the relative quality of higher education. In Table 4 it is assumed that this relative quality can be measured by taking the ratio of per-student annual expenditure on tertiary education to the per-student annual expenditure on secondary education. Using information on $m$ from Table 3, columns 3 and 6 of Table 4 obtain the values of $B-(1+m)$. A striking conclusion from Table 4 is that this condition is more easily satisfied for developing countries than for emerging and developed nations as their estimate of $B$ is higher.

Table 4: Sufficient Condition of Lemma $1^{\text {a,b }}$

| OECD <br> Countries | $B$ | $B-(1+m)$ | Partner <br> Countries | $B$ | $B-(1+m)$ |
| :--- | :---: | ---: | :--- | :---: | ---: |
| Italy | 1.027 | -0.348 | Brazil | 6.693 | 5.276 |
| Korea | 1.179 | -0.238 | Chile | 3.010 | 1.760 |
| Mexico | 2.985 | 1.610 | Estonia | 0.932 | -0.443 |
| Netherlands | 1.597 | 0.347 | Israel | 1.900 | 0.525 |
| Portugal | 1.420 | 0.003 | Russian Fed | 1.784 | 0.409 |
| Turkey | 2.534 | 1.117 | Slovenia | 1.063 | -0.354 |

Notes: (a) Parameter $B$ is obtained by dividing the per-student annual expenditure on tertiary education by the per-student annual expenditure on secondary education; (b) reference year for $B$ is 2006.
Source: Authors' own calculations using $m$ in Table 3 and OECD (2009,
Table B1.1a)

## 5. Public Funding of Higher Education and Human Capital

On the expenditure side of its balance sheet the government faces public expenditure in higher education equal to $\mu\left(A_{t}\right) g_{t}$. Enrollment in higher education is costly and requires a net payment from private sources equal to $\mu\left(A_{t}\right) z_{t}^{*}-\mu\left(A_{t}\right) g_{t}$. It is easy to verify that $\left(1-g_{t} / z_{t}^{*}\right)$ represents the share of private investment in total expenditure on higher education. A decision by schools to charge a higher tuition fee $z_{t}^{*}$ increases this share while a public subsidy will decrease it. Data in Table 5 reveal that in tertiary education the proportion funded privately varies widely across our sample of countries. In Chile and Korea for example, public funding represents only a small part of investments in tertiary education. In contrast in the Netherlands, approximately 73 percent of expenditure on higher education is public. These stylized facts raise the main issue of this section: what is the role of a government subsidy in enhancing higher education? We will focus on the impact of public funding of higher education on the aggregate stock of human capital. The impact effect at current date $t$ will be analyzed first, after which we focus on the dynamic analysis.

Table 5: Private Funding of Tertiary Education ${ }^{\text {a,b,c }}$

| OECD <br> Countries | Private <br> Funding <br> $1-g_{t} / z_{t}^{*}$ | Partner <br> Countries | Private <br> Funding <br> $1-g_{t} / z_{t}^{*}$ |
| :--- | :---: | :--- | :---: |
| Italy | 27.0 | Brazil | - |
| Korea | 76.9 | Chile | 83.9 |
| Mexico | 32.1 | Estonia | 26.9 |
| Netherlands | 26.6 | Israel | 49.9 |
| Portugal | 32.3 | Russian Fed | - |
| Turkey | - | Slovenia | 33.1 |

Notes: (a) Private funding of tertiary education as a percentage of total tertiary expenditure; (b) In 2006; (c) "-" indicates not available.

Source: OECD (2009, Table B3.2b)

It is crucial to be more precise regarding the response of the threshold parameter $\Lambda_{t}$ to a subsidy because it defines the skilled labor set, noting that the government budget must be balanced in equilibrium. For that it is important to obtain the response of $z_{t} / X_{t}^{\xi}$ to this subsidy. Note that the left-hand side of (5) is simply $\tau w_{t} H_{t}$, a useful shorthand expression for government tax revenues. Denote by $\gamma_{t}, 0 \leq \gamma_{t} \leq 1$, the fraction of government revenues allocated to compulsory schooling, thus:

$$
\begin{align*}
& X_{t}=\gamma_{t} \tau w_{t} H_{t}  \tag{21}\\
& g_{t} \mu\left(A_{t}\right)=\left(1-\gamma_{t}\right) \tau w_{t} H_{t} \tag{22}
\end{align*}
$$

Hence, when $\gamma_{t}=1$ education subsidies to higher education are zero $\left(g_{t}=0\right)$, hence the tertiary education is fully privately financed. With $g_{t}=z_{t}^{*}$, higher education is fully publicly financed. Using the above equations, we can write that:

$$
\begin{equation*}
\frac{z_{t}}{x_{t}^{\xi}}=\frac{z_{t}^{*}-\left(1-\gamma_{t}\right) \tau w_{t} H_{t} / \mu\left(A_{t}\right)}{\left(\tau \gamma_{t} w_{t} H_{t}\right)^{\xi}} \tag{23}
\end{equation*}
$$

To obtain the effect of higher expenditure in compulsory schooling in equilibrium, we derive from this expression (using some earlier conditions):

$$
\begin{equation*}
\frac{\partial\left(z_{t} / X_{t}^{\xi}\right)}{\partial \gamma_{t}}=\frac{\xi}{\gamma_{t}^{1+\xi}\left(\tau w_{t} H_{t}\right)^{\xi}}\left\{\frac{1}{\xi \mu\left(A_{t}\right)} X_{t}-z_{t}\right\}>0 \leftrightarrow X_{t} / \mu\left(A_{t}\right) z_{t}>\xi \tag{24a}
\end{equation*}
$$

Namely, the partial derivative is positive as long as $X_{t} / \mu\left(A_{t}\right) z_{t}>\xi$. This condition holds in most countries since (i) per-student public expenditure on compulsory schooling $X_{t}$ is higher than per-student private expenditure on higher education $z_{t}$, and (ii) $\mu\left(A_{t}\right)<1$ (less than 0.5 in many economies) and $\xi<1$. Using these observations we obtain a positive effect of increasing compulsory schooling expenditures on threshold parameter $\Lambda_{t}$ when the government budget is balanced:

$$
\begin{equation*}
\frac{\delta\left(\Lambda_{t}\right)}{\delta \gamma_{t}}>0 \tag{24b}
\end{equation*}
$$

Vice versa: an increase in the public funding to higher education (a decrease in $\gamma_{t}$ ) leads to a decrease in the threshold level:

Proposition 4: Assume that $X_{t} / z_{t}^{*} \mu\left(A_{t}\right)>\xi$ holds at some period $t$. Increasing the public funding $g_{t}$ leads in equilibrium to: (i) a larger set of skilled agents at date $t+1$; (ii) a lower stock of human capital $H_{t}$ used in production at date $t$ and; (iii) lower total expenditure on education at date $t$. Also, the marginal rate of substitution between investing in basic education and investing in higher education is larger than unity.

The above condition requires that the ratio of total expenditure on basic schooling to total spending on higher education is bounded from below by $\xi<1$.

Proof: For some $t$ assume that $g_{t}$ increases. Let us rewrite (5) as follows:

$$
\begin{equation*}
\tau w_{t}\left[\int h_{t}(\omega) d \mu(\omega)+m X_{t}^{\xi} \bar{\theta} \int_{\sim A_{t}} h_{t}(\omega)^{v} d \mu(\omega)\right]=X_{t}+g_{t} \mu\left(A_{t}\right) \tag{5’}
\end{equation*}
$$

since $\widetilde{\theta}_{t}(\omega)$ are i.i.d. Any increase in $g_{t}$ decreases parameter $\gamma_{t}$. By (20), as $g_{t}$ expands, $z_{t} / X_{t}{ }^{\xi}$ decreases, which clearly implies a decrease in $X_{t}$. Since $\Lambda_{t}$ declines we obtain that the set $A_{t}$ expands. From (5') we see that $H_{t}$ decreases, hence the RHS $X_{t}+g_{t} \mu\left(A_{t}\right)$ must decrease as well even though $g_{t} \mu\left(A_{t}\right)$ increases. Thus, total expenditures on education decrease. The drop in $X_{t}$ is larger than the initial increase in $g_{t}$ : the marginal rate of substitution between $X_{t}$ and $g_{t}$ is therefore larger than 1 in absolute value.

Proposition 4 shows that in equilibrium with balanced budget the opportunity cost of increasing resources in favour of higher education is larger than unity. The reason is
that some unskilled workers that previously contributed to tax revenues now become users of higher education subsidies and become skilled. ${ }^{6}$

Now let us consider the effect of increasing the public funding to higher education to enhance the formation of skilled labor (along a feasible education program). Consider the case where the government proposes two policies: either 'no public funding for higher education', i.e., $g_{t}=0$, or the policy $\left\{\bar{g}_{t}\right\}_{t=0}^{\infty}$ which indicates at each date t a 'per-student funding at a given level $\bar{g}_{\mathrm{t}}{ }^{\prime}$. At date $t$, let the set of families who opt for a 'skilled child' under the 'no funding' policy be defined by:

$$
\begin{equation*}
A_{t}^{0}=\left\{\omega \left\lvert\, \tilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v} \geq \frac{1}{(1-\tau)(B-1)}\left\{\frac{1}{\left[\frac{w_{t+1}}{1+r_{t+1}}-\frac{m}{B-1} w_{t}\right]} \frac{z_{t}^{*}}{\left[\tau w_{t} H_{t}\right]^{\xi}}\right\}=\Lambda_{t}^{0}\right.\right\} \tag{25}
\end{equation*}
$$

Let us denote the set of families at period $t$ who opt for a 'skilled child' under the 'perstudent public funding $\bar{g}_{t}{ }^{\prime}$ by:

$$
\begin{equation*}
\bar{A}_{t}=\left\{\omega \left\lvert\, \tilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v} \geq \frac{1}{(1-\tau)(B-1)}\left\{\frac{1}{\left[\frac{w_{t+1}}{1+r_{t+1}}-\frac{m}{(B-1)} w_{t}\right]} \frac{z_{t}^{*}-\bar{g}_{t}}{\bar{X}_{t}^{\xi}}\right\}=\bar{\Lambda}_{t}\right.\right\} \tag{26}
\end{equation*}
$$

We shall make in the following proposition an assmption regarding the 'sensitivity' of the set of skilled labor to changes in 'initial endowment', namely to variations in the threshold level $\Lambda_{t}$. Let us rewrite the aggregate human capital of generation $t+1$ :

$$
\begin{equation*}
\hat{H}_{t+1}=\int h_{t+1}(\omega) d \mu(\omega)=X_{t}^{\xi}\left[B \int_{A_{t}} \theta_{t+1}(\omega) h_{t}(\omega)^{v} d \mu(\omega)+\int_{\sim A_{t}} \theta_{t+1}(\omega) h_{t}(\omega)^{v} d \mu(\omega)\right] \tag{27}
\end{equation*}
$$

Does a certain level of public funding of higher education enhance the formation of human capital, and hence growth in our economy? The literature has some support for this claim [see, e.g., Bassanini and Scarpenta, 2001; Caucutt and Krishna, 2003;

[^3]Blankenau, 2005; Arcalean and Schiopu, 2008]. We show that in our framework such result depend on certain parameter values:

Proposition 5: Assume that initially there is no government intervention in financing higher education. Introducing a public funding for higher education at the level $\bar{g}_{t}$ varies the corresponding threshold levels from $\left\{\Lambda_{t}^{0}\right\}$ to $\left\{\bar{\Lambda}_{t}\right\}$. Define:

$$
\begin{equation*}
\bar{\Lambda}_{t}=\Lambda_{t}^{0}\left(1-d_{t}\right), \text { for } t=0,1,2, \ldots \tag{28}
\end{equation*}
$$

If $d_{t} \leq \bar{g}_{t} / z_{t}^{*}$ holds for all $t$, then introducing this public funding for higher education policy increases the aggregate human capital of each generation, namely, $\hat{H}_{t}^{0}<\bar{H}_{t}$ holds for all $t$.

Note that $\bar{g}_{t} / z_{t}{ }^{*}$ is the share of the public funding in the total cost of higher education $z_{t}^{*}$. Thus, if the sensitivity of the threshold level to variations in the funding level is not 'too high', hence the resulting expansion of the set of skilled agents $A_{t}$ is not 'too rapid', we obtain that higher public funding will enhance the creation of human capital. This condition depends basically on the distribution of the initial endowments $Z_{0}(\omega)$ as well as the 'smoothness' of the human capital distributions in equilibrium and the density function of the random ability distribution. Clearly, the public investments in compulsory education over time matters as well. The condition assumed in Proposition 4 compares the per-student investment in compulsory schooling with the average cost of higher education at some given date. In Proposition 5 condition (28) makes an assumption about the elasticity of the threshold levels for different levels of public funding.

Proof: Write: $z_{t}^{*}=z_{t}^{0}$ and hence, $\bar{z}_{t}=z_{t}^{*}-\bar{g}_{t}$. Thus:

$$
\frac{\overline{\bar{z}}_{t}}{\left(\bar{X}_{t}\right)^{\xi}}=\frac{z_{t}^{0}}{\left(\bar{X}_{t}\right)^{\xi}}-\frac{\bar{g}_{t}}{\left(\bar{X}_{t}\right)^{\xi}}=\frac{z_{t}^{*}}{\left(X_{t}\right)^{\xi}}\left(1-d_{t}\right)
$$

We obtain from this equation,

$$
\frac{z_{t}^{*}}{\left(X_{t}\right)^{\xi}}\left[\left(\frac{X_{t}}{\bar{X}_{t}}\right)^{\xi}-1+d_{t}\right]=\frac{\bar{g}_{t}}{\left(\bar{X}_{t}\right)^{\xi}}
$$

which yields:

$$
\left(\frac{X_{t}}{\bar{X}_{t}}\right)^{\xi}=\frac{1-d_{t}}{1-\bar{g}_{t} / z_{t}^{*}}
$$

Now, let us define $Q(\bar{g})=B \int_{A_{t}} h_{t}^{\nu}(\omega) d \mu(\omega)+\int_{\sim \mathcal{A}_{t}} h_{t}^{\nu}(\omega) d \mu(\omega)$ and write the expressions for the ratio of generational aggregate human capital:

$$
\begin{equation*}
\frac{\bar{H}_{t+1}}{\widehat{H}_{t+1}^{0}}=\left(\frac{X_{t}}{\bar{X}_{t}}\right)^{\xi} \frac{\bar{\theta} Q\left(\bar{g}_{t}\right)}{\bar{\theta} Q(0)}=\frac{1-d_{t}}{1-\bar{g}_{t} / z_{t}^{*}} \frac{Q\left(\bar{g}_{t}\right)}{Q(0)} \tag{29}
\end{equation*}
$$

Since $\bar{\Lambda}_{t}<\Lambda_{t}$ the set of skilled with the subsidy contains (strictly) the set under 0 subsidy, namely: $A_{t} \subset \bar{A}_{\tau}$, hence $Q\left(\bar{g}_{t}\right)>Q(0)$. Thus, by our assumption, using (29) we obtain that $H^{0}{ }_{t+1}<\bar{H}_{t+1}$ for all t.

## 6. Political Equilibrium

So far it is assumed that the allocation of the public education funds (hence, $\gamma_{t}$ ) is exogenously given. This assumption is questionable since the allocation of government revenues between these two types of education is likely to vary with the educational technology of early education vs. college education, market conditions at home and abroad, etc. Moreover, Table 6 that compares the shares of public expenditure on tertiary education (as a percentage of total public expenditure) reveals a large diversity in education systems. While the largest share is observed for Turkey,
that of Korea and Chile is twice as low. Clearly the latter countries rely heavily on private funding to finance higher education.

Table 6: Public Expenditure on Tertiary Education ${ }^{\text {a,b }}$

| OECD Countries | $\left(1-\gamma_{t}\right)$ | Partner Countries | $\left(1-\gamma_{t}\right)$ |
| :--- | ---: | :--- | ---: |
| Italy | 16.84 | Brazil | 16.67 |
| Korea | 14.67 | Chile | 15.06 |
| Mexico | 17.27 | Estonia | 19.44 |
| Netherlands | 27.50 | Israel | 16.79 |
| Portugal | 19.46 | Russian Fed | 22.14 |
| Turkey | 31.03 | Slovenia | 21.71 |

Notes: (a) As a percentage of total public expenditure on education; (b) In 2007.
Source: Authors' own calculations based on OECD (2009, Table B4.1)

In economies with heterogeneous agents, the choice of an 'optimal' $\gamma_{t}$ can become the outcome of some political process at each date. It is possible to establish a mapping between the set of heterogeneous agents, given their preferences regarding education, and an 'optimal' education policy determined by majority voting. Economies at different stages of development with a different composition of their labor force between skilled and low-skilled workers are then expected to reach different political equilibria.

## Preferences of agents

As we have observed earlier in (18), maximization of utility by an agent is equivalent to the maximization of his/her offspring's income. Let us therefore express individual income as a function of $\gamma_{t}$ by substituting away $X_{t}$ and $g_{t} \mu\left(A_{t}\right)$. Making use of (7), (8), (16) and (17) income of agent $\omega$ who has either a low-skilled or a skilled offspring is:

$$
\begin{aligned}
& y_{t+1}^{l}(\omega)=\left(\frac{\alpha_{3}}{\alpha_{1}+\alpha_{2}+\alpha_{3}}\right)\left(1+r_{t+1}\right)\left\{\frac{(1-\tau)\left(w_{t+1}+m w_{t}\left(1+r_{t+1}\right)\right.}{\left(1+r_{t+1}\right)} \widetilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v} \gamma_{t}^{\xi} \tau^{\xi} w_{t}^{\xi} H_{t}^{\xi}+y_{t}(\omega)\right\} \\
& y_{t+1}^{s}(\omega)=\left(\frac{\alpha_{3}}{\alpha_{1}+\alpha_{2}+\alpha_{3}}\right)\left(1+r_{t+1}\left\{\left\{\frac{(1-\tau) w_{t+1}}{\left(1+r_{t+1}\right)} B \tilde{\theta}_{t+1}(\omega) h_{t}(\omega)^{v} \gamma_{t}^{\xi} \tau^{\xi} w_{t}^{\xi} H_{t}^{\xi}+y_{t}(\omega)-z^{*}+\frac{\left(1-\gamma_{t}\right)}{\mu\left(A_{t}\right)} \tau w_{t} H_{t}\right\}\right.\right.
\end{aligned}
$$

Given the parameters at each date $t$, including $H_{t}$ and $y_{t}(\omega)$ both expressions for next generation income are strictly concave function of $\gamma_{t} \in[0,1]$. This implies that the optimal choice $\gamma_{t}(\omega)$ of each agent is unique.

Assume now that each individual votes for either 'no public funding' or for 'public funding at level $g=\bar{g}_{t}$ per-student'. The choice will be determined by comparing the income of his/her offspring under these two policies; namely, given $Z_{t+1}(\omega)$ we compare $y_{t+1}^{l}(\omega)$ under $g_{t}=0$ with $y_{t+1}^{s}(\omega)$ under $g_{t}=\bar{g}_{t}$. Denote by $\bar{\gamma}_{t}$ the fraction of the education budget assigned for compulsory schooling when higher education is publicly funded at $g_{t}=\bar{g}_{t}$. The condition that determines voting in support of $g_{t}=\bar{g}_{t}$ per-student is given by:

$$
\begin{aligned}
& \left.\frac{(1-\tau) w_{t+1}}{1+r_{t+1}} B Z_{t+1}(\omega)\left[\tau w_{t} H_{t} \bar{\gamma}_{t}\right)\right]^{\xi}-z_{t}^{*}+\bar{g}_{t}+y_{t}(\omega) \geq \\
& {\left[\frac{1+r_{t+1}}{1-\tau}\right]^{-1}\left[w_{t+1}+m w_{t}\left(1+r_{t+1}\right] Z_{t+1}(\omega)\left[\tau w_{t} H_{t}\right]^{\xi}+y_{t}(\omega)\right.}
\end{aligned}
$$

Rearrangement results in $Z_{t+1}(\omega) \geq v_{t}$, where:

$$
v_{t}=\frac{\left(z_{t}^{*}-\bar{g}_{t}\right) \frac{1}{1-\tau}\left[\tau w_{t} H_{t}\right]^{-\xi}}{\left[B \frac{w_{t+1}}{1+r_{t+1}}\left(\bar{\gamma}_{t}\right)^{\xi}-m w_{t}\right]-\frac{w_{t+1}}{1+r_{t+1}}}
$$

Expressed differently:

$$
\begin{equation*}
v_{t}=\frac{\left(z_{t}^{*}-\bar{g}_{t}\right)\left[\tau w_{t} H_{t}\right]^{-\xi}}{(1-\tau)}\left[\left(B\left(\bar{\gamma}_{t}\right)^{\xi}-1\right) \frac{w_{t+1}}{1+r_{t+1}}-m w_{t}\right]^{-1} \tag{30}
\end{equation*}
$$

This defines the set of voters who support the suggested public funding for higher education. Namely, all voters whose endowment is such that $Z_{t+1}(\omega) \geq v_{t}$ will vote in favour of public funding, all others will vote against. Thus, the threshold $v_{t}$ partitions the distribution of endowments between those who favour public funding for higher education at level $\mathrm{g}=\bar{g}_{t}$ versus the alternative policy $g_{t}=0$.

It is interesting to report the following properties, indicated from expression (30): (i) a lower wage/rental ratio at the next period (resulting from globalization and liberalization of capital markets) implies an increase in $v_{t}$ and a smaller group of individuals who support $g=\bar{g}_{t}$; (ii) In a society endowed with a larger stock of human capital $H_{t}$ more people support a higher level of public resources allocated for higher education relative to a country with lower $H_{t}$; (iii) As public education expenditures $\left(\tau w_{t} H_{t}\right)$ increase more individuals support an increase in resources for higher education; (iv) A higher value of $m$ or lower value of $\xi$ imply less support for the policy $g=\bar{g}_{t} ;(\mathrm{v})$ The result of Corollary 1 still holds in this case since $v_{t}$ does not depend on the intensity of altruism.

Following the determination of $v_{t}$ let us now characterize the voting behaviour of individuals in generation $t$ using the two claims below:

Claim 1: $v_{t}>\bar{\Lambda}_{t}$.

Proof: Let us rewrite the expression for $v_{t}$ as follows:

$$
\begin{equation*}
v_{t}=\frac{\left(z_{t}^{*}-\bar{g}_{t}\right)\left[\bar{\gamma} \tau w_{t} H_{t}\right]^{-\xi}}{(1-\tau)}\left[\left(B\left(\bar{\gamma}_{t}\right)^{\xi}-\left(\bar{\gamma}_{t}\right)^{-\xi}\right) \frac{w_{t+1}}{1+r_{t+1}}-\left(\bar{\gamma}_{t}\right)^{-\xi} m w_{t}\right]^{-1} \tag{31}
\end{equation*}
$$

From (29) and (31) we see easily that $v_{t}>\bar{\Lambda}_{t}$ holds if and only if :

$$
\left[\left(B-\left(\bar{\gamma}_{t}\right)^{-\xi}\right) \frac{w_{t+1}}{1+r_{t+1}}-\left(\bar{\gamma}_{t}\right)^{-\xi} m w_{t}\right]<(B-1) \frac{w_{t+1}}{1+r_{t+1}}-m w_{t}
$$

which holds since $\left(\bar{\gamma}_{t}\right)^{-\xi}>1$.

Claim 2: $\bar{\Lambda}_{t}<\Lambda_{t}^{0}$ holds for all $t$.

Proof: To prove this claim let us define: $h_{t}(y)=\frac{z_{t}^{*}-y}{(A-y)^{\xi}}$ where the positive constant A is $\tau w_{t} H_{t}$. By straightforward calculation we verify that $h^{\prime}(y)<0$ since $\frac{z^{*}-y}{A-y}<1$ and $\xi<1$. Thus:

$$
\frac{z_{t}^{*}}{\left(\tau w_{t} H_{t}\right)^{\xi}}>\frac{z_{t}^{*}-\bar{g}_{t}}{\left(\tau w_{t} H_{t}-\bar{g}_{t}\right)^{\xi}}>\frac{z_{t}^{*}-\bar{g}_{t}}{\left[\tau w_{t} H_{t}-\bar{g}_{t} \mu\left(\bar{A}_{t}\right)\right]^{\xi}}
$$

The following corollaries follow directly from Claim 1 and Claim 2.

Corollary 2: Some of the agents who voted against instituting public funding of higher education will invest in higher education when public funding exists.

Corollary 3: Some of the families who did not invest in higher education under the no-public funding regime will invest in higher education when public funding is introduced.

## Majority Voting

In order to reach a political equilibrium, what matters is to know the relative position of the median voter in the distribution of initial endowments. Let ' $M$ ' denote the median voter and let $Z_{t+1}(M)=\tilde{\theta}_{t+1}(M) h_{t}(M)^{v}$ be his/her initial endowment. From the above discussion we can summarize our results:

Proposition 6: When the allocation of resources invested in public education is determined by a political equilibrium, applying the median voter theorem implies that public funding is approved, i.e., $g_{t}=\bar{g}_{t}$ if and only if $Z_{t+1}(M) \geq v_{t}$. Thus the shape of the distribution of endowments matters in this case for the determination of the equilibrium.

Thus we obtain that in a society with a majority of low-skilled workers with low endowments the median voter is in favour of not allocating public resources to college education (Su, 2004; Blankenau et al., 2007). This result is clear since parents of generation $t$ knowing that their child is becoming a low-skilled worker will not benefit from supporting any level of public funding for higher education. They perceive public funds assigned for higher education as a net transfer of government resources from them to individuals who shall mostly have high income in the future. In richer economies with a majority of skilled workers the allocation of resources depends on parameters of the model.

## The Possibility of Inefficiency of Public Funding

Suppose that in a society with a majority of low-skilled workers the political equilibrium implies that no public resource is allocated to college education. Is this situation desirable or is it always the case that public funds be used to subsidize higher education? The answer may depend on the underlying features of the economy, such as wages (affected by international markets), the productivity and costs of the higher education.

To show this, consider some initial conditions of this economy and a feasible education policy $\left\{\left(X_{t}, g_{t}\right)\right\}$ with a corresponding competitive equilibrium. For each date $t$ the net value of labor, denoted by $W_{t}\left(X_{t}, g_{t}\right)$, is defined as the total labor income of generation $t$ minus the government investment in higher education of generation $t$; namely:

$$
W_{t}\left(X_{t}, g_{t}\right)=\left[m w_{t}+w_{t+1}\right] \int_{\sim A_{t}} \theta_{t+1}(\omega) h_{t}^{\nu}(\omega) X_{t}^{\xi}+B w_{t+1} \int_{A_{t}} \theta_{t+1}(\omega) h_{t}^{\nu}(\omega) X_{t}^{\xi}-g_{t} \mu\left(A_{t}\right)
$$

Given some initial conditions at $t=0$, we say that a feasible education policy $\left\{\left(X_{t}^{*}, g_{t}^{*}\right)\right\}$ dominates the feasible education policy $\left\{\left(X_{t}, g_{t}\right)\right\}$ if for any date $t$ switching from $\left(X_{t}, g_{t}\right)$ to $\left(X_{t}^{*}, g_{t}^{*}\right)$ results in:
(a) $W_{t}\left(X_{t}^{*}, g_{t}^{*}\right)>W_{t}\left(X_{t}, g_{t}\right)$
(b) At each date $k, k>t$, the government has to choose between these two education policies $\left(X_{k}^{*}, g_{k}^{*}\right)$ and $\left(X_{k}, g_{k}\right)$, then $\left(X_{k}^{*}, g_{k}^{*}\right)$ will have the higher net value of labor, namely, $W_{k}\left(X_{k}^{*}, g_{k}^{*}\right)>W_{k}\left(X_{k}, g_{k}\right)$.

Thus, from the definition we see that the policy $\left\{\left(X_{t}^{*}, g_{t}^{*}\right)\right\}$ generates more net aggregate income for each generation, given that each generation compares these two options given the distribution of human capital at the outset of the period. Let us compare now the no-public funding policy $\left\{\left(X_{t}^{0}, g_{t}^{0}\right)\right\}$ and the full-public funding policy (discussed earlier), $\left\{\left(\hat{X}_{t}, \hat{g}_{t}=z_{t}^{*}\right)\right\}$ :

Proposition 7: Assume that the following two conditions hold: (a) $X_{t}^{0} / z_{t}^{*}>\xi$ and (b) $B^{1 / 5}\left[1-z_{t}^{*} / \tau w_{t} H_{t}^{0}\right] \leq 1$ for all dates $t$. Then, the no-public funding policy dominates the full-public funding policy.

Condition (a) which has been assumed in Proposition 4 is a mild assumption. Condition (b) requires that B should not be 'too large' and/or the cost of per-student higher education is not 'small' compared to the per-student public education budget. Also, when $\xi$ is close to 1 and $B$ is not too high it helps condition (b) to be satisfied. Under these assumptions the existence of cases where the government does not allocate public funds to higher education may be better from economic efficiency point of view than the fully-funded case.

Proof: Suppose that we switch from zero-public-funding to full-public funding at date $t$. Comparing the net labor income in these two cases, the Proposition requires that:

$$
\begin{align*}
& W_{t}\left(X_{t}^{0}, 0\right)=\left[m w_{t}+w_{t+1}\right] \int_{\sim A_{t}} \theta_{t+1}(\omega) h_{t}^{v}(\omega) X_{t}^{\xi}+B w_{t+1} \int_{A_{t}} \theta_{t+1}(\omega) h_{t}^{v}(\omega) X_{t}^{\xi}>  \tag{33}\\
& W_{t}\left(\hat{X}_{t}, z_{t}^{*}\right)=B w_{t+1} \int \theta_{t+1}(\omega) h_{t}^{v}(\omega) \hat{X}_{t}^{\xi}-z_{t}^{*}
\end{align*}
$$

But the right hand side of (33) can be rewritten as follows:

$$
W_{t}\left(\hat{X}_{t}, z_{t}^{*}\right)=B w_{t+1} \hat{X}_{t}^{\xi} \int_{A_{t}} Z_{t+1}(\omega)+w_{t+1} \hat{X}_{t}^{\xi} \int_{\sim A_{t}} Z_{t+1}(\omega)
$$

Thus, the inequality in (33) holds if the following inequality holds:

$$
\begin{aligned}
& m w_{t}\left(1+r_{t+1}\right)\left(X_{t}^{0}\right)^{\xi} \int_{\sim A^{0}} Z_{t+1}(\omega)+w_{t+1}\left[\left(X_{t}^{0}\right)^{\xi}-B \hat{X}_{t}^{\xi}\right] \int_{\sim A^{0}} Z_{t+1}(\omega)> \\
& B w_{t+1}\left[\hat{X}_{t}^{\xi}-\left(X_{t}^{0}\right)^{\xi}\right] \int_{A_{c}^{0}} Z_{t+1}(\omega)-z_{t}^{*}
\end{aligned}
$$

A sufficient condition for this inequality to be satisfied is: $\left(X_{t}^{0}\right)^{\xi} \geq B \hat{X}_{t}^{\xi}$. This can be rewritten as: $X_{t}^{0} \geq B^{1 / 5} \hat{X}_{t}$. Rewritting this inequality:

$$
\begin{equation*}
\tau w_{t} H_{t}^{0} \geq B^{1 / 5}\left[\tau w_{t} \hat{H}_{t}-z_{t}^{*}\right] \tag{34}
\end{equation*}
$$

Using Proposition 4 we obtain that by increasing public funding from $g_{t}^{0}=0$ to $\hat{g}_{t}=z_{t}^{*}$ the period t stock of human capital will decline; namely, that $\hat{H}_{t}<H_{t}^{0}$. Now, from (34) we obtain:

$$
1 \geq B^{1 / s}\left[\frac{\hat{H}_{t}}{H_{t}^{0}}-\frac{z_{t}^{*}}{\tau w_{t} H_{t}^{0}}\right]
$$

Thus, we attain that condition (b) of the Proposition implies condition (34). Now, in each date $k>t$, given the initial distribution of human capital, a choice between these two public funding regimes requires the same type of comparison as we did for date $t$. Hence, when the conditions required in this Proposition hold at date $k$ we obtain the same outcome.

## 7. Concluding Remarks

The tremendous expansion of globalization in the last two decades has affected small open economies very significantly. Since the formation of human capital has assumed an important role in enhancing endogenous economic growth, studying the implications of capital mobility and other international factors on the choice of education policy and the evolution of skilled vs. low-skilled sets of workers in each country became an essential topic. The relevant theoretical literature (see, e.g., De Fraja, 2002; and many others) in economics has dealt with these issues mostly within closed economies, hence the aim of this paper is to promote our understanding of these issues in small open economies. Our model has been utilized to explore the role of international factors in affecting individual education choices as well as governmental decisions related to educational funding policies. The results we have obtained may be relevant to certain small open economies but not to others. Some of the conditions we have assumed are related to the productivity of the higher education system, the cost of attaining skills and to international factors. Also, an essential feature of our analysis is the international mobility of physical capital (and immobility
of the labor force) and its dependence on the distribution of human capital among countries.

Is it always desirable that public funds be used to subsidize higher education? The answer may depend on the underlying features of the economy, such as wages (affected by international markets), the productivity of the whole education system, and some other parameters. In some cases we have demonstrated that such public funding will enhance the formation of human capital and thus promote economic growth. On the other hand, we have derived conditions under which public financing of higher education is inefficient. In other words, in some small open economies refraining from using public resources for higher education can 'dominate' the regime in which the government fully funds higher education. Thus, using public funds to send 'low quality' students to college may be inefficient since the government can invest these resources to improve the compulsory schooling system (which is benefiting all students).

The dynamic framework we have applied has several important feature, some of them contribute to our results in a significant way. For example, we take into account the opportunity cost of attending higher education and parental altruism. It is not clear to us how robust are the results when we dispose of such assumptions. However, we feel comfortable with such assumptions since they add realism to the analysis. In this work we have abstained from studying the effects of international factors on income inequality, but this important issue should be considered in future research. In a different framework, the effect of international factors on income distribution in equilibrium has been examined by Viaene and Zilcha (2002).

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[^0]:    ${ }^{2}$ See also Su (2004), Blankenau and Camera (2006).
    ${ }^{3}$ Researchers in a number of fields have showed that investments in care and education early in children's lives carry high individual and social rates of returns. The most recent evidence is reviewed in Cunha et al. (2006). It is therefore not surprising to see increases in pre-primary enrollments. In a number of OECD countries (The Czech Republic, Germany, New Zealand and Poland) annual expenditures per student are higher on pre-primary education than on primary education (OECD, 2009, Table B1.1a).

[^1]:    ${ }^{4}$ Public funding provides only a share of investments in tertiary education. In 2006 the proportion of private funding of tertiary education ranged between $3.6 \%$ in Denmark and $83.9 \%$ in Chile (OECD, 2009, Table B3.2b). Different combinations of tuition fees and government subsidies in our model can reproduce the relative importance of private funding observed in the data.

[^2]:    ${ }^{5}$ Thus we depart from the dynastic model where the utility functions of all future generations enter this utility function.

[^3]:    ${ }^{6}$ The reasoning of Proposition 4 is modified when partial derivatives in (24a) and (24b) are either negative or zero. This is when economies devote resources to higher education that are larger than those on basic education.

