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Consolidated-Budget Rules and Macroeconomic Stability

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Fiscal policy that sets income taxes counter cyclically can cause macroeconomic instability by giving rise to multiple equilibria and consequently to fluctuations caused by self fulfilling expectations. This paper shows that consolidated budget rules with endogenous income-tax rates can be stabilizing. The size of the government, however, plays a key role. If government spending is not too large relative to private consumption, a Fisherian monetary policy [such that the real rate of interest is constant in and off the steady state] is stabilizing. Thus, small governments are more apt to carrying out optimal monetary policies along the lines of Khan, King and Wolman [Review of Economic Studies, 2003, (70)]. Calibration to the US economy shows that it is necessary that government consumption should not exceed half the size of private consumption. Furthermore, the sum of private and public consumptions should not exceed labor income. This result survives even when social returns are increasing.

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Abstract

Fiscal policy that sets income taxes counter cyclically can cause macroeconomic instability by giving rise to multiple equilibria and consequently to fluctuations caused by self fulfilling expectations. This paper shows that consolidated budget rules with endogenous income-tax rates can be stabilizing. The size of the government, however, plays a key role. If government spending is not too large relative to private consumption, a Fisherian monetary policy [such that the real rate of interest is constant in and off the steady state] is stabilizing. Thus, small governments are more apt to carrying out optimal monetary policies along the lines of Khan, King and Wolman [Review of Economic Studies, 2003, (70)]. Calibration to the US economy shows that it is necessary that government consumption should not exceed half the size of private consumption. Furthermore, the sum of private and public consumptions should not exceed labor income. This result survives even when social returns are increasing.

Key words: Fiscal Policy; Distorting Tax; Monetary Policy; Macroeconomic Stabilization; Finance Constraint;

1 Introduction

Different attempts to characterize optimal monetary policy often arrive at different conclusions. Notable recent attempts include Khan, King and Wolman (2003) [hereafter KKW] and Schmitt-Grohé and Uribe (2007) [hereafter SGU]. SGU study welfare maximizing fiscal and monetary rules in a model with sticky prices, money, and distortionary taxes where the monetary authority operates an interest rate rule. They assume that government spending is set so as to minimize costs, noting that this assumption is unrealistic. Their main findings are that the size of the inflation coefficient in the interest rate rule plays a minor role for welfare. It matters only insofar as it affects the determinacy of equilibrium. That welfare gains from interest rate smoothing are negligible and that optimal fiscal policy is passive. SGU find that optimal level of inflation is positive, though small.

In a stark contrast, KKW find that optimal monetary policy is governed by two familiar principles. First, the average level of nominal interest should be sufficiently low, as suggested by Milton Friedman, and that there should be deflation on average. Yet, Keynesian frictions imply that the optimal nominal interest rate is positive. Second, KKW emphasize that the price level should be stabilized as suggested by Irving Fisher, albeit around a deflationary trend path. KKW conclude that optimal monetary policy should choose real allocations that closely resemble those which would occur if prices were flexible and there is some tendency for the monetary authority to smooth nominal and real interest rates.

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This paper is centered on the question whether a plausible consolidated budget rule, as in SGU, is able to stabilize the economy near a long run equilibrium that has the characteristics of KKW. For that matter, and unlike most of previous literature it is assumed that the interest rate rule follows Fisherian lines, that is that the monetary authority sets the nominal rate of interest so as to maintain a constant real rate of interest. Unlike previous literature, that mostly emphasizes the Taylor principle as a guideline for a sound monetary policy, it seems that the rule under scrutiny can stabilize the economy contingent on the size of government consumption.

The present paper examines a consolidated budget setup where a fiscal authority taxes income and a monetary authority finance the primary deficit via seniorage. We follow Leeper (1991) and assume that the size of seniorage and its composition (bonds and money) are set exclusively by the monetary authority before the size of primary deficit is revealed. Only then the fiscal authority sets the rate of income tax so as to balance the consolidated budget. SGU provide evidence using a numerical model calibrated to the U.S. economy that this type of policy prescription is stabilizing. Here a formal proof is provided and the link between government consumption and macroeconomic stability is scrutinized.

Results show that a consolidated budget rule whereby the monetary authority sets the nominal interest so as to increase the real rate of interest during booms induces a determinate equilibrium. This result is consistent with the celebrated Taylor principle and with the results obtained in SGU. Unlike previous literature, results also show that with high degrees of intertemporal substitution, a small government can stabilize the economy by assuming a Fisherian monetary policy stance such that induces a constant real interest rate in and off the steady state and so capable of implementing an optimal policy in the sense of KKW. Economies that exhibit production externalities that increase with per capita capital yield a similar result. Low elasticity of intertemporal substitution, and thus a penchant to smooth consumption, will shrink government consumption in Ramsey stationary economies. Conversely, consumption smoothing allows a larger minimal size of government spending in endogenously growing economies, as long as total consumption in the economy is within the bound of labor income.

The rest of the paper is organized as follows: section 2 illustrates a model where a consolidated government runs a balanced budget. The composition of the budget and the restriction imposed on the two authorities (fiscal and monetary) are thoroughly described. The optimal program of a representative household is then scrutinized and local stability analysis of equilibrium is performed. It turns out that a policy such that imposes an increase in the expected real rate of interest during booms is sufficient to overcome the indeterminacies reported in Schmitt-Grohe and Uribe's (1997) and in Guo and Harrison (2008). Results change where the monetary authority implements a Fisherian policy. The link between monetary policy and the size of government spending is discussed in section 3. Section 4 extends the analysis to economies that exhibit production externalities associated with per capita capital. Section 5 concludes.

2 Consolidated Balanced Budget with Income Tax and Finance Constraints

We assume that the government is comprised of a fiscal authority and a monetary authority, and that the government runs a consolidated budget. Hence, assuming a monetary authority we implicitly assume the existence of money. Accordingly, money enters the economy via a cash-in-advance constraint on all transactions. The analysis is restricted to steady states where the nominal rate of interest is strictly positive. This assumption is in line with the upshot of Khan et. al. (2003) that the optimal nominal interest rate is positive. Furthermore, restricting attention to positive nominal interest rates conveniently allows us to avoid steady state multiplicity as emphasized at Benhabib et. al (2002).

2.1 The Economic Environment

2.1.1 The Government

It is assumed that the consolidated government prints money, M_t, issues nominal risk free bond, B_t, collects taxes in the amount of T_t and faces an exogenous stream of expenditure g_t. Its instantaneous dollar denominated budget constraint is given by

$$R_t B_t + P_t g_t = \dot{M}_t + \dot{B}_t + P_t T_t$$

where P_t is the level of nominal prices. It is assumed that the monetary authority implements an interest rate feedback rule. It imposes a desired interest rate,

R_t, by controlling the price of riskless nominal bonds and exchanging money for bonds at any quantities demanded at that price. In that sense, the nominal rate of interest is exogenous and M_t, B_t are endogenous.

The fiscal authority is then constrained to set T_t so as to balance the budget. It is assumed throughout the paper that $T_t = \tau_t [y_t - \delta q_t k_t]$ where τ_t denotes an income tax rate and it can vary with time, y_t is total income in the economy, and k_t denotes the stock of capital. The term $\delta q_t k_t$ represents a depratiation tax allowance where δ is a constant rate of capital depreciation and q_t denotes the market price of one unit of installed capital[1]. Accordingly, the nominal consolidated budget constraint is given by $R_t B_t + P_t g_t = \dot{M}_t + \dot{B}_t + P_t \tau_t [y_t - \delta q_t k_t]$.

let $m_t \equiv \frac{M_t}{P_t}$ and $b_t \equiv \frac{B_t}{P_t}$denote real money holdings and real bonds holdings, respectively. Also let $a_t \equiv b_t + m_t$ denote a measure of goverment liabilities denominated in consumption goods. Dividing both sides of the nominal instanteneous budget constraint by P_t and rearranging, yield that goverment liabilities evolve according to:

$$\dot{a}_t = (R_t - \pi_t) a_t - R_t m_t + [g_t - \tau_t (y_t - \delta q_t k_t)]$$

Where $\pi_t \equiv \frac{\dot{P}_t}{P_t}$ is the rate of change of nominal prices i.e. the rate of inflation.

In this economy, printing money to finance the primary deficit gives rise to inflation. As inflation erodes real liabilities it can be viewed as a source of

[1] In general, total tax revenues consist of lump sum taxation, income taxation, and revenues from firms' profits taxation. However, in our model, under perfect competition firms' profits are zero. Also, in this model and without loss of generality, I can ignore lump sum taxation as we know that it is not a source of indeterminacies.

revenue. Inflation therefore plays a role similar to that of a lump sum tax. The fiscal authority sets an exogenous stream of (real) expenditure $\{g_t\}_{t=0}^{\infty}$ that converges monotonically to a long run level denoted by g^*. Contingent on the realization of monetary policy, to be specified in following sections, the fiscal authority sets the income tax rate, τ_t, so as to balance the instantaneous budget of the consolidated government. It follows from the law of motion for a_t that goverment liabilities increase with the primary deficit, $g_t - \tau_t (y_t - \delta q_t k_t)$, and with the real rate of interest paid over outstanding debt, $(R_t - \pi_t) a_t$, and is decreasing with government income associated with money holdings, $R_t m_t$. In that sense, the opportunity cost of holding money operates as a lump sum tax.

It is assumed that monetary policy takes the form of an interest-rate feedback rule whereby the nominal interest rate is set as an increasing function of instantaneous inflation. Specifically, it is assumed that

$R_t = R(\pi_t)$ where π_t can also be interpreted as expected-inflation [2]. $R(\cdot)$ is continuous, non-decreasing and strictly positive, and there exists at least one steady-state, π^*, such that $R(\pi^*) = \rho + \pi^*$ where ρ denotes the rate of time preference of a representative household and π^* is a socially optimal inflation target and according to Khan et. al. (2003) $R(\pi^*) > 0$. It is further assumed that the monetary authority reacts to an increase (decrease) in the rate of inflation by increasing (decreasing) the nominal rate of interest.

[2] The instantaneous rate of inflation in a continuous-time setting is the right-derivative of the logged price level and thus, the discrete-time counterpart of a countinuous-time policy rule that sets the interest rate in response to the instanteneous rate of inflation is characterized by forward-looking policy that responds to expected future inflation.

2.1.2 Households

The economy is populated by a continuum of identical infinitely long-lived households, with measure one. Each household is endowed with perfect foresight and one unit of time which is supplied inelastically in the labor market. Accordingly, preferences are represented by a concave function $u(c_t)$,where c_t denotes consumption and the analysis throughout is carried out over intensive measures. It is assumed that consumption and money balances are complements. Accordingly, I impose a cash-in-advance constraint on consumption and so money enters the liquidity constraint. The representative household's lifetime utility function is given by

$$U = \int_0^\infty e^{-\rho t} u(c_t) dt$$

where $\rho > 0$ denotes the rate of time preference, c_t denotes consumption per capita, $u(\cdot)$ is twice differentiable, strictly increasing, and strictly concave. The household's wealth $w_t \equiv a_t + q_t k_t$ consists of financial assets and capital where q_t is the market price of capital in terms of the consumption good. Hence, real wealth evolves according to $\dot{w}_t = \dot{a}_t + \dot{q}_t k_t + q_t \dot{k}_t$. It is assumed that the law of motion for capital is $\dot{k}_t = I_t - \delta k_t$ where I_t denotes the flow of gross investment.

Substituting in the laws of motion for a_t and k_t yields that $\dot{w}_t = (R_t - \pi_t)\, a_t - R_t m_t + [g_t - \tau_t (y_t - \delta q_t k_t)] + \dot{q}_t k_t + q_t [I_t - \delta k_t]$.

Equilibrium in the goods market at the closed economy implies $c_t + I_t + g_t = y_t = f(k_t)$ where output per capita,$f(k_t)$, exhibits a constant returns to scale production technology and is twice differentiable, strictly increasing and strictly concave. Thus, substituting $g_t = f(k_t) - c_t - I_t$ into the law of motion of households' wealth and rearranging yields that the household's intertemporal budget constraint is:

$$\dot{w}_t = (R_t - \pi_t)\, w_t - R_t m_t + \left[\dot{q}_t - (R_t - \pi_t)\, q_t - (1-\tau_t)\delta q_t\right] k_t + (1-\tau_t) f(k_t) + [q_t - 1]\, I_t - c_t \tag{1}$$

The RHS of Eq. (1) demonstrates that essentially wealth grows at a rate that the monetary authority impose. However, holding money entails an opportunity cost that equals the nominal rate of interest. Also, capital entails an opportunity cost of $(R_t - \pi_t)\, q_t$ as well as depraeciation at an after tax rate of $(1-\tau_t)\delta q_t$. On the other hand, capital appreciates at a rate $\dot{q}_t$ and is also productive so that an after tax income $(1-\tau_t) f(k_t)$ is made available to the household. Finally, gross investment contributes a measure q to real wealth and it has an opportunity cost of -1. The intertemporal budget constraint demonstrates that from a private point of view, government expenditures are not an issue. The representative agent thus takes into account the income tax and interest rates imposed by the government policy stance. In that sense the budget constraint demonstrates that whereas the government sets its fiscal policy at the social level, this policy transforms into private level incentives via prices and taxes.

Money enters the economy via a liquidity constraint on all transactions:

$$\int_t^{t+\Gamma} [c(s) + I(s)]\, ds \leq m_t \tag{2}$$

that can be linearly approximated as [3]:

[3] This version of cash-in-advance is similar to Rebelo and Xie (1999) and Feenstra

$$\Gamma(c_t + I_t) \leq m_t \tag{3}$$

Finally, and without loss of generality, Γ is normalized to 1 and the household's lifetime maximization problem becomes

$$Max_{\{c_t, I_t, m_t, k_t\}} \int_0^\infty e^{-\rho t} u(c_t) dt$$

$s.t.$

$$\dot{w}_t = (R_t - \pi_t)\, w_t - R_t m_t + \left[\dot{q}_t - (R_t - \pi_t)\, q_t - (1-\tau_t)\delta q_t\right] k_t + (1-\tau_t) f(k_t) + [q_t - 1]\, I_t - c_t \tag{4}$$

$$c_t + I_t \leq m_t$$

With the following no-Ponzi-game condition: $Lim_{t\to\infty} w_t e^{-\rho t} = 0$.

The household's problem suggests that capital accumulation entails an opportunity cost due to a finance constraint. This specification is similar to Woodford (1986). This modeling choice was made so that conclusion drawn from this model will apply to a discrete time setup as well. In general, macroeconomic continuous time modeling could be misleading in the sense that it does not correctly approximate the behavior of the discrete time model of arbitrarily small periods. Therefore, special care should be taken with assumptions of the model that are not realistic for small period length. Carlstrom and

(1985). a Taylor series expansion gives $\int_t^{t+\Gamma} [c(s)+I(s)]ds = \Gamma[c(s)+I(s)] + \frac{1}{2}\Gamma^2[\dot{c}(t) + \dot{I}(t)] + \cdots$ and so $\Gamma(c+I) \leq m$ can be interpreted as a first-order approximation.

Fuesrt (2005) point out that modeling policy issues in continuous time could end up with conclusions that are opposite to the conclusions drawn from a discrete-time counterpart of the model. They attribute the opposite conclusions to the difference in timing in the no-arbitrage condition of investing in bonds and capital between the two settings: while the continuous-time setting entails a contemporaneous no-arbitrage condition, a similar no-arbitrage condition in the discrete-time setting involves only future variables which bring a zero eigenvalue into the linearized dynamic system. Gliksberg (2009) shows that introducing finance constraints as in Woodford (1986) is one way to overcome implausible contemporaneous features of no-arbitrage in continuous time macroeconomic models that enter at the "back door" as the period length gets shorter.

2.1.3 ***The optimal program***

Households choose sequences of $\{c_t, I_t, m_t, k_t\}$ so as to maximize lifetime utility, taking as given the initial stock of capital k_0, and the time path $\{\tau_t, R_t, \pi_t\}_{t=0}^{\infty}$ which is exogeneous from the view point of a household. The necessary conditions for an interior maximum of the household's problem are

$$u'(c_t) = \lambda_t + \zeta_t \tag{5a}$$

$$q_t - 1 = \frac{\zeta_t}{\lambda_t} \tag{5b}$$

$$\zeta_t = R_t \lambda_t \tag{5c}$$

$$\dot{q}_t = -(1-\tau_t) f'(k_t) + \left[R_t - \pi_t + (1-\tau_t)\delta\right] q_t \tag{5d}$$

$$\zeta_t (m_t - c_t - I_t) = 0; \zeta_t \geq 0 \tag{5e}$$

Where λ_t is a time-dependent co-state variable interpreted as the marginal valuation of wealth. ζ_t is a time-dependent Lagrange multiplier associated with the liquidity constraint and equation (5e) is the corresponding Kuhn-Tucker condition. Second, the co-state variables must evolve according to the law

$$\dot{\lambda}_t = \lambda_t \left[\rho + \pi_t - R_t\right] \tag{6}$$

Following Khan et. al. (2003) attention is restricted to steady states with a positive nominal interest rate. As a result, equation (5c) implies that ζ_t, the Lagrange multiplier associated with the liquidity constraint, is positive. It then follows from (5e) that $m_t = c_t + I_t$. The economic intuition is simple: near a steady state with a positive nominal interest rate holding money entails opportunity costs, and minimizing the opportunity cost of holding money implies that the liquidity constraint is binding. It then follows from equations (5a)-(5c) that near this steady state $u'(c_t) = \lambda_t(1 + R_t)$ and $q_t = 1 + R_t$

Finally, the law of motion for capital is

$$\dot{k}_t = f(k_t) - c_t - g_t - \delta k_t \tag{7}$$

and the transversality condition is $Lim_{t\to\infty}\lambda_t w_t e^{-\int_0^t [R(s)-\pi(s)]ds} = 0$.

Thus, equations (5d), (6) – (7) fully describe the optimal program of a representative household as it takes the time path $\{\tau_t, R_t, \pi_t\}_{t=0}^{\infty}$ as (exogenously) given.

2.2 General Equilibrium

In equilibrium, the goods market clear

$$f(k_t) = c_t + I_t + g_t \tag{8}$$

The gross rate of investment is set so as to equate the market price of an installed unit of capital to the opportunity cost of investing due to the finance constraint

$$q_t = 1 + R_t \tag{9}$$

Assets market clears so as to equate the marginal utility of consumption to the marginal valuation of wealth distorted by the liquidity constraint associated with consumption

$$u'(c_t) = \lambda_t(1 + R_t) \tag{10}$$

and the motion equations (5d), (6) – (7) display the evolution of $\{\lambda_t, q_t, k_t\}_{t=0}^{\infty}$

.

2.2.1 Equilibrium Dynamics

Conjecture that equilibrium in the underlying economy is a mapping of (λ, q, k).[from this point on the time notation is omitted for simplicity] In this section we will characterize the monetary-fiscal policy that induce a unique equilibrium. Note that equation (9) and the type of interst rate rule imply that

$$q = 1 + R(\pi) \tag{11}$$

it then follows that $\pi = \pi(q); \pi_\lambda = \pi_k = 0; \pi_q = \frac{1}{R'(\pi)}$ where subscripts denote partial derivatives and $R'(\pi)$ is the increment in percentage points to the nominal interest in response to a one percent increase in the rate of inflation relative to the target. Also, equations (9)-(10) imply that $u'(c) = \lambda q$ and therefore $c_\lambda = \frac{q}{u''(c)}; c_q = \frac{\lambda}{u''(c)}; c_k = 0;$

Thus, the dynamics of all the variables in the economy is a mapping in the (λ, q, k) space and the evolution of (λ, q, k) can be described by: $\dot{\lambda} = F(\lambda, q, k), \dot{q} = G(\lambda, q, k), \dot{k} = H(\lambda, q, k)$

where

$$F(\lambda, q, k) \equiv \lambda\left[\rho + \pi(q) - R(\pi(q))\right] \tag{12}$$
$$G(\lambda, q, k) \equiv -(1-\tau)f'(k) + \left[R(\pi(q)) - \pi(q) + (1-\tau)\delta\right]q \tag{13}$$
$$H(\lambda, q, k) \equiv f(k) - c(\lambda, q) - g - \delta k \tag{14}$$

and the transversality condition is $Lim_{t\to\infty} \lambda w e^{-\int_0^t [R(\pi(q))-\pi(q)]ds} = 0$

2.2.2 Equilibrium and Local Real Determinacy (LRD)

Following Evans and Guesnerie (2005) I consider only saddle-path stable solutions as macroeconomically stable.

Definition 1 *Equilibrium displays Local-Real-Determinacy (LRD) if there exists a Saddle-Path stable solution in the* (λ, q, k) *space. Otherwise equilibrium is non-LRD.*

Local-Real-Determinacy

Equations (10), (12)–(14) imply that in the steady state $R^* = \rho + \pi^*, q^* = 1+R^*, \lambda^* = \frac{u'(c^*)}{1+R^*}, f'(k^*) = \frac{(1+R^*)[\rho+\delta(1-\tau^*)]}{1-\tau^*}$. Linear approximation to equations (12)–(14) near the steady state (λ^*, q^*, k^*) is obtained through the system

$$\begin{bmatrix} \dot{\lambda} \\ \dot{q} \\ \dot{k} \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & \frac{u'(c^*)}{(1+R^*)}\frac{1-R'(\pi^*)}{R'(\pi^*)} & 0 \\ 0 & \rho+\delta(1-\tau^*)+(1+R^*)\frac{R'(\pi^*)-1}{R'(\pi^*)} & -(1-\tau^*)f''(k^*) \\ -\frac{1+R^*}{u''(c^*)} & -\frac{u'(c^*)}{u''(c^*)(1+R^*)} & f'(k^*)-\delta \end{bmatrix}}^{A} \begin{bmatrix} \lambda-\lambda^* \\ q-q^* \\ k-k^* \end{bmatrix} \tag{15}$$

where $R'(\pi^*)$ is the increment in percentage points to the nominal interest in response to a one percent increase in the rate of inflation relative to the target and τ^* is the rate of income tax that balances the consolidated budget in the steady state where π^* prevails. Specifically, τ^* is the solution to $0 = \rho a^* - (\rho+\pi^*)\, m^* + g^* - \tau^* \left[f(k^*) - \delta k^* \left(1+\rho+\pi^*\right)\right]$

Let α_i *(i=1,2,3)* denote the eigenvalues of matrix A, then,

$$\alpha_1\alpha_2\alpha_3 = \frac{R'(\pi^*)-1}{R'(\pi^*)}\left[-\frac{u'(c^*)}{u''(c^*)}\right](1-\tau^*)f''(k^*) \tag{16}$$

$$\alpha_1+\alpha_2+\alpha_3 = \rho-\delta\tau^*+f'(k^*)+(1+R^*)\frac{R'(\pi^*)-1}{R'(\pi^*)} \tag{17}$$

It is now straightforward to show the affect that fiscal and monetary policies have over the determination of allocation. In the tradition of RBC literature that discuss indeterminacy of the real variables, indeterminacy means that from the same initial conditions there exist an infinite number of trajectories all of which converge to a common stationary equilibrium. This outcome allows for the existence of sunspot equilibria - that is, equilibrium allocations influenced by purely extrinsic beliefs unrelated to the economy's fundamentals (see Cass and Shell (1983)). Such sunspot equilibria provide a modern interpretation of Keynes's hypothesis that economic fluctuations are driven by "animal spirits". It is our goal, in the current context, to provide policy guidelines so as to induce a unique determination of equilibrium allocation thus preventing expectations driven fluctuations.

Note the Euler equation (6) as it holds the key to the macroeconoomic dynamics. Note that λ denotes marginal utility of consumption distorted by the cash-in-advance constraint. It is straightforward to obtain the link between the evolutions of λ,private consumption and the valuation of capital by time deriving both sides of equation (6) . Lemma2 establishes this link:

Lemma 2 $\frac{\dot{\lambda}}{\lambda} = -\frac{1}{\sigma}\frac{\dot{c}}{c} - \frac{\dot{q}}{q}$ *where* $\frac{1}{\sigma} \equiv -\frac{u''(c)c}{u'(c)}$

Also, it follows from equations (16)–(17) that regardless of the stance of the fiscal authority, an active monetary policy - such that increases the real interest in response to an increase in expected inflation - is stabilizing (though according to Khan et.al (2003) not necessarily optimal) and a passive monetary policy is distabilizing.

Proposition 3 $R'(\pi^*) > 1 \Rightarrow$*Equilibrium is LRD.* $R'(\pi^*) < 1 \Rightarrow$*Equilibrium is non-LRD*

(Proof in Appendix A)

In what follows I illustrate the intuition that underlies proposition 3. In the absense of a monetary authority, as in Schmitt-Grohe and Uribe's (1997) balanced budget constraint, when agents become optimistic about the future of the economy and decide to work harder and invest more, the government is forced to lower the tax rate as total output rises. The countercyclical tax policy will help fulfill agents' initial optimistic expectations, thus leading to indeterminacy of equilibria and endogenous business cycle fluctuations.

However, when a consolidated budget is considered, the monetary authority has a control over the real interst rate via financial markets. Suppose that the economy shifts away from the steady state as a result of a positive shock to expected productivity. In terms of the model, the stock of capital is now below its steady state level, and the marginal product of capital is higher than its steady state level. The nominal interest rate would consequently rise because initially, the real interest rate has increased. In order to finance the increase in payments following the rise of the real interest rate, inflation tax revenues must increase which in turn further increases the nominal interest rate.

At the next instant, the stance of the monetary authority is carried out in the open market. Under the active stance, the monetary authority increases the rate of bond creation relative to the rate effective prior to the shock, thus driving the real interest rate above its steady state level. This policy effects households' allocation between investment and consumption via an arbitrage channel.

Under the active stance the real interest rate is above its steady state level, and according to the Euler equation (6) it induces a negative growth rate in λ. According to Lemma 2 this also implies that $\frac{1}{\sigma}\frac{\dot{c}}{c}+\frac{\dot{q}}{q} > 0$ thus implying that the marginal utility of consumption grows at a faster rate than the rate of growth of the marginal valuation of installed capital. This characterizes an allocation which further distances the economy from the steady state regardless of fiscal policy. It is this mechanism that prevents optimism that is not anchored in fundamentals from becoming self fulfilling.

Under the passive stance the real interest rate is below its steady state level, and according to the Euler equation and Lemma 2 this also implies that $\frac{1}{\sigma}\frac{\dot{c}}{c}+\frac{\dot{q}}{q} < 0$ thus implying that the marginal utility of consumption grows at a slower rate than the rate of decline of the marginal valuation of installed capital. This characterizes an allocation that converges to the steady state, and any such trajectory is consistent with equilibrium. Thus, optimism under a passive monetary stance is self fulfilling.

3 Fisherian monetary policy

Khan et. Al. (2003) find that optimal monetary policy is governed by two familiar principles: First, the average level of the nominal interest rate should be sufficiently low and yet that frictions imply that the optimal nominal interest rate is positive. And second, as various shocks occur to the real and monetary sectors, the price level should be largely stabilized, as suggested by Inving Fisher. Khan et. Al. (2003) conclude that although in their model economy price adjustments are costly and firms are imperfectly competitive, the monetary authority chooses real allocations that closely resemble those which occur if prices were flexible. Furthermore, in their benchmark model, there is a tendency to smooth nominal and real interest rates. In what follows, the determinants of optimal monetary policy set forth in Khan et. Al (2003) are scrutinized in a flexible prices economy. In particular, as optimal policy emphasizes real interest rate smoothing, the discussion focus is on whether a Fisherian monetary policy stance is consistent with a unique determination of equilibrium.

A Fisherian monetary policy stance, where the formal representation is $R'(\pi^*) =$

1, implies that in our model economy the real interest rate remains constant in and off the steady state and equals ρ. According to equation (16) this policy introduces a zero eigenvalue.

Accordingly, a linear approximation to equations (12)–(14) near the Fisherian policy steady state (q^*, k^*) is obtained through the system

$$
\begin{bmatrix} \dot{q} \\ \dot{k} \end{bmatrix} = \overbrace{\begin{bmatrix} \rho + \delta(1-\tau^*) & -(1-\tau^*)f''(k^*) \\ -\frac{u'(c^*)}{u''(c^*)(1+R^*)} & f'(k^*) - \delta \end{bmatrix}}^{B} \begin{bmatrix} q - q^* \\ k - k^* \end{bmatrix} \tag{18}
$$

and the stability consequences of a Fisherian monetary policy are summerized in the following proposition:

Proposition 4 *A Fisherian monetary policy stance induces an LRD equilibrium iff* $\frac{g^*}{c^*} < \sigma(c^*)\varphi(k^*) + \omega^* - 1$ *where* $\sigma(c^*) \equiv -\frac{u'(c^*)}{c^* u''(c^*)}, \varphi(k^*) \equiv -k^* \frac{f''(k^*)}{f'(k^*)}, \omega^* \equiv \frac{f(k^*) - k^* f'(k^*)}{c^*}$

(Proof in Appendix A)

$\sigma(c^*)$ and $\varphi(k^*)$ measure the intertemporal elasticity of consumption substitution and the elasticity of marginal product of capital near the steady state, respectively. ω equals labor income divided by consumption expenditure.

Consider for example an economy where $u(c_t) = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$ and where production technology is of the form $f(k_t) = Ak_t^{\alpha}$. Then, the elasticities of intertemporal substitution and marginal productivity are constant and equal $\sigma, 1-\alpha$, respectively. Thus, for this economy, a Fisherian monetary policy stance is stabilizing only if and only if $\frac{\bar{g}}{c^*} < \sigma(1-\alpha) + \omega^* - 1$.

If we combine government and private consumption, the ratio of wage income to consumption has been approximately 1 since 1890 in U.S. data. Bennett and Farmer (2000) assume that ω^* is approximately equal to 1. Basu and Kimball (2002) indicate that taking nominal wages and salaries from the U.S

National Income Accounts and dividing by nominal spending on non-durable consumption and services gives an average ratio of 0.9. Basu and Kimball (2002) use the prices as perceived by consumers so they define the ratio of wage income to consumption using after-tax wage. This excersize yields 0.8 as their preferred value. It is thus plausible to assume $\omega^* \in [0.8, 1]$. The elasticity of intertemporal substitution in consumption is usually shown to be low. Campbell (1999) and Kocherlakota (1996) suggest the interval (0.2, 0.8). Finally, the common assessment for α is approximately 0.3. Thus, in view of the ranges of structural parameters, the threshold for $\frac{g^*}{c^*}$ necessary to induce a LRD equilbrium where monetary policy is neutral is in the range [0,0.56].

4 Equilibria with Externalities

Benhabib et. al. (2000) show that a small divergence between the social and private returns in multisector growth model is sufficient for multiple equilibria. In the previous section a consolidated-budget rule was found to induce LRD in a single sector growth model where income taxes distort private returns. In what follows I carry out a similar exercise while assuming that the production technology exhibits an externality associated with per capita capital[4]. Production externalities enter the model economy as in Kehoe et al.

[4] One can extend the analysis of previous sections by the inclusion of useful government spending. Guo and Harrison (2008) show that in the absence of a monetary authority, externalities related to government spending act simply as a scaling constant in either firms' production or households' utility function. Therefore, subsequent to Guo and Harrison (2008) we should look for external effects in production that do not derive from goverment purchases.

(1992) and Rebelo and Xie (1999). Suppose that the production function, $f(k_{p,t}, k_{a,t})$, exhibits a positive externality where k_p, k_a are private capital stock and per capita capital stock in the entire economy, respectively. $f(\cdot,\cdot)$ is strictly increasing in both arguments and concave in k_p and continuously differentiable. The representative household's optimal program given the initial stock of private capital $k_{p,0}$, the per capita stock of capital $k_{a,0}$, and the time paths of $\{\tau, R, \pi\}$ maximizes the current value hamiltonian $H \equiv u(c) + \lambda\left[(R-\pi)w + \left[\dot{q} - (R-\pi)q - (1-\tau)\delta q\right]k_p + (1-\tau)f(k_p, k_a) + [q-1-R]I - (1+R)c\right]$; hence, the optimality conditions associated with the household's problem are:

$$\lambda = \frac{u'(c)}{1+R} \tag{19}$$
$$q = 1+R \tag{20}$$
$$\dot{\lambda} = \lambda\left[\rho + \pi - R\right] \tag{21}$$
$$\dot{q} = -(1-\tau)f_1(k,k) + \left[R - \pi + (1-\tau)\delta\right]q \tag{22}$$
$$\dot{k} = f(k,k) - c - g - \delta k \tag{23}$$

and the transversality condition is $Lim_{t\to\infty}\lambda w e^{-\int_0^t [R(s)-\pi(s)]ds} = 0$

Where subscripts denote partial derivatives, and the condition for a symmetric equilibrium, $k_p = k_a = k$, is substituted into equation (22) only after the derivative of H with respect to k_p is taken.

4.1 The Government

The real value of the government's liabilities evolves according to $\dot{a} = [R-\pi]a - Rm + [g - \tau[f(k,k) - \delta qk]]$ whereas the interest rate feedback rule is of the form $R = R\left[\pi(\lambda, q, k)\right]$. The government, unlike the households sector, internalizes the externality.

4.2 General Equilibrium

Consumption per capita and the rate of inflation are set in general equilibrium. Also, the fiscal and monetary policy are set so as to obtain a solution to the central planner's problem . Hence, these magnitudes are derived as if the central planner internalizes the externality. In equilibrium, the goods market clear

$$f(k, k) = c + I + g \tag{24}$$

Also, in equilibrium the rate of investment is set so as to equate the ratio between marginal valuations of financial wealth and productive capital to the opportunity cost which is the gross rate of interest, and assets market clears so as to equate the marginal utility of consumption to the marginal valuation of financial wealth. Thus, equation (9) - (10) hold and the motion equations (21) – (23) display the evolution of $\{\lambda, q, k\}_{t=0}^{\infty}$.

4.2.1 Equilibrium Dynamics

Conjecture that equilibrium in the underlying economy is a mapping of (λ, q, k). In this section we will characterize the monetary-fiscal policy that induce an LRD equilibrium. Note that equation (21) and the type of interst rate rule imply that

$$\eta = 1 + R(\pi) \tag{25}$$

it then follows that $\pi = \pi(\eta); \pi_\lambda = \pi_k = 0; \pi_\eta = \frac{1}{R'(\pi)}$ where subscripts denote partial derivatives and $R'(\pi)$ is the increment in percentage points of the nominal interest to a one percent increase in the rate of inflation relative to the target. Also, equations (19)-(20) imply that $u'(c) = \lambda q$ and therefore $c_\lambda = \frac{\eta}{u''(c)}; c_\eta = \frac{\lambda}{u''(c)}; c_k = 0;$

The dynamics of all the variables in the economy can thus be described by (λ, q, k) and the evolution of (λ, q, k) can be described by:

$$\dot{\lambda} = \lambda \left[\rho + \pi(q) - R(\pi(q))\right]$$

$$\dot{q} = -(1-\tau) f_1(k,k) + \left[R(\pi(q)) - \pi(q) + (1-\tau)\delta\right] q$$

$$\dot{k} = f(k,k) - c(\lambda, q) - g - \delta k$$

4.2.2 Equilibrium and Local Real Determinacy (LRD)

Local-Real-Determinacy

In the steady state $R^* = \rho + \pi^*, q^* = 1 + R^*, \lambda^* = \frac{u'(c^*)}{1+R^*}, f_1(k^*, k^*) = \frac{(1+R^*)[\rho+\delta(1-\tau^*)]}{1-\tau^*}$. Linear approximation near the steady state is obtained through the system

$$\begin{bmatrix} \dot{\lambda} \\ \\ \dot{q} \\ \\ \dot{k} \end{bmatrix} = Đ \times \begin{bmatrix} \lambda - \lambda^* \\ \\ q - q^* \\ \\ k - k^* \end{bmatrix}$$

where Đ≡

$$\begin{bmatrix} 0 & \frac{u'(c^*)}{1+R^*}\frac{1-R'(\pi^*)}{R'(\pi^*)} & 0 \\ \\ 0 & \rho+\delta(1-\tau^*)+(1+R^*)\frac{R'(\pi^*)-1}{R'(\pi^*)} & -(1-\tau^*)\left[f_{11}(k^*,k^*)+f_{12}(k^*,k^*)\right] \\ \\ -\frac{1+R^*}{u''(c^*)} & -\frac{u'(c^*)}{u''(c^*)(1+R^*)} & f_1(k^*,k^*)+f_2(k^*,k^*)-\delta \end{bmatrix}$$

Let β_i *(i=1,2,3)* denote the eigenvalues of matrix Đ, thus,

$$\beta_1\beta_2\beta_3 = \frac{R'(\pi^*)-1}{R'(\pi^*)}\left[-\frac{u'(c^*)}{u''(c^*)}\right](1-\tau^*)\left[f_{11}(k^*,k^*)+f_{12}(k^*,k^*)\right]$$
$$\beta_1+\beta_2+\beta_3 = \rho-\delta\tau+(1+R^*)\frac{R'(\pi^*)-1}{R'(\pi^*)}+f_1(k^*,k^*)+f_2(k^*,k^*)$$

Proposition 5 *In an economy where marginal product of capital is non-increasing in the social level, an active monetary policy stance within a consolidated-budget rule induces an LRD equilibrium.*

(Proof in Appendix A)

Consider an economy where $f(k_{p,t},k_{a,t}) = Ak_{a,t}^{1-\alpha+\varepsilon}k_{p,t}^{\alpha}$, where $Ak_{a,t}^{1-\alpha+\varepsilon}$ can be interpreted as total factor productivity that exhibits spillovers that increase with per capita capital. It is assumed that in the private level production technology exhibits constant returns to scale and that α measures the share of private capital in production. In this economy in a symmetric equilibrium $f(k,k) = Ak^{1+\varepsilon}$, accordingly $f_{11}(k,k) + f_{12}(k,k) = \alpha\varepsilon Ak^{\varepsilon-1}$ and $\beta_1\beta_2\beta_3 = \frac{R'(\pi^*)-1}{R'(\pi^*)}\left[-\frac{u'(c^*)}{u''(c^*)}\right](1-\tau^*)\alpha\varepsilon Ak^{\varepsilon-1}$.

Where $\varepsilon < 0$ the marginal product of capital in non increasing in the social level and our economy exhibits dynamics that are similar to those of the economy in section 2. Specifically, the propositions in section 2 hold where $\varepsilon < 0$.

Consider an economy where $\varepsilon = 0$. This would imply an Ak type economy. Note that $\beta_1\beta_2\beta_3 = 0$ in an Ak economy regardless of the monetary policy stance. It is however straightforward to show that in an Ak type economy a linear approximation to the dynamics near the steady state under a Fisherian monetary policy is obtained by

$$\begin{bmatrix} \dot{q} \\ \dot{k} \end{bmatrix} = \overbrace{\begin{bmatrix} \rho + \delta(1-\tau^*) & 0 \\ -\frac{u'(c^*)}{u''(c^*)(1+R^*)} & f_1(k^*,k^*) - \delta \end{bmatrix}}^{\check{\mathrm{D}}} \begin{bmatrix} q - q^* \\ k - k^* \end{bmatrix}$$

It is obvious that both eigenvalues of $\check{\mathrm{D}}$ have positive real parts which demonstraits that the economy jumps to the BGP.

Consider an economy where $\varepsilon > 0$. This would imply a Benhabib et. al. (2000) type economy, where divergence between private and social returns prevails. In such an economy an active monetary policy, that is $R'(\pi^*) > 1$, induces $\beta_1\beta_2\beta_3 > 0, \beta_1 + \beta_2 + \beta_3 > 0$. that means that either all eigenvalues are instable or that we have two stable eigenvalues and one instable eigenvalue, and it is impossible at this point to determine the types of eigenvalues based on a qualitative analysis.

Consider now an economy where $\varepsilon > 0$ and a monetary authority that implements a Fisherian monetary policy. A linear approximation to the dynamics near the steady state under a Fisherian monetary policy is obtained by

$$\begin{bmatrix} \dot{q} \\ \dot{k} \end{bmatrix} = \overbrace{\begin{bmatrix} \rho + \delta(1-\tau^*) & -(1-\tau^*)\alpha\varepsilon A k^{\varepsilon-1} \\ -\frac{u'(c^*)}{u''(c^*)(1+R^*)} & \alpha A k^{\varepsilon} - \delta \end{bmatrix}}^{\breve{G}} \begin{bmatrix} q - q^* \\ k - k^* \end{bmatrix}$$

and the stability consequences of a Fisherian monetary policy are summerized in the following proposition:

Proposition 6 *Where marginal product of capital is increasing in the social level and the monetary authority implements a Fisherian stance equilibrium is LRD iff* $\frac{g^*}{y^*} < 1 - \alpha - [1 + \sigma\varepsilon]\frac{c^*}{y^*}$ *where* y^* *denotes total output.*

(Proof in Appendix A)

As expected, in this economy ε and the elasticity of intertemporal substitution play are critical in the ability of the government to stabilize the economy. Consider for example an economy where elasticity of intertemporal substitution is sufficiently low and the returns in the social level are close to unity. Thus, at a first approximation $\sigma\varepsilon \approx 0$. The upshot for such an economy is that a consolidated nudget rule based on a Fisherian monetary policy can stabilize the economy if the share of goverment consumption relative to total output does not exceed $1 - \alpha - \frac{c^*}{y^*}$

5 Conclusion

The paper is centered on the question whether a plausible consolidated budget rule is able to stabilize the economy near a long run equilibrium that has the characteristics of Khan, King and Wolman (2003). For that matter, and unlike most of previous literature it is assumed that the interest rate rule follows Fisherian lines, that is the monetary authority sets the nominal rate of interest so as to maintain a constant real rate of interest. Unlike previous literature, that mostly emphasizes the Taylor principle as a guideline for a sound monetary policy, the rule under scrutiny stabilizes the economy contingent on the size of government consumption.

Results show that a consolidated budget rule whereby the monetary authority sets the nominal interest so as to increase the real rate of interest during booms induces a determinate equilibrium. This result is consistent with the celebrated Taylor principle and with the results obtained in Schmitt-Grohé and Uribe (2007). Unlike in previous literature, results show that with high degrees of intertemporal substitution, a small government can stabilize the economy by assuming a Fisherian monetary policy stance such that induces a constant real interest rate in and off the steady state. Economies that exhibit production externalities that increase with per capita capital yield a similar result. A plausible interpretation is that a prerequisite for stabilizing the economy along the policy line set forth by Khan, King and Wolman (2003) is that total consumption - which include private and government consumption - should not exceed labor income. Low elasticity of intertemporal substitution, and thus the desire to smooth consumption, will shrink government consumption in a Ramsey stationary economy. Conversely, consumption smoothing allows a higher minimal government size in endogenously growing economies, as long as total consumption is bounded by labor income.

Appendix A

Proof of Proposition 3

Consider an active stance, i.e. $R'(\pi^*) > 1$: Note the right hand side of equation 16. When monetary policy is active the product of eigenvalues is negative, which imply that either there is one negative eigenvalue and two eigenvalues with positive real parts, or all three eigenvalues are negative. Note equation 17. Under an active stance the sum of eigenvalues is positive which rules out the possibility that all the eigenvalues are negative. With one negative eigenvalue and one predetermined state variable the equilibrium is saddle-path stable.

Consider the passive stance: The passive policy implies the product of eigenvalues is positive which implies that either two eigenvalue are stable and one is unstable, or that all three eigenvalues are unstable. And in this case equilibrium is non-LRD. QED.

Proof of proposition 4

Consider a monetary policy stance such that $R'(\pi^*) = 1$: under this policy stance the real interest rate is constant in and off the steady state and equals ρ.Consequently, as we can see from the euler equation 6, the distorted marginal utility of consumption is constant and the time path of all the variables in the economy is spanned by $\{q, k\}$. Accordingly, the evolution of $\{\lambda, q, k\}$ is

$$\dot{\lambda} = 0$$

$$\dot{q} = -(1-\tau)f'(k) + q\left[\rho + \delta(1-\tau)\right]$$

$$\dot{k} = f(k) - c\,(q) - g - \delta k$$

equilibrium dynamics also implies that $\pi_k = 0; \pi_q = 1$ and $c_q = \frac{u'(c)}{u''(c)(1+R)}; c_k = 0;$

Linear approximation near the steady state is obtained through

$$\begin{bmatrix} \dot{q} \\ \\ \dot{k} \end{bmatrix} = \overbrace{\begin{bmatrix} \rho + \delta(1-\tau^*) & -(1-\tau^*)f''(k^*) \\ \\ -\frac{u'(c^*)}{u''(c^*)(1+R^*)} & f'(k^*) - \delta \end{bmatrix}}^{B} \begin{bmatrix} q - q^* \\ \\ k - k^* \end{bmatrix}$$

Equilibrium is LRD iff the product of eigenvalues, denoted as $\alpha_1\alpha_2$, is negative.

Note that $\alpha_1\alpha_2 = \left[\rho + \delta(1-\tau^*)\right]\left[f'(k^*) - \delta\right] - \left[(1-\tau^*)f''(k^*)\right]\frac{u'(c^*)}{u''(c^*)(1+R^*)}$

and in the steady state $\rho + \delta\,(1-\tau^*) = \frac{(1-\tau^*)f'(k^*)}{(1+R^*)}$.

Thus,

$$\alpha_1\alpha_2 = \frac{(1-\tau^*)f'(k^*)}{(1+R^*)}\left[f'(k^*) - \delta\right] - \left[(1-\tau^*)f''(k^*)\right]\frac{u'(c^*)}{u''(c^*)(1+R^*)} = \frac{(1-\tau^*)f'(k^*)}{(1+R^*)}\left[\left[f'(k^*) - \delta\right] - \frac{f''(k^*)}{f'(k^*)}\frac{u'(c^*)}{u''(c^*)}\right]$$

$$= \frac{(1-\tau^*)f'(k^*)}{(1+R^*)}\frac{c^*}{k^*}\left[\frac{k^*}{c^*}\left[f'(k^*) - \delta\right] - \frac{k^*f''(k^*)}{f'(k^*)}\frac{u'(c^*)}{u''(c^*)c^*}\right]$$

finally $\alpha_1\alpha_2 < 0 \iff \frac{k^*}{c^*}\left[f'(k^*) - \delta\right] - \frac{k^*f''(k^*)}{f'(k^*)}\frac{u'(c^*)}{u''(c^*)c^*} < 0$

observe the expression $\frac{k^*f'(k^*) - \delta k^*}{c^*}$. note that, in the steady state equals $\delta k^* =$

$f(k^*) - c^* - g^*$ which is the steady state rate of investment. accordingly, $\frac{k^* f'(k^*) - [f(k^*) - c^* - g^*]}{c^*} = 1 - \frac{f(k^*) - k^* f'(k^*)}{c^*} + \frac{g^*}{c^*}$.

hence $\alpha_1 \alpha_2 < 0 \Longleftrightarrow 1 - \frac{f(k^*) - k^* f'(k^*)}{c^*} + \frac{g^*}{c^*} < \frac{k^* f''(k^*)}{f'(k^*)} \frac{u'(c^*)}{u''(c^*) c^*}$

QED.

Proof of proposition 5

Marginal product of capital at the social level is non increasing iff $f_{11}(k^*, k^*) + f_{12}(k^*, k^*) \leq 0$. Consider an active stance, i.e. $R'(\pi^*) > 1$. When monetary policy is active the product of eigenvalues is negative, which imply that either there is one negative eigenvalue and two eigenvalues with positive real parts, or all three eigenvalues are negative. Under an active stance the sum of eigenvalues is positive which rules out the possibility that all the eigenvalues are negative. Thus, equilibrium is saddle-path stable.

Consider the passive stance: The passive policy implies the product of eigenvalues is positive which implies that either two eigenvalue are stable and one is unstable, or that all three eigenvalues are unstable. And in this case equilibrium is non-LRD. QED.

Proof of proposition 6

$Det(\breve{G}) = [\rho + \delta(1 - \tau^*)] [\alpha A k^{\varepsilon} - \delta] - [(1 - \tau^*) \alpha \varepsilon A k^{\varepsilon - 1}] \frac{u'(c^*)}{u''(c^*)(1 + R^*)}$

and in the steady state $\rho + \delta(1 - \tau^*) = \frac{(1 - \tau^*) f_1(k^*, k^*)}{(1 + R^*)}$.

Thus,

$Det(\breve{G}) = \frac{(1-\tau^*)f_1(k^*,k^*)}{(1+R^*)}\left[\alpha Ak^{\varepsilon} - \delta\right] - \left[(1-\tau^*)\alpha\varepsilon Ak^{\varepsilon-1}\right]\frac{u'(c^*)}{u''(c^*)(1+R^*)} = \frac{(1-\tau^*)\alpha Ak^{\varepsilon}}{(1+R^*)}\left[\left[\alpha Ak^{\varepsilon} - \delta\right] - \frac{\alpha\varepsilon Ak^{\varepsilon-1}}{\alpha Ak^{\varepsilon}}\frac{u'(}{u''(}\right.$

$= \frac{(1-\tau^*)\alpha Ak^{\varepsilon}}{(1+R^*)}\frac{c^*}{k^*}\left[\frac{k^*}{c^*}\left[\alpha Ak^{\varepsilon} - \delta\right] - \varepsilon\frac{u'(c^*)}{u''(c^*)c^*}\right]$

finally $Det(\breve{G}) < 0 \Longleftrightarrow \frac{\alpha Ak^{1+\varepsilon} - \delta k^*}{c^*} - \varepsilon\frac{u'(c^*)}{u''(c^*)c^*} < 0$

observe the expression $\frac{\alpha Ak^{1+\varepsilon} - \delta k^*}{c^*}$. note that, in the steady state equals $\delta k^* = f(k^*, k^*) - c^* - g^*$ and that $\alpha Ak^{1+\varepsilon} = \alpha f(k^*, k^*)$

let $y^* \equiv f(k^*, k^*)$ denote total output. Accordingly, rearranging $\frac{\alpha y^* - [y^* - c^* - g^*]}{c^*} - \varepsilon\frac{u'(c^*)}{u''(c^*)c^*}$

yields that $Det(\breve{G}) < 0 \Longleftrightarrow \frac{g^*}{y^*} < (1-\alpha) - (1+\sigma\varepsilon)\frac{c^*}{y^*}$

QED.

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