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Vote revelation: empirical content of scoring

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In this paper I consider choice correspondences defined on an extended domain: the decisions are assumed to be taken not by individuals, but by committees and, in addition to the budget sets, committee composition is observable and variable. In this setting, I establish restrictions on the choice structures that are implied by sincere scoring decision-making by rational committee members.

1 Introduction

Consider an observer trying to make sense of the goings on in a secretive committee, such as the old Soviet Politburo. Such an observer would not have any direct evidence about preferences of individual committee members, nor would he be likely to observe the rules the committee uses to make its decisions. Nevertheless, our Kremlinologist does have some information to work with. For one, he may have a reasonably good idea of the options

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the committee members are facing. He would also be able to observe the committee decision: perhaps, it would come out in the Pravda. Finally, the committee membership is public knowledge (he could determine it by observing the figures standing on the observation deck of Lenin's Mausoleum during the Revolution Day parade). What sort of deductions would it be possible to make about the unobservable preferences and preference aggregation rules within the committee from this information? In fact, not much could be said from a single observation of the committee decision. However, it turns out that, if a number of observations of decisions taken by a committee with variable membership is available, one can use the available data to test certain hypotheses about the committee functioning.

The approach I use in this paper is closely related to the ideas of revealed preference and rationalizability, that have long been standard foundations of economic analysis. Ever since Houthakker (1950) it has been known that a simple consistency condition on choices (the Strong Axiom of Revealed Preference, SARP) is a necessary and sufficient condition for being able to explain individual choices with rational preference maximization. Over the years a sizeable literature on restrictions on choices implied by various individual and group decision-making procedures developed. Thus, for instance, in the context of social choice rules, Blair *et al.* (1976) characterized such restrictions as would derive from maximizing preferences that are merely acyclic, rather than transitive. This, of course, may be interpreted as characterizing choices made by committees of rational members with some of those members exercising veto power. However, though well-established, the tradition of revealed preference approach to group decisions has not been much developed recently. In particular, I am aware of no studies establishing "signatures" imposed on collective decisions by most commonly used voting rules. It is precisely this that I attempt to do in this paper.

In fact, when in recent years concepts of choice and revealed preference have received substantial renewed attention in economics, it was in the context of individual decision-making. This attention has been derived from the new focus on "boundedly rational" decision-making procedures different from the usual rational preference maximization. In this context one might mention Manzini and Mariotti (2007) work on "sequential rationalizability" or Masatlioglu and Ok (2005) study of choice with status-quo bias, both of which attempt to establish restrictions imposed on choices by distinct decision-making procedures. Other recent studies, such as Caplin and Dean (2009), attempt to explore the restrictions that various "boundedly rational" procedures would impose on records that are somewhat more detailed than the usual choice data, though still plausibly observable.

In the situation described at the beginning of this introduction, the group decision data is, in fact, richer than usual: in addition to the record of choices from a given set of alternatives, we have the committee membership at each decision point to consider. Thus, if we want to test a given theory of how the committee works, we have more information to base our testing on. Even on incomplete data (i.e., when not all possible observations might be there), we may observe enough to do this.

In this study I concentrate on a particular class of theories about the internal committee workings. I will generally assume that each committee consists of rational members who decide using some scoring rule (a class of rules, which includes simple "first past the post" plurality, approval voting or the Borda Count), and will try to formulate the natural restrictions (so far incomplete) on my observations implied by these rules. Even when the particular scoring rule is unknown, such restrictions turn out to be non-trivial.

The scoring rules are those in which individuals are asked to provide each alternative with a numeric score (reflecting their preferences), the individual scores are added up and the alternative with the highest aggregate score is chosen. These rules have long been characterized by social choice theorists (see Smith 1973, Young 1975 or Myerson 1995).

This work is also related to the study of empirical content of sincere (vs. strategic) voting by Degan and Merlo (2009). In fact, if the formal decision rule is known, this work may be reinterpreted precisely as the test of voter sincerity: if I know how the votes are counted, violations of the conditions established here could only be interpreted as indications that the scores do not directly reflect rational individual preference. Thus, to the extent one maintains the assumption that voters are rational, sincere voting would be falsified in this case. Likewise, this paper is related to Kalandrakis (2010) work on rationalizing individual voting decisions. This paper crucially differs from both Degan and Merlo (2009) and Kalandrakis (2010), however, in that I do not assume observability of individual votes (nor do I impose anything in addition to rationality on individual preferences). Rather, individual votes are "revealed" here from the observations of the group choices. In fact, by establishing a number of "SARP-like" conditions, I hope to characterize the conditions under which revealed scores are consistent (so far this characterization is incomplete).

2 Basic Set-up

Consider a finite set $N = \{1, 2, ...n\}$ of agents and a finite set $X = \{x_1, x_2...x_m\}$ of alternatives. A set of alternatives to be considered by a *committee* $S \in 2^N \backslash \{\varnothing\}$ is $B \in 2^X \backslash \{\varnothing\}$; following the standard terminology of individual choice theory, I shall call B the *budget set*. If a committee S is offered a choice from the budget set B the committee choice is recorded as $\varnothing \neq C(B, S) \subset B$. The *committee choice structure* is defined as a pair $(\mathcal{E}, C(.,.))$ where $\mathcal{E} \subset 2^X \backslash \{\varnothing\} \times 2^N \backslash \{\varnothing\}$ is the record of which budget sets where considered by which committees and $C : \mathcal{E} \to X$, such that $C(B, S) \subset B$ is the non-empty-valued choice correspondence, recording committee choices.

In order to explain observed committee choice structures I shall, in general, assume that each agent $i \in N$ has rational (complete and transitive) preferences $\succsim_i$defined over X. The committee choice structure provides a record of observed committee choices, which may be used by an observer to deduce the preference profiles and the preference aggregation rules the committee uses. In this paper Il concentrate on a particular class of such rules: the scoring rules, a class that includes such distinct procedures as the plurality vote (in which the winner is an alternative that is chosen by the largest number of voters), the Borda Count (in which alternatives get assigned the most points for being someone's top choice, a point less for being a second choice, etc., the scores get summed up over all the voters and the alternative with the largest score wins), or the Approval Voting (in which an individual is allowed to mark alternatives as acceptable or unacceptable, and the alternative which has been marked as acceptable by the largest number of voters gets chosen). Overall, I shall assume that agents are non-strategic, in that they ignore who else is in the committee (as noted above, the conditions I am deriving here might, if the formal rule is observable, be viewed as empirical implications of sincere voting itself). However, I shall allow the votes to depend on the budget sets under consideration (as would be the case in a sincere Borda Count). Thus, if the set of alternatives B, a vote of agent $i \in S$ is a function $v_i^B : B \to \mathbb{R}$.

Given a vote from each of its members a committee S chooses an alternative that gets the highest score

$$C^{scoring}(B, S) = \arg\max_{x \in B} \sum_{i \in S} v_i^B(x)$$

where $\sum_{i \in S} v_i^B(x)$ is called the *score* received by an alternative $x \in B$ in voting by committee S. Such a choice structure is said to be generated by the scoring rule.

Following Myerson (1995) I shall allow agents to submit votes that are distinct from reporting their preference orderings. In fact, for the purposes of defining a scoring rule one does not need to assume that the votes themselves derive from rational preferences. However, as the scoring rules require agents to report a ranking of alternatives in B by means of their votes $v_i \in \mathbb{R}^k$. Though in general such a ranking may not necessarily represent a rational preference (and thus, for instance, could be inconsistent over the different budget sets B), I shall concentrate on voting that, indeed, can be viewed as a sincere representation of individual preferences. Formally, given a rational preference profile $\succsim = (\succsim_1, \succsim_2, ..., \succsim_n)$ I shall say that a committee vote v_i^B is *(weakly) consistent with preferences* if $x \succsim_i y$ implies $v_i^B(x) \geq v_i^B(y)$.

If a committee choice structure is such that for any $(B, S) \in \mathcal{E}$

$$C(B, S) = C^{scoring}(B, S)$$

where the votes are consistent with preferences for some rational preference profile $\succsim$. I shall say that $\succsim$ rationalizes $(\mathcal{E}, C(.,.))$ via a scoring rule.

It should be noted, that unless the choice structure is extended by allowing observing variations in committee membership, scoring rules would, at first glance, appear particularly unpromising from the standpoint of this research: it would seem that nearly every possible committee decision could be explained by some sort of scoring applied to an unobserved preference profile of a fixed committee. Thus, if one defines, in the spirit of Salant and Rubinstein (2008) work on the choice with frames, the choice correspondence as

$$C_c(B) = \{x : x \in C(B, S) \text{ for some committee } S\}$$

little, if anything appears to be imposed on $C_c(.)$ (some restrictions may be derived from the relative cardinalities of B and N, if the latter is observed, but that appears to be it). However, it turns out that more can be said if committee membership and its variations are observed.

3 Revealed Scoring

Supposing that committees are making their decisions using scoring rules implies that each committee produces a ranking, represented by the score in question. Of course, if committee members may change their votes arbitrarily based on either committee membership or the set of alternatives involved, not much could be done here. For this reason, at least for now, I shall assume that voters are restricted to be weakly consistent with sincere voting. With this assumption, I shall try to "reveal" as much as possible about individual votes.

I shall start by defining the direct preference revelation

- *Direct revelation.* For each $(B, S) \in \mathcal{E}$ a pair of nested binary relations $P^*_{B,S} \subset R^*_{B,S}$ on B is defined by

  (i) let $x \in C(B, S)$ then $xR^*_{B,S}y$ for any $y \in B$

  (ii) let $x \in C(B, S)$ and $y \notin C(B, S)$ for some $y \in B$ then $xP^*_{B,S}y$

This constitutes a record of direct preference revelation: if an alternative is chosen, it implies it received at least as high a score as any other feasible alternative and a strictly higher score than any feasible alternative not chosen..

The reinforcement axiom of Smith (1973) and Young (1975) provides us with a way of extending these revealed scoring relations, often even when a particular pair (B, S) is not in $\mathcal{E}$. This axiom states that if each of the two disjoint committees makes the same choice, the union of those two committees has to follow it. It is easy to see that every scoring rule would satisfy it: thus, for instance, if $C(\{a, o\}, \{1, 2\}) = C(\{a, o\}, \{3, 4\}) = \{a\}$ we may not have $C(\{a, o\}, \{1, 2, 3, 4\}) = \{o\}$. Furthermore, individual preference revelation is sometimes possible in this framework as well: if one ever observes an individual choosing an alternative when alone, this reveals his/her preference that would be unchanged even when the budget set changes. This motivates the following extension of the score revelation

- *Reinforcement.* The binary relations $P_{B,S} \subset R_{B,S}$ on B are defined by

  (i) xP^*y implies xPy, xR^*y implies xRy,

  (ii) For any $B \in 2^X \setminus \{\varnothing\}$ and any $S, T \in 2^N \setminus \{\varnothing\}$ such that $S \cap T = \varnothing$, $xR_{B,S}y$ and $xR_{B,T}y$ imply that $xR_{B,S\cup T}y$

  (iii) For any $B \in 2^X \setminus \{\varnothing\}$ and any $S, T \in 2^N \setminus \{\varnothing\}$ such that $S \cap T = \varnothing$, $xP_{B,S}y$ and $xR_{B,T}y$ imply that $xP_{B,S\cup T}y$

  (iv) For any $B \in 2^X \setminus \{\varnothing\}$ and any $S, T \in 2^N \setminus \{\varnothing\}$ such that $S \subset T (T \setminus S \neq \varnothing)$, $xP_{B,S}y$ and $yR_{B,T}x$ imply that $yP_{B,T\setminus S}x$

(v) For any $B \in 2^X \setminus \{\varnothing\}$ and any $S, T \in 2^N \setminus \{\varnothing\}$ such that $S \subset T (T \setminus S \neq \varnothing)$, $xR_{B,S}y$ and $yP_{B,T}x$ imply that $yP_{B,T\setminus S}x$

(vi) For any $B \in 2^X \setminus \{\varnothing\}$ and any $i \in N$, $xP_{B,\{i\}}y$ implies $xR_{D,\{i\}}y$ for all $D \in 2^X \setminus \{\varnothing\}$

The statements $xP_{B,S}y$ (respectively, $xR_{B,S}y$) may be understood as "x is revealed (directly or indirectly) to have obtained a higher (respectively, at least as high) score than y in a vote by a committee S over the budget set B". Of course, no matter how obtained, scoring revelation cannot be self-contradicting. Thus, for instance, if $C(\{a, o\}, \{1, 2\}) = C(\{a, o\}, \{3, 4\}) = \{a\}$ one may not have $C(\{a, o\}, \{1, 2, 3, 4\}) = \{o\}$. In fact, since the binary relations $R_{B,S}$ and $P_{B,S}$ refer to the number of votes, the relation should be transitive (if more people vote for x than for y and more people vote for y than for z more people should be voting for x than for z).

This motivates the following simple axiom:

Axiom 1 (Committee Axiom of Revealed Preference (CARP)) [1]
For any $B \in 2^X \setminus \{\varnothing\}$, any $S \in 2^N \setminus \{\varnothing\}$ and any $x_1, x_2, ...x_n \in B$,
$x_1 R_{B,S} x_2, x_2 R_{B,S} x_3 ... x_{n-1} R_{B,S} x_n$ implies $\neg (x_n P_{B,S} x_1)$

Example 1 *Consider the budget set $B = \{a, b, c\}$ and the four disjoint committees S_1, S_2, S_3 and T. Let $C(B, S_1) = a$, $C(B, S_2) = b, C(B, S_3) = c$, $C(B, S_1 \cup T) = b, C(B, S_2 \cup T) = c, C(B, S_3 \cup T) = a$. It is not hard to see that this implies that $bP_{B,T}cP_{B,T}aP_{B,T}b$ which, of course, contradicts Axiom 1: committee T should be giving alternative b a higher score than alternative c, alternative c a higher score than alternative a, and alternative a the higher score than alternative b, which is impossible.*

Of course, as noted above, one may be able to make inferences about individual preferences, for instance, from direct or indirect observations of singleton coalitions, by defining an individual revealed preference relation P_i as follows:

- *Individual preference revelation.* If $xP_{B,\{i\}}y$ for some $B \in 2^X \setminus \{\varnothing\}$ define xP_iy.

[1] I am grateful to Professor Schofield for the naming suggestion for this axiom

As I have assumed individual rationality it is clear that the standard Strong Axiom of Revealed preference must hold for individual preference as well.

Axiom 2 (Strong Axiom of Revealed Preference (SARP)) *For any* $i \in N$ *and any* $x_1, x_2, ...x_n \in B, x_1 P_i x_2, x_2 P_i x_3 ... x_{n-1} P_i x_n$ *implies* $\neg (x_n P_i x_1)$

It is clear that both CARP and SARP would have to hold if a committee of rational individuals is deciding by sincere votes using a scoring rule, since otherwise we'd have to accept either cycles in individual preferences or in group scores (as in the example above). Hence, the main result of this paper:follows immediately from the construction.

Theorem 1 *A committee choice structure* $(\mathcal{E}, C(.,.))$ *may be generated by a scoring rule strictly consistent with rational preferences only if the implied* P_i *satisfies SARP for each* i *and the implied* $R_{B,S}$ *satisfies CARP for each* (B, S)*.*

It should be stressed that this result provides only a necessary and not a sufficient condition for rationalizability with scoring. In fact, counterexamples to the converse are not hard to generate, as possibilities for indirect score revelation are by no means exhausted with application of reinforcement.

Example 2 *Consider a budget set* $B = \{a, b\}$ *and four committees* S_1, S_2, T_1 *and* T_2 *such that* $S_i \cap T_j = \varnothing$*. Suppose* $\{a\} = C(B, T_1) = C(B, T_2) = C(B, S_1 \cup T_1) = C(B, S_2 \cup T_2)$*, while* $\{b\} = C(B, S_1 \cup T_2) = C(B, S_2 \cup T_1)$*. Of course, if these decisions were arrived to by scoring, this would imply (by reinforcement) that both* S_i *would have to be choosing* b *as well. The votes of* S_1 *are sufficient to overturn the preference of* T_2*, but not of* T_1 *for* a*, so we can conclude that the advantage in votes that* T_1 *gives to* a *is strictly bigger than that given by* T_2*. However, the votes of* S_2 *overturn the choice of* T_1*, but not the choice of* T_2*, so the vote advantage of* a *in* T_2 *is strictly bigger than that in* T_1*. Clearly, this is impossible, unless the scores assigned are not independent of committee membership.*

The establishment of the exact conditions for rationalizability with scoring is, thus, for the moment, an open question.

4 Conclusions and further research

So far it has been possible to establish a set of properties of committee choice structures that are necessary consequences of sincere scoring-based committee decisions It remains to see if this could be strengthened to a concise sufficient condition for rationalizability with scoring. An interesting further extension of the model would be to consider the consequences of particular scoring rules, such as plurality, approval or the Borda Count.

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