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International security, insurance, and protection: failure of the conventional model of alliances

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Abstract

Olson and Zeckhauser (OZ, 1966) proposed a now famous theory of security alliances that has evolved into a general paradigm of group good allocation behavior. Once "security" is disaggregated into more realistic components than OZ's amorphous measure, we show the following new results. First, for a single agent we show that if market insurance is unavailable so that reliance on self-insurance alone is necessary, important new issues are raised as to the definition of "fair pricing" and inferiority with implications that are significantly different from the conventional market oriented analysis. Next, when insurance and protection are taken to be international public goods, we show how rather innocuous assumptions concerning countries' preferences lead to pervasive goods inferiority for at least self-insurance --- an unwelcome effect that becomes more severe when insurance must be self-provided rather than purchased in a market. Consequently, when "defense" or "security" is disaggregated into more realistic categories instabilities and corner solutions may more likely occur than in the conventional standard. Instable outcomes and rampant free-riding become a norm for both self insurance and self protection. We also investigate the welfare effects of economic growth and income redistribution by providing numerical examples. Our results suggest that negative income effects create conflicts and other difficulties within countries managing insurance and protection.

This paper extends parts of "National Adversity: Managing insurance and protection," presented to a Conference on "The Causes and Consequences of Conflict," Wissenschaftszentrum Berlin (WZB), Germany, March 28-29, 2008 and PET conference 2008, Seoul, Korea June 28. The authors thank Robin Boadway, Richard Cornes, Magnus Hoffman, Kai Konrad, and other conference participants for insightful comments on that earlier paper.
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International Security, Insurance, and Protection: Failure of the Conventional Model of Alliances

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Abstract

Over forty years ago Olson and Zeckhauser (OZ, 1966) proposed a now famous theory of security alliances that has evolved into a general paradigm of group good allocation behavior. Once "security" is disaggregated into more realistic components than OZ's amorphous measure, as a template for resource allocation in security alliances, we show the following new results. First, for a single agent we show that if market insurance is unavailable so that reliance on self-insurance alone is necessary, important new issues are raised as to the definition of "fair pricing" and inferiority with implications that are significantly different from the conventional market oriented analysis. Next, when insurance and protection are taken to be international public goods shared among countries, we show how rather innocuous assumptions concerning countries' preferences lead to pervasive goods inferiority for at least self-insurance --- an unwelcome effect that becomes more severe when insurance must be self-provided rather than purchased in a market. Consequently, when "defense" or "security" is disaggregated into more realistic categories instabilities and corner solutions may more likely occur than in the conventional standard. Instable outcomes and rampant free-riding become a norm for both self insurance and self protection. We also investigate the welfare effects of economic growth and income redistribution by providing numerical examples. Our qualitative and quantitative results suggest that negative income effects create conflicts and other difficulties within countries managing insurance and protection.

Keywords: self-insurance, self-protection, actuarially fair condition, inferior goods, alliances, public goods
F50, H41, H50, D81.

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1. Introduction

Among the first to examine combinations of instruments open to individuals to manage risks to their well-being were Ehrlich and Becker ("EB," 1972). EB identified several types of preparation available to expected utility maximizing agents faced with what we will call "costs of emergency." These costs consist of any mix of (a) probability of loss and (b) magnitude of loss (hereafter together referred to as "risk profile"). Among such preparations were (a) "self insurance" to compensate for or reduce the magnitude of loss (b) "self-protection" to reduce the probabilities of loss --- neither of which involved market choice or market insurance --- and (c) insurance purchased from others in a market.

It would now be generally agreed that these ideas apply to nations also to entire societies attempting to cope with diverse conflicts. When an agent provides security by spending on commercial market insurance, then a standard result states that with actuarially-fair linear pricing, complete coverage (net of premiums) is purchased from resources available in the good contingency (EB, Mossin 1968, Hirshleifer and Riley 1975). We compare this benchmark result with *self-insurance* by an entire country, and use it also to examine the effect of such self-insurance on the choice of *self-protection*. This necessitates close attention to fair pricing in a context of diminishing returns, and we discover that here the definition of actuarial fairness itself is ambiguous.

Governments need not passively accept risks that production and/or consumption decline in unwelcome situations. For them diverse instruments exist to manage national adversity including self-insurance and self-protection. Moreover, security benefits may spill over becoming public goods among countries (Olson and Zeckhauser, 1966) which then interact in their allocations of national income to international (or regional) welfare and safety. Examples of these public goods include formation of international organizations, collective military preparedness, active international diplomacy, and foreign aid which may reduce commonly shared probabilities of regional and international tension. And special trading agreements or collaborative stockpiling or common strategic defenses may provide mutual self-insurance to reduce or offset emergency losses. Nevertheless, utilization of EB to model collective improvements to the entire "risk profiles" as international public goods is sparse, except for some work on terrorism such as Sandler (1992, 1997, and 2005). In particular, economists' voluntary public good (VPG) models have not been well extended to understand incentives and behaviors of sovereign agents desiring to manage risks along multiple channels analogous to EB's self-protection and self-insurance. The purpose of this paper is to contribute to that extension.

Analysis of multiple instruments of collective risk management must extend the standard market insurance paradigm in several respects. First, sovereign agents will face *increasing costs* for self-insurance coverage in contrast to the competitive linear prices relevant to individual market insurance. Second, when instruments both for self-insurance and for self-protection are available to a single decision maker, reducing risks through protection changes the price of self-insurance and thus the two allocations will interact in an unfamiliar manner (see Ihori and McGuire, 2008a). They show that prominent among these interactions is a type of self inflicted moral hazard. Since inter-agent incentives will be transmitted by income effects in the VPG model the issue of normality vs inferiority of protection and insurance is of special importance. Therefore, we must integrate into such non-market situations known results concerning inferiority of market insurance (Mossin 1968, Hoy and

Robson 1981) and of protection (Ihori and McGuire, 2006, 2007). Third, if both self-insurance and self-protection are available to all members of a group, we must deal with problems with incentives caused by goods inferiority that can preclude determinate Nash equilibria.

In this paper, we show that if an agent insures itself and if its spending on this self-insurance is subject to diminishing returns due to decreasing productivity of insurance premiums, then (contrary to the conventional market result just mentioned) complete coverage (net of premiums) need not be purchased from resources available in the good contingency, irrespective of the ambiguity of fairness in pricing. This distinction is important when insurance is provided by a large agent, say the size of an entire nation, since such self insurance (as a national public good) is assumedly characterized by the decreasing returns. Here, we argue, nations will have access to actuarially fair returns to their insurance outlays.

The necessary inferiority of *market* insurance is well established when risk aversion diminishes with wealth, resources are allocated optimally, and insurance is *unfairly* priced (Mossin 1968, EB). As we demonstrate when returns to insurance are diminishing unfair pricing is no longer necessary for inferiority. That is we show that diminishing productivity of insurance together with actuarial fairness in returns in no way resolves the inferiority problem. Moreover, as Ihori-McGuire (2006, 2007) show, improvements in protection may easily be inferior under either decreasing or increasing risk aversion. Thus, when both types of security improvement are inferior, such income effects should lead to specialization in provision of both public goods, i.e. to Nash equilibria at corners for both insurance and protection.

Going beyond individual countries, when insurance is provided as a public good by/among nations in a group, we demonstrate a strong presumption for goods-inferiority and the problems of equilibria and instability associated therewith. If we consider multi-types of pure public goods, a second source of instability and imbalance follows from incentives discovered by Cornes and Itaya (2008). They demonstrate that when two agents in a partnership or alliance both could provide two different (pure) public goods, then at a Nash equilibrium, it is impossible (unless both agents have identical preference functions) for both agents to share in the provision of both goods. Here we develop a (more) generalized and realistic formulation that can avoid the Cornes-Itaya difficulty.

But, since we can not always exclude the possibility of inferiority (at least for self insurance), we may still find corner solutions when interior Nash equilibria are unstable. Specifically when self protection is normal but self insurance is inferior and allies' wealth rises (or economic growth occurs), we would expect that public good provision of self protection increases, while provision of self insurance declines. However, just the opposite of this result can also be argued in a multi-country framework with inferior goods. Each country may provide one type of public good, each country may free-ride with respect to either insurance or protection (while supplying all of the other good to the group) and both countries gain compared to isolation. We could explain any behavior of security spending in the real world by applying our theoretical model.

We also provide some numerical results by assuming CRRA utility functions. Then, the sign of income effect of self insurance may depend on the productivity of insurance. We show that an increase in national income of one country could have negative spillovers on the other country in the allies if the income effect is negative.

Corner solutions may more likely occur. It is also shown that income redistribution between two allied countries is not neutral any more. If self insurance is inferior, income redistribution exaggerates the original redistribution effect by forcing the giving country to provide more public goods and hence the receiving country may free ride on the public goods.

This paper consists of five sections. First, we formulate a basic analytical framework. Namely, section 2 reviews characteristics of optimization under market insurance, self-insurance, and alternatively self-protection for a single country. Then, in section 3 extending the single country model to a two country world, we comment on how the provisions of self-insurance and self-protection as public goods demonstrate an inherent potential for an “unstable conflict” leading to centralized or decentralized specialization in provision. Section 4 presents some numerical results. Finally section 5 concludes.

2. Analytical Framework for a Single Agent

Let us begin with a narrative to illustrate the concept that our models will try to capture or summarize. Imagine there is an island nation that is subject to flooding. Whenever a flood happens there is a big loss. No matter how big the flood, the loss is the same, L. To protect itself against this loss the country can reduce the frequency of flooding by building flood-barriers, dikes, channels etc. If it builds no dikes a flood happens every other year. If it builds dikes that are 6 feet tall, the country will be flooded every 6 years. If this country builds dikes 15 feet tall it will be flooded every 11 years. The frequency of flooding depends on the height of its flood barriers. The relationship between cost of dikes and frequency of flood (1-p) will be known with certainty.[1]

To prepare for this loss, the country can also stockpile food and other necessities. Ignore time discounting and assume to start a linear relationship such that that if a flood happens every other year, each year without a flood the country can set aside x pounds of goods for the next year when there is a flood and have available in that year x pound of goods. If a flood happens every 6 years then to have x pounds available during the flood, the country only needs to give up x/5 lbs during each of the 5 dry years. If a flood happens every 11 years the country can provide x lbs during the flood by giving up only x/10 lb in each of the dry years. In other words, as an initial assumption suppose the country can self-insure at an actuarially fair price, (1-p)/p, irrespective of the scale of provision, "x."

2.1 Risk Profiles and Emergency Cost

Congruent with the foregoing story we consider a single agent and two contingent states, a good state "1" and a bad state, "0". Ignoring all insurance and compensation possibilities (that is taking L as a fixed parameter) expected utility for this agent is given as:

$$W = pU^1(Y) + (1-p)U^0(Y-L) \qquad C^1 = Y; \quad C^0 = Y-L \tag{1}$$

or

$$W = W(Y, p) \tag{2}$$

[1] Let "d" be the number of dry years and "w" the number of wet years. Then p = d/(w+d) gives the frequency of dry years or of success, and (1-p) give the frequency of wet years or of failure.

where W is expected utility, C is consumption, L is loss in the bad state, and p is the chance of a good state. Our analysis will focus on the two canonical types of Ehrlich-Becker (EB) defense; (i) EB's "self-protection;" which raises p and reduces (1- p), (ii) EB's "self-insurance" which reduces L. Aside from our flood narrative, the variable "p" might be risk of trade interruption, disease outbreak, environmental calamity, or war. Later in section 3 we will assume these are shared indivisibly by two coalition partners. Utility function U( ) is assumed to be the same whether luck is good or bad. U^1 denotes realized utility if the good event happens, and U^0 if the bad event happens, and $U_Y \equiv \partial U / \partial Y > 0, U_{YY} \equiv \partial^2 U / \partial Y^2 < 0$.

To establish the incentives for a single agent Eq. (3) shows the individual budget constraint: where Y is a fixed income and m_k denotes allocations (k = 1,2) to risk reduction, $p(m_1)$, and or loss reduction $-L(m_2)$ --- here

$$Y = C + m_k \tag{3}$$

considered a variable of choice. Therefore, if our concern is with insurance only --- with -L a variable but p taken to be a parameter --- Eq. (1) can be written

$$\tilde{W} = \tilde{W}(Y, m_2) . \tag{4}$$

Eq (4) then shows it can be natural and helpful to consider m_2 rather than L to be the national security public good.

2. 2 Baseline for Comparison: Market Insurance the Standard Result

We will emphasize presently how the structure and context of self-insurance for a large entity such as an entire nation is inherently quite different from market insurance. So it is for later comparative purposes useful to set out in brief summary the standard market insurance model, where again m_2 represents quantity of coverage in bad times.

A basic feature of the standard market insurance model (which distinguishes it from self-insurance) is that the cost of insurance coverage in bad times is linear. For a linear insurance recovery function instead of $-L(m_2)$ we write (with numeraire income being consumption in good times):

$$C^1 = Y - m_1 - \pi m_2 \tag{5}$$

and

$$C^0 = Y - m_1 - (L - m_2) \tag{6}$$

where πm_2 gives the <u>expenditure</u> on market insurance in good times (measured in units of C^1), and m_2 represents the <u>amount</u> of insurance coverage purchased at price π. Entered as a parameter, m_1 gives the allocation to risk improving self-protection. Then welfare becomes

$$W = p(m_1)U^1[Y - m_1 - \pi m_2] + (1 - p(m_1))U^0[Y - m_1 - (\bar{L} - m_2)] \tag{7}$$

If instead we took m_2/π to mean the <u>amount</u> of coverage and m_2 contingency-1 expenditures, we would write

$$W = p(m_1)U^1[Y - m_1 - m_2] + (1 - p(m_1))U^0[Y - m_1 - (\bar{L} - \frac{m_2}{\pi})] \tag{8}$$

Eqs. (7) and (8) are equivalent in the standard linear case, but we will favor (7) as more conventional. Then for optimal insurance, maximizing (7) or (8) with respect to m_2 yields necessary condition (9).

$$-p\pi U_Y^1 + (1-p)U_Y^0 = 0 \tag{9}$$

If the price of insurance happens to be actuarially fair then as in (10) the FOC would entail $U_Y^1 = U_Y^0$ whence

$$\text{Actuarial fair price:} \qquad \pi = (1-p)/p \tag{10}$$

$U^1 = U^0$ and, therefore, $C^1 = C^0$ or $Y - m_1 - \pi m_2 = Y - m_1 - (\bar{L} - m_2)$. And from this it follows that at the optimum, insurance coverage purchased is

$$m_2 = p\bar{L} \tag{11}$$

so that the total cost of such fairly priced insurance at the optimum becomes

$$\pi m_2 = (1-p)\bar{L} \tag{12}$$

This standard result states that with fair linear pricing, complete coverage (net of premiums) is purchased. We will use this *market insurance* summary as the benchmark for later to comparison with *self-insurance* by an entire country, and also in examination of the effect of insurance on the choice of protection.

Fair Pricing and Non-Inferiority of Market Insurance

Eq. (11) shows that (with $\bar{L}$ fixed) at the fair insurance optimum, the amount of coverage purchased is independent of Y and income effect is zero. (But if $\bar{L}$ is an increasing function of Y, then the income effect becomes positive.) Now to summarize the conventional wisdom about linear market insurance, actuarial fairness in pricing, and goods inferiority: With p given, we consider just the utility function Eq. (7) at first, disregarding π for the moment. Along a 45^O line of equal consumptions the slope of indifference curves is (1-p)/p since C^0=C^1 implies $U^0 = U^1$ and $U^0_Y = U^1_Y$. Off the 45^O diagonal we have to write $\frac{(1-p)U_Y^0}{pU_Y^1} = -\frac{dY^1}{dY^0}$ for the slope since in these areas $U_Y^0 \neq U_Y^1$.The slope of each member of a family of indifference curves will change systematically with changes in p. For example, considering decreasing values of p (probability of good contingency) with $p^A > p^B > p^C$, the slopes of the indifference curves along the 45^O diagonal or any ray through the indifference map origin change become steeper, when going from p^A to p^B to p^C.

Now introduce opportunities to insure. If insurance is linear and fairly priced at π = (1-p)/p it will always be purchased to bring equilibrium to the 45^Odegree diagonal. This solution is independent of risk aversion --- whether it is constant, increasing, or decreasing --- and it eliminates possibility of inferiority since equilibrium entails C^0=C^1. However, if insurance is not fairly priced then the irrelevance of risk aversion changes. Now whether insurance is inferior, normal, or borderline depends on two factors (1) risk aversion and (2) whether price of insurance is greater than the "fair" price or less expensive.

Begin with the more ordinary case where prices are less favorable than actuarially fair, i.e. π > (1-p)/p. Then the consumer will trade to point short of a 45^Odegree diagonal, so that in equilibrium C^0<C^1. Suppose next that insurance is priced below actuarial fairness --- "super fairly" priced, i.e. $\pi < (1-p)/p$. Then optima will occur below the 45^{O} degree line, where allocations resulting in equilibrium $C^0 > C^1$ resemble gambles.[2]

2.3 Self Insurance

Self-insurance provided to itself by a large entity such as a nation differs in two important respects from standard market insurance. To show this, we alter notation slightly. Rather than -L(m) where L(0) was a threatened loss if nothing is spent on insurance, we write

$$-L(m) = -[\bar{L} - \mathsf{L}\ (m)] \tag{13}$$

Now the entire, total, insurance <u>benefit</u> is shown by **L,** and L(0) is given by $\bar{L}$.

2.3.1 Diminishing Returns

First of all, self-insurance differs from standard market insurance in that self-insurance function **L** should show diminishing returns or increasing costs. $\mathbf{L}' > 0$, $\mathbf{L}'' < 0$. EB make this assumption also, and refer glancingly to the role of human capital in providing for self-insurance as a source of diminishing returns. We believe (i) that scale considerations appropriate for an entire country along an extensive margin as well as (ii) other cooperating factors of production, as in EB, argue that "self-insurance" has such declining marginal productivity. National self-insurance may often involve actions like stockpiling or standby production maintenance and these surely will show diminishing returns[3]. If m_2 is very productive, -L may even conceivably be negative for high values of m_2 (recognized also by EB as "negative insurance" or as termed here, “gambling.”), so that over some region $-L(m_2) = -[\bar{L} - \mathsf{L}\ (m_2)] > 0$. However, we ignore this case as it implies a reversal between good and bad contingencies. Declining "productivity" of "m_2" thus is the first source of a distinction between sovereign self-insurance vs. lesser scale decentralized market insurance.

Such diminishing returns also will introduce issues in the formulation of inter-contingency pricing of self-insurance that are absent from market insurance --- a fact not recognized in the literature as far as we can determine.[4] Quite arguably, the self-insurance function should be written more generally as $\mathsf{L}\ (m_2, p, \pi)$ to allow for more complicated interactions between insurance, risk, and price. This would cause the definitions and formulation of "actuarial fairness" become ambiguous. But we relegate these complications to the appendix and concentrate here on the most salient formulation of pricing when insurance is non-linear with inter-contingency

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[2] If insurance is "super fair with equilibrium such that $C^0 > C^1$ below a 45^{O}degree line then if risk aversion is decreasing ---then greater income would lead to more gambling being purchased. But if risk aversion were to increase with income, then increases in income would lead to reductions in gambling purchases.

[3] In our story of flood protection and insurance, as greater quantities of consumables are set aside during dry years, their costs of preservation and delivery during good years might increase more than proportionately. For example, as an extreme case, if p = ½ setting aside $m_2 = 1$ provides 1 unit in bad times, but saving $m_2 = 8$ yields only 4 units in bad times, etc. Here $\mathbf{L} = (m_2)^{2/3}$.

[4] EB state in passing that self-insurance is independent of risk, but this is surely a mistake. Stockpiling for a seven year recurring famine is surely more expensive than for one that comes every 25 years.

price π as shown in (14):

$$W = pU^1[Y-\pi m_2]+(1-p)U^0[Y-\{\bar{L}-L(m_2)\}] : m_1 \textit{ not shown} \qquad (14)$$

where π shows the actuarial price per unit in good times necessary to yield m_2 units of resources in adversity.

2.3.2 Salience of Fair Pricing

Complications like writing $L = L(m_2, p, \pi)$ aside, a second major difference between self-insurance as provided by an entire country and ordinary market insurance is that when a whole nation provides insurance to itself, fair pricing would seem to be the standard case and not an outlier just referenced for comparison. Of course nation's can make mistakes, have imperfect information etc. But countries in this position are "bargaining with themselves" as to how much insurance and at what price to provide it. They should not in principle have to worry about adverse selection or moral hazard. So they should not give themselves deductibles, "load" prices nor impose arbitrary insurance limits to control fraud (Of course countries have corruption, rent seeking, and numerous misalignments of incentives to concern them.) Moreover, it is plausible to assume that the nation as a price maker, not as a price taker, incorporates this actuarially fair condition at its optimization.

2.3.3 The Insurance Optimization Problem

Now similar to EB's derivation, expected utility (14) is maximized with respect to m_2. This gives (15) as the first order condition. Eq. (15) shows the marginal cost of providing L,

$$\text{FOC:} \quad -p\pi U_Y^1 + (1-p)U_Y^0 L'(m_2) = 0 \qquad (15)$$

i.e. $[p\pi U_Y^1]$,equal to the marginal benefit of providing L ,i.e. $(1-p)U_Y^0 L'$, evaluated at the solution value of m_2 If this necessary condition is rewritten as in (16) then its actuarial meaning becomes clear. The RHS there gives the probability weighted marginal <u>insurance/benefit receipt</u> under adversity for the last, probability-weighted dollar of premium paid in good times

$$U_Y^1 / U_Y^0 = [(1-p)/p\pi]L'(m_2) \qquad (16)$$

2.3.4 Definitions of "Actuarial Fairness:"

If self-insurance is actuarially fair (henceforth simply "fair") as we believe should be the paradigm for an entire country then the concept must be defined. An obvious parallel to the fairness under linear market insurance is resource allocation fairness as in Eq. (10) which is the same as (17).

<u>Resource allocation fairness</u>: $$\pi = (1-p)/p . \qquad (17).$$

An alternative definition of actuarial fairness would incorporate the marginal productivity of resources as applied to the bad contingency. This we label "marginal productivity fairness," as defined by Eq. (18):

<u>Marginal productivity fairness</u>: $$\pi = L'(1-p)/p \qquad (18)$$

2.3.5 Optimal Solution Allocations

If fairness under marginal productivity ---Eq. (18) --- obtains then at the optimum $U_Y^0 / U_Y^1 = 1$ and thus $U_Y^1 = U_Y^0; U^1 = U^0; Y^1 = Y^0$ so that the optimum occurs along the 45^O and the analysis of inferiority proceeds just as in the linear case. Although this definition of actuarial fairness leads to a nice symmetry it assumes that the self-insuring agent somehow knows its optimal purchase of insurance "in advance," so as to have knowledge of L' before actually making its allocations. But this seems implausible. Moreover, under Eq. (18) one could not tell whether insurance was fairly priced until the optimum was actually chosen. This supports the first, more conventional, and we believe preferable, definition of actuarial fairness. Under fairness by that first definition Eq. (17) then the optimum simplifies to

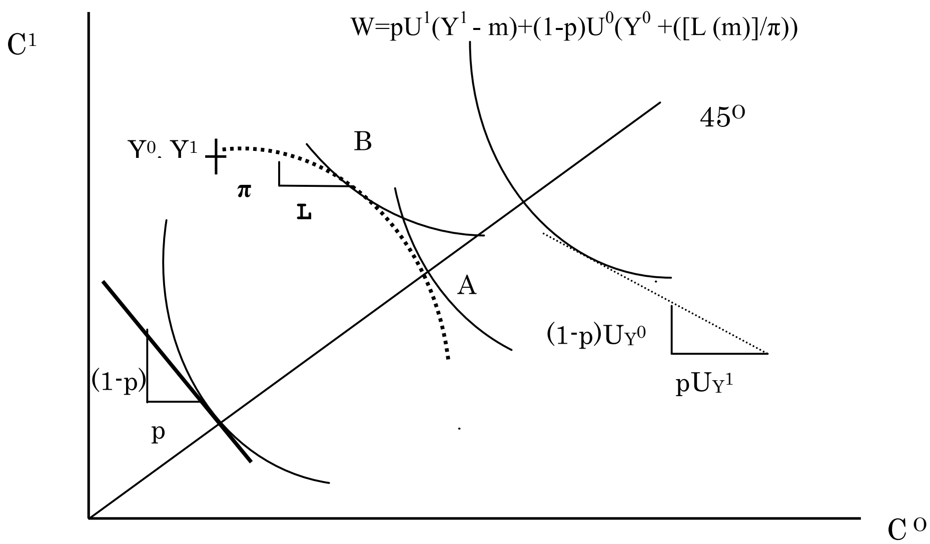
$$U_Y^1 / U_Y^0 = L'(m_2) \qquad (19)$$

Eq. (19) is simply a familiar equality of MRS and Marginal Rate of Transformation, which obtains irrespective of risks (1-p) so long as the price of insurance is fair. But note, in contrast to the linear market insurance case no equivalence is implied between U_Y^1 and U_Y^0, with significant implications, we shall see for the inferiority vs. normality of insurance.

2.3.6 Effects of Risk Aversion on Self-Insurance

Economists have long known of a systematic inter-dependence between risk aversion and provision of market "linear" insurance. Established in the theory of market insurance is that insurance will be an inferior good if (a) risk aversion is diminishing with income, (b) insurance is *not* fairly priced, and (c) its purchase is optimized as in Eqs. (14-19) (Mossin 1968, Hoy and Robson 1981). To explore when this continues to hold for self-insurance we must investigate the sign of the income effect on insurance spending.

Consider Eq. (14) as the maximand, and the FOC given by Eq. (15). Indifference curves as in Figure 1 have slopes $(1-p)U_Y^0 / pU_Y^1 = -[dY^1 / dY^0]$ which simplify to $(1-p)/p = -[dY^1 / dY^0]$ only along the diagonal where $U_Y^0 = U_Y^1$.



Figure

Considering $L(m_2)$ to be a cost or transformation function the optimum occurs at tangency between the dashed MRT and the MRS. If insurance is "resource allocation fair," that is $\pi = (1-p)/p$ then the FOC simplify to Eq. (19). If tangency occurs at A in Fig. 1 then it must be the case that at the optimum $L' = 1$. But if the tangency is at a point such as B then $L' > 1$. At A, irrespective of the risk aversion properties of the utility function, insurance is normal, but at B declining risk aversion will generate insurance as an inferior good, *even though it is fairly priced*, that is, "resource allocation fairly" priced.

Another way to state this conclusion is to repeat the optimum

$$\pi p/(1-p)L'(m) = U_Y^0 / U_Y^1 \tag{20}$$

This suggests two cases:

Case I: optimum above the 45^O $$\pi p > (1-p)L' \Rightarrow U_Y^0 > U_Y^1 \Rightarrow C^1 > C^0 \tag{21}$$

and

Case II: optimum below the 45^O $$\pi p < (1-p)L' \Rightarrow U_Y^0 < U_Y^1 \Rightarrow C^1 < C^0 \tag{22}$$

In support of the foregoing illustration, therefore, we want to derive when m_2 is an inferior and when a normal "good". Total differentiation of FOC (15) gives:

$$\partial m_2 / \partial Y \equiv M_Y = [p\pi U_{YY}^1 - (1-p)U_{YY}^0 L'] / D \tag{23}$$

where D represents the second order condition.

$$\text{SOC:}\quad D = p\pi U^1_{YY} - (1-p)L'U^0_{YY} + (1-p)U^0_Y L'' < 0 \qquad (24)$$

From the SOC we know therefore

$$p\pi U^1_{YY} - (1-p)U^0_{YY}L' < -(1-p)U^0_Y L'' > 0 \qquad (25)$$

Hence the sign of the numerator in Eq. (23) is ambiguous [5] and m_2 as self insurance *may* be inferior depending on the pricing of insurance.

Marginal Productivity Fairness

"Marginal actuarial productivity" meaning $\pi / L'(m)$ may be actuarially fair following Eq (18). Then $\pi / L'(m) = (1-p)/p$ and we would conclude that when "actuarial marginal productivity" is fairly priced, at the optimum insurance cannot be inferior since under such assumptions $U^1_Y = U^0_Y$, therefore, $Y^1 = Y^0$ and $U^1_{YY} = U^0_{YY}$. Under this condition then the first two terms in Eq. (24) vanish and thus D < 0, while the numerator of Eq. (23) = 0. This corresponds to a tangency such as point A in Figure 1.

Resource Allocation Fairness

We now consider the case of resource allocation fairness, which seems more plausible. This is where the sign of (23) is ambiguous. Specifically, if the numerator is positive, given the SOC, the sign of Eq. (23) is negative, and m_2 becomes inferior[6]. When m_2 is inferior, greater income lowers marginal cost $\Delta MC = MC_{MY} = p\pi U^1_{YY} < 0$ by more than it reduces marginal benefit $-\Delta MB = -MB_{MY} = -(1-p)U^0_{YY}L' > 0$. This required relationship between MC_{MY} and MB_{MY} can be derived from the risk aversion properties of the utility function, with absolute risk aversion R defined as

$$R = -U_{YY}/U_Y \qquad \text{or} \qquad -U_{YY} = RU_Y \qquad (26)$$

As shown in note 7, first if R is constant, then irrespective of fairness or unfairness of insurance pricing, from the FOC at an optimum the numerator of Eq. 23 vanishes and insurance is a borderline normal good. However if risk aversion is increasing ($R_1 > R_0$), H < 0 then sign of (23) is positive, and negative if risk aversion is decreasing[7].

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[5] The numerator of (24) is actually the <u>difference</u> between the effects of greater income on marginal cost $\Delta MC = MC_{MY} = p\pi U^1_{YY} < 0$ and marginal benefit $\Delta MB = MB_{MY} = (1-p)U^0_{YY}L' < 0$. Note for later use that whereas MC_{MY} has elements in the good state of the world, MB_{MY} refers exclusively to the bad state.

[6] Note if self-insurance is fair and therefore (1-p) = - pπ, then the numerator of (24) becomes $p(U^1_{YY} - U^0_{YY})$.

[7] This being another approach to Mossin's (1968) result, the numerator of (22) can be rewritten as:

$$H = -(p\pi R_1 U^1_Y - (1-p)R_0 U^0_Y L') \qquad (27a)$$

Considering the FOC, we obtain

$$H = p\pi U^1_Y (R_0 - R_1) = (1-p)U^0_Y (R_0 - R_1)L' : L' > 0 \qquad (27b)$$

Thus, if risk aversion is increasing (decreasing), m_2 is normal (inferior).

It follows then that the case for *inferiority of insurance is even stronger for self-insurance* than for market insurance since for diminishing return self-insurance fair pricing does not rule inferiority out. Generally, we should expect absolute risk aversion to decrease with wealth, so that for optimal/tangency-outcomes in the region of inferiority above the 45^O line the amount of insurance purchased will decline with wealth. Market insurance produces outcomes in this region of inferiority only if it is unfairly priced. But self-insurance leads to the region of inferiority even when it is fairly priced just provided the productivity of self-insurance is not so excessively great that $L' < 1$. This conclusion goes beyond the standard case of market insurance. Inferiority in a market insurance setting requires unfair insurance pricing to avoid the line of equalized wealth; but self insurance will prove inferior even if it is fairly priced and optimally provided since even optimal and fair self-insurance because of diminishing returns will not in general equalize incomes or marginal utilities across contingencies. Thus, (as will be the focus of Section 3), the dilemmas implicit in goods-inferiority as they apply to group allocation become more probable and serious when a group is composed of "self-insurers".

2. 4 Self Protection

Now to return to our flood protection anecdote we suppose a country can reduce the frequency of flooding by building flood-barriers, dikes, channels etc. (See Cornes, 1993). The frequency of flooding depends on the height of the flood barriers, and the relationship between cost of dikes and frequency of flood is known with certainty i.e. (1-p) in our model. Thus the second risk management instrument to consider is self-protection with m_1 spent to reduce the chance of a bad event, 1-p, i.e. to decrease what we call "baseline risk[8] of [1-p(0)]." Ihori

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With the numerator of Eq. (23) written as $MC_{MY} - MB_{MY}$ it is understood that both MC_{MY} and MB_{MY} are negative. If $|MC_{MY}| < |MB_{MY}|$ then $M_Y < 0$ and m_2 spent on self-insurance is inferior. Using this notation to write (27a) gives

$$H = [MC_{MY} - MB_{MY}]$$

And

$$\begin{aligned} MC_{MY} &= -R_1\, p\pi\, U_Y^1 \\ -MB_{MY} &= +R_0\, p\pi\, U_Y^1 \end{aligned} \tag{27c}$$

In MC_{MY} the terms p and R interact just as in the case of m_1 spent for self protection considered below. However, for self insurance when m_2 and, therefore, -L are optimized this interaction is washed out of the sum of MC_{MY} and $-MB_{MY}$. The change in marginal benefit of more insurance when Y is increased depends only on its impact in one contingency, i.e. on. $(1-p)R_0U^0_Y L'$ When m_2 for insurance is optimized, as shown in Eq. (19) U_Y^0 and U_Y^1 are balanced as per eq. 19 so that the independent effect of U_Y^0 and (1-p) are all absorbed in U_Y^1 and p or vice versa. When similar analysis is performed on self protection, the FOCs do not cancel out the interdependence between R and p, so that our conclusions for the two cases differ.

[8] Our "baseline risk" corresponds to what is sometimes referred to as "background risk" in economics of insurance analyses. Background risk distinguishes "independent" background risk where p(0) is not influenced by the value of

and McGuire (2007) demonstrated (with insurance fixed parametrically) that for self-protection the issue of normality-inferiority is substantially more involved than it has proven to be for self-insurance as analyzed here. We now desire to extend the Ihori-McGuire analysis of self-protection developed for the special case of fixed uninsured loss to the more general case of (1) variable loss and (2) self-insurance where (3) insurance benefit is non-linear, and (4) actuarial fairness interacts with diminishing returns. Whatever the risk-reduction/self-protection function, $p(m_1)$, $p' > 0$, and $p'' < 0$ are assumed throughout .[9]

To begin, we repeat (14) now including both variables m_1 and m_2. Inserting the condition for resource actuarial fairness, $\pi = [(1-p)/p]$ directly gives:

$$W = p(m_1)U^1[Y - m_1 - \frac{1-p(m_1)}{p(m_1)} m_2] + (1-p(m_1))U^0[Y - m_1 - \{\bar{L} - \text{L}\ (m_2)\}] \qquad \text{(14 repeated)}$$

The FOC for determining optimal expenditure on self-protection becomes:

$$[p'(U^1 - U^0)] - [p U_Y^1 + (1-p)U_Y^0] + [(p'/p)\, m_2 U_Y^1] = 0\,. \qquad (29)$$

We can characterize this optimality condition on the provision of self protection saying that there are "direct" marginal benefits in the form of the gain in utility $p'(U^1 - U^0)$, "direct" marginal costs $[pU^1_Y + (1-p)\ U^0_Y]$ and "indirect" benefits, $p' m_2 U_Y^1 / p$,comprised of, an unambiguous gain from the decrease in insurance premiums paid for *the same* m_2 coverage received. These indirect benefits stem from the lower price implied by lower risk (1-p). Note that since the sole variable of choice here is m_1 any possible implications of the change in $p(m_1)$ on subsequent choices of m_2 and, therefore, of insurance purchased (by the agent choosing insurance) are irrelevant.

$$\text{SOC} \qquad E = \begin{array}{l} p''(U^1 - U^0) - 2p'(U_Y^1 - U_Y^0) + [p U_{YY}^1 + (1-p)U_{YY}^0] \\ -U_{YY}^1 \frac{p' m_2}{p}(-1 + \frac{m_2 p'}{p^2}) + U_Y^1 \frac{m_2}{p^2}(pp'' - p' p') < 0 \end{array} \qquad (30)$$

Without further specification these SOCs need not always hold for mutual self protection, but we assume they are satisfied; then taking total differentiation of FOC (30) gives:

$$\frac{\partial m_1}{\partial Y} = -\frac{p'(U_Y^1 - U_Y^0) - [p U_{YY}^1 + (1-p)U_{YY}^0] + U_{YY}^1 p' \frac{m_2}{p}}{E} \qquad (31)$$

Condition (30), assuming the SOC actually obtains, determines the denominator in (31) as negative at an optimum. But again the sign of the numerator is ambiguous, and the normality or inferiority of m_1 depends on this numerator, just as in the self insurance model. Now, however, Ihori-McGuire (2006, 2007) demonstrate that there is an

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L(0) as in our model here, versus "non-independent" background risk where p(0) and L(0) are interdependent, and asks how the choice of protection or insurance varies with the independence property (See Schlesinger, 2000).

[9] Where useful we can re-write Eq. (2) as (28) where L, m_2, and L are now taken to be parameters. Differences in functions $\tilde{W}_i(\cdot)$ *and* $\hat{W}_i(\cdot)$ are implicit in each model, so the notational distinction will be omitted henceforth

$$\hat{W} = \hat{W}(C, m_1) \qquad (28)$$

.

interaction between risk aversion R and baseline probability, so the normality/inferiority of m_1 is more involved than that of m_2 in the case of self-insurance. Here if absolute risk aversion is increasing/decreasing and (1-p) is initially low/high, m_1 will be normal, while if absolute risk aversion is decreasing/increasing and (1-p) is low/high, m_1 becomes inferior[10] Thus since 1-p is presumably low initially and absolute risk aversion is presumably decreasing, m_1 is presumably inferior, However, we cannot completely exclude the possibility of a normal case simply by assuming that risk aversion decreases with wealth.

3. Mutual Provision of Public Goods

We now turn to the case where countries can provide self-insurance and self-protection mutually, as public goods for each other. We have shown (Ihori-McGuire 2008a,b) that for both decreasing returns insurance and for linear constant returns insurance --- i.e. for self-insurance and market insurance --- incomplete information sharing and (inadequate recognition of the benefit that protection improvements confer on insurance) may create extreme moral hazard as between government agents/bureaus leading to zero provision of protection. To avoid this implied inefficient bias against risk reduction in favor of insurance we will assume that all inter-bureaucratic externalities are recognized. This heroic assumption, however, as we show next will not paper over conflicts and instabilities that countries desiring to cooperate in mutual security must face due to their own incentives

3.1 The Shadow of Cornes-Itaya

If we consider multi-types of pure public goods, a second source of instability and imbalance follows from incentives discovered by Cornes and Itaya (2008). They demonstrate that when two agents in a partnership or alliance both could provide two different (pure) public goods, then at a Nash equilibrium, it is impossible (unless both agents have identical preference functions) for both agents to share in the provision of both goods.

The Cornes=Itaya result

To apply their insight to our analysis, consider the following two-country model, country A and country B.

$$W^A = p(M_1)U(C^{1A}) + (1 - p(M_1))U(C^{1A} - L(M_2)) \qquad (33\text{-}1)$$

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[10] Ihori and McGuire (2007) demonstrate that there is a critical-crossover probability p* such that if risk aversion is increasing M switches from normal to inferior while if risk aversion is decreasing M switches from inferior to normal as this value p* is crossed. In a generalized analysis of risk taking and insurance, Eeckhoudt and Gollier (2000) comment on the fact that a decline in first order stochastic risk may increase an optimizing agent's optimal exposure to risk (p. 122) considering it to be a"puzzle." This relation between RA and p accounts for one source of such a "puzzle."

One objection to this analysis might be our assumption that loss L(M) is independent of wealth. One might argue that the richer an agent/country the greater its loss from adversity. We could include this effect by replacing the loss, L(M), with a proportional reduction of income.

$$W = p(m)U^1(Y - m) + (1 - p(m))U^0(\alpha Y - m) : 0 < \alpha < 1 \qquad (32)$$

This formulation implies a single crossover probability --- not constant p* but rather p = p(α) --- such that income effects change sign when probability changes and are neutral at p(α).

$$W^B = p(M_1)U(C^{1B}) + (1 - p(M_1))U(C^{1B} - L(M_2)) \tag{33-2}$$

where the international public goods are defined as

$$M_1 = m_1^A + m_1^B$$

$$M_2 = m_2^A + m_2^B$$

Then, the first order conditions for each country are as follows

$$FOC_{\{M_1, C^{1A}\}}(M_1, C^{1A}, M_2) = 0 \tag{34-1}$$

$$FOC_{\{M_2, C^{1A}\}}(M_1, C^{1A}, M_2) = 0 \tag{34-2}$$

$$FOC_{\{M_1, C^{1B}\}}(M_1, C^{1B}, M_2) = 0 \tag{34-3}$$

$$FOC_{\{M_1, C^{1B}\}}(M_1, C^{1B}, M_2) = 0 \tag{34-4}$$

(Note that (34-1) in our insurance-plus-protection context means

$$p'[U^1 - U^0] = p\,U_Y^1 + (1 - p)U_Y^0$$

and other FOCs are interpreted similarly.)

For a two member group we also have the overall feasibility condition in the good state

$$Y^A + Y^B = M_1 + M_2 + C^{1A} + C^{1B}. \tag{35}$$

When countries are not identical with respect to preferences, we have five conditions (34-1)-(34-4) and (35) but there are four endogenous variables: M_1, M_2, C^{1A}, C^{1B}, which causes the problem to be over-determinate. One of the FOC can not hold with equality at a Nash equilibrium. On the other hand, if countries are identical, the problem --- and the amount of each good supplied by each country --- becomes indeterminate. This is the Cornes-Itaya result.

In our formulation

$$W = p(m_1)U^1[Y - m_1 - \frac{1 - p(m_1)}{p(m_1)} m_2] + (1 - p(m_1))U^0[Y - m_1 - \{\bar{L} - \mathrm{L}\ (m_2)\}] \tag{36}$$

Observe that unlike the standard constant average cost case (so conventional in the voluntary public good model of groups, including Cornes-Itaya) in this instance the cost of insurance is a non-linear function of m_1, m_2 and this is true even if insurance is fairly priced That is, with E indicating expenditure on insurance:

$$E = \frac{1-p(M_1)}{p(M_1)} m_2 \tag{37}$$

Hence, the FOC with respect to M_1, C^{1A} must also be a function of two variables m_1, m_2. This means that, we can not write the FOC as a function of the three variables M_1, C^{1A}, M_2 alone any more. Instead of (34-1)-(34-4), we now have

$$FOC_{\{M_1,C^{1A}\}}(M_1, C^{1A}, M_2, m_1^A, m_2^A) = 0 \quad (38\text{-}1)$$

$$FOC_{\{M_2,C^{1A}\}}(M_1, C^{1A}, M_2, m_1^A, m_2^A) = 0 \quad (38\text{-}2)$$

$$FOC_{\{M_1,C^{1B}\}}(M_1, C^{1B}, M_2.m_1^B, m_2^B) = 0 \quad (38\text{-}3)$$

$$FOC_{\{M_1,C^{1B}\}}(M_1, C^{1B}, M_2, m_1^B, m_2^B) = 0 \quad (38\text{-}4)$$

Note that here (38-1) corresponds to

$$[p'(U^1 - U^0)] - [pU_Y^1 + (1-p)U_Y^0] + [(p'/p)m_2U_Y^1] = 0$$

Now there are eight endogenous variables; M_1, M_2, C^{1A}, C^{1B}, $m_1^{A.}, m_1^B, m_2^A, m_2^B$ with eight conditions: i.e. four FOCs (38-1)-(38-4), plus (5-1)(5-2) and

$$Y^A = m_1^A + m_2^A + C^{1A} \quad (39\text{-}1)$$

$$Y^B = m_1^B + m_2^B + C^{1B} \quad (39\text{-}2)$$

Therefore, we can determine the interior values of all the endogenous variables uniquely.

In other words, in our formulation, the non-linear "technology" for transforming C^{1A} into M_1, M_2 using m_1^A, m_2^A for country A (of course, the same is true for country B), implies that the Cornes-Itaya result will not hold. That is, we can have an interior solution for both types of the pure public goods even if both countries are not identical.

Of course, if the public goods are not normal but instead are inferior, we may well have corner solutions. But these corner solutions are not caused by the same mechanism as caused the Cornes-Itaya result. Therefore, we may say that the Cornes-Itaya result holds for the special case of linear cost or "technology" between public good and private consumption. Our (more) generalized and we think realistic formulation may avoid the Cornes-Itaya difficulty. But, since we can not exclude the possibility of public good inferiority (at least for self insurance), we have not avoided corner solutions associated with instability of interior Nash equilibrium.

Indeed, Cornes and Schweiberger (1996) pointed out that the Cornes-Itaya results would not hold and that multiple pure public goods could be provided at a Nash solution in a general equilibrium model with nonlinear returns to scale in production or “technology”. Thus this formulation of ours in effect gives a practical example of their more abstract insight. To emphasize in summary, even if a country’s transformation curve between private consumption and insurance benefit were linear,

$$Y = C + m_k$$

The interconnections between risk, price of insurance, and resources allocated to insurance, generate non-linearities. As just one example the actuarial fair price condition:

$$\pi = (1-p)/p$$

essentially brings about a ‘non-linear’ technology between public and private goods consumption. Specifically

$$E = \frac{1-p(M_1)}{p(M_1)} m_2$$

is a non-linear function of m_1, m_2. Thus, an important contribution of this paper is that even if we assume standard linear budget relationships a non-linear “technology” in consumption still follows once we incorporate an actuarially fair (or unfair) price condition, since the effect of *protection expenditures* on the price of *insurance* is inherently *non linear* .

3.2 The case where self insurance is inferior

Our (more) generalized or realistic formulation may avoid Cornes-Itaya problems. But, since we can not always exclude public good inferiority, at least not for self insurance, this formulation may raise another difficulty: corner solutions that follow from instabilities in interior Nash equilibria. Suppose self-protection m_1 is normal but self insurance m_2 is inferior. Then if allies’ wealth rises (or economic growth occurs), we would expect M_1 to increase, and M_2 to decline.

When each country could provide both types of public good[11], as shown above each type of security can easily become inferior if absolute risk aversion to declines with wealth. And as recognized, for example, in Kerschbamer and Puppe (1998) or in Ihori and McGuire (2006), when a public good is inferior, the Nash equilibrium solution for two countries becomes unstable implying a corner solution where only one country provides the public good. The intuition is as follows. When one country (home) provides security it creates a positive externality for its partner country (foreign), and hence the partner's effective income rises. But when public goods are inferior, the partner will react to an increase in its income by decreasing its provision of the public

[11] Our simple solution to the problem of diminishing returns and distribution of infra marginal costs/gains in a public good spillover environment will be to assume a "summation *finance* aggregator," $M = \sum m$, in the provision of public good L, even though L(M), p(M), represents a "non-summation *consumption* aggregator" (e.g. $p(M) \neq \sum p_i(m_i)$) Then, importing an idea from contest theory we take primitive preferences as being over *contributions* to insurance or to risk reduction, rather than insurance coverage or risk reduction itself.

good. This generates a negative externality for home which then will react by raising its provision further. Thus, at the Nash equilibrium only the home country provides the public goods.

In a multiple-country framework with inferior goods, we could have corner solutions for variety cases. Each country may provide one type of public good, each country may free-ride with respect to either insurance or protection (while supplying all of the other good to the group) and both countries gain compared to isolation. AS shown in simulation results below, we could explain any behavior of security spending in a real world by applying our theoretical model.

To sum up, in a world where mutual collective security is feasible a natural aspiration might be that both components insurance and protection be provided by allies cooperatively and in an efficient first best amount. However, our analysis has shown that for the plausible case of decreasing risk aversion when both kinds of security are expected to be inferior goods, then at a non-cooperative Nash outcome each of those components of security will be provided by one country alone, while other countries ride free. The prevalence of free riding means that the total of security provision is too small. And the incentive configuration that gave inferiority and corner solutions makes it is difficult for both countries to cooperate for their mutual security.

4. Numerical Results[12]

In this section, we conduct numerical simulations to investigate the welfare impact of income change and the effect of income redistribution. In the next subsection, we provide some numerical results where we pay attention to the income effects on self-insurance and self-protection under the specification.

We specify the functional form of utility function as a constant relative risk aversion (CRRA) utility function. We also specify the functional form of risk reduction function as

$$p(M_1) = \frac{p_d M_1 + 1}{p_d M_1 + (1/p_0)}. \tag{41}$$

In equation (41), p_0 is the probability of the good state when no self-protection is made, and p_d is a parameter regarding the marginal risk reduction of contribution. The marginal risk reduction is given by

$$\frac{dp}{dM_1} = \frac{p_d}{(p_d M_1 + (1/p_0))^2}\left(\frac{1}{p_0} - 1\right), \tag{42}$$

which is positive but decreasing with the contribution. Unlike the Tullock's and Hirshleifer's contest success functions, equation (41) means that the sum of the contributions by allies determines the risk of a bad state.[13] Using this functional form, the risk reduction function has the following properties:

$$p_0 \leq p(M_1) < 1 \text{ and } p'(M_1) > 0 \text{ for any } M_1 \geq 0, \text{ and } \lim_{M_1 \to \infty} p(M_1) = 1,$$

We note that there is no upper bound with respect to the size of self-protection.

We specify the loss reduction function as a linear function of the sum of contributions by the two allied

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12 To conduct the numerical simulations, we used Mathematica 7.0.0.

13 For their contest success functions, please refer to Tullock(1967) and Hirshleifer (1989).

countries:

$$L(M_2) = L_d M_2, \tag{43}$$

where L_d is the loss reduction from one unit of contribution and it is assumed constant. We have $L' = L_d, L'' = 0$. In section 2 we emphasized that the marginal productivity of self insurance should be decreasing. However, we assume the special case of $L' = L_d, L'' = 0$ for simplicity.

4.1 CRRA utility function

In the numerical simulation, we use the following CRRA utility function

$$U(C) = \frac{C^{1-\theta}}{(1-\theta)}, \tag{44}$$

where $\theta(>0)$ is the constant relative risk aversion. Let us consider the welfare maximization of country A. The same reasoning is applied to country B. The FOC for country A with respect to M_2 is given by

$$U_Y^{1A} / U_Y^{0A} = L'(M_2), \tag{45}$$

where the superscript A in the LHS of (45) represents country A. Under the specification of functions, we have

$$C^{0A} / C^{1A} = (L_d)^{1/\theta}. \tag{46}$$

Since the absolute risk aversion is reciprocal to consumption under the specification of (44), combining equation (46) with our theoretical results regarding the inferiority of insurance in section 2, we get the following relation

$$\begin{aligned} &\frac{\partial m_2^A}{\partial Y^A} < 0 \text{ if } L_d < 1, \\ &\frac{\partial m_2^A}{\partial Y^A} > 0 \text{ if } L_d > 1. \end{aligned} \tag{47}$$

When the marginal productivity of self-insurance is smaller than one, the consumption at a bad state, C^{0A}, is less than the consumption at a good state, C^{1A}. Then, the absolute risk aversion in a bad state is greater than that in a good state, which leads to the inferiority of self-insurance. In other words, if we assume a CRRA utility function, the sign on income effect of self insurance may be determined by the magnitude of marginal productivity of self-insurance. Note that if L_d =1, the income effect reduces to zero, as pointed out in section 2.

Table 4.1: Parameter Values

θ	$\bar{L}$	p_0	p_d
0.9	10	0.25	1

Table 4.2: Welfare impact of change in A's income in one-country model ($L_d = 0.9$)

Y^A	m_1^A	m_2^A	m_1^B	m_2^B	$p(m_1^A)$	$L(m_2^B)$	C^{1A}	C^{0A}	W^A
20	1.443	3.990	0	0	0.449	3.591	13.657	12.148	12.905
30	1.445	3.437	0	0	0.449	3.094	24.337	21.649	13.672
40	1.447	2.884	0	0	0.449	2.596	35.017	31.149	14.179
50	1.449	2.330	0	0	0.449	2.097	45.696	40.648	14.561
60	1.451	1.776	0	0	0.450	1.598	56.375	50.147	14.870

Table 4.3: Welfare impact of change in A's income in one-country model ($L_d = 1.1$)

Y^A	m_1^A	m_2^A	m_1^B	m_2^B	$p(m_1^A)$	$L(m_2^B)$	C^{1A}	C^{0A}	W^A
20	1.496	4.954	0	0	0.454	5.449	12.551	13.953	12.953
30	1.497	5.413	0	0	0.454	5.955	22.000	24.457	13.701
40	1.498	5.873	0	0	0.454	6.461	31.449	34.962	14.200
50	1.499	6.334	0	0	0.454	6.967	40.899	45.467	14.578
60	1.500	6.794	0	0	0.455	7.473	50.348	55.973	14.884

4.2 Welfare impact of economic growth

Now, we present the impact of economic growth on security spending of countries. Table 4.1 presents the values of parameters that we choose in the simulation.[14] Regarding the value of L_d, we consider two cases: greater than one and smaller than one. As shown in the previous section, the self-insurance becomes inferior if the value of L_d is smaller than one.

As a benchmark, we conduct numerical simulation of one-country model, in which country A maximizes its welfare while the contributions by country B are set to zero.[15] The results are presented in Tables 4.2 and 4.3.[16] As shown in Section 4.1, the sign of income effect on self-insurance is determined by the sign of $L_d - 1$. Table 4.2 describes the effect of income change in country A when $L_d = 0.9$. In this table, the self-insurance contribution is decreasing with country A's income. Table 4.3 describes the effect when $L_d = 1.1$. In this case, the self-insurance

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[14] These values of the parameters are chosen rather arbitrarily. The robustness has not been tested yet.

[15] In the simulation of one-country model, we find the maximizing point of the following problem using a built-in function of the software: maximize W^A subject to

$$0 \le m_1^A \le Y^A, 0 \le m_2^A \le Y^A, 0 \le m_1^A + \frac{(1-p(M_1))}{p(M_1)} m_2^A \le Y^A, m_1^A \le Y^A - \bar{L} + L(M_2),$$

(41), (43), and (44) with respect to m_1^A and m_2^A, taking as a given m_1^B and m_2^B. This maximization gives the best response of country A. Denote the best response by a vector-valued function $\mathbf{br}(\mathbf{m^B}, Y^A)$, where $\mathbf{m^B} \equiv (m_1^B, m_2^B)$.

[16] Tables 4.2 and 4.3 present the values of function $\mathbf{br}(\mathbf{m^B}, Y^A)$ for various levels of country A's income, while the contribution made by country B is fixed to zero, i.e. $\mathbf{m^B} = \mathbf{0}$.

contribution is increasing with A's income. In contrast, the sign of income effect on the self-protection contribution is positive in both cases. In Tables 4.2 and 4.3, the self-protection slightly increases with A's income. Note that the sign of income effect on self protection is generally ambiguous. For the parameter values we assume here it becomes positive.

Then, we consider the two-country model.[17] Tables 4.4 and 4.5 present the results of numerical simulation in which country A's income increases while country B's income remain unchanged. Namely, Table 4.4 describes the case in which $L_d = 0.9$, and Table 4.5 describes the case in which $L_d = 1.1$. Each row of the tables represents the Nash equilibrium levels of economic variables. The first and second columns represent the exogenously determined incomes of countries A and B. We set the income of country B to be 40 and increase gradually the income of country A from 20 to 80.

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[17] The simulation of two-country model is conducted as follows. We use the best response function defined in footnote 15, **br**(.,.). Denote a vector of country A's contribution by $\mathbf{m^A} \equiv (m_1^A, m_2^A)$. For country B, the best response with respect to $\mathbf{m^A}$ is given by $\mathbf{br}(\mathbf{m^A}, Y_B)$. If $\mathbf{m^A}$ constitutes a Nash equilibrium, it must satisfy $\mathbf{m^A} = \mathbf{br}(\mathbf{br}(\mathbf{m^A}, Y^B), Y^A)$. We found this $\mathbf{m^A}$ using a built-in function that searches the numerical root of an equation. Then, we substitute the $\mathbf{m^A}$ in the best response function to get the Nash equilibrium level of B's contribution.

Table 4.4: Welfare impact of change in A's income in two-country model ($L_d = 0.9$)

Y^A	Y^B	m_1^A	m_2^A	m_1^B	m_2^B	$p(M_1)$	$L(M_2)$	W^A	W^B
20	40	1.102	3.116	0.000	1.280	0.412	3.957	12.972	14.295
30	40	1.046	2.474	0.000	1.522	0.406	3.596	13.720	14.279
40	40	0.980	1.831	0.001	1.750	0.398	3.223	14.219	14.262
50	40	0.949	1.320	0.150	2.031	0.412	3.016	14.603	14.249
60	40	0.944	0.845	0.283	2.312	0.426	2.841	14.913	14.238
70	40	1.031	0.421	0.307	2.565	0.438	2.688	15.170	14.232
80	40	0.000	0.002	1.446	2.883	0.449	2.596	15.417	14.179

Table 4.5: Welfare impact of change in A's income in two-country model ($L_d = 1.1$)

Y^A	Y^B	m_1^A	m_2^A	m_1^B	m_2^B	$p(M_1)$	$L(M_2)$	W^A	W^B
20	40	0.000	1.711	0.987	4.491	0.398	6.822	13.280	14.243
30	40	0.000	2.966	0.576	3.464	0.344	7.073	13.857	14.277
40	40	0.240	3.243	0.240	3.243	0.330	7.134	14.295	14.295
50	40	0.456	3.970	0.000	2.980	0.327	7.644	14.639	14.324
60	40	0.550	4.079	0.000	3.069	0.341	7.863	14.937	14.330
70	40	0.590	4.531	0.000	2.965	0.346	8.247	15.186	14.344
80	40	0.632	5.004	0.000	2.852	0.352	8.641	15.403	14.358

Let us investigate the income effect on security contributions. The third and fourth columns represent the equilibrium levels of self-protection and self-insurance by country A. The fifth and sixth columns represent the equilibrium levels by country B. In Table 4.4, the self-insurance is an inferior good. The self-insurance by country A decreases with its income growth. The decrease in A's contribution reduces country B's full-income. As a result, country B's self-insurance increases with country A's growth.

It is interesting to note that the effect on the self-protection is not monotonic. As country A's income increases, country A reduces its self-protection until its income becomes 1.5 times as great as that of B. When A's income becomes 70, country A increase its self-protection. After the level, country A does not provide self-protection at all. Although the income effect on self protection is always positive in a closed model, as shown in Table 4.2, the sign becomes ambiguous in a two country setting. This result highlights the complicated interdependence due to the presence of two public goods in a two country model.

If Y^A=Y^B=40, both countries are identical. Nevertheless, Table 4.4 shows that the equilibrium values of security contributions and welfare are different between two countries. This asymmetric outcome is caused by the multiple-equilibria where one country provides more on m_1 and m_2, while the other country provides less. In Table 4.5 we do not have the asymmetric Nash outcome. This suggests that we may obtain multiple-equilibria if the self insurance contribution is inferior..

In Table 4.4 the income growth in country A has conflicting effects on the welfare of the two countries.

The ninth and tenth columns represent the welfare of countries A and B. Country A’s welfare increases with its own income growth, while country B’s welfare slightly decreases. This decrease is to be caused by reductions in positive spillover effect from country A since country A reduces its contributions on national security.

In contrast, Table 4.5 presents the case in which self-insurances are normal goods. In this case, country A increases its self-insurance contribution with its income, while country B decreases its self-insurance contribution with its income. The self-protection by country A also increases with its income. As long as country A’s income is less than B’s income, country A does not raise its self-protection. After that, country A increases its expenditure as its income grows. Again, the response of country B is rather complicated in the multiple-country setting. The welfare of allied countries A and B is improved by the increase in A’s income, which is intuitively plausible.

Table 4.6: Welfare Effects of Income Transfer ($L_d = 0.9$)

Y^A	Y^B	m_1^A	m_2^A	m_1^B	m_2^B	$p(M_1)$	$L(M_2)$	W^A	W^B
40	40	0.980	1.831	0.001	1.750	0.398	3.223	14.219	14.262
50	30	0.000	0.908	1.207	2.845	0.424	3.378	14.652	13.701
60	20	0.000	0.043	1.432	3.960	0.448	3.603	14.966	12.907
70	10	2.000	3.000	0.000	3.462	0.500	5.816	15.167	11.995

Table 4.7: Welfare Effects of Income Transfer ($L_d = 1.1$)

Y^A	Y^B	m_1^A	m_2^A	m_1^B	m_2^B	$p(M_1)$	$L(M_2)$	W^A	W^B
40	40	0.240	3.243	0.240	3.243	0.330	7.134	14.295	14.295
50	30	0.616	3.910	0.000	2.861	0.350	7.448	14.637	13.875
60	20	0.870	4.957	0.000	2.352	0.384	8.039	14.924	13.300
70	10	0.982	5.672	0.000	1.753	0.398	8.167	15.167	12.285

4.3 Income redistribution between allies

Let us investigate the welfare effects of income transfer from country B to country A. As explained in 3.1, due to the non-linear technology (37), income redistribution has real effects even if two types of public goods are pure with respect to their own benefits. Therefore, it is meaningful to conduct a numerical calculation to investigate the outcome quantitatively.

Table 4.6 presents the result of numerical simulation in which the parameter values are set as presented in Table 4.1. As shown in the table, the welfare of country A becomes better by the transfer, while the welfare of B becomes worse. That is, the neutrality result does not hold in this model.

When the self-insurance is an inferior good, income transfer expands the welfare difference between countries. As shown in Table 4.6, the self-insurance contribution by country A decreases and that by B increases with transfer, except the case in which A’s income is seven times as large as B’s income. An income transfer decreases the contribution from the recipient and increases that from the donor. In this sense the induced changes in security contributions exaggerate the original effect of income redistribution. This is a new result with the inferiority of public goods.

In our two-public good model, income redistribution between allies has a welfare impact even if the self-insurance is a normal good. We change the sign of L_d -1 to be positive, and conduct numerical simulations. The results are presented in Table 4.7. The contributions by the recipients increase with the transfer, while the donor decreases its self-insurance and stops its self-protection. These two opposite effects on contributions are not completely canceled out. Namely, by the induced response of both countries with respect to security contributions, the original redistribution effect is partially offset although the welfare of country A improves and the welfare of country B deteriorates to some extent.

5. Conclusion

This paper has investigated two types of preparation available to expected utility maximizing agents faced with "costs of emergency", namely self-insurance and self-protection. Self-insurance provided to itself by a large entity such as a nation differs in two important respects from standard market insurance. First of all, the self-insurance function should show diminishing returns or increasing costs whereas market insurance typically has linear or piecewise linear pricing. Second, self-insuring countries far more readily than individuals should have access to actuarially fair prices, which they can then incorporate into their optimization.

If a small price taking agent provides insurance at a market price, then the standard result holds under fair and linear pricing that complete coverage (net of premiums) is purchased. We have used this standard result as a benchmark to compare with self-insurance by an entire country, and also in our examination of the effect of insurance on the choice of protection.

First, we show that the effect of decreasing productivity (ordinary diminishing returns) when a nation provides self insurance is for complete coverage (net of premiums) not to be purchased except by unlikely chance.

Second, the case for *inferiority of insurance is even stronger for self-insurance* than for market insurance since for diminishing return self-insurance fair pricing does not rule inferiority out. Inferiority in a market insurance setting requires unfair insurance pricing to avoid the line of equalized wealth between contingencies; but self insurance will prove inferior even if it is fairly priced and optimally provided since even optimal and fair self-insurance because of diminishing returns will not in general equalize incomes or marginal utilities across contingencies. Thus, the dilemmas implicit in goods-inferiority as they apply to group allocation become more probable and serious when a group is composed of "self-insurers".

Third, our model of individual self-insurance and self-protection readily extends to groups of two countries where such insurance and protection are public goods. The Cornes-Itaya result holds for the special case of linear “technology” between the public good and private consumption. Our (more) generalized or realistic formulation may avoid their difficulty. But since we cannot always exclude inferiority, at least not for self insurance, we may have another problem, corner solutions or instabilities of interior Nash equilibria. When both types of security spending are inferior, we demonstrated an inherent potential for destabilizing incentives to generate corner outcomes with complete perfect specialization in the provision of these public goods. The source of the difficulty is that both protection and insurance are rather likely to be economically inferior and economic inferiority leads to corner solutions. Since the interdependence due to two public goods in a two country setting is very complicated with the possibility of inferior public goods, we could explain any distribution of security spending in a real world by applying our theoretical model.

We then provided some numerical results by assuming CRRA utility functions. In our simulation, the sign of income effect of self insurance may depend on the marginal productivity of insurance. We showed that corner solutions are likely to occur. Even if the sign of income effect on self protection is always normal in a one-country model, it becomes ambiguous in a two-country setting. An increase in national income of one country could have negative spillovers on the other country in the allies if the income effect is negative. We also showed that income redistribution between two allied countries is not neutral any more. Actually, it exaggerates the original redistribution effect by forcing the giving country to provide more public goods and hence the receiving country may free ride on the public goods if the self insurance contribution is inferior..

Overall, we have shown that when "defense" or "security" is disaggregated into more realistic categories instabilities and corner solutions the conventional standard where instable outcomes and rampant free-riding become a norm for both self insurance and self protection. By undermining the OZ model these results suggest that negative income effects create conflicts and other difficulties within countries managing insurance and protection.

APPENDIX

Alternatives in the Structure and Price of Insurance

The foregoing text assumes one simple relationship between premiums paid, πm_2, during good times and $\mathbf{L}(m_2)$ benefits received during bad times, where the variable of choice is units of coverage m_2. We could cast the problem in terms of units of expenditure (where we define $x_2 = \pi m_2$, "x" for "expenses," and where $m_2 = x_2/\pi$) such that the variable of choice is x. Here $1/\pi$ would indicate the efficiency of resource transfer across contingencies. Thus in place of Eq. 14

$$W = pU^1[Y - \pi m_2] + (1-p)U^0[Y - \{\bar{L} - \mathrm{L}\ (m_2)\}] : m_1 \textit{ not shown} \qquad \text{(14 repeated)}$$

We could write:

$$W = pU^1[Y - x_2] + (1-p)U^0[Y - \{\bar{L} - \mathrm{L}\ (x_2/\pi)\}] : m_1 \textit{ not shown} \qquad \text{(14a)}$$

Optimization of (14a) gives the same results as derived above in the text but measured in terms of expenditure rather than quantity of coverage. However, when transfer of resources across contingencies is written as in (14a) it becomes obvious that every unit of x_2 may *not* transfer into just exactly x_2/π units to the bad contingency. Once this is recognized then the idea of "fair" pricing becomes not so clear. For example, in place of (14a) we could write (14b)

$$W = pU^1[Y - x_2] \quad + \quad (1-p)U^0[Y - \{\bar{L} - \frac{1}{\pi}\mathrm{L}\ (x_2)\}] : m_1 \textit{ not shown} \qquad \text{(14b)}$$

Eq. (14b) represents another structure of diminishing returns. There $1/\pi$ indicates not the productivity or accumulation of x_2-resources but the accumulation in bad times of $\mathbf{L}$ units of benefit. Now, in a sense, the self-insurance function operates during good times to create transfers available only under adversity; so that for the total $\mathbf{L}/\pi$ received in adversity, x_2 was set aside in the good contingency[18].

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[18] One way to think about this is to assume there is a steady state. We can then frame self-insurance in terms of changes in, or alternative parameters for this steady state. Normalizing the notation of note 1, let w = 1, d = n., Here is the steady state. Every n+1 years there is a crisis, requiring that the self-insurance accumulation be utilized. Each year from 1 to n, $\$m_2$ is set aside in anticipation of year n+1. That is "self-insurance" is fair. At year n+2 the process starts over again, and on and on. Therefore p = n/[n+1], and (1-p) = 1/[n+1]. $\pi = 1/n$ so that $m_2/\pi = nm_2$. Changes in the risk of adversity then are given by changes in n. If n is large adversity happens only rarely so that (1-p) is small, and if n is small, the risk of adversity (1-p) is high. We ignore discounting, and uncertainty.

Case I: Here the total insurance consumed in year n+1 is $n\mathrm{L}\ (m)$. Here diminishing returns apply to each year's

When insurance is supplied in the market and its marginal costs and benefit are linear, this distinction between (14a) and (14b) does not arise or it doesn't matter; but when **L** displays diminishing returns how to describe the cost of insurance seems to be ambiguous. Do diminishing returns apply both to quantities reserved and to the lapse of time between adversities, or only to the former? As for comparing (14a) vs. (14b), the difference seems to lie in "when" the resources transferred across contingencies become productive --- before (as in Eq. (14b)) or after (as in Eq. (14a)) the transfer. The novelty of this distinction, or the fact that it seems to have been overlooked, may be because when the insurance function is linear, as in market insurance, no such difference arises.

For either of these options (14a) or (14b) the idea of actuarial fairness is still applicable, but its implications with respect to outcomes vary. For example, if marginal utilities just happened to be equalized at the optimum such that **L'** = 1 then consumption would also be equalized, and this would imply $L(m_2) = (1-p)\overline{L}$, in the one case or $L(x_2/\pi) = (1-p)\overline{L}$ in the other. Although these formulas have a nice symmetry to market insurance neither of these conditions is required to obtain even though coverage is fairly priced --- not required because there is no necessity that **L'** = 1. In either case FOCs inform us of optimal insurance protection and allow us to compare configurations (14a) and (14b), assuming self-insurance to be fairly priced.[19] More generally, if the benefit in the bad contingency depends in a more complicated manner on resources set aside in good times then we should write $L = L(\pi, p, x_2)$ and the effects of fair insurance and even its definition may become ambiguous.

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insurance savings individually so that every new year the country begins a new decreasing returns process.

Case II: Here the total insurance coverage consumed in year n+1 is $L(nm)$. Here m_2-savings set aside each year generate less marginal return than the same m savings of the year before.

In both I and II, greater m_2 produces more diminishing returns, although in different manner and to different degree. Note however, that with case II, diminishing returns, in addition to being greater for larger values of m_2, are more severe the rarer the emergency (higher value of n).

[19] First, note that for π = [(1-p)/p] = 1, $L'(x_2/\pi) = L'(m_2)$, while for π > 1, $L'(x_2/\pi) > L'(m_2)$ and for π < 1, $L'(x_2/\pi) < L'(m_2)$. So if self-insurance has the form of Eq. (14a) i.e., $L(m_2)$, there is an interaction between insurance and protection not present in the case of market insurance. For π < 1 (and hence risk (1-p) low) the solution value of "m_2" will be less than it is when odds are equal. On the other hand, for π > 1 (and risk (1-p) high) optimal m_2 will be greater than the best choice of "m_2" when odds are equal (maintaining throughout these comparisons, the fair insurance equality π = (1-p)/p).

But if self-insurance has the form of Eq. (16a), i.e. $L(x_2/\pi)$, then there is an additional interaction between insurance and protection. There, when π = (1-p)/p) varies as a parameter, both the MRS between contingencies shifts and MRT shifts as well so that the overall effect of π on the optimal m_2 cannot be deduced *a priori*.

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