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Threshold public goods in ambiguity dynamics

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Agents face an ambiguous risk of biodiversity survival as well as ambiguous expected losses from its extinction. Indeed, as a collectivity, agents are faced with the option of privately funding the protection of biodiversity for biomedical research. We propose two evolutionary models of threshold public goods game and public goods option market, where we consider population dynamics with proportional fair-share contributors versus free-riders. In the first model, we find that agents contribute only if they face ambiguous risks. They contribute if their proportional fair-share is less than their expected salvage of wealth. While a common disease can induce social cooperation, a rare disease will provoke unconditional social free-riding. In the second model, in case of ambiguous survival, the public good is provided when the agents exchanging option contracts are equally divided into buyers and sellers. This is true when the proportional fair-share equals the expected payoff at the market price and holds for a specific market belief over the species' survival. Whereas rare diseases and/or low probability of survival induce negative option prices, common diseases and/or high probability of survival induce quasi-null option prices. The absence of surplus captured on the option market condemns its raison d'être.

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Threshold Public Goods in Ambiguity Dynamics

Abstract

Agents face an ambiguous risk of biodiversity survival as well as ambiguous expected losses from its extinction. Indeed, as a collectivity, agents are faced with the option of privately funding the protection of biodiversity for biomedical research. We propose two evolutionary models of threshold public goods game and public goods option market, where we consider population dynamics with proportional fair-share contributors versus free-riders. In the first model, we find that agents contribute only if they face ambiguous risks. They contribute if their proportional fair-share is less than their expected salvage of wealth. While a common disease can induce social cooperation, a rare disease will provoke unconditional social free-riding. In the second model, in case of ambiguous survival, the public good is provided when the agents exchanging option contracts are equally divided into buyers and sellers. This is true when the proportional fair-share equals the expected payoff at the market price and holds for a specific market belief over the species' survival. Whereas rare diseases and/or low probability of survival induce negative option prices, common diseases and/or high probability of survival induce quasi-null option prices. The absence of surplus captured on the option market condemns its *raison d'être*.

Keywords: biodiversity; ambiguity; threshold public goods; option markets; evolutionary game theory
JEL Classification: C73, D81, H41, Q57

1. Introduction

As a collectivity, agents are faced with the option of privately funding the protection from extinction of biodiversity. Presently, 1.7 million species have been identified, but this number is believed to be to a great extent higher. Climate change has produced shifts in the distribution and abundance of species (Thomas *et al*. 2004, Wright and Muller-Landau 2006); a number of species are likely to become extinct and some of them face extinction before being identified and studied (Schelling 1992). Yet, species conservation can be beneficial (Polasky and Solow 1995), that is, they have an important quasi-option value (Arrow and Fisher 1974) or the value of the future option made available through their preservation. Indeed, the loss of species deprives of tools for biomedical research and precludes new medicines for untreatable human diseases (Chivian and Bernstein 2004).

Are agents willing to reduce an ambiguous risk by collectively contributing to a threshold public good? To estimate the monetary value of changes in probabilities of health risks, economists mostly use contingent valuation and stated-preference methods (Acton 1973, Jones-Lee *et al*. 1985, Thompson *et al*. 1984) where the metric is the willingness-to-pay to reduce the risk (Pratt and Zeckhauser 1996). As such, Weinstein *et al*. (1980) find that the willingness-to-pay for a mortality reduction is contingent on the reduction amount and the initial probability level. Further, values do vary, depending on whether the valuation is ex ante (health insurance, environmental health) or ex post (medical care). Papers show a significant diminishment in the risk reduction value despite a positive expected value of such reductions (Viscusi *et al*. 1987, Hammitt and Graham 1999). On the one side, increased threshold uncertainty increases the equilibrium contributions if the public good's value is sufficiently high (McBride 2006) or under low social uncertainty (Wit and Wilke 1998). On the other side, ambiguity aversion affects the agents' monetary-equivalents with ambiguous mortality risks (Treich 2010). Also, agents are discouraged by environmental uncertainty and the fear that their contributions be a waste (Au 2004). This leads to the sequential collapse of contributions (Gangadharan and Nemes 2009).

In principle, rational agents have an incentive to avoid contributing and to free-ride on others' provisions, that is, they attempt to exploit the common enterprise, as contributors provide benefit to the others while inflicting a personal sacrifice. This rationale leads to the well-known social dilemmas (Hauert *et al*. 2006) and settles on underfunding and the abandonment of the public good. However, many theoretical propositions of why agents cooperate to some extent have emerged. Developed ideas of how to overcome social dilemmas include direct and indirect reciprocity (Dreber and Nowak 2008); money-back guarantee (Cadsby and Maynes 1999); personal benefit from contributing (Doebeli *et al*. 2004); punishment of free-riding (Sigmund *et al*. 2001, Hauert *et al*. 2006); coercive threat (Okada and Bingham 2008); voluntary participations (Croson and Marks 2000, Hauert *et al*. 2002); collective bad risk (Fon 1988); leadership (Zimmermann and Eguiluz 2005); sequential contributing (Erev and Rapoport 1990); private losses (Milinski *et al*. 2008, Wang *et al*. 2009), warm-glow (Andreoni 1990) or competition (Dragicevic and Meunier 2010), to name just a few.

In this paper, we are interested in the capacity of agents to jointly produce threshold public goods when they face ambiguous risks and losses, through the population dynamics in replication. Evolutionary dynamics are helpful as they introduce cooperation. Indeed, an agent has to reduce her wealth for another to increase hers (Dreber and Nowak 2008). In public goods games, population dynamics are relevant, for the reason that the intervention of the entire population is necessary to produce the threshold public good. We confront proportional fair-share contributors – under the distribution of disease in the population – who donate the minimum average amount with free-riders who provide null contributions.

First, we propose a model of threshold public goods game with ambiguous risk of the species survival and individual ambiguous expected salvage of wealth from saving the species. This work is inspired by the literature on collective social dilemmas (Bach *et al*. 2006, Milinski *et al*. 2008, Wang *et al*. 2009, Wang *et al*. 2010). In order to lower ambiguous losses, agents can jointly produce public goods if they attain the threshold level of cost to produce the public good. Specifically, public goods are provided if joint fair-share contributions equal or exceed the required threshold level of provisions; otherwise, no public good is provided.

We find that agents contribute only if they face an ambiguous probability of survival of the species. Our results show that when the probability of survival is null, free-riding is the only steady state. When the probability is ambiguous, agents contribute if the proportional fair-share is less than the expected salvage of wealth; otherwise, they free-ride. So considered, there is a unique unstable Nash equilibrium in which the trade-off rule determines the agents' collective behavior. While a common disease can induce social cooperation, a rare disease will provoke unconditional social free-riding.

Second, we propose an evolutionary game of an option or a prediction market for a public good. Prediction markets are markets where agents exchange contracts whose payoffs are tied to the outcomes of unknown events. In an efficient prediction market, the market price best predicts the event (Wolfers and Zitzewitz 2004). The issue of any market is its performance as a predictive tool. In the political domain, Berg *et al*. (2008) document that the Iowa Electronic Markets yield accurate predictions. Nevertheless, a widespread behavioral bias of agents is to trade according to their subjective beliefs and desires, rather than objective probability assessments (Forsythe *et al*. 1999). This is all the more interesting in our case, for the probability of species survival is ambiguous. By entering the market, the agent who decides to trade at a certain probability level reveals her beliefs over the threshold attainment. Finally, prediction markets can be used for policy analysis. Link and Scott (2005) show that a prediction market with private investors can be used to value the success of governmental research projects. Prediction markets can also assist public institutions in managing social risks such as environmental disasters (Arrow *et al*. 2008).

Our results show that agents buy option contracts only if they face an ambiguous probability of survival. They are net sellers if proportional fair-shares exceed expected payoffs at the market price. When equal, agents are evenly divided into buyers and sellers and provide the public good. This result holds for a certain level of market belief over the species' survival. In this case, the full market efficiency is the unique unstable Nash equilibrium. Still, the results show that no surplus can be captured at the equilibrium, which will put a stop to exchanging. Indeed, while a high probability induces quasi-null option prices, low probability induces negative option prices, that is, a willingness-to-protect oneself from the probable loss. Rare diseases unconditionally induce social free-riding. In all cases, the option market is doomed to disappear.

Section 2 introduces the threshold public goods games, both in the static and dynamic contexts. Likewise, we present a static then a dynamic model of an option market for public goods in Section 3. Conclusive remarks are given in Section 4.

2. Threshold public goods game

2.1. Compound probability

Let us first identify the subject of species' survival for biomedical research in terms of probabilities. The reasoning is more complex than it seems on the surface. In fact, the agent has to deal with three consecutive bets. The first is on the existence of the unidentified species capable of supplying medicinal substances; the second is on the survival of such a species given its uncertain survival; and the third is of whether the agent willing to fund the species' protection will ever benefit from medical treatment in her lifetime. Therefore, we consider three independent lotteries **A**, **B** and **C** which respond to the three following questions:

- Does the species exist? Lottery **A**
- Will the species survive? Lottery **B**
- Will the species be of use rapidly enough? Lottery **C**

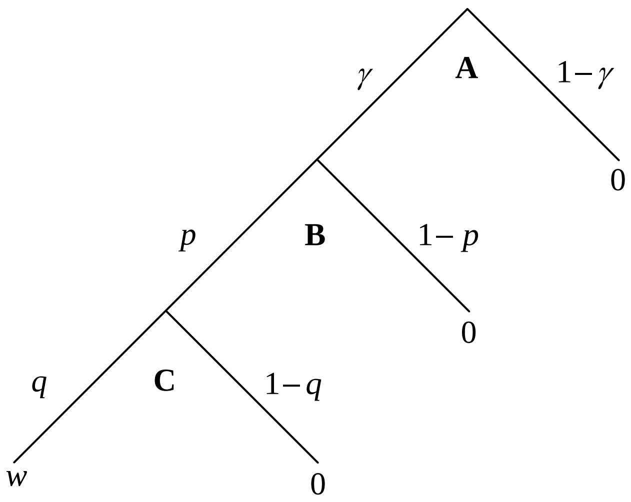

Fig. 1 Three-stage lottery

We have a three-stage lottery of $\mathbf{A}:(\mathbf{B},\gamma;0,1-\gamma)$, $\mathbf{B}:(\mathbf{C},p;0,1-p)$ and $\mathbf{C}:(w,q;0,1-q)$. If the compound probability axiom holds, we can transform this multi-stage lottery into a reduced compound lottery $\mathbf{D^3}$ with a single stage such as[1]

$$\mathbf{D^3}:\ [w,\gamma pq;\ 0,\gamma p(1-q);0,\gamma(1-p);\ 0,1-\gamma] \tag{1}$$

[1] We ignore the case of $\gamma(1-p)$ on purpose. Indeed, contrary to γ and q, the probability p of species' survival depends on whether the agent decides to contribute to the public good. This bet is under her control.

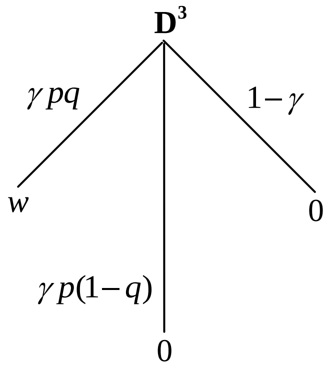

Fig. 2 Compound lottery

The probability of realization of the public good and thus the expected salvage of wealth amounts to

$$\mathbf{D^3} \equiv (1-\gamma)(0) + \gamma p(1-q)(0) + \gamma pq(w) = \gamma pq(w) = \phi(w), \qquad (2)$$

Where $\phi = \gamma pq$. Nonetheless, the probability of existence and the probability of usefulness of the species do not affect its probability of survival, so the joint probability of survival given the other events is simply *p*. So $(p \mid \gamma q) = p$, and $\phi(w)$ reduces to $p(w)$. In other words, an agent who decides to act on the species' survival and thus to affect its probability considers its existence and usefulness as granted. Otherwise, her reasoning has no sense. We consider the context of ambiguity because the subjective probability of survival remains ambiguous after the agent contributed to the public good. Indeed, the agent ignores whether there is a free-rider, who might have jeopardized the threshold attainment and thus the probability of survival of the species, among all other agents in the population.

2.2. Static game

Let $w > 0$ be the agent's endowment or amount of wealth and *g* her contribution to the public good. To satisfy Nash equilibria, her contribution in the population of size N is bounded by the constraints of efficiency $Ng = G$ and rationality $g < w$. The population is composed of contributors *n* and $N - n$ free-riders. All agents must contribute their proportional fair-share $gk^{-1} = Gk^{-1} / N > 0$ to attain the threshold. The proportional fair-share is a fair-share in view of the spread of disease $k \in [0,1]$ in the whole population. The proportional threshold is then

$Gk^{-1} = Ngk^{-1}$. When $k \to 1$, the whole population is at risk of suffering from a disease, so the proportional fair-share equals GN^{-1}. As $k \to 0$, the proportion at risk rarefies. By means of *prevalence proportion,* we know that the probability of an agent, randomly picked from the population, of being at risk is equal to the proportion of the population which is at risk. As *k* decreases, it is more and more costly for an agent to fund the public good.

Now consider the following expected payoff matrix.

	Contributor	Free-rider
Contributor	$p(w-gk^{-1})$; $p(w-gk^{-1})$	$(1-p)(w-gk^{-1})$; $(1-p)w$
Free-rider	$(1-p)w$; $(1-p)(w-gk^{-1})$	$(1-p)w$; $(1-p)w$

If all agents contribute their proportional fair-share, we have $n = N$. The threshold is attained and the survival is certain $p = 1$. In this case, the analysis ceases, since the purpose of collective contributing is to increase the probability of survival of the species.

Some agents may free-ride. In this case, we have $ng \le G \Leftrightarrow n(G/N) \le G \Leftrightarrow 0 \le N - n$. The payoffs of a contributor and a free-rider facing an uncertain probability of survival are

$$\begin{cases} \pi_c = (1-p)(w-gk^{-1}) + p(w-gk^{-1}) \\ \pi_f = (1-p)w \end{cases} . \tag{3}$$

If there is a free-rider, the threshold cannot be attained. The probability that a contributor salvages her wealth given her contribution is *p*, and $1-p$ that she does not in presence of a free-rider. An agent who free-rides runs a risk of $1-p$ to fail savaging her wealth, not without reminding that she compromises *p* for every other contributing agent in the population. Simulations of *g* given *p* and *k* are presented in Table 1.

2.2.1. Null survival

When the survival of species, *i.e.* probability of realization of the public good, is null or $p = 0$ (3) reduces to

Table 1 Simulations of fair-shares given p and k for $w = 10$: $g = pwk$

p / k	0.01	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.10	0.01	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.20	0.02	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
0.30	0.03	0.30	0.60	0.90	1.20	1.50	1.80	2.10	2.40	2.70	3.00
0.40	0.04	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20	3.60	4.00
0.50	0.05	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.60	0.06	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00
0.70	0.07	0.70	1.40	2.10	2.80	3.50	4.20	4.90	5.60	6.30	7.00
0.80	0.08	0.80	1.60	2.40	3.20	4.00	4.80	5.60	6.40	7.20	8.00
0.90	0.09	0.90	1.80	2.70	3.60	4.50	5.40	6.30	7.20	8.10	9.00
1.00	0.10	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00

$$\begin{cases} \pi_c = w - gk^{-1} \\ \pi_f = w \end{cases} . \qquad (4)$$

We see that $w > w - gk^{-1}$. Free-riding always dominates because it provides a higher expected payoff.

2.2.2. Ambiguous survival

When the survival of species is ambiguous or $p \in [0,1]$, the outcome depends on the tradeoff between the proportional fair-share and the salvage of wealth in expectation. Two outcomes arise. If $\pi_c < \pi_f \Leftrightarrow gk^{-1} > pw$, so free-riding dominates because the cost of contributing or the proportional fair-share is greater than the benefit from contributing or the expected salvage of wealth. If $\pi_c > \pi_f \Leftrightarrow gk^{-1} < pw$, so contributing dominates because the cost of contributing to the public good is less than the expected benefit from its production.

2.3. Dynamic game

We now combine game theory and population dynamics in a replicator equation. We consider infinite populations consisting of x contributors and y free-riders, where $x + y = 1$, that is, the sum denotes a normalized population density such that 0 corresponds to a null population density and 1 is the maximal population density.

According to replicator dynamics (Hofbauer and Sigmund 1998) the evolution of the system is given by the following differential equations

$$\begin{cases} \dot{x} = x(f_c - \overline{f}) \\ \dot{y} = y(f_f - \overline{f}) \end{cases} , \qquad (5)$$

The system in (5) establishes the expected payoffs of contributors f_c and free-riders f_f in time, given the average expected payoff in the population $\overline{f} = xf_c + yf_f$. This payoff is determined by the interactions in randomly formed groups of contributors and free-riders. The groups are formed by interpreting densities as probabilities for drawing either strategy. We study the interactions of model-agents or average agents issued from those random groups.

Let us set a mixed population where N agents are randomly chosen according to the Binomial probability function. Following Bailey *et al*. (2005) and Hauert *et al*. (2006), the probability that there are n contributors among the $N-1$ other agents in the population of size N in which the model-contributor or model-free-rider finds herself is determined by

$$f(n \mid N-1, x) = \binom{N-1}{n} x^n y^{N-1-n} . \tag{6}$$

This probability is independent of whether the model-agent is a contributor or a free-rider. Every model-agent encounters the same expected number of contributors, and hence the same expected payoff from others during the game. The only determinant of success in the well-mixed populations is the payoff that the model-agent herself receives.

The expected payoffs of a model-contributor and a free-rider are

$$\begin{cases} f_c = (1-p)(w-gk^{-1}) + p(w-gk^{-1})x^{N-1} \\ f_f = (1-p)w \end{cases} . \tag{7}$$

where x^{N-1} is the random variable issued from the Binomial distribution. Agents adopt the strategy of the model-agent with a probability proportional to the difference between her payoff and their own. Substituting $y = 1-x$ into the differential equations yields a single differential equation

$$\dot{x} = x(1-x)(f_c - f_f) , \tag{8}$$

So the dynamic evolution of $x(t)$ amounts to

$$\dot{x} = x(1-x)[\,p(w-gk^{-1})x^{N-1} - (1-p)gk^{-1}] . \tag{9}$$

2.3.1. Null survival

When the survival of species is null or $p=0$, we obtain

$$\dot{x} = -x(1-x)gk^{-1}. \tag{10}$$

Solving $\dot{x}=0$ gives two fixed points of the replicator dynamics which cancel out $x(1-x)$: $x=0$ and $x=1$. We now proceed to the study of stability of steady states by the Lyapunov method. The derivative of $F(x)$ is

$$F'(x) = -gk^{-1} + 2xgk^{-1}. \tag{11}$$

At $x=0$, $F'(0)<0$, that is, a stable equilibrium. Since $\dot{x}=0+F'(0)x$ if $x>0$, a deviation brings *x* back to 0. At $x=1$, $F'(1)>0$, that is, an unstable equilibrium (see Table 2). In this case, a deviation removes *x* from 1. Figure 3 illustrates the dynamics with null survival.

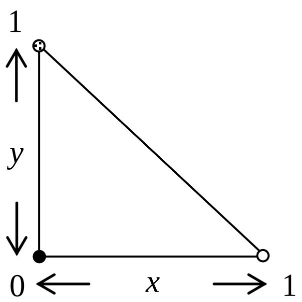

Fig. 3 Dynamics of *x* and *y* for $p=0$.

Proposition 1. *In case of null survival of the species, free-riding is the only steady state.*

This result is consistent with the dead-anyway effect (Pratt and Zeckhauser, 1996), which states that the marginal cost of contributing decreases with risk. The effect can be potentially important in magnitude when the risk tends to one (Treich 2010). Species being unlikely to survive or their risk of extinction equals one, agents expect to suffer from their certain extinction and invariably free-ride.

2.3.2. Ambiguous survival

When the survival of species is ambiguous or $0 < p < 1$, we have

$$\dot{x} = x(1-x)[p(w-gk^{-1})x^{N-1} - (1-p)gk^{-1}]. \tag{12}$$

Fixing $\dot{x} = 0$ gives two fixed points of the replicator dynamics: $x = 0$ and $x = 1$. We look at the derivative of $F(x)$ which is

$$F'(x) = (1-2x)[p(w-gk^{-1})x^{N-1} - (1-p)gk^{-1}] + x(1-x)[(N-1)p(w-gk^{-1})x^{N-2}]. \tag{13}$$

At $x = 0$, $F'(0) < 0$, that is, a stable equilibrium. At $x = 1$, $F'(1) \lesseqgtr 0$. Just as in the static framework, if $gk^{-1} < pw$, we are in presence of a steady state. If greater, the equilibrium is unstable (see Table 2).

Proposition 2. *Contributing dominates when the proportional fair-share is less than the expected salvage of wealth. Otherwise, the model-agent is better off free-riding.*

2.3.3. Existence and uniqueness of an interior equilibrium

Let us now prospect the interior equilibrium. We reduce $x(t)$ to the function $T(x) = f_c - f_f$

$$T(x) = p(w-gk^{-1})x^{N-1} - (1-p)gk^{-1}. \tag{14}$$

The interior equilibrium is the root of $T(x)$ in the interval $[0,1]$. If $gk^{-1} > pw$, $gk^{-1} - pw > 0$ but $T(1) > 0$ so there is no interior equilibrium. If $gk^{-1} < pw$, $gk^{-1} - pw < 0$ and $T(1) > 0$. Furthermore, $p \in [0,1]$ verifies $T(0) < 0$. Up to now, the fulfilled conditions are necessary but not sufficient. At last, we have

$$T'(x) = (N-1)p(w-gk^{-1})x^{N-2}. \tag{15}$$

$T'(x) > 0$ is increasing which ends the proof. There is a unique root of $T(x) = 0$ situated in $[0,1]$ and it equals

$$x^* = \left[\frac{gk^{-1} - pgk^{-1}}{pw - pgk^{-1}} \right]^{1/N} . \tag{16}$$

At $x = x^*$, $T'(x^*) > 0$, that is, an unstable equilibrium (see Table 2). Figures 4 and 5 show the ambiguous survival dynamics relative to the trade-off rule.

Proposition 3. *There is a unique unstable Nash equilibrium x^* where the trade-off between the proportional fair-share and the expected salvage of wealth determines whether agents end up contributing or free-riding.*

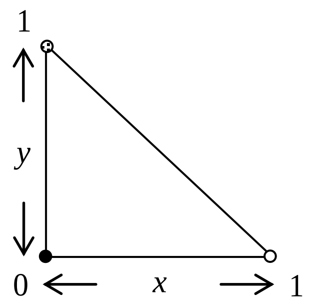

Fig. 4 Dynamics of *x* and *y* for $p \in [0,1]$ and $gk^{-1} > pw$

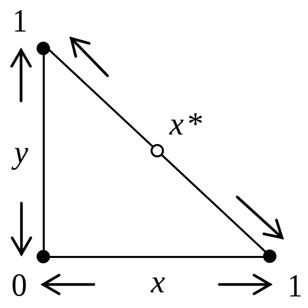

Fig. 5 Dynamics of *x* and *y* for $p \in [0,1]$ and $gk^{-1} < pw$

Table 2 Stability of equilibria

	$p = 0$	$0 < p < 1$	
		$gk^{-1} > pw$	$gk^{-1} < pw$
$x = 0$	stable [●]	stable [●]	stable [●]
$x = x^*$			unstable [○]
$x = 1$	unstable [○]	unstable [○]	stable [●]

At last, let us linger on the k parameter. When the risk is high or $k \to 1$, the trade-off $gk^{-1} \lesseqgtr pw$ depends on the probability of species' survival. As the risk turns low or $k \to 0$, the constraint against contributing gets unbounded or $gk^{-1} \to \infty$. In parallel, when $p \to 1$ we have $pw \to w$. The trade-off is conceivable only if the fair-share equals reasonable and bounded amounts of wealth, whereas low k leads to a cost of contributing greater than what the wealth constraint permits. Thenceforth, when the risk is low, the proportional fair-share exceeds the expected salvage of wealth. We have $gk^{-1} >> pw$.

Proposition 4. *Low k provokes social free-riding and disinterest in the public good.*

This result invalidates the argument defended by Olson (1968) and Marwell and Ames (1979) who state that public goods are provided by groups in which an individual has an interest in the good that is greater than the cost of the good. The high interest of an agent at risk does not suffice to cover the cost of the public good. The only case where their argument could hold water is if the model-agent is the agent at stake.

3. Public goods option market game

3.1. Static game

Now consider an option market for the public good, where agents exchange option contracts on public goods by buying and selling until they agree on a price. The option market price can reveal the social probability of species' survival and of the attainment of the threshold. We assume the market-efficiency hypothesis. For that reason, the threshold can only be attained at the market equilibrium where all possible exchanges are cleared at the market price. In this case, the proportional fair-share equals the expected payoff from salvaging wealth.

In view of the fact that exchanges are based upon predictions of the probability of species' survival, our option market is a prediction market. The prediction market can be considered a representative person with a set of expectations (Wolfers and Zitzewitz 2004). Even though the equilibrium price does not reveal the mean belief that agents hold, it yields a bound on the mean belief (Manski 2006). The equilibrium reveals the position of the model-agent. The agent is defined a buyer if she believes that $p > 0.5$, and a seller otherwise.

Yet, there is a difference between the standard prediction market and ours. The latter does not produce outcomes tied to events exogenous to the market. Indeed, option exercises depend on the number of betting exchanges cleared on the market. If agents conclude an insufficient number of option contracts, they fail to sufficiently provide the public good and the species' survival is jeopardized. In turn, buyers fall salvaging their wealth through, and sellers lose their premium from bearing the risk of species extinction.

In terms of the expected payoff matrix, we have

	Buyer	Seller
Buyer	$(1-p)(w-gk^{-1})\ ;\ (1-p)(w-gk^{-1})$	$p(w-gk^{-1})\ ;\ (1-p)w+p\theta$
Seller	$(1-p)w+p\theta\ ;\ p(w-gk^{-1})$	$(1-p)w\ ;\ (1-p)w$

Buyers willing to increase the species' probability of survival offer to buy option contracts at their willingness-to-pay, *i.e*. their proportional fair-share, whereas sellers unconvinced of the survival likelihood propose selling contracts at their willingness-to-accept. If a buyer happens to meet a buyer, their contracts are not exchanged and both face the risk of $1-p$ of losing their wealth. Likewise, if both agents propose asks, they are exposed by $1-p$ to the wealth disappearance. Otherwise, the buyer salvages her wealth at p by bidding her proportional fair-share and the seller receives a premium of $p\theta$ for her asking price, which here corresponds to the option price: the survival probability times the offer.

The market of size M is divided between m buyers and $M-m$ sellers. If all the bid-ask spreads are zero and buyers' and sellers' bids and offers match or $m = M - m \Leftrightarrow m = M/2$, the option market is fully efficient and the threshold is attained. If $m \neq M - m$ and $m \neq M/2$, the payoffs of a buyer and a seller facing an uncertain probability of species' survival are

$$\begin{cases} \pi_b = (1-p)(w-gk^{-1}) + p(w-gk^{-1}) \\ \pi_s = (1-p)w + p\theta \end{cases}. \tag{17}$$

3.1.1. Null survival

When the survival of species is null or $p=0$, (17) reduces to

$$\begin{cases} \pi_b = w - gk^{-1} \\ \pi_s = w \end{cases}. \tag{18}$$

We have $w > w - gk^{-1}$. Selling the option contract of the species' survival provides a higher expected payoff so traders are net sellers.

3.1.2. Ambiguous survival

When $p \in [0,1]$, the outcome depends on the tradeoff between the proportional fair-share and the expected salvage of wealth at the market price. We have two possible outcomes. If $\pi_b < \pi_s$ then $gk^{-1} > p(w-\theta)$, that is, the cost of contributing is greater than the expected payoff from salvaging wealth at the market price; the option price is greater than the expected payoff from contributing, so agents are net sellers. If $\pi_b > \pi_s$ we have $gk^{-1} < p(w-\theta)$. The expected payoff from contributing is less than the expected benefit from salvaging wealth at the market price so agents are net buyers. Finally, at $p\theta = pw - gk^{-1}$, buyers and sellers equalize their payoffs or $\pi_b = \pi_s$ and the market is fully efficient. Therefore, the public good will be provided by the option market for $p = gk^{-1}(w-\theta)^{-1}$.

Simulations in Table 3 show that for levels of *k* and *p* close to zero, the unbounded constraint against contributing yields negative option prices. The risk premium turns out to be negative. Despite appearances, this result is not absurd, and is known to exist in the capital asset pricing model. Analyzing premiums for public good losses resumes to studying the agents' behavior in a context where property rights are passed over. In our case, the agent has to consider herself the owner of the public good from which she loses her wealth. Asking for compensation demanded for the public loss, even in case of private fatalities, makes *de facto* selling agents creditors of the public good. And we know, for example, that risk premium can be negative with credit default option swap contracts. As a result, when agents decide to sell an option contract on the public good, the option price reveals that they ask the market to protect them from their potential loss of wealth in exchange of a premium. Buyers of option

Table 3 Simulations of static option prices given p and k for $w = 10$: $p\theta = pw - gk^{-1}$

p / k	0.01	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99	1.00
0.00	–200.00	–20.00	–10.00	–6.67	–5.00	–4.00	–3.33	–2.86	–2.50	–2.22	–2.02	–2.00
0.10	–199.00	–19.00	–9.00	–5.67	–4.00	–3.00	–2.33	–1.86	–1.50	–1.22	–1.02	–1.00
0.20	–198.00	–18.00	–8.00	–4.67	–3.00	–2.00	–1.33	–0.86	–0.50	–0.22	–0.02	0.00
0.30	–197.00	–17.00	–7.00	–3.67	–2.00	–1.00	–0.33	0.14	0.50	0.78	0.98	1.00
0.40	–196.00	–16.00	–6.00	–2.67	–1.00	0.00	0.67	1.14	1.50	1.78	1.98	2.00
0.50	–195.00	–15.00	–5.00	–1.67	0.00	1.00	1.67	2.14	2.50	2.78	2.98	3.00
0.60	–194.00	–14.00	–4.00	–0.67	1.00	2.00	2.67	3.14	3.50	3.78	3.98	4.00
0.70	–193.00	–13.00	–3.00	0.33	2.00	3.00	3.67	4.14	4.50	4.78	4.98	5.00
0.80	–192.00	–12.00	–2.00	1.33	3.00	4.00	4.67	5.14	5.50	5.78	5.98	6.00
0.90	–191.00	–11.00	–1.00	2.33	4.00	5.00	5.67	6.14	6.50	6.78	6.98	7.00
0.99	–190.10	–10.10	–0.10	3.23	4.90	5.90	6.57	7.04	7.40	7.68	7.88	7.90
1.00	–190.00	–10.00	0.00	3.33	5.00	6.00	6.67	7.14	7.50	7.78	7.98	8.00

contracts become the sellers of the protection contracts they demand. Since scarcity is highly valued on markets, the rarer the disease is, the higher the premium gets. As a result, the cost of protecting oneself on an option market, given the smallness of *k*, is exorbitant.

On the contrary, when *k* and *p* tend to 1, option prices are close to the level of wealth: buyers' expected benefit from providing the public good approximates their proportional fair-share. Therefore, the option market fails to be surplus-generating. The game is solved with public demand and supply which never meet. Given that no buyer will accept to contribute unless her benefit from the public good overpasses her cost of funding it, and given that no seller will accept to exchange at a negative price unless she asks the market to protect her from the risk of wealth loss, only a non-market provision at a fiscal capitation relative to the society's risk aversion seems feasible. Although the option market mechanism can be efficient enough to equalize expected benefits and costs and thus to produce the public good, it produces null surpluses at the equilibrium and thus fails to fulfill the role it has been assigned.

3.2. Dynamic game

We now consider infinite populations of *x* buyers and *y* sellers, where $x+y=1$. Let $z=x/y$ denote the ratio of buyers to sellers. The evolution of the system is given by the following differential equations

$$\begin{cases} \dot{x} = z(f_b - \overline{f}) \\ \dot{y} = (1-z)(f_s - \overline{f}) \end{cases}, \tag{19}$$

where *z* and $1-z$ establish the market surpluses of the demand f_b and supply f_s sides, given the average surplus in the market $\overline{f} = zf_b + (1-z)f_s$. Let us set a mixed population where *M* agents are randomly chosen. The probability that a buyer faces *m* buyers in the population of size *M* at a particular seller is given by the Binomial distribution

$$f(m \mid M-1, z) = \binom{M-1}{m} z^m (1-z)^{M-1-m}. \tag{20}$$

In the population, the probability for a model-buyer to be allocated the exchange is $(m+1)^{-1}$. Indeed, the model-seller chooses a model-buyer at random when more than one. Following the work of Bach *et al*. (2006) and Julien *et al*. (2008), the probability for a buyer to be served when selecting a seller is given by

$$[1-(1-z)^M](Mz)^{-1}. \tag{21}$$

$(1-z)^M$ is the probability that model-agents do not match, *i.e*. all the surplus is captured by the supply side, so $[1-(1-z)^M]$ is the probability that the model-buyer captures some surplus on the market by finding the right model-seller, given the population and the surplus available on the market. The average payoffs of a model-buyer and a model-seller, among $M-1$ other agents, are

$$\begin{cases} f_b = (1-p)(w-gk^{-1}) + p(w-gk^{-1})[1-(1-z)^M](Mz)^{-1} \\ f_s = (1-p)w + p\theta \end{cases}. \tag{22}$$

The differential equations yield a single formulation in form of

$$\dot{z} = z(1-z)(f_b - f_s), \tag{23}$$

So the dynamic evolution of $z(t)$ amounts to

$$\dot{z} = z(1-z)[p(w-gk^{-1})[1-(1-z)^M](Mz)^{-1} - (1-p)gk^{-1} - p\theta]. \tag{24}$$

3.2.1. Null survival

When the survival of species is null or $p=0$, we obtain

$$\dot{z} = -z(1-z)gk^{-1}. \tag{25}$$

Solving $\dot{z}=0$ gives two fixed points of the replicator dynamics which cancel out $z(1-z)$: $z=0$ and $z=1$. The derivative of $F(z)$ is

$$F'(z)=-gk^{-1}+2zgk^{-1}. \tag{26}$$

At $z=0$, $F'(0)<0$, that is, a stable equilibrium. At $z=1$, $F'(1)>0$, which implies an unstable equilibrium (see Table 5). Figure 6 points up the dynamics.

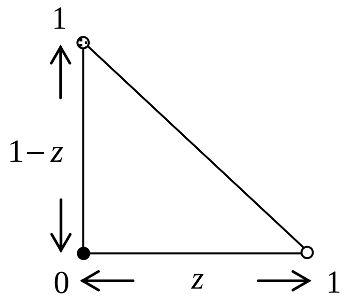

Fig. 6 Dynamics of z and $1-z$ for $p=0$.

Proposition 5. *In case of null survival of the species, agents exchanging option contracts are net sellers and the public good fails to be provided.*

3.2.2. Ambiguous survival

When the survival of species is ambiguous or $0<p<1$, we have

$$\dot{z}=z(1-z)[p(w-gk^{-1})[1-(1-z)^M](Mz)^{-1}-(1-p)gk^{-1}-p\theta]. \tag{27}$$

Fixing $\dot{z}=0$ gives two fixed points: $z=0$ and $z=1$. The derivative of $F(z)$ is

$$\begin{aligned}F'(z)&=(1-2z)[p(w-gk^{-1})[1-(1-z)^M](Mz)^{-1}-(1-p)gk^{-1}-p\theta]\\&+z(1-z)[-M(Mz)^{-1}(1-z)^{M-1}p(w-gk^{-1})-(Mz)^{-2}[1-(1-z)^M]p(w-gk^{-1})]\end{aligned}. \tag{28}$$

At $z=0$, $F'(0)<0$, that is, a stable equilibrium. At $z=1$, $F'(1)\lesseqgtr 0$. If the inequality $gk^{-1}<p(wM^{-1}-\theta)[1-p(1-M^{-1})]^{-1}$ for $p<M(M-1)^{-1}$ is verified, we are in presence of a

Table 4 Simulations of dynamic option prices given p and k for $w = 10$: $p\theta = pwM^{-1} - gk^{-1}[1 - p(M-1)M^{-1}]$

p / k	0.01	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99	1.00
0.01	–200.00	–20.00	–10.00	–6.67	–5.00	–4.00	–3.33	–2.86	–2.50	–2.22	–2.02	–2.00
0.10	–181.90	–18.10	–9.00	–5.97	–4.45	–3.54	–2.93	–2.50	–2.18	–1.92	–1.74	–1.72
0.20	–163.80	–16.20	–8.00	–5.27	–3.90	–3.08	–2.53	–2.14	–1.85	–1.62	–1.46	–1.44
0.30	–145.70	–14.30	–7.00	–4.57	–3.35	–2.62	–2.13	–1.79	–1.53	–1.32	–1.17	–1.16
0.40	–127.60	–12.40	–6.00	–3.87	–2.80	–2.16	–1.73	–1.43	–1.20	–1.02	–0.89	–0.88
0.50	–109.50	–10.50	–5.00	–3.17	–2.25	–1.70	–1.33	–1.07	–0.88	–0.72	–0.61	–0.60
0.60	–91.40	–8.60	–4.00	–2.47	–1.70	–1.24	–0.93	–0.71	–0.55	–0.42	–0.33	–0.32
0.70	–73.30	–6.70	–3.00	–1.77	–1.15	–0.78	–0.53	–0.36	–0.23	–0.12	–0.05	–0.04
0.80	–55.20	–4.80	–2.00	–1.07	–0.60	–0.32	–0.13	0.00	0.10	0.18	0.23	0.24
0.90	–37.10	–2.90	–1.00	–0.37	–0.05	0.14	0.27	0.36	0.43	0.48	0.52	0.52
0.99	–20.81	–1.19	–0.10	0.26	0.45	0.55	0.63	0.68	0.72	0.75	0.77	0.77
1.00	–19.00	–1.00	0.00	0.33	0.50	0.60	0.67	0.71	0.75	0.78	0.80	0.80

steady state (see Table 5). Knowing that $p \in [0,1]$, the opposite inequality is impossible. The market liquidity equals $1 - p(M-1)M^{-1}$ and corresponds to exchanges of proportional fair-shares without significant movements in the market price.

Proposition 6. *In case of ambiguous survival of the species, agents exchanging option contracts are net sellers* (*equally divided into buyers and sellers*) *if the proportional fair-share is greater than* (*equals*) *the expected payoff from salvaging wealth at the market price.*

3.2.3. Existence and uniqueness of an interior equilibrium

We reduce $z(t)$ to the function $S(z) = f_b - f_s$

$$S(z) = (1-p)(w - gk^{-1}) + p(w - gk^{-1})[1-(1-z)^M](Mz)^{-1} - (1-p)w - p\theta . \tag{29}$$

The interior equilibrium has a root of $S(z)$ in $[0,1]$. We have $z = 0$ or $S(0) < 0$. In parallel, at $z = 1$, $S(1) > 0$ if $gk^{-1} < p(wM^{-1} - \theta)[1 - p(1-M^{-1})]^{-1}$ for $p < M(M-1)^{-1}$. The derivative equals

$$S'(z) = -M(Mz)^{-1}(1-z)^{M-1}p(w - gk^{-1}) - (Mz)^{-2}[1-(1-z)^M]p(w - gk^{-1}) . \tag{30}$$

For $z > 0$, we have $S'(z) > 0$. By substituting $(1-z)^M$ with Z^M, the root of $S(Z) = 0$ in $[0,1]$ amounts to

$$Z^* = \left[\frac{p\theta + gk^{-1} - pgk^{-1}}{pw - pgk^{-1}} Mz + 1 \right]^{1/M} . \tag{31}$$

We have $\lim_{z\to\infty} Z^* = 1$. Furthermore, the equilibrium is stable since $S'(Z^*) < 0$ (see Table 5). At $Z^* = 1$, $p\theta = pw - gk^{-1}$ or $p = gk^{-1}(w-\theta)^{-1}$. Since $Z^* = 1 - z^*$ and $z = x / y$ the result implies that $x \propto y$. Figures 7 and 8 illustrate the dynamics depending on the trade-off rule.

Proposition 7. *There is a unique unstable Nash equilibrium* $z*$ *where the trade-off between proportional fair-share and expected payoff from salvaging wealth at the market price determines whether agents end up as net sellers or equally split between buyers and sellers. When the cost equals the expected benefit, the market is fully efficient or* $x \propto y$ *and the public good is provided. This result holds only for* $p = gk^{-1}(w-\theta)^{-1}$.

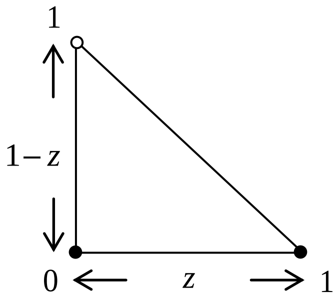

Fig. 7 Dynamics of z and $1-z$ for $p \in [0,1]$ and $gk^{-1} > \frac{p(wM^{-1}-\theta)}{1-p(1-M^{-1})}$

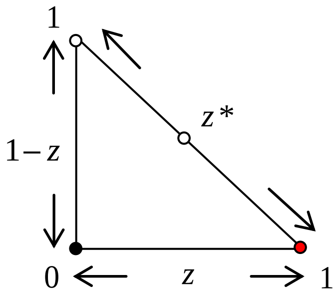

Fig. 8 Dynamics of z and $1-z$ for $p \in [0,1]$ and $gk^{-1} < \frac{p(wM^{-1}-\theta)}{1-p(1-M^{-1})}$

Table 5 Stability of equilibria

	$p=0$	$0<p<1$	
		$gk^{-1} > \frac{p(wM^{-1}-\theta)}{1-p(1-M^{-1})}$	$gk^{-1} < \frac{p(wM^{-1}-\theta)}{1-p(1-M^{-1})}$
$z=0$	stable [●]	stable [●]	stable [●]
$z=z*$			unstable [○]
$z=1$	unstable [○]	impossible [●]	stable [●]

For low levels of *k* and *p*, the dynamic simulations in Table 4 reveal that the constraint against contributing becomes unbounded, yielding null or high negative option prices. The null option price result is consistent with Plummer (1986) who shows that if the project attainment fails to change the probability of supply of the public good (and thus remains ambiguous), option price is zero. Negative option prices reveal that the proportional fair-share is much greater than the expected salvage of wealth at the market price. Sellers are willing to exchange protection contracts with buyers for running the loss by $1-p$. Like in the static game, the cost of protecting oneself for a small *k* is overpriced.

High levels of *k* and *p* induce quasi-null option prices, implying that the expected payoff from salvaging wealth at the market price equals the proportional fair-share. Therefore, buyers will exit the option market, because they do not expect a higher benefit from the public good than the cost of funding it. The option market will remain inert and therefore collapse.

Proposition 8. *Low k and/or low p induce negative option prices, i.e. sellers of the option contract demand protection for the probable loss. High k and p induce quasi-null option prices, i.e. buyers' expected payoff at the market price equals their proportional fair-share. The results imply the absence of surplus captured on the option market.*

4. Conclusion

Our threshold public goods game first reveals that agents free-ride in case of null survival of the species, a phenomenon commonly termed as the dead-anyway effect. Then, the game shows coexistence of free-riders and contributors in case of ambiguous survival of the species. Agents contribute if their proportional fair-share is less than their expected salvage of wealth. From the cost-benefit analysis, it simply states that agents are willing to contribute to the public good if their expected benefit from the public good exceeds the cost of producing it. The results show that agents tend to contribute to the public good in ambiguity, as long as they bear the risk of suffering personal losses. The result is conforming with the results by Bailey *et al*. (2005), who find less free-riding in ambiguity in presence of large populations. As a final point, we find that in case of rare diseases, social free-riding is unconditionally dominant.

As regards the option market for public goods, when the probability of survival of the species is zero, agents are net sellers. When the probability is ambiguous, the market is fully efficient, *i.e*. the public good is provided, only if the proportional fair-share is equal to the expected payoff from salvaging wealth at the market price. This result holds for a specific market belief over the species' survival. However, sellers face negative option prices, meaning that they are in demand for protection for running the probable loss of wealth. On the demand side of the market, option market prices are close to zero, signifying that the expected payoff from salvaging wealth at the market price equals the proportional fair-share. Provided the absence of surplus realized on the market, agents have no incentive to exchange contracts. The option market is doomed to disappear, which condemns the species' survival. Providing ambiguous environmental public goods by option markets is thus socially inefficient.

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Appendix

Equation (7): we know from the Binomial theorem that $\sum_{n=0}^{N-1} \binom{N-1}{n} x^n (1-x)^{N-1-n} = 1$, thus $\Pr(N-1=n) = x^{N-1}$.

Equation (8): $\dot{x} = x(f_c - \bar{f})$ thus $x[f_c - (xf_c + yf_f)]$ and $x[f_c - xf_c - (1-x)f_f]$.

Equation (9): we have $\dot{x} = x(1-x)[pw - [gk^{-1} + p(w-gk^{-1}) - p(w-gk^{-1}]x^{N-1})]$.

Proof of Proposition 1. Equation (11): $F(x) = -x(1-x)gk^{-1} = -(x-x^2)gk^{-1} = -xgk^{-1} + x^2gk^{-1}$ and $F'(x) = -gk^{-1} + 2xgk^{-1}$. At $x=0$, $F'(0) = -gk^{-1} < 0$. At $x=1$, $F'(1) = gk^{-1} > 0$. □

Proof of Proposition 2. Equation (12) yields $F'(x) = (1-2x)[p(w-gk^{-1})x^{N-1} - (1-p)gk^{-1}]$ $+x(1-x)[(N-1)x^{N-2}p(w-gk^{-1})]$. We see that at $x=0$, $F'(0) = -(1-p)gk^{-1} < 0$. At $x=1$, $F'(1) = gk^{-1} - pw \lesseqgtr 0$. □

Proof of Proposition 3. We have $T(0) = -gk^{-1} + pgk^{-1} < 0$ and $T(1) = pw - gk^{-1} \gtreqless 0$. We have $T'(x) = (N-1)p(w-gk^{-1})x^{N-2}$. If we take the two previous cases, that is $gk^{-1} < pw$ and $gk^{-1} > pw$ we have $T(1) = pw - gk^{-1}$. There is a unique equilibrium of $T(x)=0$, $x(1-x)[p(w-gk^{-1})x^{N-1} - (1-p)gk^{-1}] = 0$ $\quad x(1-x)p(w-gk^{-1})x^{N-1} = x(1-x)gk^{-1}(1-p)$ and $x^* = [(1-p)gk^{-1}]/[p(w-gk^{-1})]^{1/N} = [(gk^{-1} - pgk^{-1})/(pw - pgk^{-1})]^{1/N}$ in $[0,1]$. At $x = x^*$, $T'(x^*) = (N-1)p(w-gk^{-1})x^{N-2} > 0$. □

Proof of Proposition 4. When $k \to 0$, $gk^{-1} \to \infty$. When $p \to 1$, $pw \to w$ thus bounded. As a result, we have $gk^{-1} >> pw$ and agents end up free-riding. □

Equation (21). From the Binomial sampling, we have $\sum_{m=0}^{M-1}(m+1)^{-1}\binom{M-1}{m}z^m(1-z)^{M-1-m}$ $=\sum_{m=1}^{M-1}\frac{1}{M}\binom{M}{m+1}z^m(1-z)^{M-1-m}=\frac{1}{zM}\left[\sum_{m=0}^{M}\binom{M}{m}z^m(1-z)^{M-m}\right]$ and $[1-(1-z)^M]/zM$ $\square$

Proof of Proposition 5. Equation (25): $F(z)=-z(1-z)gk^{-1}=-(z-z^2)gk^{-1}=-zgk^{-1}+z^2gk^{-1}$ and $F'(z)=-gk^{-1}+2zgk^{-1}$. At $z=0$, $F'(0)=-gk^{-1}<0$. At $x=1$, $F'(1)=gk^{-1}>0$. $\square$

Proof of Proposition 6. Equation (27) yields $F'(z)=(1-2z)[p(w-gk^{-1})[1-(1-z)^M](Mz)^{-1}]$ $-(1-p)gk^{-1}-p\theta]+z(1-z)[-M(Mz)^{-1}(1-z)^{M-1}p(w-gk^{-1})-(Mz)^{-2}[1-(1-z)^M]p(w-gk^{-1})$ At $z=0$, $F'(0)=-(1-p)gk^{-1}-p\theta<0$. At $z=1$, $F'(1)=-pwM^{-1}+gk^{-1}[1-p(M-1)M^{-1}]$ $+p\theta \lesseqgtr 0$. $F'(1)<0$ when $gk^{-1}<p(wM^{-1}-\theta)[1-p(1-M^{-1})]^{-1}$ for $p<M(M-1)^{-1}$ $\square$

Proof of Proposition 7. When $z\to 0$, $(Mz)^{-1}\to\infty$ yielding $0(\infty)=0$ and thus $S(0)=$ $-(1-p)gk^{-1}-p\theta<0$. Equally, we have $S(1)=pw(M)^{-1}-gk^{-1}[1-p(M-1)M^{-1}]-p\theta$. We have $S(1)>0$ for $p<M(M-1)^{-1}$ when $gk^{-1}<p(wM^{-1}-\theta)[1-p(1-M^{-1})]^{-1}$ is verified. The derivative is $S'(z)=-M(Mz)^{-1}(1-z)^{M-1}p(w-gk^{-1})-(Mz)^{-2}[1-(1-z)^M]p(w-gk^{-1})<0$ and is increasing for $-M^2z(1-z)^{M-1}-(1-z)^M>1$ or $z>0$. As $\lim_{z\to\infty}Z^*=1$, the unique equilibrium for $(1-z)^M=Z$ is $Z^*=[[(p\theta+gk^{-1}-pgk^{-1})Mz+1]/(pw-pgk^{-1})]^{1/M}$. At $z=z^*$, $S'(Z^*)<0$ as $-M(Mz^*)^{-1}(1-z^*)^{M-1}<(Mz^*)^{-2}[1-(1-z)^M]$. Lastly, $p(w-\theta)=gk^{-1}$. Given that $Z^*=1-z^*\Leftrightarrow 1-z^*=1-(x/y)\Leftrightarrow z^*=x/y$, at the equilibrium the matching is full or $x\propto y$. $\square$

Proof of Proposition 8. When $k\to 0$ and/or $p\to 0$, $gk^{-1}[1-p(M-1)M^{-1}]\to+\infty$ thus $pwM^{-1}-gk^{-1}[1-p(M-1)M^{-1}]\to-\infty$ $\square$