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A Spatial Model of Common-value Elections

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JEL Classification Number D72, D82

Keywords: Voting, Elections, Median Voter, Information, Platform Convergence, Jury Theorem, Swing Voter's Curse, Turnout, Abstention, Roll-off

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1 Introduction

One of the earliest and strongest formal arguments in favor of the democratic institution of majority voting is Condorcet's (1785) classic "jury theorem": if one of two policy alternatives is better for society, in some objective sense, and voters seek independently to identify that policy, with even minimal success, then majority opinion is likely—in fact, almost certain, in a large electorate—to correctly identify the superior policy. Since public policies inevitably effect different groups of citizens differently, however, existing literature has largely dismissed this information pooling role of elections as applicable only to a few specific voting environments such as juries and small committees. On the other hand, the broad goals of many public policies—such as national defense, economic and environmental stability, and eliminating crime, poverty, and corruption—have essentially unanimous appeal; if voters base policy evaluations on societal outcomes such as these, their preferences are likely to be correlated, making the assumption of identical preferences a plausible approximation even in public elections. In this light, recent literature has reexamined voter incentives in common-values settings. Most notably, an influential paper by Feddersen and Pesendorfer (1996) points out that when voter differences are predominantly informational, uninformed citizens have a strategic incentive to abstain from voting, in deference to those with better expertise. This observation provides a possible explanation for various empirical features of public elections, such as the common practice of *roll-off* (i.e. voting in some races on a ballot, but abstaining in others, even though voting costs have already been paid, and so are no longer relevant), as well as the correlation between voter participation and information variables.

Despite this resurgent interest, information models of elections remain quite simple in certain respects. In particular, they focus exclusively on voter incentives, with no role for other political actors such as politicians beyond passively representing particular policy alternatives. Furthermore, the number of such alternatives is typically limited to two. This is in contrast, for example, to spatial models such as Downs (1957), that emphasize the incentives of candidates who must compete for votes by choosing policy from among an entire continuum of alternatives. Thus, the purpose of this paper is to explore the incentives that candidates face, and the implications of candidate behavior for voter welfare and other electoral outcomes, in an environment in which voter disagreements arise primarily from differences of opinion, rather than fundamental differences of preference.

The model analyzed below synthesizes key aspects of the earlier models mentioned above. As in Condorcet's (1785) model, one of two policies is optimal for society, depending on an unobserved state of the world. An electorate can implement either of these policies, or may implement any convex combination of the two, so that, as in Downs' (1957) model, there is an entire continuum of policy possibilities. Citizens share a common utility function over policy outcomes, and are risk-averse, preferring moderate policies in the face of uncertainty. Private opinions regarding the state of the world are modeled as independent private signals that are each correlated with the true state variable. The quality of an individual's signal depends on her[1] expertise, which varies throughout the electorate, as in McMurray's (2010) generalization of Feddersen and Pesendorfer (1996). As in the Downsian (1957) framework, candidates in this model compete for votes by proposing platform policies, and the candidate who receiving the most votes takes office, and determines policy.

As an example of a political issue that might closely resemble the model described, consider a society of individuals who all wish to exit an economic recession, but disagree over the question of whether growth will be helped or hindered by a "stimulus" policy of increasing fiscal spending. If this question were resolved, all would wish to implement either the maximum or minimum level of stimulus; in the midst of uncertainty, however, voters may prefer to hedge against the possibility of implementing precisely the wrong policy, by instead instituting a stimulus of moderate size. As an alternative example, consider two equally-costly plans for reducing pollution, crime, corruption, or poverty. In this case, imperfectly-informed citizens may prefer to partially implement both plans, even though under perfect information they would implement only the more effective of the two.[2]

Analysis of this model produces a number of insights into candidate and voter incentives. The first of these is an immediate consequence of model assumptions: conditional on private signals, voter preferences are *single-peaked* over the policy interval. This observation provides an information rationale for ideological diversity, which has been corroborated empirically, but is commonly assumed without theoretical foundation. It is also the case that the most expert decisions have the most confidence in their own opinions, and so form the strongest and most extreme views. This provides a possible explanation for Palfrey and Poole's (1987) finding that citizens who adopt extreme ideological positions tend to be better-informed than those with more moderate ideologies.

The remaining results of this paper depend crucially on candidate motivations. Following existing literature, this paper considers a number of possibilities. In the first version of the model, candidates are motivated purely by the desire to win office, and make binding commitments to implement platform policies. In this case, there is a unique equilibrium, in which citizens vote *sincerely*, or *informatively* for the candidate whose platform they prefer, but candidates adopt identical platforms at the center of the policy space. This result is reminiscent of Black's (1948) canonical median voter theorem, and arises for the

[1] Throughout this paper, feminine pronouns refer to voters and masculine pronouns refer to candidates.

[2] In business committee settings, identical logic could apply to competing plans for advertizing or sales incentives.

same reason: moving away from the center merely concedes votes to a candidate's opponent. The welfare implication of this result, however, differs starkly from previous models: rather than representing a Condorcet winner or utilitarian optimum, this equilibrium outcome is the policy that would be optimal only on the basis of prior information; if any private information were available, a superior policy could be identified.

A second version of this model assumes that candidates do not value office for its own sake, but instead seek office merely as a means of influencing public policy; like other citizens, each desires the policy outcome that is optimal for society. In equilibrium, candidates adopt distinct platforms, and citizens again vote sincerely. This is again reminiscent of a standard result by Wittman (1977, 1983), although in this model an equilibrium necessarily exists. Also, unlike standard models, divergence from the center actually enhances welfare here.

With informative voting, the candidate with the superior platform is more likely to win the election by a single vote than to lose by a single vote. Accordingly, an additional vote for that candidate is less likely to be *pivotal* (i.e. change the election outcome) than a vote for his opponent. An uninformed citizen therefore suffers from a *swing voter's curse*, as in Feddersen and Pesendorfer (1996) and McMurray (2010), and prefers to abstain rather than vote for either candidate, even if voting is costless. By reproducing this incentive, this model affirms the information rationale offered in those models for empirical phenomena such as roll-off, and the correlation between relative information variables and turnout.

A third specification of this model Follows the citizen-candidate tradition of Osborne and Slavinski (1996) and Besley and Coate (1997) in assuming that candidate commitments prior to an election are not credible: once elected, a candidate may implement the policy of his choice. For policy-motivated candidates, this introduces an opportunity for the winning candidate to update his policy choice on the basis of information reflected in vote totals, that was unavailable when his platform decision was made. Since equilibrium voting is informative, the winning candidate's *margin of victory* provides information regarding the location of the optimal policy. Specifically, a candidate who wins by a large margin will implement a policy that is more extreme than his platform, while a candidate who wins by a small margin will moderate his stance. This provides a theoretical foundation for the popular notion of an *electoral mandate*, by which large margins of victory are thought to communicate permission from voters to implement more extreme policies. Since literally every vote influences the margin of victory, this result also provides a foundation for the popular mantra that "every vote counts", contrary to standard models in which a vote has no influence unless it creates or breaks a tie.

The result that every vote influences policy outcomes implies that a rational citizen no longer needs to condition her behavior on the unlikely event in which her vote is pivotal.

This fact undermines the logic of the swing voter's curse, which arises when a voter compares the probabilities of her vote being pivotal in the two states of the world. Even though a pivot vote is no longer relevant, however, and even though voting is costless, similar logic produces a *signaling voter's curse* in this context, so that a poorly informed citizen again has incentive to abstain. Her own vote has the potential to push the eventual policy outcome in either direction; recognizing that the winning candidate will respond optimally to the superior expertise reflected by the votes of her peers, however, a poorly informed citizen again prefers to abstain.[3] Thus, this model demonstrates the empirical predictions of Feddersen and Pesendorfer (1996) and McMurray (2010) to be more general than perhaps thought. As in those models, strategic abstention here is also welfare-improving: although the private information of nonvoters is not utilized, the response to informed votes is amplified.

One standard result in spatial voting models is Duverger's (1954) Law, which essentially states that plurality rule elections foster two strong parties, and discourage the creation of smaller parties. This is because, while a vote for either of two major candidates is already unlikely to be pivotal, voting for a sure loser is less likely still to change the election outcome. The standard model therefore predicts that citizens should not vote for fringe parties, who therefore should have no incentive to run for office. Even if a minor party candidate did manage to attract strong support, he would risk splitting votes with the closer of the two major candidates, thereby inadvertently "spoiling" the election in favor of his least-favored opponent. That analysis changes in this setting, however, because eventual policy outcomes depend on each candidate's vote total. Even if he does not win office, then, voting for an extreme candidate pushes policy in the desired direction. This therefore justifies the popular notion of sending a message to a winning candidate by casting a "protest vote" for a likely loser with a more extreme policy position, in turn providing an incentive for such protest candidates to run for office in the first place.

In addition to the references above, this model shares much in common Razin's (2003) model of signaling in common-value elections. Most notably, that model demonstrates the possibility of electoral mandates, inferred by the winning candidate from his margin of victory. In that model, however, large margins of victory can also make a winning candidate more moderate, rather than more extreme. Candidates also behave deterministically, rather than strategically, and voters have homogeneous information quality. The possibility of multiple candidates is not considered, and voter abstention is not allowed. Shotts (2006) and Meirowitz and Shotts (2007) consider an alternative role for signaling in elections, which is to influence incumbent politicians' perceptions of re-election prospects.

[3] Put differently, a poorly informed citizen abstains for fear that the winning candidate will overreact to her vote, inferring it to be of average quality when in fact its quality is below average.

The remainder of this paper is organized as follows. Section 2 introduces the model, and Section 3 characterizes equilibrium, assuming the various combinations of office or policy motivation, and credible commitments or responsive candidates. Section 3.1 analyzes the case in which candidates commit to observable policy platforms, including office- and policy-motivated candidates in Sections 3.1 and 3.1 and abstention in Section 3.1. Section 3.2 considers the case of responsive candidates, including the possibilities of multiple candidates in Section 3.2 and abstention in Section 3.2, and examples in Section 3.2. Section 4 analyzes welfare for the various model specifications, and Section 6 concludes. Proofs of most formal results are presented in the Appendix.

2 The Model

A society consists of N citizens and two candidates, A and B. For technical convenience, I adopt Myerson's (1998, 2000) assumption that the precise number N of citizens is unknown, but is drawn from a Poisson distribution with mean n.[4] Together, this electorate must choose and implement a policy from the interval $[-1, 1]$ of alternatives, which will provide a common benefit to every citizen. Ex ante, it is unknown which policy alternative will be best for society. Let $Z \in \{-1, 1\}$ denote the true state of the world, which identifies the optimal policy. If policy $x \in [-1, 1]$ is implemented in state Z then each citizen receives utility $u(x, Z)$, which declines with the distance between the implemented policy and the optimal policy:

$$u(x, Z) = -(x - Z)^2 . \tag{1}$$

Thus, $Z = -1$ implies that the best policies lie at the lower end of the policy space, while $Z = 1$ implies that higher policies are better. Note, however, that $u(x, Z)$ is strictly concave in x, implying that citizens are risk-averse, and may therefore prefer policies in the interior of the policy space. Specifically, it is straightforward to show that, conditional on available information Ω, (1) is maximized when $x = E(Z|\Omega)$, as stated in (2):

$$E(Z|\Omega) = \arg\max_x E[u(x, Z)|\Omega] . \tag{2}$$

Ex ante, prior beliefs are such that either state of the world is equally likely (i.e. $\Pr(Z = 1) = \Pr(Z = -1) = \frac{1}{2}$), implying that the ex ante optimal policy is at the center of the policy interval.

[4] As Bade (2006) discusses, one important advantage of this assumption is that the numbers N_A and N_B of votes for the two candidates are independent random variables. In the numerical examples in Sections 3.2 and 3.2, N is instead fixed and known.

A citizen's private opinion regarding the optimal policy is represented by a private signal $S_i \in \{-1, 1\}$ which is positively correlated with Z. Because citizens differ in expertise, however, these signals are of heterogeneous quality. Specifically, each citizen is endowed with information quality $Q_i \in [0, 1]$, drawn independently (and independent of Z) from a common distribution F which, for technical convenience, is assumed to have a differentiable and strictly positive density f.[5] Conditional on Z and Q_i, the distribution of S_i is given as follows, for any $s, z \in \{-1, 1\}$ and $q \in [0, 1]$.

$$\Pr(S_i = s | Z = z, Q_i = q) = \frac{1}{2}(1 + zsq). \tag{3}$$

With this distribution, the correlation between S_i and Z is given simply by $Corr(S_i, Z|Q_i) = Q_i$. Thus, Q_i measures the strength of a citizen's conviction that her private opinion of the optimal policy is correct. To a perfectly informed (i.e. $Q_i = 1$) citizen, for instance, S_i reveals Z perfectly; to a perfectly uninformed (i.e. $Q_i = 0$) citizen S_i reveals nothing.

The distribution F of expertise within the population is common knowledge, but Q_i and S_i are observed only privately. Conditional on private information, the posterior distribution of Z is given by the same expression as in (3):

$$\Pr(Z = z | S_i = s, Q_i = q) = \frac{1}{2}(1 + zsq), \tag{4}$$

for any $s, z \in \{-1, 1\}$ and $q \in [0, 1]$. A citizen's expectation of the optimal policy, then, is simply $E(Z|S_i, Q_i) = S_i Q_i$. As noted above, this is the policy that maximizes the expectation of (1), conditional on private information alone. Thus, conditional on private information alone, preferences over policies are single-peaked, as in traditional models. The assumption that F has full support (i.e. that f is strictly positive) implies that the distribution of citizens' ideal points also has full support on the policy interval.

With individual citizens thus informed, candidates propose policy platforms $x_A, x_B \in [0, 1]$.[6] Observing these platforms, each citizen then votes (at no cost) for one of the two candidates. A strategy $\sigma : [0, 1] \times \{-1, 1\} \times [0, 1]^2 \to \{A, B\}$ specifies a citizen's behavior for every possible realization $(q, s) \in [0, 1] \times \{-1, 1\}$ of her private information and for every pair $(x_A, x_B) \in [0, 1]^2$ of candidate platforms.[7] Let Σ denote the set of all such

[5] The distribution F of information quality is assumed here to be exogenous. For information acquisition games that lead in equilibrium to asymmetric information, see Martinelli (2006, 2007) and Oliveros (2007).

[6] Informally, candidates can be thought of as citizens who hold private opinions, as in the citizen-candidate framework of Osborne and Slavinski (1996) and Besley and Coate (1997). Formally, however, candidates are not citizens in that they do not receive private signals of their own. This is largely for simplification; alternatively, the information structure of this model could be reinterpreted as describing updated beliefs after both candidates' signals were announced publicly.

[7] Mixed strategies could be allowed, but would be used with zero probability in equilibrium, as the analysis below makes clear.

strategies. For any strategy σ, it is also convenient to define the associated *sub-strategy* $\bar{\sigma} : [0,1] \times \{-1,1\} \longrightarrow \{A,B\}$ as the projection of σ onto the set $[0,1] \times \{-1,1\}$ of types, for a particular pair (x_A, x_B) of platform policies, and to let $\bar{\Sigma}$ denote the set of all such sub-strategies. $\bar{\Sigma}$ can then also be interpreted as the set of strategies available to a voter in the voting subgame associated with a particular pair of candidate platforms.[8] In the analysis below, I restrict attention to equilibria that are *symmetric* with respect to voter strategies, meaning that every voter plays the same strategy in equilibrium; accordingly, σ can be reinterpreted as an entire profile of strategies, with each citizen responding identically to private information and candidate platforms.[9]

Votes are cast simultaneously, and an election winner $W \in \{A,B\}$ is determined by simple majority rule, breaking a tie if necessary by a fair coin toss. Once elected, candidate j implements a policy $y_j \in [-1,1]$. If candidates are *committed*, the winner must implement his pre-election platform policy $y_j = x_j$; if candidates are *responsive*, the winning candidate's policy choice may be a function $y_j : \mathbb{Z}_+^2 \longrightarrow [-1,1]$ of realized vote totals $a, b \in \mathbb{Z}_+$. Let Υ denote the set of all such policy functions. The ultimate policy outcome $Y \in [-1,1]$ therefore depends on the strategy choices of both voters and candidates, and on the realizations of the election winner W and the numbers N_A and N_B of votes for each candidate, which in turn depend on the private information (Q_i, S_i) of each citizen, and therefore on the state Z. In choosing their strategies, citizens and *policy-motivated* candidates seek to maximize their expectations of $u(Y,Z)$, while *office-motivated* candidates maximize the probability of being elected. The analysis below characterizes *symmetric perfect Bayesian equilibrium*, which is defined below for each of the various versions of the model.

3 Equilibrium

3.1 Commited Candidates

This section assumes that candidates' campaign platform commitments are binding, so that the winning candidate must implement his platform policy $y_j = x_j$. Section 3.1 begins

[8] In Sections 3.1 through 3.1 and in Sections 3.2 and 3.2, abstention from voting is not allowed, but Sections 3.1 and 3.2 introduce voter abstention, and then Section 3.2 considers the possibility of additional candidates. Representing voter abstention as a vote for candidate 0, therefore, a citizen's set of actions expands from $\{A,B\}$ to $\{A,B,0\}$ to $\{A,B,C,D,0\}$. When using these expanded action sets, denote the set of strategies as Σ' or Σ'' and the set of sub-strategies as Σ'_{x_A,x_B} or Σ''_{x_A,x_B}.

[9] In a game of population uncertainty such as this, this is actually without loss of generality. Since the set of players is random, any asymmetric strategy profile would be outcome equivalent to a symmetric profile of mixed strategies, with mixture probabilities reflecting the fraction of potential players who take each action; the best response to any such strategy is therefore the same for every individual.

by analyzing the voting subgame associated with a particular pair (x_A, x_B) of candidate strategies. Sections 3.1 and 3.1 then proceed by backward induction to analyze candidates' incentives for platform selection, under the assumptions that candidates are office- or policy-motivated, respectively.

Voting

For any voting sub-strategy $\bar{\sigma} \in \bar{\Sigma}$, the sub-strategy $\sigma_i^* \in \bar{\Sigma}$ is a *best response* to $\bar{\sigma}$ if it maximizes $Eu\left(Y, Z|Q_i, S_i; \sigma_i, \bar{\sigma}\right)$, where σ_i denotes a citizen's own sub-strategy and $\bar{\sigma}$ is the sub-strategy of her peers. $\sigma^* \in \bar{\Sigma}$ is an equilibrium in the voting subgame if it is its own best response. Citizens seek to implement the superior policy, but which policy outcome is superior depends on the state variable: when $x_A < x_B$, x_A is superior to x_B if $Z = -1$ while x_B is superior if $Z = 1$. A citizen's expectation $E\left(Z|Q_i, S_i\right) = Q_i S_i$ of the state depends on her private information; of natural interest, therefore, is a *belief threshold strategy* σ_T, according to which citizens with high expectations vote B and citizens with low expectations vote A.

Definition 1 *The voting sub-strategy $\bar{\sigma} \in \bar{\Sigma}$ is a sincere belief threshold sub-strategy if there exists some belief threshold $T \in [-1, 1]$ such that*

$$\bar{\sigma}(q, s) = \begin{cases} A \text{ if } qs < T \\ B \text{ if } qs > T \end{cases};$$

if the inequalities are reversed then $\bar{\sigma}$ is an insincere belief threshold sub-strategy. The voting strategy $\sigma \in \Sigma$ is a (sincere/insincere) belief threshold strategy if there exists a belief threshold function $T : [-1, 1]^2 \longrightarrow [-1, 1]$ such that, for every platform pair $(x_A, x_B) \in [-1, 1]^2$, the sub-strategy $\bar{\sigma}$ associated with σ is a (sincere/insincere) belief threshold sub-strategy, with belief threshold $T(x_A, x_B)$.

Lemma 1 states that optimal voter responses to exogenous platforms $x_A < x_B$ can be characterized by a belief threshold, and that an equilibrium belief threshold exists.[10] If platforms $x_A = -x_B$ are symmetric around zero then equilibrium voting is a symmetric belief threshold strategy, meaning that voting is also *sincere* (i.e. $\sigma(q, -1) = A$ and $\sigma(q, 1) = B$).

Lemma 1 *If candidates are committed then, for any pair (x_A, x_B) of platform policies, there exists a sub-strategy $\bar{\sigma}^*$ that constitutes an equilibrium in the voting subgame. If $x_A \neq x_B$ then $\bar{\sigma}^*$ is a sincere belief threshold sub-strategy, with belief threshold T such that $T = 0$ if and only if policy platforms $x_A = -x_B$ are symmetric around zero.*

Proof. See Appendix. ■

[10] If $y_A = y_B$ then, trivially, any voting strategy constitutes an equilibrium.

Office Motivation

An *office-motivated* candidate j receives utility 1 if he wins the election, and 0 otherwise. Expected utility is therefore given simply by his probability $\Pr(W = j; x_A, x_B, \sigma)$ of winning the election. A *perfect Bayesian equilibrium* is therefore a pair (x_A^*, x_B^*) of candidate platform strategies and a voting strategy σ^* such that the voting sub-strategy $\sigma^*(x_A, x_B)$ induced by σ^* for any pair (x_A, x_B) of candidate platforms constitutes an equilibrium in the voting subgame, and x_j^* maximizes $\Pr\left(W = j; x_j, x_{-j}^*, \sigma^*\right)$, where x_{-j}^* is the equilibrium platform of candidate j's opponent.

A citizen prefers the policy platform that is nearest to her private expectation of Z; accordingly, as Lemma 1 states, citizens with high expectations vote for candidate B while citizens with low expectations vote for candidate A. Which candidate receives the larger share of votes, therefore, depends (in expectation) on which platform is closest to a larger fraction of citizens' expectations. Given the symmetry of the model, this is the platform that is closer to the zero policy (i.e. the *ex ante* median of citizens' expectations). In choosing his platform, therefore, an office-motivated candidate seeks to adopt a more moderate position than his opponent. Accordingly, as Theorem 1 now states, the unique symmetric perfect Bayesian equilibrium is such that both candidates adopt the zero policy.

Theorem 1 (Median Voter Theorem) *If candidates are commited and office-motivated then (x_A^*, x_B^*, σ^*) is a symmetric perfect Bayesian equilibrium only if $x_A^* = x_B^* = 0$ and σ^* is almost everywhere equivalent to a sincere belief threshold strategy, with threshold function T^* such that $T^*(0,0) = 0$. Furthermore, such an equilibrium exists.*

Proof. See Appendix. ∎

The candidate behavior predicted by Theorem 1 closely resembles that predicted by the well-known Median Voter Theorem introduced by Black (1948) and Downs (1952), and for the same reason: efforts to attract large fractions of the electorate push candidates toward one another, and toward the median voter's ideal point. The welfare implications in this model differ dramatically, however, from the implications in a model with fundamental differences in tastes. In that context, the median voter theorem is a positive outcome, representing a compromise between the competing desires of citizens at opposite ends of the preference spectrum; if citizens are risk-averse, the median voter's ideal policy minimizes the maximum disutility experienced by any citizen, and so may maximize a utilitarian social welfare function. In this context, by contrast, citizens unanimously prefer (ex post) a more extreme policy; the zero policy is optimal only when no information is available beyond the common prior. In this setting, then, the median voter theorem represents a complete failure to utilize citizens' private information.

The result that candidate platforms converge in equilibrium has sometimes been viewed as an empirical failing of rational voting models, both because candidate platforms in real-world elections appear to differ substantially, and because identical platforms would give citizens no incentive to vote. As in previous literature, however, the prediction here of platform convergence depends on candidate motivation; in the following section, equilibrium candidate platforms do not coincide.

Policy Motivation

In this section, candidates are assumed to be ordinary citizens, choosing policy platforms to maximize the expectation of (1). The optimal policy for such a candidate is his expectation of the state, conditional on available information. Though candidate j must commit to a policy platform before observing the election outcome, he can condition his expectation on the event $W = j$ in which he wins the election, since only then will his platform policy be implemented. Theorem 2 now states that equilibrium exists, and that equilibrium voting is informative, implying that candidates anticipate learning different pieces of information upon winning the election, and that policy platforms diverge accordingly. In particular, an equilibrium exists in which voting is also sincere.

Theorem 2 (Policy divergence) *If candidates are committed and policy-motivated then* (x_A^*, x_B^*, σ^*) *is a symmetric perfect Bayesian equilibrium only if candidate platforms are given by* $x_j^* = E(Z|W = j) \equiv \hat{z}_j$ *for* $j = A, B$*, with* $x_A^* \neq x_B^*$*, and the voting strategy* σ^* *is almost everywhere equivalent to a sincere belief threshold strategy. Furthermore, such an equilibrium exists, with platforms* $x_A^* = -x_B^*$ *symmetric around zero.*

Proof. See Appendix. ■

Abstention

The analysis of Section 3 assumes that every citizen must vote. In most real-world election environments, however, voters are allowed to abstain; indeed, in most democracies, abstention rates tend to be fairly high. The set of voting strategies can now be denoted as $\Sigma' = \left\{\sigma : [0,1] \times \{-1,1\} \times [0,1]^2 \to \{A, B, 0\}\right\}$, and the set of voting sub-strategies induced by Σ' can now be denoted as $\bar{\Sigma}'$. With this modification, Lemma 2 repeats the voting subgame analysis of Section 3.1 for a given pair $x_A < x_B$ of policy platforms. As before, citizens who strongly believe the state to be high or low will have strong preferences for y_B or y_A, respectively. Now, however, a *belief threshold strategy* σ_{T_1,T_2}, must be redefined using two thresholds instead of one, to allow for the possibility that some citizens abstain altogether from voting.

Definition 2 *The subgame voting strategy $\bar{\sigma} \in \Sigma'$ is a sincere belief threshold sub-strategy if there exist belief thresholds $T_1, T_2 \in [-1, 1]$ such that*

$$\bar{\sigma}(q, s) = \begin{cases} A \text{ if } qs \leq T_1 \\ 0 \text{ if } T_1 < qs < T_2 \\ B \text{ if } qs \geq T_2 \end{cases} ;$$

if the inequalities are reversed then $\bar{\sigma}$ is an insincere belief threshold sub-strategy. The voting strategy $\sigma \in \Sigma'$ is a (sincere/insincere) belief threshold strategy if there exists a belief threshold function $T : [-1, 1]^2 \longrightarrow [-1, 1]^2$ such that, for every platform pair $(x_A, x_B) \in [-1, 1]^2$, the sub-strategy $\bar{\sigma}$ associated with σ is a (sincere/insincere) belief threshold sub-strategy, with belief thresholds $T_1(x_A, x_B)$ and $T_2(x_A, x_B)$.

Since voting is costless, and since each citizen's private signal induces a strict preference ordering over the two policy outcomes, it may seem unlikely that allowing abstention will alter equilibrium behavior. However, as Lemma 1 states, $T_1 < T_2$ in equilibrium, implying positive abstention. The logic behind this result is the swing voter's curse (Feddersen and Pesendorfer, 1996): because citizens' opinions are correlated with the truth, and voting is informative, the candidate with the superior policy is more likely to win the election by one vote than to lose by one vote. A vote for the inferior candidate is therefore more likely to be pivotal than is a vote for the superior candidate, so a citizen who is indifferent—or, by continuity, almost indifferent—between voting for the two candidates strictly prefers to abstain.

Lemma 2 (Swing voter's curse) *If candidates are committed and abstention is allowed then, for any pair (x_A, x_B) of platform policies, there exists a sub-strategy $\bar{\sigma}^* \in \Sigma'$ that constitutes an equilibrium in the voting subgame. If $x_A \neq x_B$ then $\bar{\sigma}^*$ is a sincere belief threshold sub-strategy, with belief threshold functions such that $T_1 < T_2$. Also, $T_1 = -T_B$ if and only if $x_A = -x_B$.*

Proof. See Appendix. ■

The last part of Lemma 2 points out that if policy outcomes are symmetric around the zero policy then equilibrium voting behavior may exhibit the same symmetry. In that case, whether a citizen votes or not depends only on her whether her information quality Q_i exceeds the threshold T^*. Like Lemma 1, Lemma 2 characterizes equilibrium responses to exogenous policy outcomes. Proposition 1 now treats the case in which policy outcomes are determined by campaign platforms, which are chosen by policy-motivated (i.e. citizen) candidates before the election. Like Theorem 2, Proposition 1 predicts that campaign platforms will diverge in equilibrium. Partial equilibrium voting behavior is given by Lemma 2, implying positive abstention in equilibrium. As in Lemma 2, equilibrium voting behavior and candidate platforms may be symmetric around the zero policy.

Proposition 1 *[Restate as (1) characterization, (2) existence, with additional characterization.]If policy platform commitments are binding, candidates are policy-motivated, and abstention is allowed then there exists a symmetric perfect Bayesian equilibrium* (x_A^*, x_B^*, σ^*) *that satisfies the following conditions:*

1. Candidate platforms are given by $x_j^* = E(Z|W = j) \equiv \hat{z}_j$ *for* $j = A, B$*, and are symmetric around zero (i.e.* $x_A^* = -x_B^*$*).*

2. σ^* *is a sincere belief threshold strategy, with belief threshold functions* $T_1^* < T_2^*$ *that are symmetric around zero in equilibrium (i.e.* $T_1^*(x_A^*, x_B^*) = -T_2^*(x_A^*, x_B^*)$*).*

Proof. See Appendix. ■

Like Theorem 2, Proposition 1 predicts that candidate platforms will diverge in equilibrium. The reason for this is that the two candidates learn different information from voters: candidate A is more likely to be elected in state -1 and candidate B is more likely to be elected in state 1. Conditional on being elected, therefore, A's expectation of Z is lower than B's. It may be, however, that the identity of the election winner is not a sufficient statistic for the information conveyed through voting. For example, with informative voting, if candidate B wins the election in state -1 it will likely be by only a few votes, whereas in state 1 he may win by a landslide. Put differently, the election winner is determined by the sign of the difference $N_B - N_A$ in vote totals for the two candidates; it may be the case that the magnitude $|N_B - N_A|$ of this difference carries informational content as well. If this is the case, a candidate who observes vote totals may wish to deviate from the campaign policy platform that he committed to before the election. In this section, such deviations are prohibited. In real-world elections, however, pre-election commitments may be quite difficult or even impossible to enforce. Accordingly, Section 3.2 relaxes the assumption that campaign commitments are binding.

3.2 Responsive Candidates

Voting

In this section, as in Section 3.1, candidates are policy-motivated, seeking to maximize the expectation of (1), just like ordinary citizens.[11] Unlike Section 3.1, however, a win-

[11] Since voting behavior can no longer be conditioned a candidate's policy choice, an office-motivated candidate has no basis for choosing policy, conditional on winning the election. Since policy-motivated behavior is optimal from voters' perspective, as Section 4 shows, an office-motivated candidate would presumably wish simply to mimic a policy-motivated candidate.

ning candidate is no longer required to implement the platform policy that he adopted before the election. Thus, a perfect Bayesian equilibrium $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ includes a pair $(x_A^*, x_B^*) \in [-1,1]^2$ of platform policies, a pair $(y_A^*, y_B^*) \in \Upsilon^2$ of policy functions, and a voting strategy σ^* such that $\left(x_j^*, y_j^*\right)$ maximizes $Eu\left(X, Z; x_j, y_j, x_{-j}^*, y_{-j}^*, \sigma^*\right)$, where x_j and y_j are the platform and policy function adopted by candidate j and x_{-j} and y_{-j} are the platform and policy function adopted by j's opponent, and the sub-strategy $\bar{\sigma}^*$ induced by σ^* maximizes $Eu\left[X, Z; \left(x_j^*, y_j^*\right)_{j=A,B}, \bar{\sigma}\right]$ for any pair (x_A, x_B) of platform policies.

In equilibrium, as Lemma 3 now states, a candidate merely implements his expectation of Z. Because policy is implemented after vote totals are observed, he conditions his expectation on this information. Voters' equilibrium behavior is again characterized by a belief threshold strategy. For such a strategy, each candidates' beliefs regarding the true state increase in the number of B votes and decrease in the number of A votes. By voting A, therefore, a voter essentially pushes the eventual policy outcome slightly to the left; by voting B, she pushes it to the right.

Lemma 3 *If candidates are responsive and policy-motivated then* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ *is a symmetric perfect Bayesian equilibrium only if the following are true:*

1. σ^* is a belief threshold strategy.

2. For all $a, b \in \mathbb{Z}_+$ and for $j = A, B$, policy functions are given by $y_j^*(a,b) = \hat{z}_{a,b}$ and either $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ or $\hat{z}_{a,b+1} < \hat{z}_{a,b} < \hat{z}_{a+1,b}$.

Proof. See Appendix. ■

If her fellow-citizens vote according to a particular belief threshold strategy, and the winning candidate responds as Lemma 3 predicts, a citizen who is sufficiently confident that $Z = -1$ prefers to vote for candidate A, thereby pushing the ultimate policy outcome to the left. Similarly, a citizen who is sufficiently confident that $Z = 1$ prefers to vote for candidate B. Thus, the best response to a belief threshold strategy is another belief threshold strategy. Accordingly, Theorem 3 identifies a symmetric perfect Bayesian equilibrium, in which citizens vote according to a sincere belief threshold strategy σ^*, sincerely reporting their private signals, and candidates respond by implementing their expectations of Z, given vote totals, as in Lemma 3. Since platform commitments are not binding, they lack credibility, and are ignored by voters in equilibrium. Thus any pair of platforms can be consistent with equilibrium. In particular, Part 1 of Theorem 3 assumes that candidates platforms reflect candidates' *ex ante* expectations of the true state, as would be the case if platforms were binding. Indeed, these platforms would be optimal if, for example, platorm commitments were binding with some positive probability.

Theorem 3 (Signaling Equilibrium) *If candidates are responsive and policy-motivated then a symmetric perfect Bayesian equilibrium* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ *exists, which exhibits the following properties:*

1. Candidate platforms are given by $x_j^* = \hat{z}_j$ *for* $j = A, B$, *and are symmetric around zero (i.e.* $x_A^* = -x_B^*$*).*

2. σ^* *is a sincere belief threshold strategy, and every sub-strategy* $\bar{\sigma}^*$ *associated with* σ^* *is characterized by the belief threshold* $T^* = 0$.

3. For all $a, b \in \mathbb{Z}_+$ *and for* $j = A, B$, *policy functions are given by* $y_j^*(a, b) = \hat{z}_{a,b}$ *and are both monotonic (i.e.* $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$*) and symmetric around zero (i.e.* $\hat{z}_{a,b} = -\hat{z}_{b,a}$*).*

Proof. See Appendix. ■

In Sections 3.1 through 3.1, as in standard voting models, an individual vote has influence only in the extremely unlikely event that it is "pivotal", either making or breaking a tie. In the equilibrium of Theorem 3, this is no longer the case; instead, *every* vote is pivotal, in the sense that every vote influences the ultimate policy outcome, by pushing the policy-maker's expectations one way or the other. In this setting, then, the popular mantra that "every vote counts" in public elections can be taken quite literally.

The candidate platforms $\hat{z}_j$ specified for the equilibrium of Theorem 3 can be viewed as weighted averages of the policy outcomes $\hat{z}_{a,b}$ associated with particular pairs (a, b) of vote totals. Accordingly, policy outcomes will sometimes be higher, and sometimes lower, than candidates' policy platforms. Specifically, a policy outcome turns out to be more extreme than the winning candidate's policy platform (i.e. $\hat{z}_{a,b} < \hat{z}_A$ or $\hat{z}_{a,b} > \hat{z}_B$) when that candidate wins by a higher margin of victory than expected, and is less extreme (i.e. $\hat{z}_A < \hat{z}_{a,b} < \hat{z}_B$) when the margin of victory was more narrow than expected. This fact illustrates the popular notion of electoral "mandates", by which candidates who win by large margins are expected to implement more extreme policies. The larger the margin of victory, the larger the mandate, in the sense that policy outcomes become more extreme as the margin of victory increases.

Multiple Candidates

In this section, the set $\{A, B, C, D\}$ of candidates is expanded from two to four. Thus, the set of voting strategies is given by $\Sigma'' = \left\{\sigma : [0,1] \times \{-1, 1\} \times [0,1]^2 \to \{A, B, C, D, 0\}\right\}$ and the set of induced sub-strategies denoted as $\bar{\Sigma}''$. As in Section 3.2, candidates are responsive; as in Section 3.2, abstention is allowed. Definition 3 redefines the concept of a belief threshold strategy for this setting, using four belief thresholds instead of two. Under such a strategy, citizens with strong private opinions vote for candidates A or D, citizens with moderate opinions vote for candidates B or C, and citizens with only weak opinions abstain.

Definition 3 *The symmetric sub-strategy $\bar{\sigma} \in \bar{\Sigma}''$ is a sincere belief threshold sub-strategy if there exist belief thresholds $T_1, T_2, T_3 \in [-1, 1]$ such that*

$$\bar{\sigma}(q,s) = \begin{cases} A \text{ if } qs \in (-1, T_1) \\ B \text{ if } qs \in (T_1, T_2) \\ C \text{ if } qs \in (T_2, T_3) \\ D \text{ if } qs \in (T_3, 1) \end{cases};$$

if the inequalities are reversed then $\bar{\sigma}$ is an insincere belief threshold sub-strategy. The voting strategy $\sigma \in \Sigma''$ is a (sincere /insincere) belief threshold strategy if there exists a belief threshold function $T : [-1, 1]^4 \longrightarrow [-1, 1]^3$ such that, for every platform quadruple $(x_A, x_B, x_C, x_D) \in [-1, 1]^4$, the sub-strategy $\bar{\sigma}$ associated with σ is a (sincere/insincere) belief threshold sub-strategy, with belief thresholds $T_1(x_A, x_B, x_C, x_D)$, $T_2(x_A, x_B, x_C, x_D)$, and $T_3(x_A, x_B, x_C, x_D)$.

Theorem 4 now states the existence of a perfect Bayesian equilibrium, characterized by belief threshold voting. As prescribed by Lemma 3, the winning candidate implements his expectation of the state, conditional on vote totals; as in Lemma 3, the effect of a single vote is to push the policy outcome in one direction or another. Because more extreme citizen types vote for candidates A and D than B and C, votes for these two candidates have a greater impact on the winning candidate's beliefs. Thus, voting for an extreme candidate pushes policy by more than voting for a moderate candidate.

Theorem 4 *If candidates A, B, C, and D are responsive and abstention is allowed then there exists a symmetric perfect Bayesian equilibrium $\left[\left(x_j^*, y_j^*\right)_{j=A,B,C,D}, \sigma^*\right]$, which exhibits the following properties:*

1. Candidate platforms are given by $x_j^* = \hat{z}_j$ for $j = A, B, C, D$, and are symmetric around zero (i.e. $x_A^* = -x_D^*$, $x_B^* = -x_C^*$).

2. σ^* is a sincere belief threshold strategy, and every sub-strategy $\bar{\sigma}^*$ associated with σ^* is characterized by the same symmetric quadruple $(-T^*, 0, T^*)$ of belief thresholds, where $0 < T^* < 1$.

3. For all $a, b, c, d \in \mathbb{Z}_+$, policy functions are given by $y_j^*(a, b, c, d) = \hat{z}_{a,b,c,d}$ for $j = A, B, C, D$ and $\hat{z}_{a+1,b,c,d} < \hat{z}_{a,b+1,c,d} < \hat{z}_{a,b,c,d} < \hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1}$.

Proof. See Appendix. ■

One noteworthy comparative static result from this section is that the intensity of a citizen's political preference is positively related to her information quality. That is, citizens with poor information quality tend not to support extreme candidates. This is despite the modeling assumption that the ideal policy is commonly known to lie at one of the extremes of the policy spectrum: if information were perfect, every citizen would be an extremist. In essence, the same "signaling voter's curse" that in Section 3.2 caused citizens with the poorest information to abstain altogether from voting causes moderately informed citizens to vote for a moderate rather than an extreme candidate.

Abstention

By allowing the winning candidate to implement any policy of his choice, Theorem 5 resembles Theorem 3. Like that theorem, Theorem 5 predicts that candidates respond identically to vote totals. This implies, however, that a citizen do not actually care who wins the election, and therefore no longer restricts her attention to the rare case in which her vote changes the identity of the election winner. The logic of the swing voter's curse, therefore, no longer applies.

Since voting is costless and pivotal votes are no longer of concern, it may seem unlikely that citizens will abstain from voting in equilibrium—even a minimally informed citizen's signal, after all, is more likely to be Z than $-Z$. To the contrary, however, Theorem 5 states that equilibrium belief thresholds diverge, implying positive abstention. The logic of this result is as follows: in equilibrium, the winning candidate interprets vote totals as indicative of voters' private information. Each A vote, therefore, lowers his expectation of Z, while each B vote raises his expectation of Z. Since individual signals are correlated with the truth and voting is informative, the winning candidate's policy expectations will likely be pushed in the true direction of Z. An additional vote in the proper direction, therefore, has less marginal impact than an additional vote in the wrong direction. This makes a perfectly uninformed citizen—and, by continuity, a poorly informed citizen—prefer to abstain. Perhaps more intuitively, a perfectly uninformed citizen prefers, given her fellow-citizens' vote totals a and b, to implement the policy $E\left(Z|a,b;\sigma^*\right) \equiv \hat{z}_{a,b}$. Since (by Lemma 3) this is precisely the choice made by the winning candidate, the uninformed citizen achieves her optimum by abstaining.[12]

Theorem 5 (Signaling voter's curse) *If candidates are responsive and policy-motivated and voter abstention is allowed then there exists a symmetric perfect Bayesian equilibrium* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$, *which exhibits the following properties:*

[12] By voting, a citizen shifts the policy response to either $\hat{z}_{a+1,b}$ or $\hat{z}_{a,b+1}$.

1. Candidate platforms are given by $x_j^* = \hat{z}_j$ for $j = A, B$, and are symmetric around zero (i.e. $x_A^* = -x_B^*$).

2. σ^* is a sincere belief threshold strategy, and every sub-strategy $\bar{\sigma}^*$ associated with σ^* is characterized by the same symmetric pair $(-T^*, T^*)$ of belief thresholds, where $0 < T^* < 1$.

Proof. See Appendix. ■

The results of this section exhibit the same behavioral prediction: whether policy outcomes are exogenous (as in Lemma 2), determined by binding platform commitments (as in Proposition 1), or chosen ex post by the winning candidate (as in Theorem 5) informed citizens vote in equilibrium and uninformed citizens abstain. This prediction is consistent with the empirical evidence, reviewed in McMurray (2010), that voter turnout is correlated with information variables such as education, and age. As Feddersen and Pesendorfer (1996) point out, it also provides an explanation for voter abstention when voting is costless, such as roll-off. Turnout is also highest among those with extreme policy preferences, consistent with evidence from Palfrey and Poole (1987).

Examples

Signaling Equilibrium Propositions 2 and 3 illustrate the equilibrium identified in Theorem 3 with simple examples, assuming N to be fixed and known, and F to be uniform. Since voting behavior is based on expectations of policy outcomes, and because the number of policy outcomes is equal to the number of electoral outcomes, which grows with the number of citizens, these examples limit attention to very small electorates. Nevertheless, the examples here are typical of the types of behavior that apply more generally.

The electorate in Proposition 2 consists of only two citizens. In equilibrium, candidates first propose platform policies $x_A^* = -0.5$ and $x_B^* = 0.5$. If both citizens vote for A or for B, the winning candidate then implements -0.8 or 0.8, respectively, instead of the platform policy; if the election is tied, the winning candidate implements 0 instead.

Proposition 2 *Let F be a uniform distribution and let $N = 2$. If candidates are responsive then there exists an equilibrium $(x_A^*, x_B^*, \sigma^*, y_A^*, y_B^*)$ such that candidate platforms are $x_B^* = -x_A^* = 0.5$, voting $\sigma^* = \sigma_0$ is sincere, and policy outcomes are as follows: $y_A^*(1,1) = y_B^*(1,1) = 0$ and $y_B^*(0,2) = -y_A^*(2,0) = 0.8$.*

Proof. With sincere voting, a citizen votes B if $S_i = 1$ and votes A otherwise, so vote probabilities reduce from (5) to $\phi_{-1}(B) = \int_0^1 \frac{1}{2}(1-q)\,dq = \frac{1}{4}$ and $\phi_1(B) = \int_0^1 \frac{1}{2}(1+q)\,dq = \frac{3}{4}$, and expectations are given by $\hat{z}_{0,2} = \frac{-\frac{1}{2}(1/4)^2 + \frac{1}{2}(3/4)^2}{\frac{1}{2}(1/4)^2 + \frac{1}{2}(3/4)^2} = 0.8$ and $\hat{z}_{1,1} = \frac{-\frac{1}{2}[2(1/4)(3/4)] + \frac{1}{2}[2(1/4)(3/4)]}{\frac{1}{2}[2(1/4)(3/4)] + \frac{1}{2}[2(1/4)(3/4)]} = 0$. Candidate B can win the election either by receiving both citizens' votes or by receiving one vote and winning the tie-breaking coin toss. Conditional only on winning the election, therefore, his expectation of Z is given by $\hat{z}_B = \frac{-\frac{1}{2}\left[(1/4)^2+.5*2(1/4)(3/4)\right]+\frac{1}{2}\left[(3/4)^2+.5*2(1/4)(3/4)\right]}{\frac{1}{2}\left[(1/4)^2+.5*2(1/4)(3/4)\right]+\frac{1}{2}\left[(3/4)^2+.5*2(1/4)(3/4)\right]} = 0.5$. Probabilities and expectations for candidate A are determined symmetrically. That $\sigma^* = \sigma_0$, $x_j^* = \hat{z}_j$, and $y_j^*(a,b) = \hat{z}_{a,b}$ together constitute an equilibrium follows from Theorem 3. ■

Proposition 3 illustrates the same basic behavior, but with three citizens instead of two (therefore avoiding the possibility of a tie). In equilibrium, candidates propose platform policies ± 0.57 but then implement either ± 0.5 or ± 0.93.

Proposition 3 *Let F be a uniform distribution and let $N = 3$. If candidates are responsive then there exists an equilibrium $(x_A^*, x_B^*, \sigma^*, y_A^*, y_B^*)$ such that candidate platforms are $x_B^* = -x_A^* \approx 0.5652$, voting $\sigma^* = \sigma_0$ is sincere, and policy outcomes are as follows: $y_B^*(1,2) = -y_A^*(2,1) = 0.5$ and $y_B^*(0,3) = -y_A^*(3,0) \approx .9286$.*

Proof. As in Proposition 2, sincere voting produces vote probabilities $\phi_{-1}(A) = \phi_1(B) = \frac{3}{4}$ and $\phi_1(A) = \phi_{-1}(B) = \frac{1}{4}$. Upon winning, therefore, candidate B's expectation of Z is either $\hat{z}_{1,2} = \frac{-\frac{1}{2}\left[3(3/4)(1/4)^2\right]+\frac{1}{2}\left[3(1/4)(3/4)^2\right]}{\frac{1}{2}\left[3(3/4)(1/4)^2\right]+\frac{1}{2}\left[3(1/4)(3/4)^2\right]} = 0.5$ or $\hat{z}_{0,3} = \frac{-\frac{1}{2}(1/4)^3+\frac{1}{2}(3/4)^3}{\frac{1}{2}(1/4)^3+\frac{1}{2}(3/4)^3} = \frac{26}{28} \approx .9286$. Conditional only on candidate B winning the election, the expectation of Z is given by $\hat{z}_B = \frac{-\frac{1}{2}\left[3(3/4)(1/4)^2+(1/4)^3\right]+\frac{1}{2}\left[3(1/4)(3/4)^2+(3/4)^3\right]}{\frac{1}{2}\left[3(3/4)(1/4)^2+(1/4)^3\right]+\frac{1}{2}\left[3(1/4)(3/4)^2+(3/4)^3\right]} = \frac{26}{46} \approx 0.5652$. Probabilities and expectations for candidate A are determined symmetrically. That $\sigma^* = \sigma_0$, $x_j^* = \hat{z}_j$, and $y_j^*(a,b) = \hat{z}_{a,b}$ together constitute an equilibrium follows from Theorem 3. ■

Propositions 2 and 3 illustrate basic equilibrium behavior for an electorate with responsive candidates. They also demonstrate how the announcement of an election outcome might alter a candidate's beliefs about the true state of the world. This behavior is consistent with popular assessment of actual candidate behavior in real-world elections. Observers have long noted an empirical tendency for winning candidates to moderate their political stances (relative to campaign platforms) after close-shave elections, and to move toward extreme policy options after landslide victories, interpreting such as "mandates" from voters. In this setting, the notion of a mandate can be interpreted quite literally: large margins of victory communicate strong evidence in favor of extreme policy moves.

Abstention Proposition 4 retains the assumption of $N = 2$ from Proposition 2 but now allows for the possibility of voter abstention. As Theorem 5 predicts, poorly informed citizens abstain. In fact, equilibrium voter turnout is quite low: only 42%. If campaign platforms reflect candidates' expectations of the state conditional only on winning, as in Section 3.1, then they will diverge (to ± 0.5242), as in Theorem 2. Once election results are known, policy will then adjust, as in Theorem 3. A tie will cause either candidate to moderate his policy choice (to 0), while a slight majority will push his policy choice in the opposite direction (to ± 0.7907), and a large majority will make the policy even more extreme (± 0.9730). More often than not (i.e. with probability 0.63), then, the ultimate policy outcome that is implemented is more extreme than either candidate's campaign platform policy.

Proposition 4 *Let $N = 2$ be known and let F be uniform on $[0,1]$. If candidates are responsive then $(x_A^*, x_B^*, \sigma_{-T^*,T^*}, y_A^*, y_B^*)$ is a perfect Bayesian equilibrium, where $x_B^* = -x_A^* \approx 0.5242$, $y_B^*(0,0) = y_A^*(0,0) = 0$, $y_B^*(0,1) = -y_A^*(1,0) \approx 0.7907$, and $y_B^*(0,2) = -y_A^*(2,0) \approx 0.9730$, and where σ_{-T^*,T^*} is a symmetric belief threshold strategy with $T^* \approx 0.5814$. In this equilibrium, expected turnout is approximately 42%.*

Proof. See Appendix. ■

Proposition 5 next demonstrates that the logic of voter abstention applies even to an "electorate" comprised of only a single citizen (i.e. $N = 1$). As the sole voter, this citizen has complete control over the voting outcome. Nevertheless, she abstains in equilibrium with 0.33 probability. Before the election, candidates adopt platforms at ± 0.44; if she abstains, they implement the 0 policy instead; if she votes, they respond by implementing ± 0.67.

Proposition 5 *Let $N = 1$ be known and let F be uniform on $[0,1]$. If candidates are responsive then there is a unique belief threshold strategy σ_{-T^*,T^*} such that $(x_A^*, x_B^*, \sigma_{-T^*,T^*}, y_A^*, y_B^*)$ is a symmetric perfect Bayesian equilibrium. In this equilibrium, $T^* = \frac{1}{3}$, $x_B^* = -x_A^* \approx 0.44$, $y_j^*(0,0) = 0$, and $y_j^*(0,1) = -y_j^*(1,0) \approx 0.67$, and expected turnout is approximately 67%.*

Proof. See Appendix. ■

This exaggerated example elucidates the logic behind the signaling voter's curse: because the winning candidate does not know the citizen's type, he interprets her vote as though her information quality is average. When it is below average, therefore, the candidate will overreact to her vote, implementing a policy more extreme than her information merits; by abstaining, she achieves a more moderate policy outcome.

Multiple candidates Like Proposition 5, Proposition 6 focuses on an electorate comprised on only a single citizen. Now, however, there are four candidates, instead of two, thereby illustrating the equilibrium identified in Theorem 4. In that equilibrium, all candidates expect a positive vote share, and the citizen also abstains with positive probability (expected turnout is 80%).

Proposition 6 *Let $N=1$ be known, and let F be uniform on $[0,1]$. If candidates A, B, C, and D are responsive then (x^*,σ^*,y^*) is a perfect Bayesian equilibrium for the symmetric belief threshold voting strategy $\sigma^*=\sigma_{-.6,-.2,.2,.6}$, the vector $y^*=\left(y_j^*\right)_{j\in\{A,B,C,D\}}$ of policy responses defined by $y_j^*(0,0,0,1)=-y_j^*(1,0,0,0)=0.8$, $y_j^*(0,0,1,0)=-y_j^*(0,1,0,0)=0.4$, $y_j^*(0,0,0,0)=0$, and any vector $x^*=\left(x_j^*\right)_{j\in\{A,B,C,D\}}$ of candidate platforms. In this equilibrium, expected voter turnout is* 80%*.*

Proof. See Appendix. ■

Voting behavior in Proposition 6 is similar to that in Proposition 5, in that the type space is divided in equilibrium into equal segments—this time five instead of three. Applied here, the logic of Theorem 6 suggests that this addition of candidates improves welfare.

4 Welfare

The Condorcet (1785) jury theorem states that, as an electorate grows large, the majority decision identifies the better of two alternatives with probability approaching one. As originally stated, this result assumed sincere voting with no abstention; in this model, voting is instead strategic. Nevertheless, the same result is obtained in Theorem 6, for each of the above specifications of the model.

Theorem 6 (Jury theorem) *1. If candidates are committed and, for a population size parameter n, $Eu(Y,Z)$ is maximized by the symmetric voting sub-strategy $(\bar{\sigma}^*)_n\in\bar{\Sigma}$, then (a) $(\bar{\sigma}^*)_n$ constitutes an equilibrium in the voting subgame, and (b) the associated sequence of equilibrium policy outcomes $\left(\bar{Y}^*\right)_n$ approaches $p\lim_{k\to\infty}\left[\left(\bar{Y}^*\right)_n|Z\right]=\begin{cases}\min(x_A,x_B) & \text{if } Z=-1\\ \max(x_A,x_B) & \text{if } Z=1\end{cases}$.*

2. If candidates are committed and policy-motivated and, for a population size parameter n, $Eu(Y,Z)$ is maximized by the strategy combination $\left[\left(x_j^*\right)_{j=A,B},\sigma^*\right]_n\in[-1,1]^2\times\Sigma$, then (a) $\left[\left(x_j^*\right)_{j=A,B},\sigma^*\right]_n$ constitutes a perfect Bayesian equilibrium and (b) the associated sequence of equilibrium policy outcomes Y_n^* approaches $p\lim_{n\to\infty}(Y_n^*|Z)=Z$.

3. If candidates are responsive and policy-motivated and, for a population size parameter n, $Eu(Y,Z)$ is maximized by the strategy combination $\left[\left(x_j^*,y_j^*\right)_{j=A,B},\sigma^*\right]_{n_k}\in([-1,1]\times\Upsilon)^2\times\Sigma$, then (a) $\left[\left(x_j^*,y_j^*\right)_{j=A,B},\sigma^*\right]_n$ constitutes a perfect Bayesian equilibrium and (b) the associated sequence of equilibrium policy outcomes Y_n^* approaches $p\lim_{n\to\infty}(Y_n^*|Z)=Z$.

4. Claim 3 remains true if the number of candidates is increased.

5. Claims 1 through 4 remain true if abstention is allowed.

Proof. See Appendix. ■

Theorem 6 has a number of implications for institutional design. For example, the lack of credibility underlying campaign promises is often bemoaned for introducing uncertainty about candidates' future behavior. Part 3 of Theorem 6 implies, however, that responsive candidates will utilize information gleaned from electoral results to implement the socially optimal policy; policies that bind candidates to campaign platform policies will therefore only inhibit welfare.

A related implication of Theorem 6 is that, when campaign platform commitments are binding, welfare is higher when candidates are policy-motivated than when they are office-motivated. Specifically, Lemma 1 implies identical voter behavior regardless of candidate motivation; given this behavior, Part 2 of Theorem 6 implies that the policy platforms adopted by policy-motivated candidates (which differ in equilibrium from those adopted by office-motivated candidates, by Theorems 1 and 2) are socially optimal. This result may have relevance for determining optimal financial rewards for office holders.

The result that large electorates do well at selecting good policies might motivate popular "get out the vote" efforts to encourage voter participation. Some nations have gone as far as to make voting mandatory, levying fines on non-voters. Similar policies have been recommended for the United States (e.g. Lijphart, 1997). In an environment such as this, such policies might seem particularly useful, since every citizen possesses valuable private information; by allowing abstention, a voluntary election fails to utilize this information. On the other hand, it is also sometimes argued that voters who lack information should be somehow excluded from voting. An implication of Theorem 6, however, is that equilibrium voter abstention is socially optimal. Specifically, Part 4 of Theorem 6 states that, allowing voter abstention in each version of the model, the optimal combination of voter and candidate behavior constitutes a Bayesian equilibrium. As discussed in Section 3.2, any such equilibrium involves voter abstention. One way to understand this result is that, as McMurray (2010) points out, an optimal election mechanism would place greater weight on the votes of citizens with high-quality information than on those of poorly informed citizens; allowing abstention is a crude way of accomplishing this. With responsive candidates, an alternative intuition comes from viewing voters and candidates as senders and receivers in a "cheap talk" game (a la Crawford and Sobel, 1982). Within that framework, allowing abstention amounts to expanding the size of the message space from two messages to three. This interpretation is immediately evident in Proposition 5, where the single citizen divides her type space into three equal segments, voting according to $\sigma_{-\frac{1}{3},\frac{1}{3}}$ in equilibrium.

5 Evidence

In the model analyzed above, information leads a citizen both to favor extreme policies, and to vote. Thus, information, ideological extremeness, and voter participation should be jointly correlated. This is precisely the finding of Palfrey and Poole (1987). Information aside, however, the connection between ideology and voter participation has long been recognized. Traditionally, this correlation has been attributed to voting costs: in a standard spatial model, a citizen whose ideal point lies between the two candidates' policy platforms perceives only a small difference between the two candidates, while a citizen whose ideal point is more extreme than either platform strongly prefers the closer candidate of the two (e.g. Davis, Hinich, and Ordeshook, 1970); if voting is costly, therefore, then only citizens with extreme ideology should be willing to vote. As Feddersen and Pesendorfer (1996) emphasize, however, the logic of strategic abstention applies in costless voting environments, such as the decision of whether or not to continue voting in state and local elections, after having already voted for president, in addition to the original decision of whether to pay voting costs or not. In addition to predicting a correlation between ideology and turnout, therefore, the present model predicts a correlation between ideology and roll-off. That is, a strong ideology should make an individual not only more likely to vote, but also more likely to continue voting, once begun.

To test this prediction, this section analyzes data from the American National Election Studies (years 1952-2000), in states that held elections for senator or governor in the same year as a presidential election. In addition to reporting their voting behavior, survey respondents were asked to locate their own political views on a seven-point ideological spectrum with labels "extremely liberal", "liberal", "slightly liberal", "moderate", "slightly conservative", "conservative", and "extremely conservative", or report that they do not know where their views fall. Excluding this last group, the strength of an individual's ideology can be taken to be her distance from the center, producing four categories of ideological strength: "moderate", "slightly liberal or conservative", "liberal or conservative", and "extremely liberal or conservative".

Column 1 of Table 1 reports the results of a probit regression, in which the dependent variable is an indicator of whether a citizen voted or not, and the independent variables consist of the measure of ideological strength described above, as well as fixed election effects for every state-year pair. According to this analysis, an increase by one category of ideological strength increases an individual's propensity to vote by nearly four percent. Column 3 reports an analogous result for the case in which the dependent variable is an indicator of whether or not a citizen continued voting for senator or governor, after having voted for president.

Probit Regression Results

	Dependent Variable:			
	Voted		Continued voting	
Ideology	0.0377***		0.0113**	
+ uncertain		0.0780***		0.0255***
Std. Err.	0.0052	0.0048	0.0057	0.0054
z-statistic	7.29	16.01	1.97	4.71
n	14,443	17,399	9,422	11,177
pseudo-R2	0.0588	0.0678	0.3391	0.3579

Note : Source is ANES elections for president and senator/governor, 1952-2004. Table entries are marginal effects, evaluated at means of data. Election fixed effects are included but not shown. *, **, and *** indicate significance at the 10%, 5%, and 1% levels.

Table 1: Relationship between ideology, voting, and roll-off

6 Conclusion

Downs' spatial model and Condorcet's information model represent two of the most fundamental paradigms through which voting and elections can be understood. This paper takes a first step toward synthesizing the two, by extending the Condorcet environment to allow a continuum of policy alternatives and a substantive role for political candidates. In some respects, behavior that arises in this model strongly resembles that of its predecessors. Policy choices diverge to the extent that candidates are policy-motivated, while competition between office-motivated candidates drives both to the political center; poorly informed citizens abstain from voting, in deference to those with better expertise; and the candidate with the superior policy platform is likely to win the election.

In addition to its standard results, however, this model predicts additional political incentives that are useful for understanding empirical observations. Despite the model's symmetry, elections need not end in expected ties. The winning candidate infers a "mandate" from voters from his margin of victory, and responds with a more extreme policy position. This creates a signaling incentive for voters, who no longer perform the strategic calculus that generates the swing voter's curse, but may nevertheless abstain to avoid the "signaling voter's curse" of pushing policy in the wrong direction. The signaling role of votes provides a possible explanation for the persistence of minor candidates, who are unlikely to win office, and the voters who support them.

The private beliefs that arise naturally from heterogeneous expertise provide a theoretical justification for the one-dimensional ideological spectrum that is commonly assumed. This perspective, however, reverses the conventional welfare analysis. In standard models, competitive pressure that drives candidates to the center of the political spectrum benefits voters, by compromising between extremes to maximize total welfare, in a utilitarian sense. Evidence that candidates do not fully converge is bemoaned as a political failing, perhaps blamed on the passivity of centrist voters, who allow major parties to be controlled by more extreme interests. Here, the reverse is true: centrist voters remain deliberately passive, for lack of expertise; policy-motivated candidates pursue policies that are ultimately optimal for society; and pressure to converge to the center in pursuit of votes distracts candidates from this task. In terms of the examples cited in the introduction, political competition may lead to moderate attempts at fiscal stimulus, even when it is commonly known that either small or large stimulus is optimal, or to partial support for a variety of programs, one of which is ultimately superior.

This model is biased against moderate policies, of course, by its assumption that the optimal policy lies at one of the two extreme ends of the policy space. In many applications, a more reasonable extension may be to allow the optimal possibility to lie anywhere in the policy space. In that case, voting behavior is likely to be similar to this model, but abstention in that case may reflect an opinion that the optimal policy is indeed moderate, rather than reflecting a lack of information. Another useful extension of this work would be to consider multiple policy dimensions. In that case, competition between two candidates would be insufficient to identify the optimal policy perfectly; multiple candidates may therefore play an important role in eliciting additional dimensions of voter information. It may also be possible for candidates to learn from other political races, allowing voters to send messages to a sitting president, for example, by how they vote in midterm elections.

A Appendix: Proofs

Lemma 1 *If candidates are committed then, for any pair (x_A, x_B) of platform policies, there exists a sub-strategy $\bar{\sigma}^*$ that constitutes an equilibrium in the voting subgame. If $x_A \neq x_B$ then $\bar{\sigma}^*$ is a sincere belief threshold sub-strategy, with belief threshold T such that $T = 0$ if and only if policy platforms $x_A = -x_B$ are symmetric around zero.*

Proof. Given candidate platforms $x_A \neq x_B$ and a subgame voting strategy $\bar{\sigma} \in \bar{\Sigma}$, let $\phi_z(j) \equiv \Pr[\sigma(Q_i, S_i, x_A, x_B) = j | Z = z]$ denote the expected fraction of citizens who vote for candidate $j \in \{A, B\}$ in state $z \in \{-1, 1\}$:

$$\phi_z(j) = \sum_{s=1,-1} \int_{q:\bar{\sigma}(q,s)=j} \frac{1}{2}(1 + zsq)\, dF(q). \tag{5}$$

By the decomposition property of Poisson random variables (see Myerson, 1998), the numbers N_A and N_B of A and B votes in state z are independent Poisson random variables with means $n\phi_z(A)$ and $n\phi_z(B)$. Accordingly, let $\psi_z(a, b) \equiv \Pr(N_A = a, N_B = b | Z = z)$ denote the probability in state z of a particular pair (a, b) of vote totals.

$$\psi_z(a, b) = \frac{e^{-n\phi_z(A)}}{a!}[n\phi_z(A)]^a \frac{e^{-n\phi_z(B)}}{b!}[n\phi_z(B)]^b. \tag{6}$$

These probabilities determine the probability $\pi_z^j(m) \equiv \Pr(N_j = N_{-j} + m | Z = z)$ with which candidate $j \in \{A, B\}$ wins the election by a margin of exactly $m \in \mathbb{Z}$ votes (where $m < 0$ denotes losing by $|m|$ votes).

$$\begin{aligned} \pi_z^A(m) &= \textstyle\sum_{k=\min(0,-m)}^{\infty} \psi_z(k + m, k) \\ \pi_z^B(m) &= \textstyle\sum_{k=\min(0,-m)}^{\infty} \psi_z(k, k + m). \end{aligned} \tag{7}$$

By definition, of course, $\pi_z^A(m) = \pi_z^B(-m)$.

With this notation, candidate j wins the election in state z with probability $\Pr_z(W = j) \equiv \Pr(W = j | Z = z)$:

$$\Pr_z(W = j) = \sum_{m=1}^{\infty} \pi_z^j(m) + \frac{1}{2}\pi_z^j(0). \tag{8}$$

Of particular interest are events in which a single additional vote for one candidate would be *pivotal*, reversing the election outcome. Specifically, a vote for candidate $j \in \{A, B\}$ is pivotal when either the candidates tie and j loses the tie-breaking coin toss, or j wins the coin toss but loses the election by exactly one vote. In terms of $\pi_z^j(m)$, the probability of one of these events occurring is simply $P_{z,j}$, defined as follows.

$$P_{z,j} \equiv \frac{1}{2}\pi_z^j(0) + \frac{1}{2}\pi_z^j(-1). \tag{9}$$

By the environmental equivalence property of Poisson games (see Myerson, 1998), an individual citizen from within the game reinterprets N_A and N_B as the numbers of A and B votes cast by her peers; by voting herself, she can add one to either total. By voting for candidate j, she increases that candidate's probability $\Pr(W = j)$ of winning from $\sum_{m=1}^{\infty} \pi_z^j(m) + \frac{1}{2}\pi_z^j(0)$ to $\sum_{m=0}^{\infty} \pi_z^j(m) + \frac{1}{2}\pi_z^j(-1)$, a difference of $\frac{1}{2}\pi_z^j(0) + \frac{1}{2}\pi_z^j(-1) = P_{z,j}$. The benefit of her vote depends on the utility difference between the two policies, which can be written as follows,

$$\begin{aligned} u(x_B, z) - u(x_A, z) &= -(x_B - z)^2 + (x_A - z)^2 \\ &= 2(x_B - x_A)(z - \bar{x}), \end{aligned}$$

where $\bar{x} \equiv \frac{x_A+x_B}{2}$ is the midpoint between the two policies. Given platforms x_A and x_B, therefore, a citizen with information quality $q \in [0,1]$ and signal $s \in \{-1,1\}$ expects the (possibly negative) difference in utility $\Delta_{AB}(q,s)$ from voting for B instead of A to be as follows,

$$\begin{aligned}
\Delta_{AB}(q,s) &= \sum_{z=1,-1} \{[u(x_B,z) - u(x_A,z)]\, P_{z,B} - [u(x_A,z) - u(x_B,z)]\, P_{z,A}\} \frac{1}{2}(1+zqs) \\
&= \sum_{z=1,-1} \{[u(x_B,z) - u(x_A,z)]\, [P_{z,B} + P_{z,A}]\} \frac{1}{2}(1+zqs) \\
&= 2(x_B - x_A)(1-\bar{x})(P_{1,B} + P_{1,A}) \frac{1}{2}(1+qs) \\
&\quad +2(x_B - x_A)(-1-\bar{x})(P_{-1,B} + P_{-1,A}) \frac{1}{2}(1-qs) \\
&= (x_B - x_A) \left\{ \begin{array}{c} qs \left[\begin{array}{c} \bar{x}(-P_{1,B} - P_{1,A} + P_{-1,B} + P_{-1,A}) \\ +(P_{1,B} + P_{1,A} + P_{-1,B} + P_{-1,A}) \end{array} \right] \\ - \left[\begin{array}{c} \bar{x}(P_{1,B} + P_{1,A} + P_{-1,B} + P_{-1,A}) \\ +(-P_{1,B} - P_{1,A} + P_{-1,B} + P_{-1,A}) \end{array} \right] \end{array} \right\}.
\end{aligned} \tag{10}$$

$\Delta_{AB}(q,s)$ is positive if and only if qs exceeds T_{AB}, defined by (11) below.

$$T_{AB} = \frac{\bar{x}(P_{1,B} + P_{1,A} + P_{-1,B} + P_{-1,A}) + (-P_{1,B} - P_{1,A} + P_{-1,B} + P_{-1,A})}{\bar{x}(-P_{1,B} - P_{1,A} + P_{-1,B} + P_{-1,A}) + (P_{1,B} + P_{1,A} + P_{-1,B} + P_{-1,A})}. \tag{11}$$

Therefore, $\bar{\sigma}^*$ is a best response to $\bar{\sigma}$ in the voting subgame only if $\bar{\sigma}^*$ is a sincere belief threshold strategy.

The best-response belief threshold T_{AB} is a function of P_z and $\tilde{P}_z$, which in turn are functions of $\pi_z^j(m)$, $\psi_z(a,b)$, and $\phi_z(j)$, which ultimately depends on the subgame voting strategy $\bar{\sigma}$ used by other voters. If $\bar{\sigma}$ is itself a belief threshold strategy, say with threshold T, then $T_{AB}(T)$ can be viewed as a continuous mapping from the compact set $[-1,1]$ of possible belief thresholds into itself. Brouwer's theorem therefore guarantees the existence of a fixed point $T^* = T_{AB}(T^*)$, which can be interpreted as characterizing a belief threshold sub-strategy $\bar{\sigma}^*$ that is its own best response—an equilibrium strategy in the voting subgame. The above logic only applies when $x_A \neq x_B$ but, of course, if $x_A = x_B$ then voters of all types are indifferent between election outcomes, so *any* sub-strategy constitutes an equilibrium in the subgame. $T = 0$ is special in that a threshold at zero implies symmetric voting behavior with respect to signals and candidates, which implies symmetric behavior with respect to the state variable Z. Thus, $\phi_z(A) = \phi_{-z}(B)$, $\psi_z(a,b) = \psi_{-z}(b,a)$, $\pi_z^j(m) = \pi_{-z}^j(-m)$, $P_z = P_{-z}$, and $\tilde{P}_z = \tilde{P}_{-z}$, implying that (11) reduces to $T_{AB} = \bar{x}$. $T^* = 0$ is therefore a fixed point of $T_{AB}(T)$ if and only if $\bar{x} = 0$ or, equivalently, if policy outcomes $x_A = -x_B$ are symmetric around zero. ∎

Theorem 1 (Median Voter Theorem) *If candidates are commited and office-motivated then* (x_A^*, x_B^*, σ^*) *is a symmetric perfect Bayesian equilibrium only if* $x_A^* = x_B^* = 0$ *and* σ^* *is almost everywhere equivalent to a sincere belief threshold strategy, with threshold function* T^* *such that* $T^*(0,0) = 0$*. Furthermore, such an equilibrium exists.*

Proof. As the proof of Lemma 1 shows, the best response to any symmetric voting strategy (given candidate platform strategies $x_A \neq x_B$), is a belief threshold strategy, characterized by the belief threshold T_{AB} defined in (11). In terms of the probabilities $P_z \equiv \frac{1}{2}P_{z,A} + \frac{1}{2}P_{z,B}$ of being pivotal in state z and the midpoint $\bar{x} = \frac{x_A + x_B}{2}$ between candidate platforms, this can be rewritten as

$$T_{AB} = \frac{\bar{x}\left(P_1 + P_{-1}\right) + \left(-P_1 + P_{-1}\right)}{\bar{x}\left(-P_1 + P_{-1}\right) + \left(P_1 + P_{-1}\right)}. \tag{12}$$

The proof of Lemma 1 emphasizes that, since in particular the best response to a belief threshold strategy is another belief threshold strategy, $T_{AB}(T)$ can be reinterpreted as a function from an arbitrary belief threshold to a best-response belief threshold, holding candidate platforms fixed. With this formulation, Lemma 1 also points out that $T_{AB}(0) = 0$ if and only if $\bar{x} = 0$, which occurs when candidate platforms are symmetric around zero.

The first step of this proof is to show for the case of symmetric platform strategies (i.e. $\bar{x} = 0$) that zero lies between T and $T_{AB}(T)$. That is, one of the following must be true: either $T < 0 < T_{AB}(T)$, $T = 0 = T_{AB}(T)$, or $T > 0 > T_{AB}(T)$. To see this, consider the case of $T > 0$. (The case of $T < 0$ follows symmetric reasoning.) In that case, citizens with information quality $Q_i < T$ all vote for candidate A, and citizens with quality $Q_i > T$ vote for the candidate who seems superior (i.e. for S_i), reducing expected vote shares ϕ_z from (5) to the following:

$$\begin{array}{ll} \phi_{-1}(A) = F(T) + \int_T^1 \frac{1+q}{2} dF(q) & \phi_{-1}(B) = \int_T^1 \frac{1-q}{2} dF(q) \\ \phi_1(A) = F(T) + \int_T^1 \frac{1-q}{2} dF(q) & \phi_1(B) = \int_T^1 \frac{1+q}{2} dF(q). \end{array} \tag{13}$$

From these, it is straightforward to verify that $\phi_{-1}(A)\,\phi_{-1}(B) < \phi_1(A)\,\phi_1(B)$, which implies that a k-vote tie is more likely in state 1 than state -1,

$$\begin{aligned} \psi_1(k,k) - \psi_{-1}(k,k) &= \frac{e^{-\phi_1(A)n - \phi_1(B)n}\left[\phi_1(A)\right]^k \left[\phi_1(B)\right]^k}{k!k!} - \frac{e^{-\phi_{-1}(A)n - \phi_{-1}(B)n}\left[\phi_{-1}(A)\right]^k \left[\phi_{-1}(B)\right]^k}{k!k!} \\ &= \frac{e^{-n}}{k!k!}\left\{\left[\phi_1(A)\,\phi_1(B)\right]^k - \left[\phi_{-1}(A)\,\phi_{-1}(B)\right]^k\right\} > 0, \end{aligned}$$

implying in turn that a vote is more likely to be pivotal in state 1 than state -1:

$$\begin{aligned}
P_1 - P_{-1} &\equiv \frac{1}{2}P_{1,B} + \frac{1}{2}P_{1,A} - \frac{1}{2}P_{-1,A} - \frac{1}{2}P_{-1,B} \\
&= \left\{\frac{1}{2}\left[\frac{1}{2}\pi_1^A(0) + \frac{1}{2}\pi_1^A(1)\right] + \frac{1}{2}\left[\frac{1}{2}\pi_1^B(0) + \frac{1}{2}\pi_1^B(1)\right]\right\} \\
&\quad - \left\{\frac{1}{2}\left[\frac{1}{2}\pi_{-1}^A(0) + \frac{1}{2}\pi_{-1}^A(1)\right] + \frac{1}{2}\left[\frac{1}{2}\pi_{-1}^B(0) + \frac{1}{2}\pi_{-1}^B(1)\right]\right\} \\
&= \frac{1}{4}\sum_{k=0}^{\infty}\left\{\begin{array}{c} [\psi_1(k,k) + \psi_1(k+1,k) + \psi_1(k,k) + \psi_1(k,k+1)] \\ -[\psi_{-1}(k,k) + \psi_{-1}(k+1,k) + \psi_{-1}(k,k) + \psi_{-1}(k,k+1)] \end{array}\right\} \\
&= \frac{1}{4}\sum_{k=0}^{\infty}\left\{\begin{array}{c} \psi_1(k,k)\left[1 + \frac{p_1(A)}{k+1} + 1 + \frac{p_1(B)}{k+1}\right] \\ -\psi_{-1}(k,k)\left[1 + \frac{p_{-1}(A)}{k+1} + 1 + \frac{p_{-1}(B)}{k+1}\right] \end{array}\right\} \\
&= \frac{1}{4}\sum_{k=0}^{\infty}\left\{\psi_1(k,k)\left(2 + \frac{1}{k+1}\right) - \psi_{-1}(k,k)\left(2 + \frac{1}{k+1}\right)\right\} \\
&= \frac{1}{4}\sum_{k=0}^{\infty}\left[\psi_1(k,k) - \psi_{-1}(k,k)\right]\left(2 + \frac{1}{k+1}\right) > 0.
\end{aligned}$$

From (12), then, it is easy to see that $\bar{x} = 0$ and $T > 0$ implies $T_{AB}(T) < 0$.

The second step of this proof is to show that, for any voting strategy σ, the best-response belief threshold function $T_{AB}(T)$ increases with $\bar{x}$ (for any value of T). This can be seen most easily by differentiating (12) with respect to $\bar{x}$:

$$\begin{aligned}
\frac{\partial T_{AB}}{\partial \bar{x}} &= \frac{(P_1 + P_{-1})[\bar{x}(-P_1 + P_{-1}) + (P_1 + P_{-1})] - [\bar{x}(P_1 + P_{-1}) + (-P_1 + P_{-1})](-P_1 + P_{-1})}{[\bar{x}(-P_1 + P_{-1}) + (P_1 + P_{-1})]^2} \\
&= \frac{(P_1 + P_{-1})^2 - (-P_1 + P_{-1})^2}{[\bar{x}(-P_1 + P_{-1}) + (P_1 + P_{-1})]^2} \\
&= \frac{2P_1P_{-1} + 2P_1P_{-1}}{[\bar{x}(-P_1 + P_{-1}) + (P_1 + P_{-1})]^2} > 0.
\end{aligned}$$

The result that 0 lies between T and $T_{AB}(T)$ whenever $\bar{x} = 0$, together with the result that $T_{AB}(T)$ increases with $\bar{x}$, implies that $T < 0 < T_{AB}(T)$ whenever $T < 0 < \bar{x}$, and that $T > 0 > T_{AB}(T)$ whenever $\bar{x} < 0 < T$. A fixed point $T^* = T_{AB}(T^*)$, therefore, cannot be negative when $\bar{x} > 0$ or positive when $\bar{x} < 0$; in other words, T^* and $\bar{x}$ must have the same sign.

If $T^* > 0$ then it is straightforward to confirm from (13) that $\phi_{-1}(A) > \phi_1(B)$ and $\phi_1(A) > \phi_{-1}(B)$ (and $\psi_1(k,k) > \psi_{-1}(k,k)$, as shown above), which implies that Candidate A's expected vote share $\frac{1}{2}\phi_{-1}(A) + \frac{1}{2}\phi_1(A)$ exceeds B's expected vote share (and exceeds $\frac{1}{2}$). This implies that A is more likely to win by a margin of m votes than to lose by a margin of m votes,

$$
\begin{aligned}
& \left[\frac{1}{2}\pi_{-1}^{A}(m)+\frac{1}{2}\pi_{1}^{A}(m)\right]-\left[\frac{1}{2}\pi_{-1}^{A}(-m)+\frac{1}{2}\pi_{1}^{A}(-m)\right] \\
= & \ \frac{1}{2}\sum_{k=0}^{\infty}\left[\psi_{-1}(k+m,k)+\psi_{1}(k+m,k)-\psi_{-1}(k,k+m)-\psi_{1}(k,k+m)\right] \\
= & \ \frac{1}{2}\sum_{k=0}^{\infty}\frac{e^{-n}}{k!\,(k+m)!}\left[\begin{array}{c}\phi_{-1}^{k+m}(A)\,\phi_{-1}^{k}(B)+\phi_{1}^{k+m}(A)\,\phi_{1}^{k}(B) \\ -\phi_{-1}^{k}(A)\,\phi_{-1}^{k+m}(B)-\phi_{1}^{k}(k+m)\,\phi_{1}^{k+m}(B)\end{array}\right] \\
> & \ \frac{1}{2}\sum_{k=0}^{\infty}\frac{e^{-n}}{k!\,(k+m)!}\left[\begin{array}{c}\phi_{1}^{k+m}(B)\,\phi_{-1}^{k}(B)+\phi_{1}^{k+m}(A)\,\phi_{1}^{k}(B) \\ -\phi_{-1}^{k}(A)\,\phi_{-1}^{k+m}(B)-\phi_{-1}^{k}(B)\,\phi_{1}^{k+m}(B)\end{array}\right] \\
= & \ \frac{1}{2}\sum_{k=0}^{\infty}\frac{e^{-n}}{k!\,(k+m)!}\left[\psi_{1}(k,k)\,\phi_{1}^{m}(A)-\psi_{-1}(k,k)\,\phi_{-1}^{m}(B)\right]>0,
\end{aligned}
$$

and is therefore more likely than candidate B to win the election:

$$
\begin{aligned}
& \Pr(W=A)-\Pr(W=B) \\
= & \ \frac{1}{2}\left[\sum_{m=0}^{\infty}\pi_{-1}^{A}(m)+\frac{1}{2}\pi_{-1}^{A}(0)\right]+\frac{1}{2}\left[\sum_{m=0}^{\infty}\pi_{1}^{A}(m)+\frac{1}{2}\pi_{1}^{A}(0)\right] \\
& -\frac{1}{2}\left[\sum_{m=0}^{\infty}\pi_{-1}^{B}(m)+\frac{1}{2}\pi_{-1}^{B}(0)\right]+\frac{1}{2}\left[\sum_{m=0}^{\infty}\pi_{1}^{B}(m)+\frac{1}{2}\pi_{1}^{B}(0)\right] \\
= & \ \frac{1}{2}\sum_{m=0}^{\infty}\left[\pi_{-1}^{A}(m)-\pi_{-1}^{B}(m)\right]+\frac{1}{2}\sum_{m=0}^{\infty}\left[\pi_{1}^{A}(m)-\pi_{1}^{B}(m)\right] \\
= & \ \sum_{m=0}^{\infty}\left[\frac{1}{2}\pi_{-1}^{A}(m)+\frac{1}{2}\pi_{1}^{A}(m)-\frac{1}{2}\pi_{-1}^{B}(m)+\frac{1}{2}\pi_{1}^{B}(m)\right] \\
= & \ \sum_{m=0}^{\infty}\left\{\left[\frac{1}{2}\pi_{-1}^{A}(m)+\frac{1}{2}\pi_{1}^{A}(m)\right]-\left[\frac{1}{2}\pi_{-1}^{A}(-m)+\frac{1}{2}\pi_{1}^{A}(-m)\right]\right\} \\
> & \ 0.
\end{aligned}
$$

Since $\Pr(W=A)+\Pr(W=B)=1$, $\Pr(W=A)>\Pr(W=B)$ implies that $\Pr(W=A)>\frac{1}{2}$. Similarly, $\Pr(W=B)>\frac{1}{2}$ if x_A and x_B are such that $T^*<0$.

Together, the above results imply that $\Pr(W=A)>\frac{1}{2}$ if $x_A=0<x_B$ (since $\bar{x}>0$, implying that $T^*>0$) and $\Pr(W=B)>\frac{1}{2}$ if $x_A<0=x_B$ (since $\bar{x}<0$, implying that $T^*<0$). Thus, either candidate is penalized for deviating from $(0,0)$, which results in a tie, so $(0,0,\sigma^*)$ is a symmetric perfect Bayesian equilibrium as long as σ^* is a belief threshold strategy (which can be constructed by choosing an equilibrium belief threshold sub-strategy $\bar{\sigma}^*$ for each pair of candidate strategies, which is shown to exist by Lemma 1), and candidates win with equal probability in equilibrium (i.e. $T^*(0,0)=0$). That no equilibrium (x_A, x_B, σ) exists with $x_A \neq 0$ and $x_B \neq 0$ follows because a candidate who loses with at least $\frac{1}{2}$ probability can deviate to 0, and win with greater than $\frac{1}{2}$ probability instead. ■

Theorem 2 (Policy Divergence) *If candidates are committed and policy-motivated then (x_A^*, x_B^*, σ^*) is a symmetric perfect Bayesian equilibrium only if candidate platforms are given by $x_j^* = E(Z|W=j) \equiv \hat{z}_j$ for $j = A, B$, with $x_A^* \neq x_B^*$, and the voting strategy σ^* is almost everywhere equivalent to a sincere belief threshold strategy. Furthermore, such an equilibrium exists, with platforms $x_A^* = -x_B^*$ symmetric around zero.*

Proof. A candidate's expected utility $Eu(x_j, Z|W=j) = \sum_{z=-1,1}\left[-(x_j - z)^2 \Pr(Z=z|W=j)\right]$ conditional on winning is uniquely maximized at $x_j^* = E(Z|W=j) \equiv \hat{z}_j$, implying that (x_A^*, x_B^*, σ^*) is a symmetric perfect Bayesian equilibrium only if $x_j^* = \hat{z}_j$ for $j = A, B$. If (x_A^*, x_B^*, σ^*) is a symmetric perfect Bayesian equilibrium and $x_A^* < x_B^*$ then, by Lemma 1, the sub-strategy $\bar{\sigma}^*$ associated with σ^* on the equilibrium path is a belief threshold sub-strategy. This implies that $\hat{z}_A < \hat{z}_B$. To see this, first note that for a belief threshold strategy, (5) reduces such that $\phi_{-1}(A) > \phi_1(A)$ and, symmetrically, $\phi_1(B) > \phi_{-1}(B)$:

$$\phi_{-1}(A) - \phi_1(A) = \begin{cases} \left[F(T) + \int_T^1 q dF(q)\right] - \left[F(T) + \int_T^1 (1-q)\right] dF(q) > 0 & \text{if } T \geq 0 \\ \int_{|T|}^1 q dF(q) - \int_{|T|}^1 (1-q) dF(q) > 0 & \text{if } T < 0 \end{cases}.$$

This implies that $\pi_{-1}^A(m) > \pi_1^A(m)$ and, symmetrically, $\pi_1^B(m) > \pi_{-1}^B(m)$ for any $m \geq 0$,

$$\begin{aligned} \pi_{-1}^A(m) - \pi_1^A(m) &= \frac{e^{-n}}{(k+m)!k!}\left[\phi_{-1}^{k+m}(A)\phi_{-1}^k(B) - \phi_1^{k+m}(A)\phi_1^k(B)\right] \\ &> \frac{e^{-n}}{(k+m)!k!}\left[\phi_{-1}^{k+m}(A)\phi_1^k(B) - \phi_{-1}^{k+m}(A)\phi_1^k(B)\right] = 0, \end{aligned}$$

which in turn implies that $\Pr_{-1}(W=A) > \Pr_1(W=A)$ and, symmetrically, $\Pr_z(W=B) > \Pr_{-1}(W=B)$:

$$\begin{aligned} \Pr_{-1}(W=A) - \Pr_1(W=A) &= \left[\sum_{m=1}^{\infty}\pi_{-1}^A(m) + \frac{1}{2}\pi_{-1}^A(0)\right] - \left[\sum_{m=1}^{\infty}\pi_1^A(m) + \frac{1}{2}\pi_1^A(0)\right] \\ &= \sum_{m=1}^{\infty}\left[\pi_{-1}^A(m) - \pi_1^A(m)\right] + \frac{1}{2}\left[\pi_{-1}^A(0) - \pi_1^A(0)\right] > 0. \end{aligned}$$

These results imply, finally, that $\hat{z}_A < \hat{z}_B$:

$$\begin{aligned}
\hat{z}_B - \hat{z}_A &= E(Z|W=B) - E(Z|W=A) \\
&= \frac{\frac{1}{2}\Pr_1(W=B) - \frac{1}{2}\Pr_{-1}(W=B)}{\frac{1}{2}\Pr_1(W=B) + \frac{1}{2}\Pr_{-1}(W=B)} \\
&\quad - \frac{\frac{1}{2}\Pr_1(W=A) - \frac{1}{2}\Pr_{-1}(W=A)}{\frac{1}{2}\Pr_1(W=A) + \frac{1}{2}\Pr_{-1}(W=A)} \\
&> 0 - 0 = 0.
\end{aligned}$$

If candidate platforms $x_A = -x_B$ are symmetric around zero then, as Lemma 1 states, $T^*(x_A, x_B) = 0$ in equilibrium. This implies that (5) simplifies such that $\phi_{-1}(A) = \phi_1(B) = \int_0^1 q dF(q) = E(Q_i)$ and $\phi_1(A) = \phi_{-1}(B) = \int_0^1 (1-q) dF(q) = 1 - E(Q_i)$, (6) simplifies such that $\psi_{-1}(a,b) = \psi_1(b,a)$, (7) simplifies such that $\pi_z^A(m) = \pi_{-z}^B(m)$, and (8) simplifies such that $\Pr_z(W=A) = \Pr_{-z}(W=B)$, so that $T_{AB}(x_A, x_B) = 0$ and $\hat{z}_A = -\hat{z}_B$:

$$\begin{aligned}
\hat{z}_A &= \frac{\frac{1}{2}\Pr_1(W=A) - \frac{1}{2}\Pr_{-1}(W=A)}{\frac{1}{2}\Pr_1(W=A) + \frac{1}{2}\Pr_{-1}(W=A)} \\
&= \frac{\frac{1}{2}\Pr_{-1}(W=B) - \frac{1}{2}\Pr_1(W=B)}{\frac{1}{2}\Pr_{-1}(W=B) + \frac{1}{2}\Pr_1(W=B)} = -\hat{z}_B.
\end{aligned}$$

Thus, if $x_A = -x_B$ are symmetric around zero then voting is symmetric, leading to symmetric best response platforms $\hat{z}_A = -\hat{z}_B$. Therefore, $\hat{z}_B$ can be interpreted as mapping platform pairs $(-x_B, x_B)$ that are symmetric around zero into best-response pairs $(-\hat{z}_B, \hat{z}_B)$ that are also symmetric around zero. Since such pairs can be completely characterized by the platform of candidate B, this interpretation makes $\hat{z}_B$ a continuous function from $[0,1]$ into itself. By Brouwer's theorem, a fixed point x_B^* exists; the platform pair $(-x_B^*, x_B^*)$, together with the voting strategy identified in Lemma 1, constitute a symmetric perfect Bayesian equilibrium.

To see that $x_A = x_B$ is inconsistent with perfect Bayesian equilibrium, suppose that $x_A = x_B > 0$. (Symmetric reasoning applies to the case of $x_A = x_B < 0$.) Candidate A can then deviate to $x_A = -x_B$, symmetric from his opponent. This increases A's utility by the following amount,

$$\begin{aligned}
&\frac{1}{2}\left[u(-x_B,-1) - u(x_B,-1)\right]\Pr_{-1}(W=A) + \frac{1}{2}\left[u(-x_B,1) - u(x_B,1)\right]\Pr_1(W=A) \\
=\ &\frac{1}{2}\left[u(-x_B,-1) - u(x_B,-1)\right]\left[\Pr_{-1}(W=A) - \Pr_{-1}(W=B)\right] > 0,
\end{aligned}$$

because $u(-x_B,-1) > u(x_B,-1)$ and because (by Lemma 1) the equilibrium sub-strategy $\bar{\sigma}$ associated with $x_A \neq x_B$ is a belief threshold sub-strategy, implying that $\Pr_{-1}(W=A) > \Pr_1(W=B)$. ■

Lemma 2 (Swing Voter's Curse) *If candidates are committed and abstention is allowed then, for any pair (x_A, x_B) of platform policies, there exists a sub-strategy $\bar{\sigma}^* \in \Sigma'$ that constitutes an equilibrium in the voting subgame. If $x_A \neq x_B$ then $\bar{\sigma}^*$ is a sincere belief threshold sub-strategy, with belief threshold functions such that $T_1 < T_2$. [did I prove this part?:] Also, $T_1 = -T_B$ if and only if $x_A = -x_B$.*

Proof. Given a symmetric voting sub-strategy $\bar{\sigma}$ in the subgame associated with candidate platforms $x_A < x_B$, the difference $\Delta_{0B}(q,s)$ in expected utility for a citizen of information quality $q \in [0,1]$ and signal $s \in \{-1,1\}$ between voting for candidate B and abstaining is given by the following, where $\bar{x} = \frac{x_A+x_B}{2}$ denotes the midpoint between the two policy outcomes, as before:

$$\begin{aligned} \Delta_{0B}(q,s) &= \sum_{z=1,-1} [u(x_B,z) - u(x_A,z)] P_{z,B} \frac{1}{2}(1+zqs) \\ &= 2(x_B - x_A)(1-\bar{x}) P_{1,B} \frac{1}{2}(1+qs) \\ &\quad +2(x_B - x_A)(-1-\bar{x}) P_{-1,B} \frac{1}{2}(1-qs) \\ &= (x_B - x_A) \left\{ \begin{array}{l} qs[\bar{x}(-P_{1,B} + P_{-1,B}) + (P_{1,B} + P_{-1,B})] \\ -[\bar{x}(P_{-1,B} + P_{1,B}) + (-P_{1,B} + P_{-1,B})] \end{array} \right\}. \end{aligned}$$

A citizen prefers voting B to abstaining if and only if $\Delta_{0B}(q,s)$ is positive; when $x_A \neq x_B$, this occurs if and only if qs exceeds the belief threshold T_{0B}, defined by (16) below. Similarly, a citizen prefers abstaining to voting A if and only if qs is above the threshold T_{A0}, as defined in (14), and prefers voting B to voting A if and only if qs exceeds T_{AB}, defined above in (11) of Lemma 1 and rewritten here as (15).

$$T_{A0} = \frac{\bar{x}(P_{1,A} + P_{-1,A}) - (P_{1,A} - P_{-1,A})}{-\bar{x}(P_{1,A} - P_{-1,A}) + (P_{1,A} + P_{-1,A})} \tag{14}$$

$$T_{AB} = \frac{\bar{x}(P_{1,A} + P_{1,B} + P_{-1,B} + P_{-1,A}) + (P_{1,A} + P_{1,B} - P_{-1,B} - P_{-1,A})}{\bar{x}(P_{1,A} + P_{1,B} - P_{-1,B} - P_{-1,A}) + (P_{1,A} + P_{1,B} + P_{-1,B} + P_{-1,A})} \tag{15}$$

$$T_{0B} = \frac{\bar{x}(P_{-1,B} + P_{1,B}) + (-P_{1,B} + P_{-1,B})}{\bar{x}(-P_{1,B} + P_{-1,B}) + (P_{1,B} + P_{-1,B})}. \tag{16}$$

Setting $T_1^* = \min\{T_{A0}, T_{AB}\}$ and $T_2^* \equiv \max\{T_{0B}, T_{AB}\}$ therefore defines a sincere belief threshold sub-strategy $\bar{\sigma}^*$ that is a best response to $\bar{\sigma}$.

The above reasoning implies that $\bar{\sigma}^*$ constitutes an equilibrium in the voting subgame only if it is a sincere belief threshold sub-strategy. Also, the best-response belief thresholds T_1^* and T_2^* are functions of $P_{z,j}$, and therefore implicitly depends on $\pi_z^j(m)$, $\psi_z(a,b)$, and $\phi_z(j)$, which ultimately depend on the subgame voting strategy $\bar{\sigma}$ used by other voters. If

$\bar{\sigma}$ is itself a sincere belief threshold strategy, say with thresholds $T_1 \leq T_2$, then $T_1^*(T_1, T_2)$ and $T_2^*(T_1, T_2)$ can be viewed together as a single continuous function from the compact set $\{(T_1, T_2) : 0 \leq T_1 \leq T_2 \leq 1\}$ of possible belief threshold pairs into itself. With this interpretation, Brouwer's theorem guarantees the existence of a fixed point, which is a pair (T_1^*, T_2^*) of belief thresholds such that the associated sincere belief threshold sub-strategy is its own best response, and thus an equilibrium in the voting subgame.

To see that $T_1^* < T_2^*$ in equilibrium, consider a sincere belief threshold sub-strategy with belief thresholds $T_1 = T_2 \equiv T \geq 0$. (A symmetric argument applies if $T_1 = T_2 \leq 0$.) Following such a sub-strategy, a citizen votes for B (i.e. $qs \geq T$) only if she receives a high-quality positive signal; citizens with low-quality or negative signals all vote for A (i.e. $qs < T$). Voting probabilities therefore reduce from (5) to the following,

$$\begin{aligned}\phi_z(A) &= F(T) + \frac{1}{2}\int_T^1 (1 - zq) f(q)\, dq \\ \phi_z(B) &= \frac{1}{2}\int_T^1 (1 + zq) f(q)\, dq,\end{aligned}$$

yielding the following inequalities:

$$\phi_{-1}(A)\,\phi_1(B) > \phi_1(A)\,\phi_{-1}(B) \tag{17}$$

$$\phi_{-1}(A) > F(T) + \phi_{-1}(B) \tag{18}$$

$$F(T) + \phi_1(B) > \phi_1(A) \tag{19}$$

$$\phi_1(A)\,\phi_1(B) > \phi_{-1}(A)\,\phi_{-1}(B). \tag{20}$$

These imply that $P_{1,A}P_{-1,B} > P_{1,B}P_{-1,A}$, which is algebraically equivalent to $T_{A0} < T_{0B}$, implying that $T_1^*(T,T) < T_2^*(T,T)$. This can be seen as follows:

$$\begin{aligned}
& P_{-1,B}P_{1,A} - P_{-1,A}P_{1,B} \\
= \; & \frac{1}{2}\left[\pi_{-1}^{B}(0) + \pi_{-1}^{B}(1)\right]\frac{1}{2}\left[\pi_{1}^{A}(0) + \pi_{1}^{A}(1)\right] \\
& -\frac{1}{2}\left[\pi_{-1}^{A}(0) + \pi_{-1}^{A}(1)\right]\frac{1}{2}\left[\pi_{1}^{B}(0) + \pi_{1}^{B}(1)\right] \\
= \; & \frac{1}{4}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\psi_{-1}(j,j)\left[1 + \frac{n\phi_{-1}(A)}{j+1}\right]\psi_{1}(k,k)\left[1 + \frac{n\phi_{1}(B)}{k+1}\right] \\
& -\frac{1}{4}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\psi_{-1}(j,j)\left[1 + \frac{n\phi_{-1}(B)}{j+1}\right]\psi_{1}(k,k)\left[1 + \frac{n\phi_{1}(A)}{k+1}\right] \\
> \; & \frac{1}{4}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\psi_{-1}(j,j)\,\psi_{1}(k,k)\left[\frac{nF(T)}{j+1} - \frac{nF(T)}{k+1}\right] \\
= \; & \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{F(T)\,e^{-n}n^{2j+2k+1}}{4j!\,(j+1)!k!\,(k+1)!}\phi_{-1}^{j}(A)\,\phi_{-1}^{j}(B)\,\phi_{1}^{k}(A)\,\phi_{1}^{k}(B)\,(k-j) \\
= \; & \sum_{j=0}^{\infty}\sum_{k=j+1}^{\infty}\frac{F(T)\,e^{-n}n^{2j+2k+1}}{4j!\,(j+1)!k!\,(k+1)!}\phi_{-1}^{j}(A)\,\phi_{-1}^{j}(B)\,\phi_{1}^{k}(A)\,\phi_{1}^{k}(B)\,(k-j) \\
& -\sum_{k=0}^{\infty}\sum_{j=k+1}^{\infty}\frac{F(T)\,e^{-n}n^{2j+2k+1}}{4j!\,(j+1)!k!\,(k+1)!}\phi_{-1}^{j}(A)\,\phi_{-1}^{j}(B)\,\phi_{1}^{k}(A)\,\phi_{1}^{k}(B)\,(j-k) \\
= \; & \sum_{j=0}^{\infty}\sum_{k=j+1}^{\infty}\frac{F(T)\,e^{-n}n^{2j+2k+1}}{4j!\,(j+1)!k!\,(k+1)!}\phi_{-1}^{j}(A)\,\phi_{-1}^{j}(B)\,\phi_{1}^{j}(A)\,\phi_{1}^{j}(B)\times \\
& \left[\phi_{1}^{k-j}(A)\,\phi_{1}^{k-j}(B) - \phi_{-1}^{k-j}(A)\,\phi_{-1}^{k-j}(B)\right](k-j) \\
> \; & 0,
\end{aligned}$$

where the first inequality follows from (17) through (19) and the final inequality follows from (20). Thus, the belief thresholds $T_1 = T_2 = T \geq 0$ do not characterize their own best response. ■

Proposition 1 *If policy platform commitments are binding, candidates are policy-motivated, and abstention is allowed then there exists a symmetric perfect Bayesian equilibrium* (x_A^*, x_B^*, σ^*) *that satisfies the following conditions:*

1. Candidate platforms are given by $x_j^* = E(Z|W=j) \equiv \hat{z}_j$ *for* $j = A, B$*, and are symmetric around zero (i.e.* $x_A^* = -x_B^*$*).*

2. σ^* *is a sincere belief threshold strategy, with belief threshold functions* $T_1^* < T_2^*$ *that are symmetric around zero in equilibrium (i.e.* $T_1^*(x_A^*, x_B^*) = -T_2^*(x_A^*, x_B^*)$*).*

Proof. The optimality of $x_j^* = \hat{z}_j$ follows from the fact that $\hat{z}_j \equiv E(Z|W=j)$ uniquely maximizes $E\left[-(x_j - Z)^2 |W=j\right]$. For any threshold $T \in [0,1]$, the sincere belief threshold sub-strategy characterized by symmetric thresholds $(-T, T)$ induces symmetry in (5) through (9) and (14) through (16) so that $\phi_z(A) = \phi_{-z}(B)$, $\psi_z(a,b) = \psi_{-z}(b,a)$, $\pi_z^A(m) = \pi_{-z}^B(m)$, $\Pr_z(W=A) = \Pr_{-z}(W=B)$, $P_{z,j} = P_{-z,-j}$, $T_{AB} = 0$, and $T_{A0} = -T_{0B}$. This implies symmetric expectations $\hat{z}_A = -\hat{z}_B$ for candidates (as in condition 1) and symmetric best-response belief thresholds $T_1^* = -T_2^*$, as well. In other words, the best response to the sincere belief threshold sub-strategy characterized by $(-T, T)$ is the sincere belief threshold sub-strategy characterized by $(-T_2^*, T_2^*)$. Through its dependence on $P_{z,j}$, $\pi_z^j(m)$, $\psi_z(a,b)$, and $\phi_z(j)$, then, T_2^* can be interpreted as an implicit function of T. Since $T_2^*(T)$ is a continuous function from the compact set $[0,1]$ of thresholds into itself, Brouwer's theorem guarantees the existence of a fixed point $T^* = T_2^*(T^*)$. The sincere belief threshold sub-strategy characterized by $(-T^*, T^*)$ therefore constitutes an equilibrium in the voting subgame associated with the candidate platforms formulated based on equilibrium expectations $x_A^* = \hat{z}_A = -\hat{z}_B = x_B^*$. Lemma 2 guarantees the existence of voting sub-strategies that constitute equilibria in other voting subgames (off the perfect Bayesian equilibrium path); the equilibrium voting strategy σ^* can be constructed by selecting one such equilibrium for every pair (x_A, x_B) of candidate platforms; the result in Lemma 2 that all such equilibria are sincere belief threshold sub-strategies implies that σ^* is a sincere belief threshold strategy. ■

Lemma 3 *If candidates are responsive and policy-motivated then* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ *is a symmetric perfect Bayesian equilibrium only if the following are true:*

1. σ^* is a belief threshold strategy.

2. For all $a, b \in \mathbb{Z}_+$ and for $j = A, B$, policy functions are given by $y_j^*(a,b) = \hat{z}_{a,b}$ and either $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ or $\hat{z}_{a,b+1} < \hat{z}_{a,b} < \hat{z}_{a+1,b}$.

Proof. Condition 2 follows because a candidate's expected utility $Eu(y_j, Z|N_A = a, N_B = b) = \sum_{z=-1,1}\left[-(y_j - z)^2 \Pr(Z = z|N_A = a, N_B = b)\right]$ conditional on a particular electoral outcome $(N_A, N_B) = (a,b)$ is uniquely maximized at $y_j^* = E(Z|N_A = a, N_B = b) \equiv \hat{z}_{a,b}$, implying that $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ is a symmetric perfect Bayesian equilibrium only if $y_j^*(a,b) = \hat{z}_{a,b}$ for $j = A, B$ and for all $a, b \in \mathbb{Z}_+$, where $\hat{z}_{a,b}$ is given as follows.

$$\hat{z}_{a,b} = \frac{(-1)\,\psi_{-1}(a,b) + (1)\,\psi_1(a,b)}{\psi_{-1}(a,b) + \psi_1(a,b)}. \tag{21}$$

If $y_j^*(a,b) = \hat{z}_{a,b}$ then, given any symmetric voting strategy $\sigma \in \Sigma$ and its associated sub-strategy $\bar{\sigma} \in \bar{\Sigma}$ in the voting subgame associated with any particular pair (x_A, x_B) of platform policies, an individual voter's best response $\bar{\sigma}^*$ to $(y_A^*, y_B^*, \bar{\sigma})$ is given by a belief threshold sub-strategy. In particular, $\bar{\sigma}^*$ is a sincere belief threshold sub-strategy if $\bar{\sigma}$ is such that $\phi_{-1}(A) > \phi_1(A)$ and $\phi_1(B) > \phi_{-1}(B)$ (as shown below) and is an insincere belief threshold sub-strategy otherwise (by similar reasoning). This implies that $(y_A^*, y_B^*, \bar{\sigma}^*)$ is a perfect Bayesian equilibrium in the voting subgame only if $\bar{\sigma}^*$ is a belief threshold sub-strategy, implying in turn that $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ is a symmetric perfect Bayesian equilibrium only if σ^* is a belief threshold strategy.

To see that $\bar{\sigma}^*$ is a sincere belief threshold sub-strategy when $\bar{\sigma}$ is such that $\phi_{-1}(A) > \phi_1(A)$ and $\phi_1(B) > \phi_{-1}(B)$, first note from (6) that $\psi_z(a, b+1) = \frac{n\phi_z(B)}{b+1}\psi_z(a,b)$ and $\psi_z(a+1, b) = \frac{n\phi_z(A)}{b+1}\psi_z(a,b)$, which implies that the expectation $\hat{z}_{a,b}$ of Z given $N_A = a$ and $N_B = b$ increases from (21) with an additional B vote,

$$\begin{aligned}
\hat{z}_{a,b+1} &= \frac{(-1)\,\psi_{-1}(a,b+1) + (1)\,\psi_1(a,b+1)}{\psi_{-1}(a,b+1) + \psi_1(a,b+1)} \\
&= \frac{(-1)\,\phi_{-1}(B)\,\psi_{-1}(a,b) + (1)\,\phi_1(B)\,\psi_1(a,b)}{\phi_{-1}(B)\,\psi_{-1}(a,b) + \phi_1(B)\,\psi_1(a,b)} \\
&> \frac{(-1)\,\phi_1(B)\,\psi_{-1}(a,b) + (1)\,\phi_1(B)\,\psi_1(a,b)}{\phi_1(B)\,\psi_{-1}(a,b) + \phi_1(B)\,\psi_1(a,b)} \\
&= \hat{z}_{a,b},
\end{aligned}$$

and (by similar reasoning) decreases with each additional A vote (i.e. $\hat{z}_{a+1,b} < \hat{z}_{a,b}$). Given that $y_j^*(a,b) = \hat{z}_{a,b}$ and $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$, the benefit $\Delta_{AB}(q,s)$ to a citizen of type $(Q_i, S_i) = (q,s)$ of voting B instead of A is given simply by the following, instead of (10):

$$\begin{aligned}
\Delta_{AB}(q,s) &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left[-\left(\hat{z}_{a,b+1} - z\right)^2 + \left(\hat{z}_{a+1,b} - z\right)^2\right]\psi_z(a,b)\,\frac{1}{2}\,(1 + zqs) \\
&= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1} - \hat{z}_{a+1,b}\right) \times \\
&\qquad \left(z - \frac{\hat{z}_{a+1,b} + \hat{z}_{a,b+1}}{2}\right)\psi_z(a,b)\,(1 + zqs)\,. \qquad (22)
\end{aligned}$$

Since $-1 \leq \hat{z}_{a+1,b} < \hat{z}_{a,b+1} \leq 1$, this is strictly increasing in qs:

$$\begin{aligned}
\frac{d\Delta_{AB}(q,s)}{d(qs)} &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1} - \hat{z}_{a+1,b}\right)\left(z - \frac{\hat{z}_{a+1,b} + \hat{z}_{a,b+1}}{2}\right)\psi_z(a,b)\,z \\
&= \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1} - \hat{z}_{a+1,b}\right)\left(1 - \frac{\hat{z}_{a+1,b} + \hat{z}_{a,b+1}}{2}\right)\psi_1(a,b) \\
&\quad - \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1} - \hat{z}_{a+1,b}\right)\left(-1 - \frac{\hat{z}_{a+1,b} + \hat{z}_{a,b+1}}{2}\right)\psi_{-1}(a,b) \\
&> 0.
\end{aligned}$$

Thus, the benefit $\Delta_{AB}(q,s)$ to voting B instead of A is positive if and only if qs exceeds some belief threshold $T \in [-1,1]$, implying that $\bar{\sigma}^*$ is a best response to $(y_A^*, y_B^*, \bar{\sigma})$ in the voting subgame associated with (x_A, x_B) only if it is a sincere belief threshold sub-strategy.
■

Theorem 3 (Signaling Equilibrium) *If candidates are responsive and policy-motivated then a symmetric perfect Bayesian equilibrium* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ *exists, which exhibits the following properties:*

1. Candidate platforms are given by $x_j^* = \hat{z}_j$ *for* $j = A, B$*, and are symmetric around zero (i.e.* $x_A^* = -x_B^*$*).*

2. σ^* *is a sincere belief threshold strategy, and every sub-strategy* $\bar{\sigma}^*$ *associated with* σ^* *is characterized by the belief threshold* $T^* = 0$*.*

3. For all $a, b \in \mathbb{Z}_+$ *and for* $j = A, B$*, policy functions are given by* $y_j^*(a,b) = \hat{z}_{a,b}$ *and are both monotonic (i.e.* $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$*) and symmetric around zero (i.e.* $\hat{z}_{a,b} = -\hat{z}_{b,a}$*).*

Proof. That $y_j^*(a,b) = \hat{z}_{a,b}$ is optimal for candidate j, given platform and voting strategies, is demonstrated in Lemma 3. If σ^* is the sincere belief threshold strategy for which $T(x_A, x_B) = 0$ for all platform pairs $(x_A, x_B) \in [-1,1]^2$ then, in the voting subgame associated with any platform pair, (5) reduces such that $\phi_{-1}(A) = \phi_1(B) = \int_0^1 q\,dF(q) = E(Q)$ and $\phi_1(A) = \phi_{-1}(B) = \int_0^1 (1-q)\,dF(q) = 1 - E(Q)$. It is clear that $\phi_{-1}(A) > \phi_1(A)$ and $\phi_1(B) > \phi_{-1}(B)$; as Lemma 3 shows, this implies both that $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ and that the best response to σ^* in a particular subgame is a sincere belief threshold sub-strategy. In fact, the symmetry of σ^* further implies that (6) and (21) reduce such that $\psi_z(a,b) = \psi_{-z}(b,a)$ and $\hat{z}_{a,b} = -\hat{z}_{b,a}$. This implies that a citizen for whom $qs = 0$ is exactly indifferent between voting A and voting B in response to $\left[\left(x_j, y_j^*\right)_{j=A,B}, \sigma^*\right]$:

$$\begin{aligned}
\Delta_{AB}(0,s) &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=a+1}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a+1,b}\right)\left(z-\frac{\hat{z}_{a+1,b}+\hat{z}_{a,b+1}}{2}\right)\psi_z(a,b) \\
&\quad+\sum_{z=1,-1}\sum_{b=0}^{\infty}\sum_{a=b+1}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a+1,b}\right)\left(z-\frac{\hat{z}_{a+1,b}+\hat{z}_{a,b+1}}{2}\right)\psi_z(a,b) \\
&= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=a+1}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a+1,b}\right)\left(z-\frac{\hat{z}_{a+1,b}+\hat{z}_{a,b+1}}{2}\right)\psi_z(a,b) \\
&\quad+\sum_{z=1,-1}\sum_{b=0}^{\infty}\sum_{a=b+1}^{\infty}\left(-\hat{z}_{b+1,a}+\hat{z}_{b,a+1}\right)\left(z-\frac{-\hat{z}_{b,a+1}-\hat{z}_{b+1,a}}{2}\right)\psi_{-z}(b,a) \\
&= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=a+1}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a+1,b}\right)\left(z-\frac{\hat{z}_{a+1,b}+\hat{z}_{a,b+1}}{2}\right)\psi_z(a,b) \\
&\quad-\sum_{\tilde{z}=-1,1}\sum_{c=0}^{\infty}\sum_{d=c+1}^{\infty}\left(-\hat{z}_{c+1,d}+\hat{z}_{c,d+1}\right)\left(\tilde{z}-\frac{\hat{z}_{c,d+1}\hat{z}_{c+1,d}}{2}\right)\psi_{\tilde{z}}(c,d) \\
&= 0.
\end{aligned}$$

Thus, in any voting subgame, the belief threshold characterizing the best response to σ^* is simply $T(x_A, x_B) = 0$. In other words, σ^* induces an equilibrium in every voting subgame. Since the equilibrium conditions for voting and policy subgames do not vary with the campaign platform pair adopted by candidates at the beginning of the game, any choice of platform policies can be consistent with equilibrium, including the platforms described in condition 1. It is straightforward to confirm that $\psi_z(a,b) = \psi_{-z}(b,a)$ implies $\hat{z}_A = -\hat{z}_B$, as claimed. ■

Theorem 4 *If candidates A, B, C, and D are responsive and abstention is allowed then there exists a symmetric perfect Bayesian equilibrium $\left[\left(x_j^*, y_j^*\right)_{j=A,B,C,D}, \sigma^*\right]$, which exhibits the following properties:*

1. Candidate platforms are given by $x_j^* = \hat{z}_j$ for $j = A,B,C,D$, and are symmetric around zero (i.e. $x_A^* = -x_D^*$, $x_B^* = -x_C^*$).

2. σ^* is a sincere belief threshold strategy, and every sub-strategy $\bar{\sigma}^*$ associated with σ^* is characterized by the same symmetric quadruple $(-T^*, 0, T^*)$ of belief thresholds, where $0 < T^* < 1$.

3. For all $a,b,c,d \in \mathbb{Z}_+$, policy functions are given by $y_j^*(a,b,c,d) = \hat{z}_{a,b,c,d}$ for $j = A,B,C,D$ and $\hat{z}_{a+1,b,c,d} < \hat{z}_{a,b+1,c,d} < \hat{z}_{a,b,c,d} < \hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1}$.

Proof. That $y_j^*(a,b,c,d) = \hat{z}_{a,b,c,d}$ is optimal for a winning candidate follows from (2), letting $\Omega = \cap_{j\in\{A,B,C,D\}}(N_j = j)$ reflect the vote totals for each of the four candidates.

When voting follows a sincere belief threshold sub-strategy $\bar{\sigma} \in \bar{\Sigma}''$, logic identical to that of Lemma 3 shows that policy outcomes satisfy $\hat{z}_{a+1,b,c,d} < \hat{z}_{a,b+1,c,d} < \hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1}$. In responding to $\bar{\sigma}$, let $\Delta_{jj'}(q,s)$ denote the expected benefit to an individual of type (q,s) of voting for candidate j' instead of candidate j, where $j, j' \in \{A, B, C, D\}$. For example, the benefit $\Delta_{CD}(q,s)$ of voting for candidate D instead of candidate C is given by

$$\begin{aligned}\Delta_{CD}(q,s) &= \sum_{z=1,-1} \sum_{(a,b,c,d)\in\mathbb{Z}_+^4} 2\left(\hat{z}_{a,b,c,d+1} - \hat{z}_{a,b,c+1,d}\right) \times \\ &\quad \left(z - \frac{\hat{z}_{a,b,c+1,d} + \hat{z}_{a,b,c,d+1}}{2}\right) \psi_z(a,b,c,d) \frac{1}{2}(1+zqs).\end{aligned}$$

Since $\hat{z}_{a,b,c,d}$ is bounded between -1 and 1 and $\hat{z}_{a,b,c+1,d} < \hat{z}_{a,b,c,d+1}$, $\Delta_{CD}(q,s)$ is increasing in qs:

$$\begin{aligned}\frac{d\Delta_{CD}}{d(qs)} &= \sum_{(a,b,c,d)\in\mathbb{Z}_+^4} \left(\hat{z}_{a,b,c,d+1} - \hat{z}_{a,b,c+1,d}\right) \times \\ &\quad [\left(1 - \frac{\hat{z}_{a,b,c+1,d} + \hat{z}_{a,b,c,d+1}}{2}\right) \psi_1(a,b,c,d) \\ &\quad - \left(-1 - \frac{\hat{z}_{a,b,c+1,d} + \hat{z}_{a,b,c,d+1}}{2}\right) \psi_{-1}(a,b,c,d)] > 0.\end{aligned}$$

By a similar derivation, $\Delta_{jj'}(q,s)$ is increasing in qs whenever j precedes j' in the ordering $\{A, B, C, D\}$. This implies the existence of thresholds $T_{AB} \leq T_{BC} \leq T_{CD}$ characterizing a sincere belief threshold sub-strategy $\bar{\sigma}^* \in \bar{\Sigma}''$ that is a best response to $\bar{\sigma}$.

If the belief thresholds that characterize $\bar{\sigma}$ are symmetric around zero (i.e. if $T_1 = -T_3$ and $T_2 = 0$) then (5) and (6) reduce such that $\phi_z(A) = \phi_{-z}(D)$, $\phi_z(B) = \phi_{-z}(C)$, and $\psi_z(a,b,c,d) = \psi_{-z}(d,c,b,a)$, so that expectations $\hat{z}_{a,b,c,d} = -\hat{z}_{d,c,b,a}$ are symmetric as well.

The symmetry of $\hat{z}_{a,b,c,d}$ implies that voting benefits are symmetric as well. For example,

$$\begin{aligned}
\Delta_{CD}(q,s) &= \sum_{(a,b,c,d)\in\mathbb{Z}_+^4} \left(\hat{z}_{a,b,c,d+1}-\hat{z}_{a,b,c+1,d}\right)\times \\
&\quad \left[\begin{array}{c} \left(1-\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right)\psi_1(a,b,c,d)\frac{1}{2}(1+qs) \\ +\left(-1-\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right)\psi_{-1}(a,b,c,d)\frac{1}{2}(1-qs) \end{array}\right] \\
&= \sum_{(a,b,c,d)\in\mathbb{Z}_+^4} \left(-\hat{z}_{d+1,c,b,a}+\hat{z}_{d,c+1,b,a}\right)\times \\
&\quad \left[\begin{array}{c} \left(1-\frac{-\hat{z}_{d+1,c,b,a}-\hat{z}_{d,c+1,b,a}}{2}\right)\psi_{-1}(d,c,b,a)\frac{1}{2}(1+qs) \\ +\left(-1-\frac{-\hat{z}_{d,c+1,b,a}-\hat{z}_{d+1,c,b,a}}{2}\right)\psi_1(d,c,b,a)\frac{1}{2}(1-qs) \end{array}\right] \\
&= \sum_{(d,c,b,a)\in\mathbb{Z}_+^4} \left(\hat{z}_{a,b+1,c,d}-\hat{z}_{a+1,b,c,d}\right)\times \\
&\quad \left[\begin{array}{c} \left(1+\frac{\hat{z}_{a+1,b,c,d}+\hat{z}_{a,b+1,c,d}}{2}\right)\psi_{-1}(a,b,c,d)\frac{1}{2}(1+qs) \\ +\left(-1+\frac{\hat{z}_{a+1,b,c,d}+\hat{z}_{a,b+1,c,d}}{2}\right)\psi_1(a,b,c,d)\frac{1}{2}(1-qs) \end{array}\right] \\
&= -\Delta_{AB}(q,-s).
\end{aligned}$$

Similarly, $\Delta_{BC}(q,s)=-\Delta_{BC}(q,-s)$. This symmetry implies that belief thresholds $T_{AB}=-T_{CD}$ and $T_{BC}=0$ are symmetric around zero. In other words, the sincere belief threshold strategy characterized by $(-T_3,0,T_3)$ is the sincere belief threshold strategy characterized by $(-T_{CD},0,T_{CD})$. Therefore, T_{CD} can be viewed continuous function from the compact interval $[0,1]$ of possible thresholds into itself. Interpreted this way, Brouwer's theorem guarantees the existence of a fixed point T^*, such that the sincere belief threshold sub-strategy characterized by $(-T^*,0,T^*)$ is its own best response, and therefore an equilibrium in the voting subgame.

To see that $T^*<1$, observe that a perfectly informed citizen strictly prefers to vote for the most extreme candidate available,

$$\begin{aligned}
\Delta_{CD}(1,1) &= \sum_{z=1,-1}\sum_{(a,b,c,d)\in\mathbb{Z}_+^4} 2\left(\hat{z}_{a,b,c,d+1}-\hat{z}_{a,b,c+1,d}\right)\times \\
&\quad \left(z-\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right)\psi_z(a,b,c,d)\frac{1}{2}(1+z) \\
&= \sum_{(a,b,c,d)\in\mathbb{Z}_+^4} \left(\hat{z}_{a,b,c,d+1}-\hat{z}_{a,b,c+1,d}\right)\times \\
&\quad \left(1-\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right)\psi_1(a,b,c,d)(1+1)>0;
\end{aligned}$$

by continuity, so do sufficiently informed citizens. To see that $T^*>0$, observe that a perfectly uninformed citizen strictly prefers to vote for the least extreme candidate available,

$$\begin{aligned}\Delta_{CD}(0,s) &= \sum_{z=1,-1}\sum_{(a,b,c,d)\in\mathbb{Z}_+^4} 2\left(\hat{z}_{a,b,c,d+1}-\hat{z}_{a,b,c+1,d}\right)\times \\ &\quad \left(z-\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right)\psi_z(a,b,c,d)\frac{1}{2} \\ &= -\sum_{(a,b,c,d)\in\mathbb{Z}_+^4}\left(\hat{z}_{a,b,c,d+1}-\hat{z}_{a,b,c+1,d}\right)\left(\frac{\hat{z}_{a,b,c+1,d}+\hat{z}_{a,b,c,d+1}}{2}\right)\psi_1(a,b,c,d)<0;\end{aligned}$$

by continuity, so do sufficiently uninformed citizens.

Since the best response thresholds do not depend on the platform pair (x_A, x_B), there also exists a sincere belief threshold strategy σ^* that induces the same equilibrium belief threshold sub-strategy in every subgame. $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ is thus a perfect Bayesian equilibrium for any pair (x_A^*, x_B^*) of candidate platforms, and in particular for the platforms $x_j^* = \hat{z}_j$ specified in condition 1. The symmetry of $\psi_z(a,b,c,d) = \psi_{-z}(d,c,b,a)$ implies that $\hat{z}_A = -\hat{z}_B$, as claimed. ■

Theorem 5 (Signaling Voter's Curse) *If candidates are responsive and policy-motivated and voter abstention is allowed then there exists a symmetric perfect Bayesian equilibrium* $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$, *which exhibits the following properties:*

1. Candidate platforms are given by $x_j^* = \hat{z}_j$ *for* $j = A, B$, *and are symmetric around zero (i.e.* $x_A^* = -x_B^*$*).*

2. σ^* *is a sincere belief threshold strategy, and every sub-strategy* $\bar{\sigma}^*$ *associated with* σ^* *is characterized by the same symmetric pair* $(-T^*, T^*)$ *of belief thresholds, where* $0 < T^* < 1$.

3. For all $a, b \in \mathbb{Z}_+$, *policy functions are given by* $y_j^*(a,b) = \hat{z}_{a,b}$ *for* $j = A, B$, *and* $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$.

Proof. That $y_j^*(a,b) = \hat{z}_{a,b}$ is optimal for a winning candidate follows from the fact $Eu\left[-(y_j - Z)^2 | N_A = a, N_B = b\right]$ is uniquely maximized at $y_j^* = E(Z|N_A = a, N_B = b) \equiv \hat{z}_{a,b}$. Lemma 3 shows that policy outcomes $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ are decreasing in the number of A votes and increasing in the number of B votes, in the subgame associated with candidate platforms (x_A, x_B) for which voting follows a sincere belief threshold sub-strategy $\bar{\sigma}$. In such a subgame, the expected benefit $\Delta_{0B}(q,s)$ to an individual of type (q,s) of voting B instead of abstaining is given by the following:

$$\begin{aligned}\Delta_{0B}(q,s) &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left[-\left(\hat{z}_{a,b+1}-z\right)^2+\left(\hat{z}_{a,b}-z\right)^2\right]\psi_z(a,b)\frac{1}{2}(1+zqs) \\ &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a,b}\right)\left(z-\frac{\hat{z}_{a,b}+\hat{z}_{a,b+1}}{2}\right)\psi_z(a,b)(1+zqs).\end{aligned}$$

Since $\hat{z}_{a+1,b} < \hat{z}_{a,b} < \hat{z}_{a,b+1}$ and $\hat{z}_{a,b+1}$ and $\hat{z}_{a,b}$ are bounded between -1 and 1, $\Delta_{0B}(q,s)$ is strictly increasing in qs,

$$\begin{aligned}
\frac{d\Delta_{0B}(q,s)}{d(qs)} &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a,b}\right)\left(z-\frac{\hat{z}_{a,b}+\hat{z}_{a,b+1}}{2}\right)\psi_z(a,b)\,z \\
&= \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a,b}\right)\left(1-\frac{\hat{z}_{a,b}+\hat{z}_{a,b+1}}{2}\right)\psi_1(a,b) \\
&\quad -\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a,b}\right)\left(-1-\frac{\hat{z}_{a,b}+\hat{z}_{a,b+1}}{2}\right)\psi_{-1}(a,b) > 0,
\end{aligned}$$

implying the existence of a belief threshold T_{0B} such that a citizen prefers voting for B to abstaining if and only if $qs \geq T_{0B}$. By similar reasoning, there exist thresholds T_{A0} and T_{AB} such that the benefits $\Delta_{A0}(q,s)$ or $\Delta_{AB}(q,s)$ of abstaining or voting B instead of voting A are positive if and only if $qs \geq T_{A0}$ or $qs \geq T_{AB}$, respectively. Setting $T_1^* = \min\{T_{AB}, T_{A0}\}$ and $T_2^* = \max\{T_{AB}, T_{0B}\}$ then defines a sincere belief threshold sub-strategy that is a best response to $\bar{\sigma}$.

If the belief thresholds that characterize $\bar{\sigma}$ are symmetric around zero (i.e. if $T_1 = -T_2$) then (5) and (6) reduce such that $\phi_z(A) = \phi_{-z}(B)$ and $\psi_z(a,b) = \psi_{-z}(b,a)$, so that expectations $\hat{z}_{a,b} = -\hat{z}_{b,a}$ are symmetric as well. This implies symmetric differences $\Delta_{AB}(q,-s) = -\Delta_{AB}(q,s)$ and $\Delta_{0B}(q,-s) = \Delta_{A0}(q,s)$ in expected utility and therefore thresholds $T_{AB} = 0$ and $T_{A0} = -T_{0B}$,

$$\begin{aligned}
\Delta_{0B}(q,-s) &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left[-\left(\hat{z}_{a,b+1}-z\right)^2+\left(\hat{z}_{a,b}-z\right)^2\right]\psi_\sigma(a,b|z)\,\frac{1}{2}(1-zsq) \\
&= \sum_{\tilde{z}=-1,1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left[-\left(-y^*_{b+1,a}+\tilde{z}\right)^2+\left(-y^*_{b,a}+\tilde{z}\right)^2\right]\times \\
&\quad \psi_\sigma(b,a|\tilde{z})\,\frac{1}{2}(1+\tilde{z}sq) \\
&= \sum_{\tilde{z}=-1,1}\sum_{b=0}^{\infty}\sum_{a=0}^{\infty}\left[-\left(\hat{z}_{a+1,b}-\tilde{z}\right)^2+\left(\hat{z}_{a,b}-\tilde{z}\right)^2\right]\psi_\sigma(a,b|\tilde{z})\,\frac{1}{2}(1+\tilde{z}sq) \\
&= \Delta_{A0}(q,s),
\end{aligned}$$

so that the belief thresholds $T_1^* = -T_2^*$ characterizing the best response to $\bar{\sigma}$ are symmetric around zero as well. In other words, the best response to the sincere belief threshold sub-strategy characterized by $(-T, T)$ is the sincere belief threshold sub-strategy characterized by $(-T_2^*, T_2^*)$. Therefore, $T_2^*(T)$ can be viewed as a continuous function from the compact set $T \in [0,1]$ of possible thresholds into itself. Interpreted this way, Brouwer's theorem guarantees the existence of a fixed point $T^* = T_2^*(T^*)$ such that the sincere belief threshold sub-strategy characterized by $(-T^*, T^*)$ is its own best response, and therefore an equilibrium in the voting subgame. Since the best response thresholds do not depend on the platform pair (x_A, x_B), there also exists a sincere belief threshold strategy σ^* that induces the same equilibrium belief threshold sub-strategy in every subgame. $\left[\left(x_j^*, y_j^*\right)_{j=A,B}, \sigma^*\right]$ is thus a perfect Bayesian equilibrium for any pair (x_A^*, x_B^*) of candidate platforms, and in particular for the platforms $x_j^* = \hat{z}_j$ specified in condition 1. The symmetry of ψ_z implies that $\hat{z}_A = -\hat{z}_B$, as claimed.

That $T^* < 1$ follows because a perfectly-informed citizen strictly prefers to vote:

$$\begin{aligned}\Delta_{0B}(1,1) &= \sum_{z=1,-1}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a,b}\right)\left(z-\frac{\hat{z}_{a,b}+\hat{z}_{a,b+1}}{2}\right)\psi_\sigma(a,b|z)\frac{1}{2}(1+z)\\ &= \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\hat{z}_{a,b+1}-\hat{z}_{a,b}\right)\left(1-\frac{\hat{z}_{a,b}+\hat{z}_{a,b+1}}{2}\right)\psi_\sigma(a,b|z) > 0.\end{aligned}$$

It remains to show that $T^* > 0$, which implies that a positive fraction of the electorate abstain in equilibrium. To see this, note first that an individuals' utility $u(Y,Z)$ depends on both the unknown state of the world Z as well as the unknown policy outcome Y. Given the equilibrium policy functions specified in condition 3, however, the policy outcome $Y = y(N_A, N_B)$ is a deterministic function of the numbers N_A and N_B of votes for either candidate. By the environmental equivalence property of Poisson games (see Myerson, 1998), an individual citizen from within the game reinterprets N_A and N_B as the numbers of A and B votes cast *by her peers*; by voting herself, she can add one to either total. By voting A, abstaining, or voting B, therefore, a citizen effectively chooses whether the policy function y should be $\hat{z}_{a+1,b}$, $\hat{z}_{a,b}$, or $\hat{z}_{a,b+1}$, respectively. Of these, a perfectly uninformed citizen (i.e. $Q_i = 0$) prefers $\hat{z}_{a,b}$ because her posterior beliefs about Z are the same as her prior, so that expected utility reduces as follows:

$$\begin{aligned}E_Z\left(E_{N_A,N_B}\left\{u\left[y(N_A,N_B),Z\right]|Z\right\}|q,s\right) &= E_Z\left(E_{N_A,N_B}\left\{u\left[y(N_A,N_B),Z\right]|Z\right\}\right)\\ &= E_{N_A,N_B}\left(E_Z\left\{u\left[y(N_A,N_B),Z\right]|N_A,N_B\right\}\right).\end{aligned}$$

The inner component $E_Z\left\{u\left[y(N_A,N_B),Z\right]|N_A,N_B\right\}$ of this expression is identical to candidates' objective function, and is therefore maximized at $y^*(N_A = a, N_B = b) = \hat{z}_{a,b}$. Since $\hat{z}_{a,b}$ maximizes expected utility for every pair (N_A, N_B) of vote totals, it also maximizes the expectation $E_{N_A,N_B}\left(E_Z\left\{u\left[y(N_A,N_B),Z\right]|N_A,N_B\right\}\right)$. Thus, an uninformed citizen—and, by continuity, citizens who are sufficiently uninformed—prefers to abstain from voting, implying that $T^* > 0$ in equilibrium, as claimed. ■

Proposition 4 *Let $N = 2$ be known and let F be uniform on $[0,1]$. If candidates are responsive then $(x_A^*, x_B^*, \sigma_{-T^*,T^*}, y_A^*, y_B^*)$ is a perfect Bayesian equilibrium, where $x_B^* = -x_A^* \approx$*

0.5242, $y_B^*(0,0) = y_A^*(0,0) = 0$, $y_B^*(0,1) = -y_A^*(1,0) \approx 0.7907$, *and* $y_B^*(0,2) = -y_A^*(2,0) \approx 0.9730$, *and where* σ_{-T^*,T^*} *is a symmetric belief threshold strategy with* $T^* \approx 0.5814$. *In this equilibrium, expected turnout is approximately* 42%.

Proof. Given a symmetric belief threshold $\sigma_{-T,T}$ and conditional on the state, an individual votes with the following probabilities,

$$\begin{aligned}\phi_{\sigma_{-T,T}}(A|-1) &= \phi_{\sigma_{-T,T}}(B|1) = \int_T^1 \frac{1}{2}(1+q)\,dq \\ &= \frac{1}{2}\left(1 - T + \frac{1-T^2}{2}\right) = \frac{1}{4}(1-T)(T+3)\end{aligned}$$

and

$$\begin{aligned}\phi_{\sigma_{-T,T}}(A|1) &= \phi_{\sigma_{-T,T}}(B|-1) = \int_T^1 \frac{1}{2}(1-q)\,dq \\ &= \frac{1}{2}\left(1 - T - \frac{1-T^2}{2}\right) = \frac{1}{4}(1-T)^2,\end{aligned}$$

and abstains with probability $\phi_{\sigma_{-T,T}}(0|z) = \int_0^T dq = T$. Given any voting outcome (a,b), these probabilities determine the winning candidate's expectation $\hat{z}_{a,b}$ of Z. As Lemma 3 states, $\hat{z}_{a,b}$ is the winning candidate's optimal policy response to (a,b).

If no one votes, or if the two citizens vote for opposite candidates, then the election winner is determined by a coin toss, and implements the zero policy:

$$\begin{aligned}\hat{z}_{00} &= \frac{-\phi^2_{\sigma_{-T,T}}(0|-1) + \phi^2_{\sigma_{-T,T}}(0|1)}{\phi^2_{\sigma_{-T,T}}(0|-1) + \phi^2_{\sigma_{-T,T}}(0|1)} = 0 \\ \hat{z}_{11} &= \frac{-2\phi_{\sigma_{-T,T}}(A|-1)\,\phi_{\sigma_{-T,T}}(B|-1) + 2\phi_{\sigma_{-T,T}}(A|1)\,\phi_{\sigma_{-T,T}}(B|1)}{2\phi_{\sigma_{-T,T}}(A|-1)\,\phi_{\sigma_{-T,T}}(B|-1) + 2\phi_{\sigma_{-T,T}}(A|1)\,\phi_{\sigma_{-T,T}}(B|1)} = 0.\end{aligned}$$

If both citizens vote B then candidate B wins the election and implements

$$\begin{aligned}\hat{z}_{02} &= \frac{-\phi^2_{\sigma_{-T,T}}(B|-1) + \phi^2_{\sigma_{-T,T}}(B|1)}{\phi^2_{\sigma_{-T,T}}(B|-1) + \phi^2_{\sigma_{-T,T}}(B|1)} \\ &= \frac{-\frac{1}{16}(1-T)^4 + \frac{1}{16}(1-T)^2(T+3)^2}{\frac{1}{16}(1-T)^4 + \frac{1}{16}(1-T)^2(T+3)^2} = \frac{4T+4}{T^2+2T+5};\end{aligned}$$

if one citizen votes B while the other abstains then candidate B wins and implements

$$\begin{aligned}\hat{z}_{01} &= \frac{-2\phi_{\sigma_{-T,T}}(0|-1)\,\phi_{\sigma_{-T,T}}(B|-1) + 2\phi_{\sigma_{-T,T}}(0|1)\,\phi_{\sigma_{-T,T}}(B|1)}{2\phi_{\sigma_{-T,T}}(0|-1)\,\phi_{\sigma_{-T,T}}(B|-1) + 2\phi_{\sigma_{-T,T}}(0|1)\,\phi_{\sigma_{-T,T}}(B|1)} \\ &= \frac{-\phi_{\sigma_{-T,T}}(B|-1) + \phi_{\sigma_{-T,T}}(B|1)}{\phi_{\sigma_{-T,T}}(B|-1) + \phi_{\sigma_{-T,T}}(B|1)} = \frac{-(1-T)^2 + (1-T)(T+3)}{(1-T)^2 + (1-T)(T+3)} = \frac{T+1}{2}.\end{aligned}$$

Symmetrically, $\hat{z}_{20} = -\hat{z}_{02}$ and $\hat{z}_{10} = -\hat{z}_{01}$.

Given these values, the benefit Δ_{0B} for an individual of type (q, s) is given by the following,

$$\begin{aligned}\Delta_{0B}\left(q;\sigma_{-T,T}\right) &= \left(\hat{z}_{11}-\hat{z}_{10}\right)\left[\begin{array}{c}\left(1-\frac{\hat{z}_{11}+\hat{z}_{10}}{2}\right)(1+qs)\,\phi_{\sigma_{-T,T}}(A|1)\\ +\left(-1-\frac{\hat{z}_{11}+\hat{z}_{10}}{2}\right)(1-qs)\,\phi_{\sigma_{-T,T}}(A|-1)\end{array}\right]\\ &\quad+\left(\hat{z}_{01}-\hat{z}_{00}\right)\left[\begin{array}{c}\left(1-\frac{\hat{z}_{01}+\hat{z}_{00}}{2}\right)(1+qs)\,\phi_{\sigma_{-T,T}}(0|1)\\ +\left(-1-\frac{\hat{z}_{01}+\hat{z}_{00}}{2}\right)(1-qs)\,\phi_{\sigma_{-T,T}}(0|-1)\end{array}\right]\\ &\quad+\left(\hat{z}_{02}-\hat{z}_{01}\right)\left[\begin{array}{c}\left(1-\frac{\hat{z}_{02}+\hat{z}_{01}}{2}\right)(1+qs)\,\phi_{\sigma_{-T,T}}(B|1)\\ +\left(-1-\frac{\hat{z}_{02}+\hat{z}_{01}}{2}\right)(1-qs)\,\phi_{\sigma_{-T,T}}(B|1)\end{array}\right].\end{aligned}$$

Solving $\Delta_{0B}\left(T;\sigma_{-T,T}\right) = 0$ numerically yields $T^* \approx 0.58$, implying that $\sigma_{-0.58,0.58}$ (together with candidate platforms and $y_j^*(a,b) = \hat{z}_{a,b}$) is a perfect Bayesian equilibrium. Evaluating the formulas above for $T^* \approx 0.58$ yields $\hat{z}_{01} = -\hat{z}_{10} \approx 0.7907$, and $\hat{z}_{02} = -\hat{z}_{20} \approx 0.9730$. Expected turnout is $1 - T^* \approx 0.42$.

In this equilibrium, candidate B wins with probability

$$\begin{aligned}\Pr(W=B|-1) &= \left[\phi^2_{\sigma_{-T^*,T^*}}(B|-1)+2\phi_{\sigma_{-T^*,T^*}}(0|-1)\,\phi_{\sigma_{-T^*,T^*}}(B|-1)\right]\\ &\quad+\frac{1}{2}\left[\phi^2_{\sigma_{-T^*,T^*}}(0|-1)+2\phi_{\sigma_{-T^*,T^*}}(A|-1)\,\phi_{\sigma_{-T^*,T^*}}(B|-1)\right]\\ &= \frac{1}{16}(1-T^*)^4+2T^*\frac{1}{4}(1-T^*)^2\\ &\quad+\frac{1}{2}\left[T^{*2}+2\frac{1}{4}(1-T^*)(T^*+3)\frac{1}{4}(1-T^*)^2\right]\\ &\approx 0.2379\end{aligned}$$

in state -1 and probability

$$\begin{aligned}\Pr(W=B|1) &= \left[\phi^2_{\sigma_{-T^*,T^*}}(B|1)+2\phi_{\sigma_{-T^*,T^*}}(0|1)\,\phi_{\sigma_{-T^*,T^*}}(B|1)\right]\\ &\quad+\frac{1}{2}\left[\phi^2_{\sigma_{-T^*,T^*}}(0|1)+2\phi_{\sigma_{-T^*,T^*}}(A|1)\,\phi_{\sigma_{-T^*,T^*}}(B|1)\right]\\ &= \frac{1}{16}(1-T^*)^2(T^*+3)^2+2T^*\frac{1}{4}(1-T^*)(T^*+3)\\ &\quad+\frac{1}{2}\left[T^{*2}+2\frac{1}{4}(1-T^*)^2\frac{1}{4}(1-T^*)(T^*+3)\right]\\ &\approx 0.7621\end{aligned}$$

in state 1. Conditional only on winning, therefore, candidate B's expectation of Z is given by

$$\begin{aligned}\hat{z}_B &= \frac{-\Pr(W=B|-1)+\Pr(W=B|1)}{\Pr(W=B|-1)+\Pr(W=B|1)}\\ &\approx 0.5242\end{aligned}$$

and candidate A's expectation is symmetric $\hat{z}_A = -\hat{z}_B$.

In equilibrium, four out of five policy outcomes (i.e. ± 0.7907 and ± 0.9730, but not 0) are more extreme than the campaign platforms $x_j^* = \hat{z}_j \approx \pm 0.5242$. The probability of a tie occurring in equilibrium is

$$\begin{aligned}
& \frac{1}{2}\left[\phi^2_{\sigma_{-T,T}}(0|-1) + 2\phi_{\sigma_{-T,T}}(A|-1)\,\phi_{\sigma_{-T,T}}(B|-1)\right] \\
& +\frac{1}{2}\left[\phi^2_{\sigma_{-T,T}}(0|1) + 2\phi_{\sigma_{-T,T}}(A|1)\,\phi_{\sigma_{-T,T}}(B|1)\right] \\
= & \frac{1}{2}\left[T^2 + 2\frac{1}{4}(1-T)(T+3)\frac{1}{4}(1-T)^2\right] \\
& +\frac{1}{2}\left[T^2 + 2\frac{1}{4}(1-T)^2\frac{1}{4}(1-T)(T+3)\right] \\
= & T^2 + \frac{1}{8}(T+3)(1-T)^3 \approx 0.3696.
\end{aligned}$$

■

Proposition 5 *Let $N = 1$ be known and let F be uniform on $[0,1]$. If candidates are responsive then there is a unique belief threshold strategy σ_{-T^*,T^*} such that $(x_A^*, x_B^*, \sigma_{-T^*,T^*}, y_A^*, y_B^*)$ is a symmetric perfect Bayesian equilibrium. In this equilibrium, $T^* = \frac{1}{3}$, $x_B^* = -x_A^* \approx 0.44$, $y_j^*(0,0) = 0$, and $y_j^*(0,1) = -y_j^*(1,0) \approx 0.67$, and expected turnout is approximately 67%.*

Proof. With only a single citizen, there are only three possible voting outcomes: $(1,0)$, $(0,0)$, and $(0,1)$. As in Proposition 4, a symmetric belief threshold voting strategy $\sigma_{-T,T}$ leads the citizen to vote with probabilities $\phi_{\sigma_{-T,T}}(A|-1) = \phi_{\sigma_{-T,T}}(B|1) = \frac{1}{4}(1-T)(T+3)$ and $\phi_{\sigma_{-T,T}}(A|1) = \phi_{\sigma_{-T,T}}(B|-1) = \frac{1}{4}(1-T)^2$, and to abstain with probability $\phi_{\sigma_{-T,T}}(0|1) = \phi_{\sigma_{-T,T}}(0|-1) = F(T) = T$. The winning candidate's expectation $\hat{z}_{a,b}$ of Z is therefore given by the following:

$$\begin{aligned}
\hat{z}_{0,0} &= \frac{-T+T}{T+T} = 0 \\
\hat{z}_{0,1} &= -\hat{z}_{1,0} = \frac{-\frac{1}{4}(1-T)^2 + \frac{1}{4}(1-T)(T+3)}{\frac{1}{4}(1-T)^2 + \frac{1}{4}(1-T)(T+3)} = \frac{T+1}{2}.
\end{aligned}$$

Conditional only on winning, his expectation $\hat{z}_j$ is given by

$$\begin{aligned}
\hat{z}_B &= -\hat{z}_A = \frac{-\left[\frac{1}{4}(1-T)^2 + \frac{1}{2}T\right] + \left[\frac{1}{4}(1-T)(T+3) + \frac{1}{2}T\right]}{\left[\frac{1}{4}(1-T)^2 + \frac{1}{2}T\right] + \left[\frac{1}{4}(1-T)(T+3) + \frac{1}{2}T\right]} \\
&= \frac{-(1-T)^2 + (1-T)(T+3)}{(1-T)^2 + (1-T)(T+3) + 4T} = \frac{1-T^2}{2}.
\end{aligned}$$

As Lemma 3 states, the winning candidate prefers to implement his expectation $\hat{z}_{a,b}$ of the state. Anticipating such a policy response, the citizen perceives the benefit $\Delta_{0B}(q,s)$ of voting B rather than abstaining, given her type (q,s):

$$\begin{aligned}
\Delta_{0B}(q,s) &= (y_{0,1}-y_{0,0})\left[\begin{array}{c} \left(1-\frac{y_{0,0}+y_{0,1}}{2}\right)(1+qs) \\ +\left(-1-\frac{y_{0,0}+y_{0,1}}{2}\right)(1-qs) \end{array}\right] \\
&= 2y_{0,1}\left(qs-\frac{y_{0,1}}{2}\right),
\end{aligned}$$

which is positive if and only if $qs \geq \frac{y_{0,1}}{2}$. Thus, the optimal voting response to $(y_{1,0}, y_{0,0}, y_{0,1})$ is a quality threshold strategy with $T = \frac{y_{0,1}}{2}$. Solving $T = \frac{y_{0,1}}{2}$ and $y_{0,1} = \frac{T+1}{2}$ simultaneously yields a unique solution at $T^* = \frac{1}{3}$, $y^*_{0,1} = \frac{2}{3} \approx 0.67$ (and, by symmetry, $y^*_{1,0} = -\frac{2}{3}$), implying that the citizen votes with probability $1-F(T^*) = 1-T^* = \frac{2}{3} \approx 0.67$. If candidate platforms reflect ex ante expectations of the state, as in Lemma 2, then $x^*_B = -x^*_A = \frac{4}{9} \approx 0.44$.

In equilibrium, two out of three policy outcomes (i.e. ± 0.67, but not 0) are more extreme than the campaign platforms (i.e. $x^*_j = \hat{z}_j \approx \pm 0.5242$). The probability of a tie occurring in equilibrium is $\frac{1}{2}\phi_{\sigma_{-T,T}}(0|1) + \frac{1}{2}\phi_{\sigma_{-T,T}}(0|-1) = T = \frac{1}{3}$. ■

Proposition 5 *Let $N=1$ be known, and let F be uniform on $[0,1]$. If candidates A, B, C, and D are responsive then (x^*, σ^*, y^*) is a perfect Bayesian equilibrium for the symmetric belief threshold voting strategy $\sigma^* = \sigma_{-.6,-.2,.2,.6}$, the vector $y^* = \left(y^*_j\right)_{j\in\{A,B,C,D\}}$ of policy responses defined by $y^*_j(0,0,0,1) = -y^*_j(1,0,0,0) = 0.8$, $y^*_j(0,0,1,0) = -y^*_j(0,1,0,0) = 0.4$, $y^*_j(0,0,0,0) = 0$, and any vector $x^* = \left(x^*_j\right)_{j\in\{A,B,C,D\}}$ of candidate platforms. In this equilibrium, expected voter turnout is 80%.*

Proof. According to Lemma 3, a responsive candidate's optimal response to vote totals (a,b,c,d) is to implement his expectation $\hat{z}_{a,b,c,d}$ of Z. If the citizen votes according to the symmetric belief threshold strategy $\sigma_{-T_2,-T_1,T_1,T_2}$ then the winning candidate's expectations are as follows:

$$\begin{aligned}
\hat{z}_{1,0,0,0} &= E\left(Z|qs \in [-1,-T_2]\right) = -\frac{T_1+T_2}{2} \\
\hat{z}_{0,1,0,0} &= E\left(Z|qs \in [-T_2,-T_1]\right) = -\frac{T_2+1}{2} \\
\hat{z}_{0,0,0,0} &= E\left(Z|qs \in [-T_1,T_1]\right) = 0 \\
\hat{z}_{0,0,1,0} &= E\left(Z|qs \in [T_1,T_2]\right) = \frac{T_1+T_2}{2} \\
\hat{z}_{0,0,0,1} &= E\left(Z|qs \in [T_2,1]\right) = \frac{T_2+1}{2}.
\end{aligned}$$

The benefit $\Delta_{0C}(q,s)$ to a citizen of type (q,s) of voting for C instead of abstaining is therefore given by

$$\begin{aligned}\Delta_{0C}(q,s) &= \sum_{z=1,-1} 2\left(\hat{z}_{0,0,1,0}-0\right)\left(z-\frac{0+\hat{z}_{0,0,1,0}}{2}\right)\frac{1}{2}(1+zqs)\\ &= 2\hat{z}_{0,0,1,0}\left(qs-\frac{\hat{z}_{0,0,1,0}}{2}\right),\end{aligned}$$

which is positive if and only if $qs \geq \frac{\hat{z}_{0,0,1,0}}{2} \equiv T_1\left(\sigma_{-T_2,-T_1,T_1,T_2}\right)$. Similarly, the benefit $\Delta_{CD}(q,s)$ of voting D instead of C is given by

$$\begin{aligned}\Delta_{CD}(q,s) &= \sum_{z=1,-1} 2\left(\hat{z}_{0,0,0,1}-\hat{z}_{0,0,1,0}\right)\left(z-\frac{\hat{z}_{0,0,1,0}+\hat{z}_{0,0,0,1}}{2}\right)\frac{1}{2}(1+zqs)\\ &= \left(\hat{z}_{0,0,0,1}-\hat{z}_{0,0,1,0}\right)\left(qs-\frac{\hat{z}_{0,0,1,0}+\hat{z}_{0,0,0,1}}{2}\right),\end{aligned}$$

which is positive if and only if $q \geq \frac{\hat{z}_{0,0,1,0}+\hat{z}_{0,0,0,1}}{2} \equiv T_2\left(\sigma_{-T_2,-T_1,T_1,T_2}\right)$. Solving $T_1 = \frac{\hat{z}_{0,0,1,0}}{2}$, $T_2 = \frac{\hat{z}_{0,0,1,0}+\hat{z}_{0,0,0,1}}{2}$, $\hat{z}_{0,0,1,0} = \frac{T_1+T_2}{2}$, and $\hat{z}_{0,0,0,1} = \frac{T_2+1}{2}$ simultaneously yields $T_1^* = 0.2$, $T_2^* = 0.6$, $\hat{z}_{0,0,1,0} = 0.4$, $\hat{z}_{0,0,0,1} = 0.8$. Abstention is given by $1 - F\left(T_1^*\right) = T_1^* = 0.2$, so turnout is 80%. ■

Theorem 6 (Jury Theorem) *1. If candidates are committed and, for a population size parameter n, $Eu(Y,Z)$ is maximized by the symmetric voting sub-strategy $(\bar{\sigma}^*)_n \in \bar{\Sigma}$, then (a) $(\bar{\sigma}^*)_n$ constitutes an equilibrium in the voting subgame, and (b) the associated sequence of equilibrium policy outcomes $\left(\bar{Y}^*\right)_n$ approaches $p\lim_{k\to\infty}\left[\left(\bar{Y}^*\right)_n|Z\right] = \begin{cases}\min(x_A,x_B) & \text{if } Z=-1\\ \max(x_A,x_B) & \text{if } Z=1\end{cases}$.*

2. If candidates are committed and policy-motivated and, for a population size parameter n, $Eu(Y,Z)$ is maximized by the strategy combination $\left[\left(x_j^*\right)_{j=A,B},\sigma^*\right]_n \in [-1,1]^2\times\Sigma$, then (a) $\left[\left(x_j^*\right)_{j=A,B},\sigma^*\right]_n$ constitutes a perfect Bayesian equilibrium and (b) the associated sequence of equilibrium policy outcomes Y_n^* approaches $p\lim_{n\to\infty}\left(Y_n^*|Z\right) = Z$.

3. If candidates are responsive and policy-motivated and, for a population size parameter n, $Eu(Y,Z)$ is maximized by the strategy combination $\left[\left(x_j^*,y_j^*\right)_{j=A,B},\sigma^*\right]_{n_k} \in \left([-1,1]\times\Upsilon\right)^2\times \Sigma$, then (a) $\left[\left(x_j^*,y_j^*\right)_{j=A,B},\sigma^*\right]_n$ constitutes a perfect Bayesian equilibrium and (b) the associated sequence of equilibrium policy outcomes Y_n^* approaches $p\lim_{n\to\infty}\left(Y_n^*|Z\right) = Z$.

4. Claim 3 remains true if the number of candidates is increased.

5. Claims 1 through 4 remain true if abstention is allowed.

Proof. If candidates are committed then the voting subgame associated with a particular pair (x_A,x_B) of platform policies is a symmetric common interest game, meaning that citizens have identical preferences and face identical strategy options. In settings such as this, McLennan (1998) demonstrates that the strategy profile that maximizes the common utility function constitutes an equilibrium strategy profile, as does the symmetric strategy that maximizes utility. The latter of these results establishes claim 1(a), since $(\bar{\sigma}^*)_n$ is defined to be optimal in the set $\bar{\Sigma}$ of symmetric sub-strategies (which are strategies in the voting subgame).

If candidates are policy-motivated then they share the same utility function that voters seek to maximize. Claims 2(a) and 3(a) therefore follow from an extension of McLennan's logic: the strategy combination that jointly maximizes $Eu(Y, Z)$ constitutes an equilibrium, whether in the case of committed or responsive candidates, because deviations by candidates (holding voting behavior fixed) cannot increase $Eu(Y, Z)$ above its maximum, and the symmetric voting strategy that responds optimally to candidate behavior constitutes equilibrium in the voting subgame. This logic remains valid if the number of candidates is increased, as in claim 4, or abstention is allowed, as in claim 5.

To see part (b) of claims 1 through 3, consider first the (not necessarily optimal) strategy σ_{sv} of sincere voting, with no abstention (i.e. $\sigma_{sv}(q, -1) = A$ and $\sigma_{sv}(q, 1) = B$ for all q). Following this strategy, citizens are likely to vote A in state -1 and to vote B in state 1 (i.e. $\phi_{-1}(A) = \phi_1(B) = EQ > \phi_1(A) = \phi_{-1}(B) = E(1-Q)$). As the number of voters grows large, actual vote shares approach expected vote shares (i.e. $p\lim_{n\to\infty}[N_j - n\phi_z(j)|Z = z] = 0$). This implies that candidate A is almost sure to win the election in state -1 and candidate B is almost sure to win the election in state 1 (e.g. $\lim_{n\to\infty}\Pr(W = A|Z = -1) = \lim_{n\to\infty}\Pr(N_A > N_B|Z = -1) = 1$). It also implies that candidate A's and candidate B's beliefs regarding the optimal policy, conditional on any election outcome in which they win the election, approach -1 and 1, respectively (i.e. $\lim_{n\to\infty}\hat{z}_{a,b} = \begin{cases} -1 & \text{if } a > b \\ 1 & \text{if } a < b \end{cases}$, implying that $\lim_{n\to\infty}\hat{z}_A = -1$, $\lim_{n\to\infty}\hat{z}_B = 1$). If candidates are committed, these results together imply that x_A is implemented in state -1 and x_B is implemented in state 1, as in claim 1(b), and also that equilibrium policy platforms converge to -1 and 1, as in claim 2(b). If candidates are responsive, these results instead imply that voting and policy reactions are such that the ultimate policy outcome converges to the optimal policy, as in claim 3(b). None of this changes if the number of candidates is increased or abstention is allowed, as in claims 4 and 5.

So far, these results are for the possibly sub-optimal strategy σ_{sv} of sincere voting. The jump to the actual claims of the theorem can be made, however, using McLennan's insight again: if σ_{sv} induces a policy outcome that converges in probability to Z, it delivers utility approaching $p\lim_{n\to\infty}\left[u\left(\tilde{Y}_n, Z\right)\right] = 0$. The optimal strategy σ^* can do no worse, and so $p\lim_{n\to\infty}[u(Y_n^*, Z)] = 0$ as well, implying that $p\lim_{n\to\infty}(Y_n^*|Z) = Z$, as claimed in 2(b) and 3(b). Claim 1(b) follows from similar reasoning: the sincere sub-strategy $(\bar{\sigma}_{sv})_n$ delivers utility approaching $p\lim_{n\to\infty}\left\{-\left[\left(\bar{Y}_{sv}\right)_n - Z\right]^2 |Z\right\} = \begin{cases} -\left[x_A - (-1)\right]^2 & \text{if } Z = -1 \\ -\left[x_B - (-1)\right]^2 & \text{if } Z = 1 \end{cases}$ and the optimal sub-strategy $(\bar{\sigma}^*)_n$ can do no worse, implying that $p\lim_{n\to\infty}\left\{-\left[\left(\bar{Y}^*\right)_n - Z\right]^2 |Z\right\} = \begin{cases} -\left[x_A - (-1)\right]^2 & \text{if } Z = -1 \\ -\left[x_B - (-1)\right]^2 & \text{if } Z = 1 \end{cases}$, and therefore $p\lim_{k\to\infty}\left[\left(\bar{Y}^*\right)_n |Z\right] = \begin{cases} \min\left(x_A, x_B\right) & \text{if } Z = -1 \\ \max\left(x_A, x_B\right) & \text{if } Z = 1 \end{cases}$. Once again, this logic remains intact if the number of candidates is increased or abstention is allowed, as in claims 4 and 5. ■

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