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Keywords: land price; unfunded social security; population aging; population shrinkage

JEL classifications: H55; J11; Q15

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1. Introduction

Population aging and/or shrinkage have been observed, or even expected, in most developed economies in recent decades. Such demographic changes imply decreases in the younger working population, causing financial problems due to unfunded-based social security systems.[1] On the other hand, unlike physical capital stock, the land size remains constant, and portfolio choices of individuals will be affected by such changes in the economic condition. The purpose of the present study is to analyze the time paths of the land price and physical capital stock in an economy with population shrinkage, and then to examine the effect of pay-as-you-go public pensions on these time paths.

Since Feldstein's (1977) pioneering work, the effects of tax policies have been analyzed by authors in dynamic settings. Feldstein (1977) used an overlapping generations model to show that taxes on land rents may raise both the net rental and the price of land as opposed to static Ricardian analyses. Contrastingly, Calvo et al. (1979) asserted that the results in Feldstein (1977) depend on his model, showing that they do not necessarily hold when considering operative bequests à la Barro (1974). On the other hand, Chamley and Wright (1987) analyzed the dynamic effects of land taxes, wage taxes and intergenerational transfers and showed that if the long-term equilibrium is unique and stable, the results of Feldstein hold qualitatively, that the long-term upper limit of the land price rise is less than half of the tax revenue and that an unfunded social security program induces a negative impact on the value of land owned by the older generation. Ihori (1990a) investigated transition effects of bond- and tax-financed government spending in the model of Chamley and Wright (1987) to show that even in the case of bond finance there may be temporary crowding-in effects on capital accumulation.

However, the studies cited above assumed a constant population size and did not consider the effects of demographic change, especially population shrinkage.[2] In the

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[1] For decades since around 1980, the total fertility rates have been less than two in countries such as Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, Germany, Greece, Hungary, Italy, Japan, Luxembourg, Norway, Portugal, Spain, Switzerland, The Netherlands, and the United Kingdom in OECD countries (before 2009), and their social security systems have been unfunded.

2 Rhee (1991) showed the possibility that dynamic inefficiency occurs when there is a non-reproducible factor of production, e.g., land. He assumed the positive growth of population. Land can also serve as collateral for loans and affects capital accumulation and economic growth. For such an aspect of land, see, for example, Kiyotaki and Moore (1997), a pioneering work, among others. On the other hand, Galor and Moav (2009) analyzed the effect of the distribution of landownership on educational expenditure and hence economic growth. However, we do not consider these aspects of

present study, we investigate the transition effects of a shrinking population on land price and per worker stock of capital through consumption-saving and portfolio decisions of individuals. Since the population size will approach zero asymptotically as time passes, we focus on the transition paths converging to the long-term equilibrium rather than the long-term equilibrium itself. Then, we examine the effects of social security reform, i.e., a switch from a pay-as-you-go (thereafter PAYG) system to a fully funded one, on the transition path.[3] In the present study, a growth rate of population of less than one can be considered as population aging since younger generations become smaller than the older generations.[4] An overlapping generations model populated by two-period lived-individuals, working in the first and retired in the second, is assumed, while the growth rate of population is exogenously given as less than one. For example, Figure 1 shows the relationship between the rates of changes in the land price (for commercial areas) and the workforce, respectively, in Japan from 1975 to 2009. There is a negative correlation between them, and the rate of change in land price was negative for sufficiently small growth rates in the workforce. During the period the total fertility rate was below two (i.e., declining from 1.91 in 1975 to 1.34 in 2007), and the social security system has been rather on the unfunded and defined-benefit side in Japan.[5]

The next section introduces the overlapping generations model with land as a production input. The dynamics of the system with less than one growth rate of population will be examined in Section 3, and then the effect of pay-as-you-go public pension is examined in Section 4. Section 5 shows the results of a calibration analysis, in which the effect of the social security reform will be investigated. The final section concludes the paper.

2. Model

2.1 Production sector

Aggregate production technology of the economy is assumed to be written as follows:

$$Y_t = F(K_t, N_t, T) \tag{1}$$

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land here.

[3] 17 of the 30 OECD countries (before 2009) had defined-benefit plans in 2005 (OECD, Pensions at a Glance, 2005).

[4] Such a definition of aging is employed, for example, by Naito and Zhao (2009). Some authors such as Ehrlich and Lui (1991) defined it as an increase in the survival probability in old age.

[5] The rate of change in the land price is negative when the rate of change in the workforce is sufficiently small (i.e., smaller than 0.7 on average in Figure 1).

where K_t is aggregate capital stock in period t, N_t is aggregate labor in period t, T is the total size of land and Y_t denotes the output level in period t. Assuming that the production technology shows constant returns to scale in capital, labor and land, it follows that:

$$Y_t = N_t F(k_t, 1, a_t) \equiv N_t f(k_t, a_t)$$

where $k_t = K_t / N_t$ and $a_t = T / N_t$.

Assuming perfectly competitive factor markets, we will have

$$r_t = f_k(k_t, a_t) = r(k_t, a_t) \tag{2}$$

$$m_t = f_a(k_t, a_t) = m(k_t, a_t) \tag{3}$$

$$w_t = f(k_t, a_t) - k_t f_k(k_t, a_t) - a_t f_a(k_t, a_t) = w(k_t, a_t) \tag{4}$$

where r_t denotes the interest factor (i.e., 1 plus interest rate), m_t is the land rent and w_t is the wage rate, respectively, in period t. Subscripts attached to the functions denote the partial differentials with respect to the variable.

For expositional simplicity, we assume the decreasing marginal productivities for each factor, the Edgeworth technological complementarities among capital, labor and land, i.e.,

$$r_1, m_2 < 0, \quad w_1, w_2 > 0 \quad \text{and} \quad r_2, m_1 > 0, \tag{5a}$$

and the Inada conditions:

$$\lim_{x \to 0} f_x(k, a) = \infty \quad \text{and} \quad \lim_{x \to \infty} f_x(k, a) = 0 \tag{5b}$$

where $x = k, a$.

2.2 Consumption-savings sector

Regarding the household sector, we consider overlapping generations whose lifetime consists of two periods, working and retired. We call the working generation in period t generation t. Population grows at a constant (gross) rate $n(>0)$ from generation to generation. The lifetime utility function of a representative individual of generation t is assumed to be given as

$$u_t = u(c_t, c_{t+1}). \tag{6}$$

Each individual is assumed to supply a unit of labor in his working period. The budget constraints of the individual in the working and retired periods, respectively, are

$$w_t - \tau_t = c_t + s_t \tag{7a}$$

$$c_{t+1} = r_{t+1} s_t + \beta \tag{7b}$$

where c_t and c_{t+1} denote consumption during the working (t) and retired ($t+1$) period, s_t is the asset stock he holds at the end of period t, τ_t is the lump-sum social security contribution in period t, and β stands for public pension benefits in his

retirement (i.e., period $t+1$). We assume a defined-benefit public pension in the present study.

The individual chooses savings (i.e., the amount of asset holdings) for retirement so as to maximize his lifetime utility (6) subject to the budget constraints (7). The optimal savings plan can be written as

$$s_t = s(w_t - \tau_t, w_t - \tau_t + \frac{\beta}{r_{t+1}}, r_{t+1}) \tag{8}$$

where $w_t - \tau_t$ denotes after-contribution income in the working period and $w_t - \tau_t + \frac{\beta}{r_{t+1}}$ is the lifetime income under the public pension scheme. Note that the savings function (8) differs slightly from the usual form because of public pensions. We assume that the savings function satisfies the following properties: $\partial s_t / \partial(w_t - \tau_t) = s_1 + s_2 = 1 - \partial c_t / \partial(w_t - \tau_t + \frac{\beta}{r_{t+1}}) > 0$, and $\partial s_t / \partial r_{t+1} = s_2(-\frac{\beta}{r_{t+1}^2}) + s_3 = -\partial c_t / \partial(w_t - \tau_t + \frac{\beta}{r_{t+1}}) \cdot \frac{\beta}{r_{t+1}^2} + \partial s / \partial r_{t+1}$. For expositional convenience, we also assume that[6]

$$s_r [= \partial s / s r_{t+1}] \geq 0 . \tag{9}$$

2.3 Social security sector

The social security authority collects contributions from the working generation and pays the benefits to the retirees in each period. Assuming that the authority chooses the contribution rate so as to balance the budget under the defined-benefit scheme, the budget constraint of the authority can be written as:

$$\beta N_{t-1} = \tau_t N_t . \tag{10}$$

Since the growth rate of population is

$$N_t / N_{t-1} = n , \tag{11}$$

the budget constraint (10) can be rewritten in per worker terms as

$$\tau_t = \beta / n \quad \text{or} \quad \beta = \tau_t n . \tag{12}$$

With the balancing of the authority budget, net returns of PAYG social security, $\beta(1/r - 1/n)$, is negative when $r > 1 \geq n$, and the absolute value is greater when β is greater.

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[6] This assumption implies that we rule out the cases of Figures 1(ii) and 2(ii) in Ihori (1990a) and that we are concerned with the case in which the elasticity of supply of savings is positive ($\eta > 0$ in his notation).

2.4 Asset markets

In saving for his retirement, an individual chooses his portfolio of capital (claims) and land. The (non-)arbitrage condition between them can be given as:[7]

$$r_{t+1} = \frac{m_{t+1} + p_{t+1}}{p_t}. \tag{13}$$

The left-hand side of (13) is the rate of return to capital (claims), i.e., the interest factor, while the right-hand side is the gross rate of return from land. The first and the second term of the numerator on the right-hand side are income gain, i.e., the land rent, and capital gain (which is capital loss when minus), respectively.

The equilibrium condition in the asset market is given as:

$$s_t N_t = K_{t+1} + p_t T \tag{14}$$

where p_t is the land price at the end of period t.

2.5 Dynamic system

From the savings function (8) and the growth rate of population (11), the equilibrium condition (14) can be rewritten as

$$s(w_t - \tau_t, w_t - \tau_t + \frac{\beta}{r_{t+1}}, r_{t+1}) = nk_{t+1} + p_t a_t. \tag{15}$$

Making use of (4), we can rewrite (15) as

$$k_{t+1} = \frac{1}{n} s(w(k_t, a_t) - \frac{\beta}{n}, w(k_t, a_t) + \beta(\frac{1}{r(k_{t+1}, a_{t+1})} - \frac{1}{n}), r(k_{t+1}, a_{t+1})) - \frac{a_t}{n} p_t. \tag{16}$$

On the other hand, taking (3) into account and rewriting the (non-)arbitrage condition in the asset market (13), we obtain

$$p_{t+1} = p_t r(k_{t+1}, a_{t+1}) - m(k_{t+1}, a_{t+1}). \tag{17}$$

Since the growth rate of population and the land scale are constant, we have

$$a_{t+1} = \frac{1}{n} a_t. \tag{18}$$

Thus, the dynamic system of this model is given by three difference equations, (16), (17) and (18), for three variables, k_t, p_t and a_t.

Taking into account that equation (18) is independent of the other two, we can write the equation (16) in a general form as:

$$k_{t+1} = K(k_t, p_t; a_t). \tag{19}$$

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[7] Ihori (1990b) considered portfolios of capital (claims), land and money to analyze the effects of land taxes (the rent tax, land price tax and capital gain tax).

Inserting (19) into (17), we have

$$p_{t+1} = P(k_t, p_t; a_t). \tag{20}$$

Therefore, if a_t is constant, that is, if the growth rate of population is one ($n = 1$), the two equations (19) and (20) compose a system of two difference equations for the two variables (k_t, p_t). In this case, the system is the same as those in Chamley and Wright (1987) and Ihori (1990a).

3. Dynamic paths

For our purpose we assume $n < 1$ in this section. As can be readily seen from (16) to (18), the long-term equilibrium is given as $(k_\infty, p_\infty, a_\infty) = (\infty, 0, \infty)$, if all the land is used in production.[8] It is difficult to examine the stability of the long-term equilibrium and the transition path directly from the system. Although we will calibrate the transition path in the next section, we briefly analyze the path algebraically as follows.

Since the dynamics of per worker land a_t, (18), is independent of the dynamics of land price and per worker capital stock, we may roughly consider the dynamics of the system by two steps as follows. First, following Ihori (1990a), we confine our attention to the cases in which the sub-system, (19) and (20), of (k_t, p_t) has a saddle-pint stable equilibrium for a given per worker land, a_t. The land price and per capita stock of capital in the next period may be chosen to be on the saddle-point stable arms corresponding to the per worker land size. Next, since the population growth rate is smaller than one, the expectation will be revised in period $t+1$ and, therefore, the land price will jump from a point on a saddle-point stable path corresponding to a population size, a_t, to a point on the other path corresponding to the decreased population size, a_{t+1}. Such a jump will occur from period to period since population shrinks from period to period.

In the following sub-sections, we first examine saddle-point equilibrium of the system (19) and (20), (k_t, p_t), assuming per worker land (a_t) constant, and then analyze the effects of changes in the per worker land size on land price and per worker stock of capital.

3.1 Saddle paths[9]

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[8] When $n < 1$, we can immediately see from (18) that $a_t \to \infty$ as $t \to \infty$. Since per worker stock of capital k_t must be non-negative, it holds from (16) that $p_t \to 0$ as $t \to \infty$. Thus, in order for (16) to be satisfied, we must have $k_t \to \infty$ as $t \to \infty$.

[9] When $n = 1$, the analysis in this sub-section is basically the same as those in

First, for a given per worker land, a_t, we let the combinations of (k_t, p_t) satisfying $k_t = k_{t+1}$ in (19) be the KK curve and those satisfying $p_{t+1} = p_t$ in (20) be the PP curve.

From (16) we have the KK curve:

$$k = \frac{1}{n} s(w(k, a_t) - \frac{\beta}{n}, w(k, a_t) + \beta(\frac{1}{r(k, a_t / n)} - \frac{1}{n}), r(k, a_t / n)) - \frac{a_t}{n} p \quad (21)$$

from which the slope of KK curve is given as

$$\frac{dp}{dk}\Big|_{KK} = -\frac{n - [(s_1 + s_2)w_1 - (s_2 \frac{\beta}{r^2} - s_3)r_1]}{a_t}. \quad (22)$$

We can not determine the sign of (22) *a priori*. When $n - [(s_1 + s_2)w_1 - (s_2 \frac{\beta}{r^2} - s_3)r_1] > 0$, the KK curve is monotonically downward sloping in the (k_t, p_t) plane, while when $n - [(s_1 + s_2)w_1 - (s_2 \frac{\beta}{r^2} - s_3)r_1] < 0$, it is upward sloping. From (16) we have

$$\frac{\partial k_{t+1}}{\partial p_t}\Big|_{KK} = -\frac{a_t}{n + (s_2 \frac{\beta}{r^2} - s_3)r_1}. \quad (23)$$

Since $n + (s_2 \frac{\beta}{r^2} - s_3)r_1 > 0$ under assumption (9), we have $k_{t+1} - k_t > 0$ below the KK curve and $k_{t+1} - k_t < 0$ above the KK curve, respectively.

On the other hand, we obtain the PP curve by inserting (19) into (17):

$$p_{t+1} = p_t[1 + r(K(k_t, p_t, a_t), a_t / n)] - m(K(k_t, p_t, a_t), a_t / n). \quad (24)$$

The relation between p and k satisfying $p_{t+1} = p_t$ for a given a_t can be obtained as:

$$p = pr(K(k, p, a_t), a_t / n) - m(K(k, p, a_t), a_t / n) \quad (25)$$

from which the slope of the PP curve is

$$\frac{dp}{dk}\Big|_{PP} = \frac{(pr_1 - m_1)K_1}{1 - r - (pr_1 - m_1)K_2}. \quad (26)$$

Since $n + (s_2 \frac{\beta}{r^2} - s_3)r_1 > 0$ from assumption (9), we have

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Chamley and Wright (1987) and Ihori (1990a). Following them, we assume the existence of such saddle-point stable equilibrium.

$$K_1 \equiv \frac{\partial k_{t+1}}{\partial k_t} = \frac{(s_1 + s_2)w_1}{n + (s_2 \frac{\beta}{r^2} - s_3)r_1} > 0 \tag{27}$$

$$K_2 \equiv \frac{\partial k_{t+1}}{\partial p_t} = -\frac{a_t}{n + (s_2 \frac{\beta}{r^2} - s_3)r_1} < 0. \tag{28}$$

Therefore, the sign of (26) is positive and the PP curve is monotonically upward sloping. Since we have from (24)

$$\frac{\partial p_{t+1}}{\partial k_t}\Big|_{PP} = (pr_1 - m_1)K_1 < 0, \tag{29}$$

It follows that $p_{t+1} - p_t < 0$ on the right-hand side of the curve PP and $p_{t+1} - p_t > 0$ on the left-hand side.

From the above analysis, we have three cases for the dynamics of (k_t, p_t) for a given a_t. These are illustrated in Figure 2 (i), (ii) and (iii). We focus on the cases of stable saddle-point, (i) and (ii), and exclude the unstable case (iii) from our consideration. Since both curves are monotonically sloping, both the saddle-point equilibrium and the convergence path are unique.

3.2 Changes in population size

In this sub-section we examine how the shrinkage of the population size affects the saddle-point equilibrium analyzed in the previous sub-section.

From (16), the effect of changes in the population size on the KK curve in the neighborhood of the saddle-point equilibrium can be obtained as follows:

$$\frac{\partial p}{\partial a_t}\Bigg|_{\substack{KK \\ kconst}} = \frac{s_1 w_2 + s_2(w_2 - \frac{\beta}{r^2 n} r_2) + s_3 \frac{1}{n} r_2 - p}{a_t}. \tag{30}$$

Whether a decrease in population and hence an increase in a_t shifts the KK curve upward or downward is ambiguous *a priori*. On the other hand, we obtain from (24)

$$\frac{\partial p}{\partial a_t}\Bigg|_{\substack{PP \\ kconst}} = \frac{\frac{1}{n}(pr_2 - m_2) + (pr_1 - m_1)K_3}{1 - r - (pr_1 - m_1)K_2} \tag{31}$$

where, from (16),

$$K_3 \equiv \frac{\partial k_{t+1}}{\partial a_t} = \frac{s_1 w_2 + s_2 (w_2 - \frac{\beta}{r^2 n} r_2) + s_3 \frac{1}{n} r_2 - p}{n + (s_2 \frac{\beta}{r^2} - s_3) r_1}. \tag{32}$$

Whether the PP curve shifts upward or downward is also ambiguous *a priori*. However, it seems plausible to assume that an increase in the per worker land size raises the per worker stock of capital, increasing savings marginally through changes in factor prices (i.e., the interest and wage rates). In this case, the following inequality will hold:

$$s_1 w_2 + s_2 (w_2 - \frac{\beta}{r^2 n} r_2) + s_3 \frac{1}{n} r_2 - p < 0. \tag{33}$$

We assume that the condition (33) holds in the following.[10] In this case, we have $K_3 < 0$ and hence

$$\left.\frac{\partial p}{\partial a_t}\right|_{\substack{KK \\ kconst}} < 0 \quad \text{and} \quad \left.\frac{\partial p}{\partial a_t}\right|_{\substack{PP \\ kconst}} < 0. \tag{34}$$

Both the KK curve and the PP curve shift downward as per worker land increases.

We can consider the following two cases depending on whether the KK curve is upward- or downward-sloping:

(i) In the case of downward-sloping KK curve, the saddle-point equilibrium also shifts downward and, therefore, the equilibrium land price monotonically declines as time elapses. However, it is not clear whether per worker stock of capital increases or decreases. If the downward pressure on land price from the capital market equilibrium is stronger (weaker) than the downward pressure from the portfolio arbitrage, the per worker stock of capital will become smaller (greater, respectively).

(ii) When the KK curve is upward-sloping, we can not rule out the possibility that land price moves upward as the population size decreases, if the downward pressure on the land price from the portfolio arbitrage is sufficiently strong. The fact that the downward pressure on land price from the capital market equilibrium is weak means that relatively more land (and, therefore, less capital) is demanded despite the shrinking population. Since increases in per worker land reduce the marginal productivity of land, such rises in the land price may be considered as land price "bubbles" in a sense. The bubble may expand only when the marginal productivity of land largely decreases and the marginal productivity of capital largely increases.

[10] For the plausibility, see footnote 16.

In this case, per capita stock of capital will also increase, since increases in the per worker land size itself raise the demand for capital. However, whether the land price moves upward continuously or not is ambiguous. It may turn downward on the dynamic path. Contrastingly, if increases in the per worker land size do not reduce the demand for capital sufficiently, the land price falls. We rule out the case of infinite expansion of bubbles as an unstable case in the following.[11]

While the movements of the saddle-point equilibrium corresponding to changes in per worker size of land may be given in the above analysis, the land price and per worker stock of capital will be still on the convergence arms to each saddle-point equilibrium. From the above analysis, we can posit transition movements of the economy as follows: Since the stable arms move with the equilibrium corresponding to per worker land, the transition land price in each period tends to move monotonically downward as the population size decreases, as in the above case (i).[12] In the case (ii), although the transition land price may rise in the short term, it also tends to decline in the long term as long as the system is stable.[13] It should be noted that in the present study, by the term of the stable path we mean a transition path that converges to the land price of zero as the population size moves close to zero.[14]

Next, in order to focus on changes in per worker stock of capital, we examine the sign of the following equation, obtained from (30) and (31):

$$\left.\frac{\partial p}{\partial a_t}\right|_{\substack{KK \\ kconst}} - \left.\frac{\partial p}{\partial a_t}\right|_{\substack{PP \\ kconst}}$$

[11] Since individuals know that decreases in the population size eventually make land a less-scarce factor, the case of infinite expansion of the bubbles seems implausible. However, even if individuals know that the population size eventually converges to zero, they are not willing to increase the number of their children. For explanation of the fact, see, for example, Cigno (1989) and Sinn (2004). They suggested the externality in having children and/or the existence of pay-as-you-go social security.

[12] In particular, when the per worker stock of capital decreases, the land price will fall.

[13] In the present study, we assume that all the land will be used in production. In reality, however, when the population size becomes sufficiently small, land becomes redundant and may not be all used because of, for example, location. In fact, as a population ages, increases in forfeited arable lands are observed in Japan, although it might be caused partly by the changing industrial structure.

[14] When the land price is zero and the marginal productivity of land approaches zero, the interest rate will also be near zero. This implies that per worker stock of capital becomes approximately infinity.

$$= \frac{[n+(s_2\frac{\beta}{r^2}-s_3)r_1]\{(1-r)[s_1w_2+s_2(w_2-\frac{\beta}{r^2 n}r_2)+s_3\frac{1}{n}r_2-p]-\frac{a_t}{n}(pr_2-m_2)\}}{a_t\{(1-r)[n+(s_2\frac{\beta}{r^2}-s_3)r_1]+a_t(pr_1-m_1)\}}. \tag{35}$$

If the sign of the right-hand side of (35) is positive (negative), the saddle-point-stable per worker stock of capital, k_{t+1}, increases (decreases, respectively) at the per worker level of land, a_t. Whether or not the transitional per worker stock of capital moves qualitatively in a similar way to the saddle-point equilibrium value as the population size decreases can not be determined *a priori*. However, from the calibration result in the next section, they are likely to move similarly.[15] In the long term, since per worker land approaches infinity and the land price becomes closer to zero, the right-hand side of (35) becomes positive, that is, the downward shift of the PP curve becomes relatively great, and, therefore, per worker stock of capital increases as the population decreases.

4. Introduction of pay-as-you-go public pensions

We briefly analyze the effect of an increase or introduction of pay-as-you-go public pensions on the land price and per worker stock of capital when the growth rate of the workforce is constant, i.e., when $n=1$. It should be noted that, in contrast to Chamley and Wright (1987) and Ihori (1990), we assume here that the land size/population ratio, a_t, is not necessarily equal to 1. From (21) and (25), we obtain

$$\left.\frac{\partial p}{\partial \beta}\right|_{\substack{KK\\ kconst}} = \frac{-\frac{1}{n}(s_1+s_2)+s_2\frac{1}{r}}{a_t} < 0 \tag{36}$$

$$\left.\frac{\partial p}{\partial \beta}\right|_{\substack{PP\\ kconst}} = \frac{1}{r}(m_1-pr_1)\frac{\partial k_{t+1}}{\partial \beta} < 0 \tag{37}$$

where, from (16), we have

$$\frac{\partial k_{t+1}}{\partial \beta} = \frac{-\frac{1}{n}(s_1+s_2)+s_2\frac{1}{r}}{n+(s_2\frac{\beta}{r^2}-s_3)r_1} < 0. \tag{38}$$

An increase or introduction of PAYG public pensions exerts negative effects on the land

[15] The result is available from the author upon request.

price if the KK curve is downward-sloping. This result is consistent with those in Chamley and Wright (1987) in the long term. However, when the KK curve is upward-sloping, the land price does not necessarily fall with an increase or the introduction of PAYG pensions. From (36) and (37), we obtain

$$\left.\frac{\partial p}{\partial \beta}\right|_{\substack{KK \\ kconst}} - \left.\frac{\partial p}{\partial \beta}\right|_{\substack{PP \\ kconst}} = [-s_1 \frac{1}{n} + s_2 (\frac{1}{r} - \frac{1}{n})][\frac{1}{a_t} - \frac{\frac{1}{r}(m_1 - pr_1)}{n + (s_2 \frac{\beta}{r^2} - s_3) r_1}]. \tag{39}$$

The first term on the right-hand side of (33) is negative, while the sign of the second term is ambiguous. If the downward shift of the PP curve is relatively and sufficiently great (small), both land price and per worker stock of capital increases (decreases, respectively). The result of Chamley and Wright (1987) tends to be true only when per worker land, a_t, is sufficiently small. The difference between Chamley and Wright (1987) and us derives from the fact that we assume that per worker size of land varies as the workforce changes, while they assumed that it is always equal to one. However, after the population size becomes sufficiently small, i.e., for sufficiently great a_t, the second term on the right-hand side of (33) becomes negative. Therefore, after the population size becomes sufficiently small, the downward shift of the PP curve will be relatively great and, therefore, both land price and per worker stock of capital will be smaller.

When the KK curve is upward-sloping, we can not rule out the possibility that the land price may rise as a consequence of an increase or introduction of PAYG pensions. If the land price does not fall, the retired generation in the period of social security policy change obtains not only increased pension benefits but also capital gains from land sales. In this case, the welfare of the retired generation will be largely increased. However, after a_t becomes sufficiently great, the downward shift of the PP curve will be relatively great, and hence the saddle-point equilibrium land price will fall.

5. Calibration analysis: Transition path of the system, $\{k_t, p_t, a_t\}_{t=0}^{\infty}$

We first analyze the transition path of the dynamic system of (16), (17) and (18) for three variables, k_t, p_t and a_t, respectively, by calibration, assuming $n < 1$; and then examine the effects of social security reform under population shrinking.

5.1 Transition path

By specifying the utility function and the production function, we show the calibration results on the transition time path of land price and per worker stock of capital in this sub-section.

We assume a log-linear utility function:

$$u_t = \ln c_t + \rho \ln c_{t+1} \tag{40}$$

where ρ stands for the subjective discount factor, and a Cobb-Douglas production function:

$$F(K_t, L_t, T) = AK_t^{\alpha} N_t^{\gamma} T^{1-\alpha-\gamma} \tag{41}$$

where $0 < \alpha, \gamma, 1-\alpha-\gamma < 1$ denote the capital-, labor- and land-elasticity of output. The log-linear utility function has often been assumed in the literature on social security (e.g., van Groezen et al. (2003) and Zhang et al. (2001)), while the Cobb-Douglas production function is used in the literature on rent taxation as in Feldstein (1977) and Koethenbuerger and Poutvaara (2009).

Under these specifications, the full dynamic system can be written as[16]

$$k_{t+1} = \frac{1}{n}[\frac{\rho}{1+\rho}(1-\alpha-\gamma)Ak_t^{\alpha}a_t^{\gamma} - \frac{\beta}{1+\rho}(\frac{\rho}{n} + \frac{1}{\alpha Ak_{t+1}^{\alpha-1}a_{t+1}^{\gamma}}) - a_t p_t] \tag{42}$$

$$p_{t+1} = p_t \alpha Ak_{t+1}^{\alpha-1}a_{t+1}^{\gamma} - \gamma Ak_{t+1}^{\alpha}a_{t+1}^{\gamma-1} \tag{43}$$

$$a_{t+1} = a_t / n. \tag{18}$$

Assuming the parameters given in Table 1, we have the stable transition paths, which are illustrated in Figure 3. Figure 3 shows the time paths with and without PAYG pension whose benefit is equal to one.[17] We can easily show that the KK curve is upward-sloping and the PP curve is downward-sloping with these parameters and

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[16] Under the specifications, assumption (33) can be written as $pT > \gamma[\frac{\rho}{1+\rho}w + \frac{\beta}{(1+\rho)r}]N$. The left-hand side is the total value of land, while the sum in parentheses is the sum of savings from wage income and the present value of pension benefits/$(1+\rho)$. The aggregated sum over all the population is considered to be smaller than the GDP. The total value of land in Japan in 2005 was ¥1830 trillion (The National Tax Administration Agency), while GDP in 2005 was ¥503 trillion. Therefore, the condition (33) seems to be plausibly satisfied.

[17] At the saddle-point equilibrium when $n = 1$, per capita consumption in the working and retirement period are $(c_t, c_{t+1}) = (5.953, 5.320)$ without pensions and $(c_t, c_{t+1}) = (5.026, 5.662)$ with pensions, respectively.

that the equilibrium of the sub-system (42) and (43) for a given per capita land ($a_t = na_{t+1}$) is saddle-point stable. It should be noted that although changes in a_t are not explicitly illustrated in the (k_t, p_t) plane of Figure 3, the per worker size of land a_t varies at the same rate from period to period in both cases with and without PAYG pensions.

We assume here that one period consists of 25 years. In the initial period, defined as period 0, the economy is at the saddle-point equilibrium corresponding to constant population, $n = 1$. The initial equilibria with and without PAYG pensions are $(k, p, a) = (0.970, 0.116, 5)$ and $(k, p, a) = (1.322, 0.1982, 5)$, respectively, where we assume the initial per capita land equal to 5. That is, the introduction or increase in PAYG pension benefits reduces not only the saddle-point equilibrium land price but also per capita stock of capital.[18] Then, we suppose that the population growth rate unexpectedly falls to $n = 0.816$ and remains at that level thereafter.[19] The interesting results may be summarized as follows:

(i) Regardless of PAYG pensions, the land price falls as population shrinks, while per worker stock of capital increases. Increases in per capita land size raise the marginal productivity of capital and hence accelerate capital formation. Consequently, the land rent and the interest rate decline, while the wage rate rises because of the increased scarcity of labor.

(ii) The land price and per worker stock of capital on the transition path are lower in the case with PAYG pensions than without it for the same per worker size of land.

(iii) In the first period, when the fertility rate falls, the transition per worker stock of capital does not change both in the cases with and without PAYG pensions, but in the second period, in which the workforce size shrinks, the transition land price falls in the case with PYAG pensions but not in the case without it. This is because workers alter the demand for land through changes in their savings and portfolios in response to the changes in the return rate of PAYG pensions caused by the shrinkage of the workforce.

(iv) The lifetime utility of the generation whose fertility rate first falls decreases in the

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[18] Assuming complementarity between land and capital and the Hicksian stability, this case corresponds to (ii) in Theorem 4 of Chamley and Wright (1987).

[19] While $n = 1$ corresponds to the total fertility rate of 2, $n = 0.816$ in the present paper in which a period consists of 25 years corresponds to the total fertility rate of 1.5 for unisex individuals since the total fertility rate is the average number of children that women have from 15 years old to 49 years old during their lifetime, i.e., $2 - (2 - 0.816 \times 2) \times 34/25 = 1.5$.

case with PAYG pensions but does not change in the case without PAYG pensions, although the land price declines in both cases. In both cases, after that period, the lifetime utility increases from generation to generation and per capita output increases from period to period (see Table 2).

(v) The transitional time paths approach asymptotically the horizontal axis as the population decreases in both cases with and without PAYG pensions. In this sense, therefore, the effect of PAYG public pensions becomes smaller with time.

(vi) Along the transition path, the capital/land ratios (k / pa) in the individual portfolio are higher in the case without PAYG pensions than with them. This derives from the fact that the lower interest rate exerts stronger downward pressure on the land price through the arbitrage conditions in the asset market in the case without PAYG pensions (see Figure 4).

5.2 Social security reform

In this sub-section, we investigate the effects of a social security reform under population shrinking. In particular, we consider an unexpected switch from an unfunded to a fully-funded system during a period with such shrinkage. As shown in Samuelson (1975), funded-based social security does not change total capital formation in the economy as a whole since the social security fund flows into the capital market.[20] Therefore, we may consider unexpected abandonment of the PAYG public pensions as a social security reform. The retired generation at the period of the policy change will lose benefits (since they only made contributions), while savings of the economy will be greater without PAYG social security contributions. Thus, the economy will jump from the point on the transition path with PAYG pensions to the point at which per capita stock of capital is higher. Assuming the uniqueness and saddle-point stability of the transition path without PAYG pensions, the system may become unstable since the land price consistent with the market equilibrium conditions will not be on the transition path without PAYG pensions at the levels of per capita stock of capital and per worker land size, respectively. It should be recalled here that the properties of equilibrium will hold even at the long-term equilibrium, i.e., in the limit of a_t as t approaches infinity.

Supposing that the PAYG social security is abandoned in period t, we have savings in that period without social security contribution:

[20] We assume here that the liquidity constraint is not binding in the working period of individuals.

$$\hat{s}_{t|\beta=0} = \frac{\rho}{1+\rho} w_{t|\beta=1} \tag{44}$$

where the hat (^) denotes variables in the period of policy change, and the subscript $|\beta=1$ and $|\beta=0$ denote variables with and without PAYG pensions, respectively. Since per worker land is historically given as $a_{t|\beta=0} = a_{t|\beta=1}$, per worker stock of capital in period $t+1$ and land prices in periods t and $t+1$ must satisfy the condition:

$$r_{t+1|\beta=0} = \frac{m_{t+1|\beta=0} + p_{t+1|\beta=0}}{p_{t|\beta=0}} \tag{13'}$$

in order for the economy to be on the unique transition path. However, such per worker stock of capital and land price will not generally satisfy the capital market condition:

$$\hat{s}_{t|\beta=0} = nk_{t+1|\beta=0} + p_{t|\beta=0} a_{t|\beta=0}. \tag{45}$$

Thus, it is impossible to jump from the transitional path with PAYG pensions to the unique transition path without PAYG pensions in a given period. In other words, since per worker stock of capital deviates from the transition convergence path (i.e., $\hat{k}_{t|\beta=0} \equiv (\hat{s}_{t|\beta=0} - \hat{p}_{t|\beta=1} a_{t|\beta=0})/n$ where $\hat{p}_{t|\beta=1}$ is the land price determined in period $t-1$) and the change in per worker land is historically given, the land price $p_{t|\beta=0}$ can not by itself put the system on the convergence path generally. In contrast to the case of constant population, this implies that when the population is shrinking, the policy authority has to control the economy so as to retain the transitional convergence path (and stabilize the dynamics of the economy) for periods, minimizing the welfare loss during the periods, after the PAYG social security scheme is unexpectedly abandoned.[21]

This is an important difference from the case of constant population, in which the economy jumps from an equilibrium point with PAYG pensions to a point on the stable

[21] For example, assuming that the pension scheme is abandoned in period 5 unexpectedly without taking other measures, we have, from Table 2, $\hat{s}_{5|\beta=0} = [\rho/(1+\rho)] w_{5|\beta=1} = 2.427$, while $nk_{6|\beta=0} + p_{5|\beta=0} a_{5|\beta=0} = 2.644$. Thus, the condition (45) fails to be satisfied.

arms converging to the equilibrium without social security, as analyzed in Section 4.[22] Even if the transition is stabilized with some measure, the retired generation, and possibly also some succeeding generation(s), may not necessarily be fully compensated. It should be noted that this is apart from the viewpoint of intergenerational income distributional effects of reforms.[23]

6. Concluding remarks

We have shown that when population size remains constant, an introduction or increase in PAYG pension benefits tends to lower the land price when per worker size of land is sufficiently great, as suggested in the case of constant population by Chamley and Wright (1987). However, when per worker land is sufficiently small because of a small workforce size, the land price may rise in response to the social security policy, as opposed to them. Along the transition path on which population is shrinking, the land price falls and approaches approximately zero, and per worker stock of capital increases infinitely as long as all the land is used in production. When a population shrinks, a social security reform from an unfunded to fully funded system may destabilize the transition path. The policy authority will be required both to stabilize the system and to be concerned with intergenerational equity in the reform.

A shrinking and greying workforce is an issue not only in Japan but also in other countries facing this dismal prospect.[24] The calibration results in the present study predict that the welfare of the future generation of a smaller population will be higher. However, while we did not consider technical progress, the workforce size *per se* may affect the speed of the productivity growth of capital.[25] If population size has a kind of scale effect, the welfare may be negatively affected by the slowdown in technical progress due to the population shrinkage. This may be an issue for future research.

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[22] However, Section 4 is concerned with a jump in the opposite direction.

[23] For social security reforms with non-negative population growth, see, for example, Breyer (1989), Feldstein (1998), van Groezen et al. (2003) and Hirazawa and Yakita (2009). The literature suggests that a switch from a PAYG system to a fully funded system can not achieve Pareto improvements in intergenerational income distribution.

[24] See The Economist (November 20th 2010, p. 9), which headlined the issue as "Into the unknown." Although we considered a once-and-for-all decline in the growth rate of population in the text, the real problem in Japan is that of declines in the speed of population aging and shrinking. See also footnote 1.

[25] There may be some peer effects different from the so-called scale effect, although Jones (1995) finds it questionable.

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Table 1　Assumption on parameters

Parameters	
α	0.25(1)
γ	0.1(2)
A	10
ρ	0.375 [$\approx 1/(1+0.04)^{25}$](3)
T	5
β	0 or 1

Notes :

(1) The land-elasticity in output production (=0.1) from Feldstein (1977)

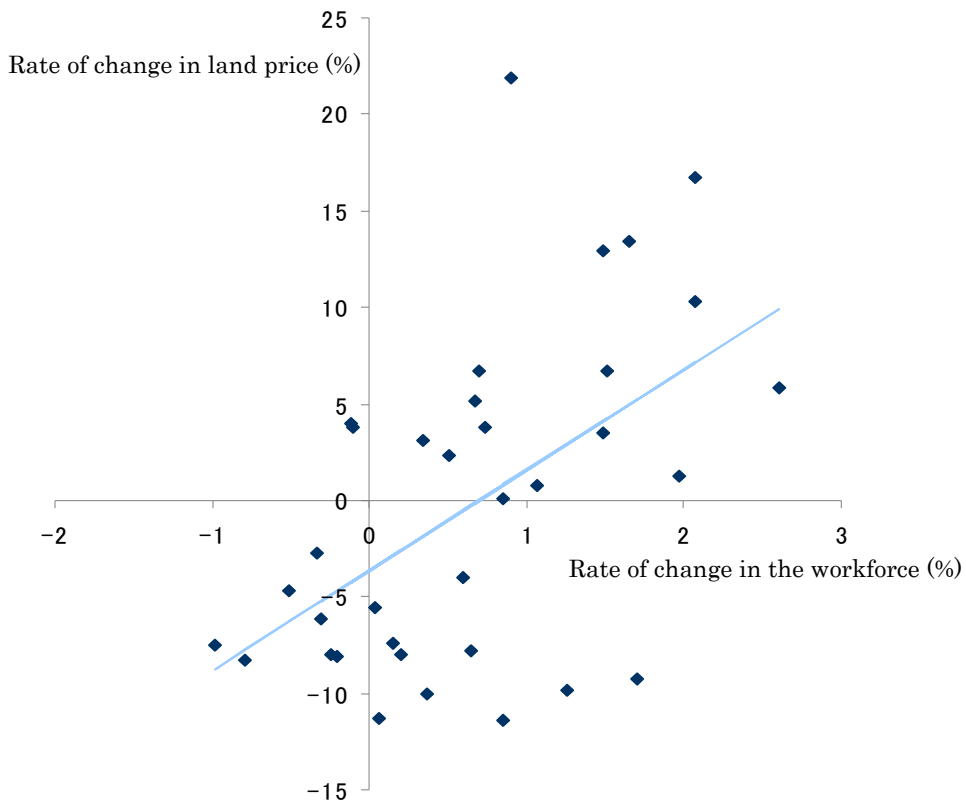
(2) The capital-elasticity in output production (=0.25) different from 0.2 in Feldstein (1977)

(3) The time preference rate in annual rate term (=0.04) corresponds to a quarterly subjective discount factor of 0.96, slightly smaller than 0.99 in Jouvet et al. (2010)

Table 2 Welfare and output levels at saddle-point stable equilibrium and the transition path

period	−1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
TFR	2	2	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
n	1.000	1.000	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816
N	1.000	1.000	1.000	0.816	0.666	0.544	0.444	0.362	0.296	0.241	0.170	0.161	0.131	0.107	0.087	0.071	0.058	0.048	0.039	0.032	0.026	0.021
β	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a (β=0)	5	5	5	6.126	7.506	9.196	11.268	13.805	16.915	20.724	25.392	31.111	38.118	46.703	57.222	70.110	85.900	105.24	128.95	157.99	193.57	237.17
k (β=0)	1.322	1.322	1.322	1.620	1.739	1.806	1.861	1.914	1.966	2.021	2.076	2.133	2.192	2.252	2.314	2.377	2.442	2.510	2.579	2.654	2.742	2.882
p (β=0)	0.182	0.182	0.182	0.160	0.135	0.114	0.096	0.080	0.067	0.056	0.047	0.040	0.033	0.028	0.023	0.020	0.016	0.014	0.012	0.010	0.008	0.006
r (β=0)	2.382	2.382	2.382	2.087	2.020	2.003	1.999	1.998	1.998	1.998	1.998	1.998	1.998	1.998	1.998	1.997	1.997	1.997	1.997	1.995	1.986	1.953
m (β=0)	0.252	0.252	0.252	0.221	0.187	0.157	0.132	0.111	0.093	0.078	0.065	0.055	0.046	0.039	0.032	0.027	0.023	0.019	0.016	0.013	0.011	0.009
w (β=0)	8.187	8.187	8.187	8.790	9.131	9.408	9.673	9.940	10.213	10.494	10.782	11.078	11.382	11.694	12.015	12.345	12.684	13.033	13.392	13.764	14.162	14.633
u (β=0)	2.411	2.411	2.411	2.459	2.499	2.537	2.575	2.612	2.649	2.686	2.724	2.761	2.798	2.835	2.873	2.910	2.947	2.984	3.022	3.059	3.096	3.135
f (β=0)	12.595	12.595	12.595	13.523	14.048	14.473	14.881	15.292	15.713	16.144	16.587	17.043	17.511	17.991	18.485	18.993	19.514	20.050	20.603	21.176	21.788	22.512
β	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
a (β=1)	5	5	5	6.126	7.506	9.196	11.268	13.805	16.915	20.724	25.392	31.111	38.118	46.703	57.222	70.110	85.900	105.24	128.95	157.99	193.57	237.17
k (β=1)	0.970	0.970	0.973	1.172	1.230	1.282	1.332	1.384	1.436	1.490	1.546	1.602	1.661	1.721	1.783	1.846	1.911	1.979	2.049	2.124	2.220	2.397
p (β=1)	0.116	0.116	0.113	0.097	0.083	0.071	0.060	0.051	0.043	0.037	0.031	0.027	0.023	0.019	0.016	0.014	0.012	0.010	0.008	0.007	0.005	0.003
r (β=1)	3.004	3.004	2.996	2.660	2.618	2.591	2.568	2.548	2.528	2.510	2.492	2.475	2.495	2.444	2.429	2.414	2.401	2.387	2.374	2.357	2.327	2.243
m (β=1)	0.233	0.233	0.233	0.204	0.172	0.144	0.121	0.102	0.086	0.072	0.061	0.051	0.043	0.036	0.030	0.025	0.021	0.018	0.015	0.013	0.011	0.009
w (β=1)	7.578	7.578	7.584	8.108	8.374	8.634	8.897	9.166	9.442	9.725	10.015	10.313	10.620	10.934	11.257	11.589	11.930	12.281	12.642	13.019	13.433	13.974
u (β=1)	2.265	2.265	2.220	2.287	2.332	2.375	2.418	2.461	2.503	2.545	2.587	2.690	2.670	2.712	2.753	2.794	2.835	2.876	2.917	2.958	2.999	3.045
f (β=1)	11.658	11.658	11.668	12.474	12.883	13.283	13.688	14.102	14.526	14.961	15.408	15.867	16.338	16.822	17.319	17.830	18.540	18.894	19.449	20.020	0.667	21.499

Figure 1 Relationship between changes in land prices and the workforce in Japan during 1975 to 2009



Notes:

1) $(\Delta p/p) = -3.62723 + 5.203713(\Delta n/n)$ where $\bar{R}^2 = 0.263828$; and ($\Delta p/p$) and
(−2.32274) (3.631097)

($\Delta n/n$) denote the percentage rate of changes in the land price for commercial areas and the workforce (i.e., working population between age 15 to 64) in January from 1975 to 2009.

2) Source: Ministry of International Affairs and Communication (http://www.stat.go.jp/data/roudou/longtime/03roudou.htm; accessed on Jan 5, 2011) and Ministry of Land, Infrastructure, Transport and Tourism (http://tochi.mlit.go.jp/chika/kouji/20090323/77.html; accessed on Jan 5, 2011).

Figure 2 (i) Downward-sloping *KK* curve and upward-sloping *PP* curve

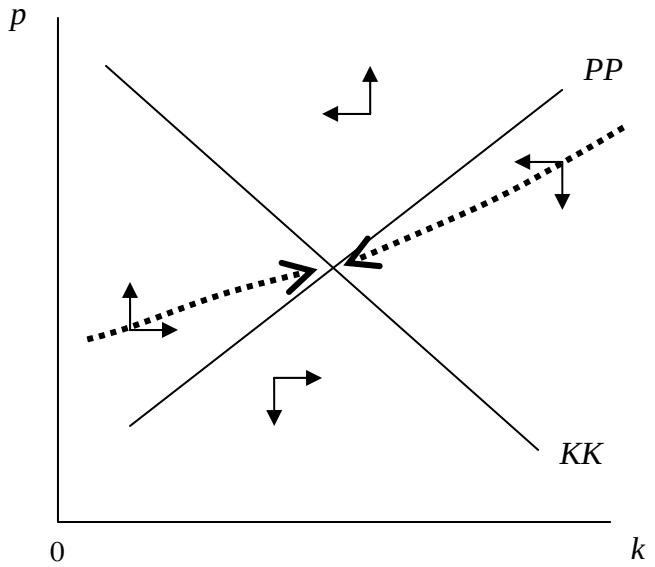


Figure 2 (ii) Upward-sloping *KK* and *PP* curves: Steeper *PP* curve

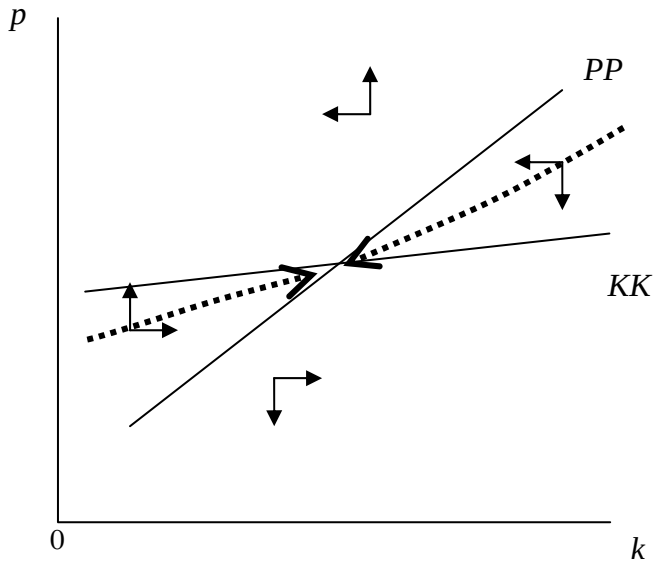


Figure 2 (iii)　　Upward-sloping *KK* and *PP* curves: Steeper *KK* curve

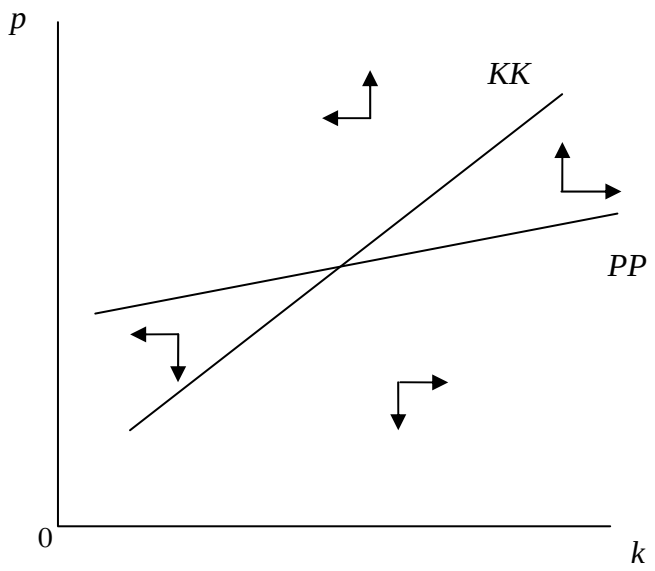


Figure 3　Transition path converging to the long-term equilibrium with without PAYG public pensions

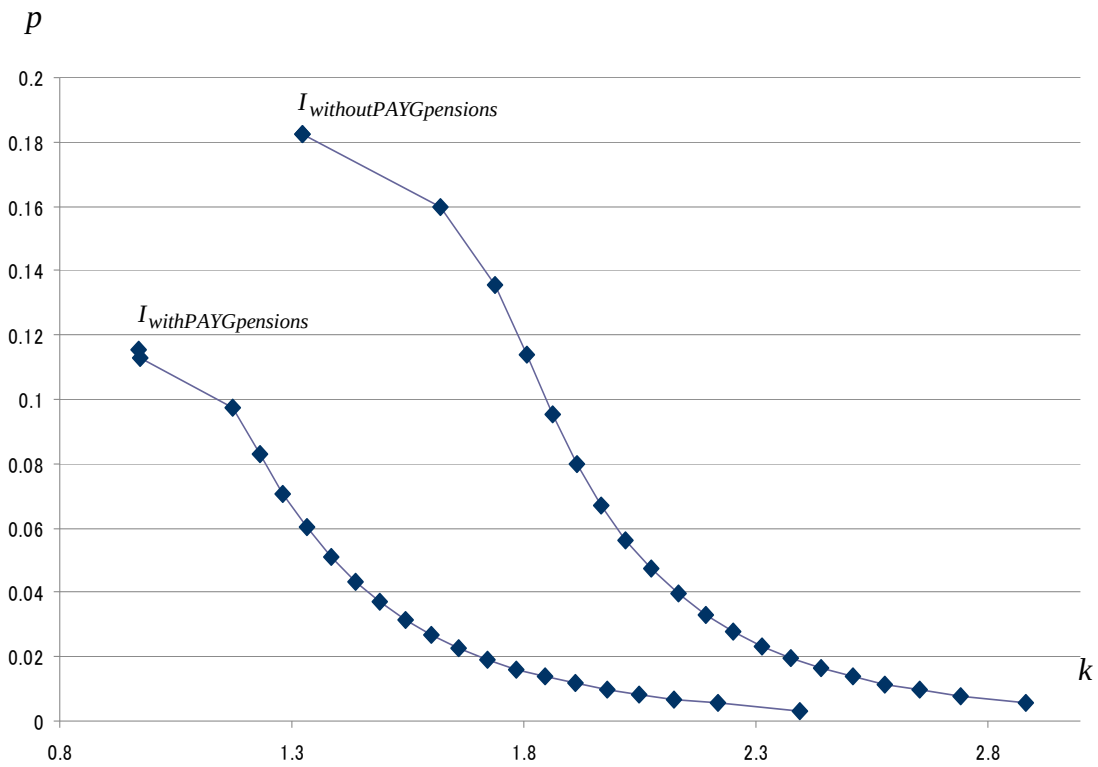


Figure 4 Ratios between capital and land in individual portfolios on the transition path