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Abstract

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Key words: Rent-seeking, public goods, group size, wealth inequality, group asymmetries.
JEL classification: D31, D70, D72, D74

1. Introduction

Many public goods or facilities are allocated in societies according to the efforts expended by different groups in trying to win these prizes. Some examples of this situation are cities or neighborhoods competing for different kinds of a public facility (hospitals, parks, libraries, etc.) or for a public project, industries struggling for government support, etc. Studies of allocations of this type are well represented in the rent-seeking literature. Although there is a vast literature in this field, there are still some open issues regarding the effect of between-group asymmetries on group success probabilities and aggregate rent-seeking efforts. We build a model depicting the rent-seeking for a pure public good between two groups, one that allows us to get away from most of the neutrality results found in the literature and thus offer new and sensible results regarding the effect of group heterogeneity on rent-seeking efforts.

The seminal contribution on rent-seeking for public goods comes from Katz et al. (1990). Their most important results can be summarized as follows. First, neither the total sum of rent-seekers nor their between-group distribution affect aggregate rent-seeking efforts. Second, regardless of the group-size, a richer group always invests more effort into rent-seeking than a poorer group. Consequently, richer groups are always more successful than poorer groups. This happens precisely because of the group size neutrality result. Finally, although they do not study the effect of wealth inequality on aggregate efforts, it can be inferred from their model that a redistribution of wealth from a richer to poorer group always increases the aggregate rent-seeking effort.

The first result noted above is quite surprising, and contradicts earlier works on collective action wherein it was suggested that group size matters, both with respect to group rent-seeking efforts and success probabilities (Olson, 1965; McGuire, 1974). The second result seems intuitively implausible. For instance, it eliminates the possibility that group-size might compensate for the level of wealth. There are several situations wherein large groups, although poorer, are more successful than richer groups. For instance, one might consider the changes in environmental legislation in many countries, forced through by citizen groups to the detriment of large enterprises. The last result is in line with previous findings in the rent-seeking literature—i.e., the more homogeneous the contestants in a static rent-seeking model, the ‘greater’ the aggregate rent-seeking efforts (Baye et al., 1993; Che and Gale, 1998; Szymanski and Valletti, 2005; Amegashie and Kutsoati, 2006; Epstein and Nitzan, 2006; and Fu, 2006). In this paper, we reconsider these findings.

Staying within the framework of rent-seeking for pure public goods, other authors have obtained alternative results in these respects.[1] Riaz et al. (1995) analyze the same rent-seeking situation studied by Katz et al. (1990), but expand individual consumption bundles to include preferences over the public good and a private good. Imposing certain restrictions on the slopes of the between-group reaction functions, they find that an increment in group-size positively affects aggregate rent-seeking efforts. They also find that group rent-seeking efforts are positively related to wealth.

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[1] Other authors have also obtained alternative results by introducing certain private characteristics to the contested rents (Nitzan, 1991; Katz and Tokatlidu, 1996; Esteban and Ray, 2001).

Cheikbossian (2008) concentrates on between-group asymmetries, and finds that the distribution of rent-seekers across groups is non-neutral. More specifically, he finds that aggregate rent-seeking efforts diminish as group-size asymmetries increase. He also finds that, regardless of a group's valuation of a public good, those groups with fewer members are always more successful. The characteristic of the model driving these results is that the rent sizes being contested are not fixed but change with group size. Group size then not only affects the relative influence of each group but also the sizes of the rents being contested. As a result, group collective action worsens as group size increases.

We consider a common type of rent-seeking situation wherein two groups formed by risk neutral individuals engage in lobbying activities to win a uniquely pure (within-group) public good. There are two dimensions by which the groups are differentiated in our framework—wealth and size (i.e., the number of members in each group). Following the literature on rent-seeking contests, we assume that each group's success probability depends on the relative amount of resources spent on rent-seeking by its members. However, unlike what we generally find in the standard literature, we measure the utility cost of rent-seeking, not directly in terms of individual efforts, but in terms of the loss in private consumption an individual faces when he or she contributes to rent-seeking. This strategy introduces an interesting feature into the model—namely, that the marginal cost of rent-seeking changes with the level of private consumption. Actually, this characteristic will drive most of our results.

Therefore, similar to Katz et al. (1990) and Riaz et al. (1995), and unlike Cheikbossian (2008), our model assumes that the rents being contested are fixed. Moreover, similar to Katz et al. (1990), but unlike Riaz et al. (1995), our model assumes that the valuation of a public good is independent of private consumption. Nevertheless, unlike with those studies, we measure the cost of rent-seeking in terms of the individual loss in private consumption. This allows us to avoid the neutrality results noted above, and go further in terms of analyzing between-group asymmetries, with a special emphasis on wealth inequalities.

We begin by analyzing the comparative static of the model. We find that group size and group average wealth positively affect group rent-seeking efforts, aggregate rent-seeking efforts, and group success probabilities. The effect of group size on aggregate efforts represents an escape from the group size neutrality result suggested by Katz et al. (1990). Moreover, it is much more general than the result presented by Riaz et al. (1995), which only holds for the particular case noted above. Notice that, unlike the case with previous studies, our model not only predicts that the aggregate effort of a group increases as its size increases, but also that the group success probability increases as its size increases.

Our result, that the level of wealth positively affects group rent-seeking efforts and group success probability, is also reported by Katz et al. (1990) and Riaz et al. (1995). Our main contribution in this regard concerns the manner in which the interaction between group size and group wealth affects group success probability. Our model predicts that it is possible to observe a poorer group being more successful than a richer group because of a larger group size. This result is more sensible than that of Katz et al. (1990) in that there are several examples in which large groups of relatively low average wealth have been more successful than small groups with higher levels of wealth. As far as we know, this result has not been anticipated by any previous study.

We begin our study of between-group asymmetries by first analyzing the distribution of the population across groups. We do this by isolating wealth asymmetries—i.e., for the case where both groups have the same average wealth. Our main result is that between-group size asymmetries do affect aggregate rent-seeking efforts. Thus, unlike with Katz et al. (1990), we find that the between-group distribution of rent-seekers in not neutral. More specifically, we find that aggregate rent-seeking efforts increase as between-group size asymmetries decrease if the change in the marginal cost of the rent-seeking of the smaller group is greater than the change in the marginal cost of that of the larger group. The intuition behind this result is simple. The total rent-seeking contribution of the larger group decreases as the between-group size asymmetries decrease. The opposite happens with respect to the total rent-seeking contribution of the smaller group. Whether the decrease in the former is dominated by the later increment depends on the between-group relative change in the cost of rent-seeking.

We next analyze asymmetries in wealth. As before, our study begins with the case where there are no asymmetries in group-size. Under these circumstances, we show that wealth asymmetries affect aggregate rent-seeking efforts, and that fewer asymmetries do not necessarily imply more aggregate efforts. Once again, the key element in this result is how the marginal cost of rent-seeking changes across groups. If the change in the marginal cost of the rent-seeking of the poorer group is greater than the change in the marginal cost of that of the richer group, then aggregate rent-seeking efforts increases. However, aggregate rent-seeking efforts decrease when the opposite is true. This result is at odds with the commonly held notion that less between-group asymmetries imply more aggregate rent-seeking efforts, and demonstrates how it depends on the between-group relative change in the marginal cost of rent-seeking.

Continuing with our analysis of wealth inequalities, we analyze the case wherein, not only are there between-group wealth asymmetries, there are also between-group size asymmetries. Here, our results depend not only on the between-group relative change in the marginal cost of rent-seeking but also on the relative group size. Relative group size (i.e., the asymmetries in group-size) matters with respect to relative wealth transfer. For instance, when the poorer group is smaller in size than the richer group, a progressive transfer of wealth implies that the increase in the average wealth of the poorer group will be relatively higher than the decrease in the average wealth of the richer group. The opposite occurs when the poorer group is larger than the richer one. Under these circumstances, we find that if the between-group relative change in the cost of rent-seeking is greater than the relative transfer, then less wealth inequality implies more aggregate rent-seeking efforts. The opposite happens when the between-group relative change in the cost of rent-seeking is smaller than the relative transfer. Once again, this result departs from the standard result—that less between-group asymmetries implies more aggregate rent-seeking efforts—and demonstrates the importance of group-size asymmetries when evaluating the effects of wealth asymmetries.

We conclude our wealth inequalities analysis by examining the case where the change in the marginal cost of rent-seeking always decreases as private consumption increases. This property is satisfied by widespread, strictly concave, utility functions, and allows us to link our results with the initial group success probability. We find that if the poorer group is smaller in size (and thus less successful) than the richer group, then less wealth asymmetry implies more aggregate rent-seeking efforts. However, when the poorer group is larger in size and more successful than the richer group, then less wealth asymmetry implies fewer aggregate rent-seeking efforts. This result is driven by the possibility that poorer but larger groups can be more successful than richer but smaller groups.

The remainder of the paper is as follows. In sections 2 and 3, we present the model and characterize the respective equilibrium. Section 4 presents and comments on the main result. The conclusions are presented in the last section. The appendix contains all our proofs.

2. The model

Let us consider two groups (*g=1,2*), both of which are competing for the allocation of a public good. The good is indivisibly allocated in the sense that only the group that receives the allocation can enjoy it. We might think of this good as a public facility or a public project (a hospital, park, library, etc.), a special law or support that might favor a particular economic sector, and so forth. The two groups must engage in rent-seeking activities so as to influence the allocation of the good in their favor. This situation is referred to in the literature as rent-seeking for pure public goods.

Accordingly, only the group that is granted the prize receives utility from the allocation. We fix this gain at one. Thus, if the prize is allocated to group *g*, each individual *i* who belongs to this group receives an extra unit of utility, and each individual who belongs to the other group receives zero utility. Therefore, individual valuation of the public good is totally symmetric within and between groups.

The number of people in each group (group-size) is n_g. Each individual *i* has exogenous wealth w_i and spends a non-negative amount of resources r_i on rent-seeking so as to maximize his or her expected utility. We assume that individuals are risk neutral and cannot borrow, and that individual wealth is public information.

Let us define $c_i = w_i - r_i$. In our framework, c_i has at least two interpretations. On the one hand, it could be understood as the individual wealth net of contribution. On the other hand, it could also be understood as the individual consumption of a private good the price of which has been fixed at one. We use the second interpretation. As in the literature on public goods, in our framework, each individual derives utility from the consumption of both the public and the private good. The expected utility of an individual belonging to group *g* is given by:

$$EU_i = p_g + f(c_i) \qquad (1)$$

where p_g is the success probability of group g, and $f(.)$ is assumed to be a continuous, increasing and strictly concave function, with $\lim_{c_i \to 0} f'(c_i) = \infty$.[2] The concavity of $f(.)$ implies that the marginal utility of consumption is decreasing.

This pay-off function allows for another interpretation. Standard rent-seeking models divide the individual payoff between the expected benefit of the prize and the rent-seeking costs. These costs are directly related to the individual efforts spent on lobbying (in our framework, r_i). Equation 1 also distinguishes between benefits and costs. However, contrary to the standard models, our pay-off function does not measure the utility cost in terms of individual efforts, but rather in terms of the loss in private consumption that individuals face when contributing to rent-seeking. Accordingly, we refer to $f(.)$ as the rent-seeking cost.

Equation 1 also introduces an interesting feature to the model, namely that the marginal cost of rent-seeking can decrease with the level of private consumption if $f''(.) < 0$, as we assume. This characteristic will drive most of our results.

Each group's success probability depends on the relative amount of resources spent on lobbying by its members. We assume the following, quite standard, functional form for success probability:

$$p_g = \frac{R_g}{R} \qquad (2)$$

for $g=1,2$, provided that $R>0$, and where R_g is the total amount of resources contributed by group g to rent-seeking (i.e., $R_g = \sum_{i \in g} r_i$), and R is the total amount of resources expended by the two groups on rent-seeking (i.e. $R = R_1 + R_2$). If $R=0$, then the respective success probabilities are given by an arbitrary vector, $\{\tilde{p}_1, \tilde{p}_2\}$, which is contained in the interior of the simplex. We refer to R as the aggregate rent-seeking effort.

3. Equilibrium

In our framework, each individual in each group takes as a given the efforts contributed by everyone else in the society, and chooses $r_i \geq 0$ to maximize equation 1, subject to equation 2. The resources spent by individual i from group g is described by the following conditions:

$$\frac{1}{R}(1 - p_g) = f'(c_i) \quad \text{if} \qquad f'(w_i) < \frac{R_{-g}}{R_{-i}^2} \qquad (3a)$$

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[2] A necessary condition for equilibrium existence in our framework is that $f''(.) \leq 0$ (See the appendix). However, as we will see later on, the most interesting case is when $f''(.) < 0$.

$$r_i = 0 \qquad \text{if} \qquad f'(w_i) \geq \frac{R_{-g}}{R_{-i}^2} \tag{3b}$$

where $R_{-i} = R - r_i$ and $R_{-g} = R - R_g$. Equations 3a and 3b implicitly describe the Nash equilibrium contribution of each individual. Under an interior solution, equation 3a describes the usual equilibrium condition according whereby the marginal utility of the contribution must be equal to its marginal disutility.

It is possible to redefine the equilibrium based on the success probabilities and the aggregate rent-seeking efforts, rather than on personal efforts. Given that $f'(.)$ decreases monotonically, from equations 3a and 3b, the equilibrium condition can be written as:

$$r_i = Max\left\{0, w_i - f'^{-1}\left(\frac{1-p_g}{R}\right)\right\} \tag{4}$$

Combining equations 2 and 4, we get:

$$p_g = \frac{1}{R}\sum_{i\in g} Max\left\{0, w_i - f'^{-1}\left(\frac{1-p_g}{R}\right)\right\} \tag{5}$$

The equilibrium can now be interpreted as a vector, $\langle p_1, p_2 \rangle$, of the success probabilities (such that $p_g \geq 0 \;\forall g$, and $\Pi = p_1 + p_2 = 1$) and a positive scalar R, such that equation 5 is satisfied for every group. In the appendix, we show that an equilibrium always exists, and that this is unique.

4. Analysis

From now on, let us assume that there exists an interior solution for every individual. If this is the case, equation 5 reduces to:

$$p_g = \frac{1}{R}\sum_{i\in g}\left(w_i - f'^{-1}\left(\frac{1-p_g}{R}\right)\right) \tag{6}$$

Given the properties of $f'(.)$, after some algebraic manipulation, equation 6 can be written as:

$$\frac{1}{R}(1-p_g) = f'(c_g^*) \tag{7}$$

where $c_g^* = \overline{w}_g - p_g R / n_g$, and $\overline{w}_g$ is the average wealth of group *g*. Note that equations 7 and 3a represent the same condition. However, we now know that, at equilibrium, all individuals belonging to the same group have exactly the same level of private consumption.[3] Additionally, from equation 7, it follows that, at equilibrium, p_g and R are completely defined by $\overline{w}_g$ and n_g. Inasmuch then as, at equilibrium, individual wealth is irrelevant, we can conclude that within-group wealth inequality affects neither p_g nor R (this replicates the so called Neutrality theorem - War, 1983; Bergstrom et al., 1986). We will use equation 7 for our analysis.

Group size and wealth

We first analyze the comparative static properties at equilibrium. Our strategy consists of examining how success probabilities change over the cross-section of groups (i.e., how $\Pi = p_1(\overline{w}_1, n_1, R) + p_2(\overline{w}_2, n_2, R)$ changes) when either n_g or $\overline{w}_g$ changes, while keeping the level of aggregate rent-seeking efforts constant. Since Π decreases as R increases (see the appendix), once we know how Π changes, we can infer how R must move to recover the equilibrium (i.e., to recover $\Pi = 1$).

Proposition 1: Let us assume that for everyone there is an interior solution. Both the aggregate rent-seeking effort and the success probability of group g strictly increase as the group-size of group g increases.

It can be shown that there is individual free-riding in our model. However, the result in proposition 1 implies that the reduction in an individual's own contribution when n_g increases is more than compensated by the contributions of new members in the group. As a result, both group rent-seeking efforts and aggregate rent-seeking efforts increase. This result allows us to move away from the group size neutrality result suggested by Katz et al. (1990).

Notice that the result in proposition 1 is much more general than that presented by Riaz et al. (1995), which is restricted for certain values of the slope of the between-group reaction functions. Moreover, our result goes further, in the sense that it also predicts that group size positively affects the respective group's success probability.

Our findings are entirely driven by the assumption that the marginal cost of rent-seeking is not constant. Notice that if $f(.)$ is assumed to be linear, then n_g affects neither aggregate rent-seeking efforts nor success probabilities. (See the appendix.) Actually, this is the same case examined by Katz et al. (1990).

The result in proposition 1 implicitly requires that a group's average wealth remain unchanged as its size increases. Thus, if the addition of new members negatively affects a group's average wealth, the result can be different. In the next proposition, we analyze the effect of a change in the group's average wealth when its size remains constant.

[3] Actually, all individuals belonging to the same group obtain exactly the same level of utility. This characteristic has been analyzed by Itaya et al. (1997).

Proposition 2. Let us assume that for everyone there is an interior solution. Both the aggregate rent-seeking effort and the success probability of group *g* strictly increase as the average wealth of group *g* increases.

The result in proposition 2 is also reported by Katz et al. (1990) and Riaz et al. (1995). However, our main contribution in this regard concerns the manner in which the interaction between group size and group average wealth affects group success probabilities. We say that a group is poorer than another group if the average wealth of the former is smaller than the average wealth of the later.[4] From propositions 1 and 2, the following corollary can be obtained.

Corollary 1: It is possible to observe a poor group being more successful than a rich group because of a higher group size or vise versa.

The possibility suggested in corollary 1 can be illustrated through a simple example. Let $f(c_i) = \ln(c_i)$, $\overline{w}_b = 98.499$, $\overline{w}_s = 100$, and $n_s = 10$. When $n_b = 13$, then, at equilibrium, $\langle p_b, p_s \rangle = \langle 0.4976, 0.5024 \rangle$. When $n_b = 26$, then, $\langle p_b, p_s \rangle = \langle 0.5, 0.5 \rangle$. Finally, when $n_b = 57$, then, $\langle p_b, p_s \rangle = \langle 0.5013, 0.4987 \rangle$.

As indicated above, Katz et al. (1990) maintain that a richer group is always more successful that a poorer one, regardless of respective group sizes. This is due to their group size neutrality result. However, there are several situations–for instance, with respect to environmental legislation—where larger groups with low average wealth are more successful than smaller groups with relatively higher average wealth. This is exactly what our model is able to predict, and (as far we know) previous models cannot. The result in corollary 1 is exploited in order to study between group asymmetries, especially asymmetries in wealth.

Between group asymmetries

We now analyze the effect of between-group asymmetries in size and wealth on aggregate rent-seeking efforts. As commented in the introduction, Katz et al. (1990) suggest that asymmetries in group-size are totally neutral and do not affect aggregate rent-seeking efforts. On the other hand, although they do not study between-group inequalities in wealth, it can be inferred from their model that fewer between-group asymmetries in wealth increases aggregate rent-seeking efforts. Against this, our model predicts that group size asymmetries affect aggregate rent-seeking efforts, and that less wealth inequality does not necessarily imply an increase in said efforts—as the standard literature suggest. We present our results in this section.

Before continuing with our analysis, we first state in lemma 1 an additional characteristic of our model that will be useful for understanding our subsequent results.

[4] This is exactly the same definition of poorer group used by the authors mentioned above.

Lemma 1: If $p_1 > p_2$ at equilibrium, then $f'(c_1^*) < f'(c_2^*)$. From the characteristics of $f(.)$, $p_1 > p_2$ also implies that $c_1^* > c_2^*$.

Lemma 1 follows immediately from equilibrium condition 7. It says that, when the success probability of group 1 is larger than the success probability of group 2, then, at equilibrium, the marginal cost of rent-seeking for individuals belonging to group 1 is smaller than that for individuals belonging to group 2. This also implies that, at equilibrium, the private consumption of group 1 is larger than the private consumption of group 2.

Let us first analyze how between-group size asymmetries affect aggregate rent-seeking efforts. The effect of redistributing people from a larger group (group *b*) to a smaller group (group *s*) – i.e. the effect of a reduction in group size asymmetries - can be analyzed by looking at how success probabilities change over the cross-section of groups following the transfer of people, and assuming that *R* remains constant. Doing this, we can infer the manner in which *R* must change to adjust the sum of success probabilities to equal one. The change in Π when there is a redistribution of people from group *b* to group *s* is given by:

$$\Delta\Pi^{size}\Big|_R = \frac{\partial p_s}{\partial n_s}\Big|_R - \frac{\partial p_b}{\partial n_b}\Big|_R \tag{8}$$

From the comparative static derived above, we already know the derivatives implied in equation 8. Replacing these terms, we obtain:

$$\Delta\Pi^{size}\Big|_R = R^2\left(\frac{f''\left(c_s^*\right)p_s}{n_s\left(R^2 f''\left(c_s^*\right)-n_s\right)} - \frac{f''\left(c_b^*\right)p_b}{n_b\left(R^2 f''\left(c_b^*\right)-n_b\right)}\right) \tag{9}$$

If expression in equation 9 is positive (negative), the redistribution of people from group *b* to group *s* makes Π higher (smaller) than one; in order then to recover equilibrium conditions, *R* must increase (decrease) whenever p_g and *R* are negatively related.

Whether $f''(c_s^*)$ is larger, equal, or smaller than $f''(c_b^*)$ is crucial in determining the sign of equation 9. As we know, $f''(.)$ measures the change in the marginal cost of rent-seeking. Thus, the effect of a reduction in group size asymmetries will depend on the between-group relative change in the marginal cost of rent-seeking. In order to isolate the effect of group size asymmetries on rent-seeking efforts, we concentrate on the case where both groups have exactly the same average wealth.

Proposition 3: Let us assume that $\overline{w}_s = \overline{w}_b$. If $f''(c_s^*) \le f''(c_b^*)$, then fewer between-group asymmetries in size will positively affect aggregate rent-seeking efforts. Otherwise the effect is ambiguous.

When we redistribute people from the larger group to the smaller one, then rent-seeking efforts will be greater in the former group, and fewer in the latter. Proposition 3 states that, in order to observe an increase in aggregate rent-seeking efforts, it is enough if the change in the marginal cost of the smaller group is greater than the change in that of the larger group.[5] This result departs from the neutrality result for group-size asymmetries.

Let us now analyze the effect of wealth inequality on the aggregate rent-seeking effort. More precisely, we wish to analyze the effect of a between-group progressive transfer of wealth (hereafter, BGPT) on the aggregate rent-seeking effort. By this we refer to a case wherein a richer group (group *h*) transfers part of its total wealth to a poorer group (group *l*) such that the sum of the total wealth of the two groups remains constant.[6]

As before, the effect of a BGPT can be analyzed by looking at how success probabilities change over the cross-section of groups following the transfer. Notice that if we transfer one unit of money from the average wealth of group *h* ($\overline{w}_h$) to the average wealth of group *l* ($\overline{w}_l$), the latter increases by n_h / n_l . Taking this into account, we calculate the change in Π when there is a BGPT as:

$$\Delta\Pi^{wealth}\Big|_R = \frac{\partial p_l}{\partial \overline{w}_l}\bigg|_R \frac{n_h}{n_l} - \frac{\partial p_h}{\partial \overline{w}_h}\bigg|_R \tag{10}$$

From the comparative static derived in proposition 1, we already know the derivatives implied in equation 10. Replacing these terms and manipulating the equation algebraically, we obtain:

$$\Delta\Pi\Big|_R = Rn_h\left(\frac{1}{R^2 - \Omega_l} - \frac{1}{R^2 - \Omega_h}\right) \tag{11}$$

where $\Omega_g = n_g \big/ f''(c_g^*) < 0$ for g=h,l. Thus, when $\Omega_l<\Omega_h$ ($\Omega_l>\Omega_h$), the transfer makes Π smaller (higher) than one; in order then to recover equilibrium conditions, R must decrease (increase) whenever p_g and R are negatively related. Notice that $\Omega_l<\Omega_h$ if and only if $\dfrac{n_l}{n_h} > \dfrac{f''(c_l^*)}{f''(c_h^*)}$. The opposite is true if $\Omega_l>\Omega_h$. We start our analysis by studying the effect of a BGPT when there are only asymmetries in wealth but not in group size—i.e., when n_l=n_h.

Proposition 4. Let us assume that for everyone there is an interior solution, and n_l=n_h. A BGPT then will generate an increase in the aggregate rent-seeking effort if $f''(c_l^*) < f''(c_h^*)$. When this inequality is reversed, the BGPT generates a decrease in the

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[5] Based on propositions 1 and 2, it follows that $p_b > p_s$. From lemma 1 then, it must be the case that $c_b^* > c_s^*$. Therefore, to observe $f''(c_s^*) \le f''(c_b^*)$, it is necessary that $f'''(.) \ge 0$. This property is satisfied by widespread concave utility functions like $f(c_i) = c_i^\alpha$ with $\alpha \in (0,1)$, and $f(c_i) = \ln c_i$.

[6] Because there is within-group neutrality, we do not restrict this transfer so as to maintain the within-group wealth distribution.

aggregate rent-seeking effort. If both terms are equal, the BGPT does not have any effect on the aggregate rent-seeking effort.

The proof of proposition 4 follows immediately, once we replace $n_l / n_h = 1$ in the inequality involving Ω_l and Ω_h. Much as with proposition 3, proposition 4 states that in order to observe an increase in aggregate rent-seeking efforts as between-group asymmetries in wealth are reduced, it is enough if the change in the marginal cost of the poorer group is greater than the change in that of the richer group.[7] However, in this case, it is also possible to state without ambiguity that when the change in the marginal cost of the poorer group is smaller than the change in that of the richer group, aggregate rent-seeking efforts will decrease. This result runs against the commonly held notion that fewer between-group asymmetries implies more aggregate rent-seeking efforts, and demonstrates that it depends on the between-group relative change in the marginal cost of rent-seeking.

Let us now consider the effect of a BGPT on the aggregate rent-seeking effort when there not only exist between-group wealth asymmetries, but also between-group size asymmetries. Some interesting results emerge.

Proposition 5: Let us assume that for everyone there is an interior solution. A BGPT then will generate an increase in the aggregate rent-seeking effort if $\frac{n_l}{n_h} < \frac{f''(c_l^*)}{f''(c_h^*)}$. When this inequality is reversed, the BGPT generates a decrease in the aggregate rent-seeking effort. If both terms are equal, the BGPT does not have any effect on the aggregate rent-seeking effort.

There are two elements involved in the inequality of proposition 5, the relative group size (n_l / n_h) and the relative change in the marginal cost of rent-seeking ($f''(c_l^*) / f''(c_h^*)$). On the one hand, the relative group size will define the magnitude of the relative transfer. For instance, when the poorer group is smaller in size than the richer group, a progressive transfer will imply that the increase in $\overline{w}_l$ is relatively higher than the decrease in $\overline{w}_h$. Thus, the relative transfer becomes larger as n_l / n_h decreases. On the other hand, as we have already commented, the ratio of second derivatives measures the between-group relative change in the marginal cost of rent-seeking at equilibrium. Thus, the change in the marginal cost of the poorer group is greater than the change in that of the richer group as $f''(c_l^*) / f''(c_h^*)$ increases.

Therefore, proposition 5 states that if the relative change in the marginal cost of rent-seeking is greater than the relative transfer, then the decrease in the rent-seeking effort of the richer group will be dominated by the increase in the rent-seeking effort of the poorer group. As a result, aggregate rent-seeking efforts will increase. When the opposite happens,

[7] We already know from lemma 1 that at equilibrium, $c_h^* > c_l^*$. As commented in footnote 5, a sufficient condition for observing $f''(c_l^*) < f''(c_h^*)$ is that $f'''(.) > 0$.

individuals in the poorer group will be better-off relatively increasing their private consumption. As a result, there will be a reduction in aggregate rent-seeking efforts. Once again, the result in proposition 5 shows that fewer between-group asymmetries do not necessarily imply more aggregate rent-seeking efforts. However, in this case, the result depends not only on the relative change in the marginal cost, but also on the magnitude of the relative transfer.

To conclude this section, we concentrate on the case where the marginal cost of rent-seeking decreases more quickly for lower than for higher levels of private consumption—i.e., $f''' > 0$.[8] Such an analysis is interesting for at least three reasons. First, as is noted in footnote 5, this property is satisfied by widespread, strictly concave, utility functions. Second, from our discussion of proposition 4, we know that when n_l=n_h and $f''' > 0$, then less wealth inequality will imply more rent-seeking aggregate efforts–this is actually the most standard result in the literature. What we want then is to see how the introduction of between group-size asymmetries affects the result. Third, this analysis allows us to link our results to the initial group success probability.

Using the results obtained in propositions 1 and 2, we already know that when n_l<n_h, then p_l<p_h. However, when n_l>n_h, the relationship between the equilibrium probabilities is no longer clear. It might be that p_l>p_h if the number of members in the poorer group is high enough to offset the negative effect due to its smaller average wealth. If this is not the case, then it must again be that p_l<p_h. Keeping in mind these facts, we can state our results in proposition 6.

Proposition 6: Let us assume that for everyone there is an interior solution, and $f'''(.) > 0$.

a) If n_l<n_h, then a BGPT will increase the aggregate rent-seeking effort.
b) If n_l>n_h and $p_l \geq p_h$ (i.e., the number of members in the poorer group is high enough to compensate for the group's smaller average wealth), then a BGPT will reduce the aggregate rent-seeking effort.
c) If n_l>n_h and p_l<p_h (i.e., the number of members in the poorer group is not high enough to compensate for the group's smaller average wealth), then the effect of a BGPT on the aggregate rent-seeking effort will be ambiguous.

Proposition 6 demonstrates that wealth equality does not necessarily increase aggregate rent-seeking efforts, even when we assume $f'''(.) > 0$. For instance, when the poorer group has a higher success probability (i.e., when the group's size compensates for its small average wealth), wealth redistribution reduces the aggregate rent-seeking effort. This result shows how, when there are asymmetries not only in wealth but also in group-size, it is not necessarily the case that greater wealth equality increases aggregate rent-seeking efforts.

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[8] A similar analysis can be done when $f''' \leq 0$. For instance, when $f'''(.) = 0$, then $f''(c_l^*) / f''(c_h^*) = 1$. From proposition 5, it follows that: (1) if n_l=n_h, then a BGPT does not affect the aggregate rent-seeking effort; (2) if n_l<n_h, then a BGPT generates an increase in the aggregate rent-seeking effort, and; (3) if n_l>n_h, then a BGPT generates a decrease in the aggregate rent-seeking effort.

5. Conclusions

This paper studies how group size and group wealth heterogeneity affects aggregate rent-seeking efforts when two groups are lobbying for a pure public good. Our analysis allows us to get away from most of the neutrality results found in the literature and thus offer new and sensible results regarding the effect of group heterogeneity on rent-seeking efforts. The key element in our model is that it measures the utility cost of rent-seeking in terms of the loss in private consumption that individuals face when contributing to this activity. This introduces an interesting feature into the model—namely, that the marginal cost of rent-seeking can decrease with the level of private consumption.

Our model predicts that larger and wealthier groups are more successful that smaller and poorer groups. Two conclusions can be obtained from this result. First, the sum of total rent-seekers positively affects the aggregate rent-seeking effort; second, it is possible to observe a poor group being more successful than a rich group because of a higher group size or vise versa. The first result represents a break from the neutrality result of group-size. The second result is more sensible than previous results, which have suggested that, regardless of the between-group size composition, richer groups are always more successful than poorer groups.

We perform an exhaustive study of the effect of between-group asymmetries on aggregate rent-seeking efforts. We begin by studying the case where there is only one between-group dimension asymmetry—either that related to group size or to group wealth. The general result is that fewer between-group asymmetries in size (wealth) implies more aggregate efforts, if the change in the marginal cost of rent-seeking of the smaller (poorer) group is larger than the change in that of the larger (richer) group. If the opposite happens, then greater homogeneity in wealth implies fewer aggregate rent-seeking efforts. This last result shows that fewer between-group asymmetries does not necessarily imply more aggregate rent-seeking efforts, if the marginal cost of rent-seeking changes with the level of private consumption.

We also study the effect of wealth inequalities on the aggregate rent-seeking effort when there exist not only between-group wealth asymmetries, but also between-group size asymmetries. Some interesting results emerge. We find that if the relative change in the marginal cost of rent-seeking is greater than the relative transfer—measured by the relative group-size—then less wealth inequality implies more aggregate rent-seeking efforts. If the opposite is true, then less wealth inequality implies fewer aggregate rent-seeking efforts. Once again, these results show that fewer between-group asymmetries does not necessarily imply fewer aggregate rent-seeking efforts. However, in this case, the result depends not only on the relative change in the marginal cost, but also on the magnitude of the relative transfer. This result shows how the effect of wealth asymmetries on aggregate rent-seeking efforts is influenced by between-group size asymmetries.

Appendix

Individual Optimal Contributions and equilibrium existence. Plugging equation 2 into equation 1, and taking as a given the contribution of the rest of individuals, each individual i in group g will maximize $EU_i = R_g \big/ R + f(c_i)$ over r_i. It can be verified that EU_i is strictly concave in r_i. From the first order condition, we get $(R - R_g) \big/ R^2 = f'(c_i)$. Reorganizing the terms and using the success probability function, we get equation 3a. Since $lim_{c_i \to 0} f'(c_i) = \infty$, then, at equilibrium, $r_i < w_i$. On the other hand, note that $\partial EU_i / \partial r_i \big|_{r_i=0} = R_{-g} \big/ R_{-i}^2 - f'(w_i)$. This marginal utility is positive if and only if $f'(w_i) < R_{-g} \big/ R_{-i}^2$. When this inequality holds, the total amount of resources spent on rent-seeking by individual i will be strictly positive and is implicitly described by equation 3a. When $f'(w_i) \geq R_{-g} \big/ R_{-i}^2$, the marginal utility is not positive, and the individual i's best response is $r_i = 0$.

Let us now prove existence and uniqueness of equilibrium**.** To do this, we use equation 5. Equation 5 implicitly defines p_g as a function of R. Moreover, it can be readily verified that p_g is a continuous function of R. Using the implicit function theorem, it can be shown that when $p_g > 0$, it is strictly decreasing in R. From equation 5, we get the following:

$$\frac{\partial p_g}{\partial R} = -\frac{\frac{1}{R^2}\sum_{i\in g}\left(w_i - f'^{-1}\left(\frac{1-p_g}{R}\right)\right) - \frac{n_g}{R^3}(f'^{-1})'\left(\frac{1-p_g}{R}\right)(1-p_g)}{1 - \frac{n_g}{R^2}(f'^{-1})'\left(\frac{1-p_g}{R}\right)}$$

(1A)

Since $f''(.) \leq 0$ and $\sum_{i\in g}\left(w_i - f'\left(\frac{1-p_g}{R}\right)\right) > 0$ under an interior solution, both the numerator and the denominator in 1A are positive. It then follows that $\partial p_g \big/ \partial R < 0 \;\; \forall g$.

Consider the function $\Pi = \frac{1}{R}\sum_g \sum_{i\in g} Max\left\{0, w_i - f'^{-1}\left(\frac{1-p_g}{R}\right)\right\}$. At equilibrium, R must correspond to $\Pi = 1$ and $p_g \geq 0 \;\; \forall g$. Note that Π strictly decreases in R, and approaches zero as R goes to infinity. On the other hand, when R approaches zero, then $p_g > 0$ and Π approaches infinity. It follows then that there must be some R for which $\Pi = 1$. Furthermore, it is unique.

Proof of proposition 1. Assume that for every i, there is an interior solution. Keeping R constant in equation 7, and using the implicit function theorem, we get

$\left.\frac{\partial p_g}{\partial n_g}\right|_R = \frac{R^2 f''(.)p_g}{n_g(R^2 f''(.)-n_g)} > 0$. Since $\left.\frac{\partial p_g}{\partial n_g}\right|_R = \left.\frac{\partial \Pi}{\partial n_g}\right|_R$, then an increase in p_g will make $\Pi > 1$. Since p_g and R are negatively related, it follows that R must increase to recover the equilibrium. This proves that R increases as the size of group g increases.

Let us now consider the complete effect of n_g on p_g. Until now, the change in n_g has not affected the success probability of group $-g$, and has affected p_g positively. Inasmuch as R increased, the success probabilities must go down to recover the equilibrium condition, $\Pi = 1$. To assure that this is the case, at the new equilibrium the final p_g must be larger than the initial p_g (likewise, p_{-g} will be smaller). This proves that the success probability of group g increases as the size of group g increases.

Notice that if $f(.)$ is assumed to be linear, then $\partial p_g / \partial n_g|_R = 0$; this implies that group-size affects neither p_g nor R.

Proof of proposition 2. Assume that for every i, there is an interior solution. As before, we keep R constant in equation 7. Again, using the implicit function theorem, we get $\left.\frac{\partial p_g}{\partial \overline{w}_g}\right|_R = \frac{n_g R f''(.)}{R^2 f''(.)-n_g} > 0$. Since $\left.\frac{\partial p_g}{\partial \overline{w}_g}\right|_R = \left.\frac{\partial \Pi}{\partial \overline{w}_g}\right|_R$, then an increase in p_g will make $\Pi > 1$.

Following the same argument as above, we can prove that both the success probabilities and the total rent-seeking effort increase as group size increases. Notice that if $f(.)$ is assumed to be linear, then $\partial p_g / \partial \overline{w}_g|_R = 0$; this implies that the average wealth of group g affects neither p_g nor R.

Proof of lemma 1. See the proof in the text.

Proof of proposition 3. From equation 9, it follows that $\Delta\Pi^{size}|_R > 0$ if and only if $R^2 f''(c_s^*) f''(c_b^*)(n_b p_s - n_s p_b) + (n_s^2 p_b f''(c_b^*) - n_b^2 p_s f''(c_s^*)) > 0$. The first term on the left-hand side of this inequality is positive if $n_b p_s - n_s p_b > 0$. Since $\overline{w}_s = \overline{w}_b$, then $p_s < p_b$. From lemma 1, it follows that $c_s^* < c_b^*$. From this inequality, it is easy to demonstrate that $n_b p_s - n_s p_b$ is always positive. We might now consider the second term on the left-hand side of the first inequality equation. From our previous analysis, it follows that $n_s^2 p_b < n_b^2 p_s$. Thus, this term is always positive if $f''(c_s^*) \leq f''(c_s^*)$.

Proof of proposition 4. See the proof in the text.

Proof of proposition 5. See the proof in the text.

Proof of proposition 6. Let us assume that for every *i*, there is an interior solution, and $f'''(.) > 0$.

a) When $n_l<n_h$, it must be the case that $p_l<p_h$. It follows from lemma 1 then that $f'(c_l^*) > f'(c_h^*)$; additionally that $c_l^* < c_h^*$, and $f''(c_l^*)/f''(c_h^*) > 1$. Thus, we conclude that $f''(c_l^*)/f''(c_h^*) > n_l/n_h$. Proposition 5 closes the proof.

b) When $n_l>n_h$ and $p_l \geq p_h$, it follows from lemma 1 that $f'(c_l^*) \leq f'(c_h^*)$, and so $c_l^* \geq c_h^*$, and $f''(c_l^*)/f''(c_h^*) \leq 1$. Thus, we conclude that $f''(c_l^*)/f''(c_h^*) < n_l/n_h$. Proposition 5 closes the proof.

c) When $n_l>n_h$ and $p_l < p_h$, it follows from lemma 1 that $f'(c_l^*) > f'(c_h^*)$; additionally, $c_l^* < c_h^*$, and $f''(c_l^*)/f''(c_h^*) > 1$. Since $n_l/n_h > 1$, the effect of a BGPT on the total rent-seeking effort is ambiguous.

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