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Optimal nondiscriminatory auctions with favoritism

Leandro Arozamena
Universidad Torcuato Di Tella

Nicholas Shunda
University of Redlands

Federico Weinschelbaum
Universidad de San Andres

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In many auction settings, there is favoritism: the seller's welfare depends positively on the utility of a subset of potential bidders. However, laws or regulations may not allow the seller to discriminate among bidders. We find the optimal nondiscriminatory auction in a private value, single-unit model under favoritism. At the optimal auction there is a reserve price, or an entry fee, which is decreasing in the proportion of preferred bidders and in the intensity of the preference. Otherwise, the highest-valuation bidder wins.

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Leandro Arozamena

Universidad Torcuato Di Tella and CONICET

Nicholas Shunda

University of Redlands

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In many auction settings, there is favoritism: the seller's welfare depends positively on the utility of a subset of potential bidders. However, laws or regulations may not allow the seller to discriminate among bidders. We find the optimal nondiscriminatory auction in a private value, single-unit model under favoritism. At the optimal auction there is a reserve price, or an entry fee, which is decreasing in the proportion of preferred bidders and in the intensity of the preference. Otherwise, the highest-valuation bidder wins.

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1 Introduction

It is frequently the case when auctions are used that the seller is not indifferent as to which of the bidders will be the winner. She uses an auction to enhance competition among bidders, but at the same time, given a selling price, she would prefer some of the bidders to win rather than others. This may occur when some of the bidders' welfare positively influences the seller's welfare. For example, in a government-run auction, domestic firms may generate more tax revenue than their foreign rivals. Alternatively, the seller and some of the bidders may be firms in the same conglomerate. We say that there is favoritism when the seller has such a preference for some bidders over others.

Favoritism usually motivates the design of discriminatory auctions.² Since the bidder's identities are relevant to the seller, the rules of the auction are specified in such a way that not only the bids made matter, but also who makes them. For example, price preferences are frequently introduced: to win, a non-preferred bidder may have to beat the highest bid made by a preferred bidder by at least a given, previously specified margin. Another usual way to discriminate, known as *right of first refusal*, is giving one of the preferred bidders the right to match the highest bid that any of her rivals may submit.³

However, in many situations discrimination is not possible. This happens quite often in public procurement, where laws and regulations sometimes forbid favoring some bidders over others to keep the field levelled and thus foster competition. There may be higher-level regulations that explicitly prevent local authorities to favor local firms. In general, this constraint may be interpreted as one imposed by a principal on an agent who is in charge of the auction.

Our aim here is then to examine the auction design problem faced by a seller who places positive weight on some of the bidders' welfare, but faces a non-discrimination constraint. We conclude that the seller will choose an auction where the highest-valuation bidder wins unless her valuation is too low. There will thus be a reserve price or an entry fee adequately chosen to exclude lower valuations, just as in the standard, revenue-maximizing auction. However, we

²The justification for discrimination that we examine, which derives from the fact that the seller values some of the bidders' utilities, is not the only possible one. With a fixed number of bidders, biasing the auction against strong bidders raises revenue, as shown by optimal auction theory. With endogenous entry, discriminating against strong bidders may also be optimal for the seller, since it could encourage the entry of weak bidders. Those arguments require asymmetry among bidders, while our model is (but for the possibly unequal consideration of bidders' utilities by the seller) symmetric.

³This right has been studied in Walker (1999), Burguet and Perry (2009), Bikhchandani et al. (2005), Arozamena and Weinschelbaum (2006), Choi (2009) and Lee (2008).

find that the set of valuations excluded is smaller when there is favoritism: there will be a lower reserve price or entry fee. Furthermore, the set of excluded valuations becomes smaller when the weight attached to the utility of any favored bidder grows. Hence, favoritism generates a more efficient auction.

There is a literature on favoritism in auctions. Laffont and Tirole (1991) and Vagstad (1995) study the case of multidimensional auctions, where favoritism may appear when the auctioneer assesses product quality. McAfee and McMillan (1989), Branco (1994), Naegelen and Mougeot (1998) examine single-dimensional auctions, where price-preferences may be used. The basic result is that the optimal allocation rule follows from comparing the maximum valuation of the preferred bidders with the maximum “virtual” valuation of the non-preferred bidders. Arozamena and Weinschelbaum (2011) extend the analysis of the single-dimensional case to a situation when the number of bidders is endogenous, and conclude that the optimal auction in that setting is nondiscriminatory. In all these papers, however, the seller is allowed to use discriminatory mechanisms.

In the following section we present the model and the basic results.

2 The model and the optimal mechanism

The owner of a single, indivisible object is selling it through an auction.⁴ For simplicity, we assume the seller attaches no value to the object. There are N bidders whose valuations for the object are given by v_i , $i = 1, \dots, N$. Each v_i is bidder i ’s private information. These valuations are distributed identically and independently according to the c.d.f. F with support on the interval $[\underline{v}, \bar{v}]$ and a density f that is positive and bounded on the whole support. The context is, then, one of independent private values. All parties to the auction are risk neutral, and we assume that the virtual valuation of any bidder, $J(v) = v - \frac{1-F(v)}{f(v)}$, is increasing in her actual valuation.⁵

Our aim is to characterize a selling mechanism that maximizes the utility of a seller who, in addition to her own expected revenue, values positively the welfare of a subset of the set of bidders. Specifically, we assume that the seller’s objective function follows from adding to the seller’s revenue each bidder’s utility, where bidder i ’s utility is weighted according to an exogenous parameter α_i , $i = 1, \dots, N$. We assume as well that $\alpha_i \in [0, 1]$ for all i . That is, the seller attaches a weakly positive weight to each bidder’s utility, but cannot value the latter

⁴All of our results, however, are applicable as well to the case of procurement auctions.

⁵This is what the literature calls a “regular case.”

more than her own, “private” utility (i.e. her revenue). Note that if $\alpha_i = 0$ for all i we have a standard, revenue-maximizing seller, whereas if $\alpha_i = 1$ for all i the seller will maximize the joint surplus of all participants in the auction –i.e. she will pursue efficiency rather than revenue maximization.

As described so far, our problem is a slight modification of the standard optimal auction problem with independent private values.⁶ Let $H_i(v_1, \dots, v_N)$ ($P_i(v_1, \dots, v_N)$) be the probability that bidder i gets the object (respectively, the price bidder i has to pay to the seller) if bidder valuations are given by (v_1, \dots, v_N) . Then, the seller has to choose a mechanism $\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N$ such that, for all (v_1, \dots, v_N) , $0 \leq H_i(v_1, \dots, v_N) \leq 1$ for all i and $\sum_{i=1}^N H_i(v_1, \dots, v_N) \leq 1$. In addition, let $h_i(v_i)$ ($p_i(v_i)$) be the expected probability that bidder i gets the object (respectively, the expected price she pays) when her valuation is v_i , and the valuations of all other bidders are unknown.

Bidder i ’s expected utility when her valuation is v_i and she announces that it is v'_i is

$$\tilde{U}_i(v_i, v'_i) = h_i(v'_i)v_i - p_i(v'_i).$$

In addition, let

$$U_i(v_i) = \tilde{U}_i(v_i, v_i) = h_i(v_i)v_i - p_i(v_i)$$

Then, our problem is

$$\max_{\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N} \sum_{i=1}^N \left(\int_{\underline{v}}^{\bar{v}} p_i(v_i) f_i(v_i) dv_i + \alpha_i \int_{\underline{v}}^{\bar{v}} U_i(v_i) f(v_i) dv_i \right)$$

subject to the standard incentive compatibility and participation constraints

$$\begin{aligned} U_i(v_i) &\geq \tilde{U}_i(v_i, v'_i) && \text{for all } i, \text{ for all } v_i, v'_i \\ U_i(v_i) &\geq 0 && \text{for all } i, \text{ for all } v_i \end{aligned}$$

This problem has been studied before, for instance, in Naegelen and Mougeot (1998).⁷ However, as mentioned above, we are interested in the case where the seller *cannot* discriminate among bidders. Hence, we add a new constraint on the set of mechanisms $\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N$ that the seller can select.

As explained above, though, we are interested in the case where the seller cannot select a discriminatory mechanism. Therefore, we add the following constraint to the seller’s problem.

⁶See Myerson (1981) and Riley and Samuelson (1981).

⁷It can be thought of as an extension to the N-bidder context of a particular case of the analysis in Naegelen and Mougeot (1998), when there is no consumer surplus and the shadow cost of public funds is zero.

No discrimination: *The seller has to choose a mechanism $\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N$ that, for any permutation $\pi : \{1, \dots, N\} \longrightarrow \{1, \dots, N\}$, satisfies*

$$\begin{aligned} H_i(v_{\pi(1)}, \dots, v_{\pi(N)}) &= H_{\pi(i)}(v_1, \dots, v_N) \\ P_i(v_{\pi(1)}, \dots, v_{\pi(N)}) &= P_{\pi(i)}(v_1, \dots, v_N) \end{aligned}$$

Once discrimination is ruled out, we must have

$$h_i(v_i) = h(v_i) \text{ and } p_i(v_i) = p(v_i) \text{ for all } i.$$

We follow the usual steps in the literature. Let $\tilde{v}_i(v_i)$ be the valuation that bidder i announces optimally when her true valuation is v_i . Clearly, by incentive compatibility, it has to be true that $\tilde{v}_i(v_i) = v_i$ and $U_i(v_i) = \tilde{U}_i(v_i, \tilde{v}_i(v_i))$. The envelope theorem then implies that

$$U'_i(v_i) = \frac{\partial}{\partial v_i} \tilde{U}_i(v_i, \tilde{v}_i(v_i)) = h(v_i).$$

Therefore, $U_i(v_i) = \int_{\underline{v}}^{v_i} h(s)ds + U_i(\underline{v})$. Stated in a way that is more convenient to us in what follows, and noting that, in the solution to our problem, $U_i(\underline{v}) = 0$ for all i such that $\alpha_i < 1$,⁸ we have

$$p(v_i) = h(v_i)v_i - \int_{\underline{v}}^{v_i} h(s)ds$$

for all i . Replacing in the objective function yields

$$\sum_{i \neq 1} \left[\int_{\underline{v}}^{\bar{v}} \left[h(v_i)v_i - \int_{\underline{v}}^{v_i} h(s)ds \right] f(v_i)dv_i + \alpha_i \int_{\underline{v}}^{\bar{v}} h(v_i)v_i f(v_i)dv_i \right].$$

Integrating by parts, we have

$$\sum_i \int_{\underline{v}}^{\bar{v}} h(v_i) \left[v_i - (1 - \alpha_i) \frac{1 - F(v_i)}{f(v_i)} \right] f(v_i)dv_i$$

Then, the seller should solve

$$\max_{\{H_i(\cdot)\}_{i=1}^N} E_{v_1, \dots, v_N} \left[\sum_i H_i(v_i) \left[v_i - (1 - \alpha_i) \frac{1 - F(v_i)}{f(v_i)} \right] \right]$$

⁸Note that $U_i(\underline{v})$ may be zero or positive for those i with $\alpha_i = 1$ in a solution to our problem. Given that we are adding the expected utilities of the seller and these bidders, how much they pay (as long as incentive compatibility holds) does not affect the objective function. There is a solution, however, where $U_i(\underline{v}) = 0$ for all i .

subject to the no-discrimination constraint.

The fact that the seller cannot discriminate among bidders, however, allows us to reexpress this problem in a more convenient way.

Remark 1 *Let $v_{(n)}$ be the n th order statistic associated to the vector of valuations (v_1, \dots, v_N) . For any vector $v = (v_1, \dots, v_N)$ we may define a function $\gamma_v : \{1, \dots, N\} \longrightarrow \{1, \dots, N\}$ that assigns to each position $1, \dots, N$ the identity of the bidder whose valuation ranks in that position. That is $\gamma(n) = i$ if $v_{(n)} = v_i$.⁹ The no-discrimination constraint implies that for any two vectors (v_1, \dots, v_N) , (v'_1, \dots, v'_N) such that $(v_{(1)}, \dots, v_{(N)}) = (v'_{(1)}, \dots, v'_{(N)})$ we must have*

$$H_{\gamma_v(n)}(v_1, \dots, v_N) = H_{\gamma_{v'}(n)}(v'_1, \dots, v'_N), \quad n = 1, \dots, N.$$

To show this, assume $H_{\gamma_v(n^*)}(v_1, \dots, v_N) \neq H_{\gamma_{v'}(n^*)}(v'_1, \dots, v'_N)$ for some n^* . Let $\tilde{\pi} : \{1, \dots, N\} \longrightarrow \{1, \dots, N\}$ be such that $\tilde{\pi}(i) = \gamma_{v'}(\gamma_v^{-1}(i))$. The functions γ_v and $\gamma_{v'}$ are permutations, so $\tilde{\pi}$ is a permutation as well. But then

$$H_{i^*}(v_1, \dots, v_N) \neq H_{\tilde{\pi}(i^*)}(v_{\tilde{\pi}(1)}, \dots, v_{\tilde{\pi}(N)})$$

for $i^* = \gamma_v(n^*)$, which violates the no-discrimination constraint.

Thus, if for any two vectors of valuations the corresponding vectors of order statistics coincide, then the seller has to allocate the good with the same probability to those bidders that occupy each ordered position in the vectors of order statistics. In other words, the probability that any given bidder wins has to depend only on the vector of order statistics and on her valuation's position in that vector. This, in turn, implies the following lemma.

Lemma 1 *The seller's problem can be expressed in terms of order statistics: she has to choose an allocation function*

$$\{H_n(v_{(1)}, \dots, v_{(N)})\}_{n=1}^N$$

That is, all valuation vectors that generate the same vector of order statistics have to be treated equally. Then, we can focus only on which allocations the seller chooses when the vector of valuations is ordered. Allocations in all other cases follow from the no-discrimination constraint.

The seller, though, cares about the identities of the bidders. Given a vector of order statistics $(v_{(1)}, \dots, v_{(N)})$, since valuations are independently drawn from the same distribution, the

⁹Since we are using continuous distributions, ties will occur with probability zero, and how positions are allocated by this function when there is a tie is irrelevant as long as the no-discrimination constraint is satisfied.

probability that bidder i 's valuation ranks in position n is the same of all bidders. Then, for that vector of order statistics, the seller's objective function will take the following expected value

$$\sum_{n=1}^N \frac{1}{N} H_n(v_{(1)}, \dots, v_{(N)}) \left[\sum_{i=1}^N \left[v_{(n)} - (1 - \alpha_i) \frac{1 - F(v_{(n)})}{f(v_{(n)})} \right] \right]$$

or

$$\sum_{n=1}^N \frac{1}{N} H_n(v_{(1)}, \dots, v_{(N)}) \left[N v_{(n)} - \frac{1 - F(v_{(n)})}{f(v_{(n)})} \sum_{i=1}^N (1 - \alpha_i) \right]$$

The seller's problem, then, becomes

$$\max_{\{H_n(v_{(1)}, \dots, v_{(N)})\}_{n=1}^N} E_{v_{(1)}, \dots, v_{(N)}} \left\{ \sum_{n=1}^N \frac{1}{N} H_n(v_{(1)}, \dots, v_{(N)}) \left[N v_{(n)} - \frac{1 - F(v_{(n)})}{f(v_{(n)})} \sum_{i=1}^N (1 - \alpha_i) \right] \right\}$$

The solution to this problem is simple. Since $J(v) = v - \frac{1-F(v)}{f(v)}$ is increasing, it is easy to show that $N v_{(n)} - \frac{1-F(v_{(n)})}{f(v_{(n)})} \sum_{i=1}^N (1 - \alpha_i)$ takes its highest value for $n = 1$. The seller should therefore allocate the object with probability 1 to the bidder with the highest valuation whenever $N v_{(1)} - \frac{1-F(v_{(1)})}{f(v_{(1)})} \sum_{i=1}^N (1 - \alpha_i) > 0$. Otherwise, she should keep the object. We therefore have the following result.

Proposition 1 *The optimal allocation rule is¹⁰*

$$H_i(v_1, \dots, v_N) = \begin{cases} 1 & \text{if } v_i > \max_{j \neq i} v_j \text{ and } N v_i - \frac{1-F(v_i)}{f(v_i)} \sum_{j=1}^N (1 - \alpha_j) > 0 \\ 0 & \text{otherwise} \end{cases}$$

This direct mechanism can be implemented by any efficient auction with an adequately chosen reserve price or entry fee. For example, the seller may choose a first-price or a second-price auction with reserve price r such that $N r - \frac{1-F(r)}{f(r)} \sum_{i=1}^N (1 - \alpha_i) = 0$. Note as well that if $\alpha_i = 0$ for all i , the optimal mechanism for the seller described in Proposition 1 coincides with the standard, revenue-maximizing direct mechanism: the object is awarded to the highest-valuation bidder and all valuations below r such that $r - \frac{1-F(r)}{f(r)} = 0$ are excluded. At the same time, if $\alpha_i = 1$ for all i , then $r = 0$, no valuations are excluded and the seller chooses an efficient auction.

Therefore, for any vector of weights $(\alpha_1, \dots, \alpha_N)$ that the seller attaches to the bidder's utilities, she chooses a mechanism that is neither efficient nor revenue-maximizing. Furthermore,

¹⁰In order to satisfy the no-discrimination constraint, we assume that, if there is a tie, all bidders with the highest valuation win with the same probability.

she selects a mechanism that falls in between the extreme cases provided by efficiency and revenue maximization.

It is then interesting to examine the effect of a change in that vector of weights on the mechanism selected by the seller and on the welfare of each of the parties involved in the auction. First, notice that r , the minimum valuation that is not excluded from the mechanism, is decreasing in α_i for any i . If the seller places a larger weight on a given bidder's welfare, the only instrument she has to enhance that bidder's welfare is to reduce the reserve price or entry fee that she employs in any auction that implements the optimal mechanism. Doing so benefits not only the bidder whose corresponding weight has risen, but all other bidders as well. Therefore, all bidder's expected utilities are increasing in any α_i .

It is not the case, though, that the seller is "sharing" her gains from having a higher α_i . There are actually two effects. First, for any given value of r , the seller's utility straightforwardly grows with α_i . Second, the seller increases her utility by reducing r . This second effect raises the utilities of all bidders, too.

References

- [1] Arozamena, L. and F. Weinschelbaum, "A Note on the Suboptimality of Right-of-First-Refusal Clauses," *Economics Bulletin*, 4, No.24, 1-5, 2006.
- [2] Arozamena, L. and F. Weinschelbaum, "On favoritism in auctions with entry," *Economics Letters*, 110, 265-267, 2011.
- [3] Bikhchandani, S., S. Lippman and R. Ryan, "On the Right-of-First-Refusal," *Advances in Theoretical Economics*, 1, No.1, Article 4, 2005.
- [4] Branco, F., "Favoring Domestic Firms in Procurement Contracts," *Journal of International Economics*, 37, 65-80, 1994.
- [5] Burguet, R. and M. Perry, "Preferred Suppliers in Auction Markets," *RAND Journal of Economics*, 40, 283-295, 2009.
- [6] Choi, A., "A Rent Extraction Theory of Right of First Refusal," *Journal of Industrial Economics*, 57 Iss. 2, Pages 252 - 262, 2009.

- [7] Laffont, J. and J. Tirole, "Auction Design and Favoritism," *International Journal of Industrial Organization*, 9, 9-42, 1991.
- [8] Lee, J., "Favoritism in Asymmetric Procurement Auctions," *International Journal of Industrial Organization*, 26, 1407-1424, 2008.
- [9] McAfee, R.P. and J. McMillan, "Government Procurement and International Trade," *Journal of International Economics*, 26, 291-308, 1989.
- [10] Myerson, R. "Optimal Auction Design" *Mathematics of Operations Research*, **6**, 58-73, 1981.
- [11] Naegelen, F. and M. Mougeot, "Discriminatory Public Procurement Policy and Cost Reduction Incentives," *Journal of Public Economics*, 67, 349-367, 1998.
- [12] Riley, J. and W. Samuelson "Optimal Auctions" *American Economic Review*, 71, 381-392, 1981.
- [13] Vagstad, S., "Promoting Fair Competition in Public Procurement," *Journal of Public Economics*, 58, 283-307, 1995.
- [14] Walker, D. "Rethinking Rights of First Refusal" *Stanford Journal of Law, Business and Finance*, **5**, 1-58, 1999.